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*Research Article*

# Novel Concepts of Domination in Vague Graphs with Application in Medicine

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VG can manage the uncertainty relevant to the inconsistent and indeterminate information of all real-world problems, in which FGs possibly will not succeed in bringing about satisfactory results. The previous definitions' restrictions in FGs have made us present new definitions in VGs. A wide range of applications have been attributed to the domination in graph theory for several fields such as facility location problem, school bus routing, modeling biological networks, and coding theory. Therefore, in this research, we study several concepts of domination, such as restrained dominating set (RDS), perfect dominating set (PDS), global restrained dominating set (GRDS), total $k$-dominating set, and equitable dominating set (EDS) in VGs and also introduce their properties by some examples. Finally, we try to represent the application and importance of domination in the field of medical science and discuss the topic in today's world, namely, the corona vaccine.

## 1. Introduction

Graph theory began its adventure from the well-known "Konigsberg bridge problem." This problem is frequently believed to have been the beginning of graph theory. In 1739, Euler finally elucidated this problem using graphs. Even though graph theory is an extraordinarily old concept, its growing utilization in operations research, chemistry, genetics, electrical engineering, geography, sociology, and so forth has reserved it fresh. In recent times, graph principle has been utilized in communication system (mobile, internet, etc.), computer layout, and so forth. In graph theory, it is far considered that the nodes, edges, weights, and so on are definite. To be exact, there may be no question concerning the existence of these objects. However, the real world sits on a plethora of uncertainties, indicating that, in some conditions, it is believed that the nodes, edges, and weights may additionally be or may not be certain. For instance, the vehicle travel time or vehicle capacity on a road network may not be identified or known exactly. To embody such graphs, Rosenfeld [1] brought up the idea of the "fuzzy graph" in 1975. Similar to set theory, the historical past of FG theory is the fuzzy set theory advanced by Zadeh [2] in 1965. Roy and Biswas investigated the importance of interval-valued fuzzy sets on medical diagnosis [3].

The notion of vague set theory, generalization of Zadeh's fuzzy set theory, was introduced by Gau and Buehrer [4] in 1993. The concepts of rough set, soft set, bipolar soft set, and neutrosophic set were introduced in [5–9]. Kauffman [10] represented FGs based on Zadeh's fuzzy relation [11]. Mordeson et al. [12–14] described some results in FGs. Akram et al. [15–17] developed several concepts and results on FGs. Samanta et al. [18–21] represented FCGs and some remarks on BFGs. Shao et al. [22–28] investigated new concepts in VGs and fuzzy graphs. VG notion was defined by Ramakrishna in [29]. Borzooei and Rashmanlou [30–34] analyzed new concepts of VGs. Rashmanlou et al. [35–40] investigated new results in VGs. Ghorai and Pal [41] studied regular product vague graphs and product vague line graphs. A VG is referred to as a generalized structure of an FG that delivers more exactness, adaptability, and compatibility to a system when matched with systems running on FGs. Also, a PVG is able to concentrate on determining the uncertainty coupled with the inconsistent and indeterminate information of any real-world problem, where FGs may not lead to adequate results.

Domination in VGs theory is one of the most widely used topics in other sciences, including psychology, computer science, nervous systems, artificial intelligence, decision-making theory, and combinations. Although the dominance of FGs has been stated by some researchers, due to the fact that VGs are wider and are more widely used than FGs, it is observed today that they are used in many branches of engineering and medical sciences. Likewise, they have been used in many applications for the formulation and solution of many problems in various areas of science and technology exemplified by computer networks, combinatorial analyses, physics, and so forth. In 1962, Ore [42] represented "domination" for undirected graphs, and he described the definition of minimum-DSs of nodes in a graph. A. Somasundaram and S. Somasundaram [43] introduced the DS and IDS in FGs. Gani et al. [44, 45] represented the fuzzy-DS and independent-DS notion utilizing strong arcs. The IDN and IR-DN in graphs are defined by Cockayne et al. [46] and Haynes et al. [47]. Parvathi and Thamizhendhi [48] described domination in intuitionistic fuzzy graphs. Jan et al. [49–51] investigated new concepts in interval-valued fuzzy graphs and cubic bipolar fuzzy graphs. Talebi et al. [52–54] introduced some results of domination in VGs, as well as new concepts in interval-valued intuitionistic fuzzy competition graph. So, in this research, we introduce different concepts of domination, such as RDS, PDS, GRDS, EDS, and total $k$-dominating set in VGS. In the end, an application of domination in medical immunization is introduced.

## 2. Preliminaries

In this section, some basic concepts of VGs are reviewed to facilitate the next sections.

A graph denotes a pair $G^* = (V, E)$ satisfying $E \subseteq V \times V$. The elements of $V$ and $E$ are the nodes and edges of the graph $G^*$, correspondingly.

An FG has the form of $\xi = (\gamma, \nu)$, where $\gamma\colon V \longrightarrow [0,1]$ and $\nu\colon V \times V \longrightarrow [0,1]$ as is defined by $\nu(ab) \leq \gamma(a) \wedge \gamma(b)$, $\forall a, b \in V$, and $\nu$ is a symmetric fuzzy relation on $\gamma$ and $\wedge$ denotes the minimum.

*Definition 1* (see [4]). A *VS A* is a pair $(t_A, f_A)$ on set $V$ where $t_A$ and $f_A$ are used as real valued functions which can be defined on $V \longrightarrow [0,1]$, so that $t_A(a) + f_A(a) \leq 1$, $\forall a \in V$. The interval $[[t_A(a), 1 - f_A(a)]$ is considered as the vague value of $a$ in $A$.

*Definition 2* (see [29]). A pair $\xi = (A, B)$ is said to be a VG on a crisp graph $G^*$, where $A = (t_A, f_A)$ is a VS on $V$ and $B = (t_B, f_B)$ is a VS on $E \subseteq V \times V$ such that $t_B(ab) \leq \min(t_A(a), t_A(b))$ and $f_B(ab) \geq \max(f_A(a), f_A(b))$, for each edge $ab \in E$.

*Definition 3* (see [35]). A VG $\xi$ is called complete VG if $t_B(ab) = \min(t_A(a), t_A(b))$ and $f_B(ab) = \max(f_A(a), f_A(b))$, $\forall a, b \in V$.

*Definition 4* (see [35]). The complement of a VG $\xi = (A, B)$ is a VG $\overline{\xi} = (\overline{A}, \overline{B})$, where $\overline{A} = A = (\overline{t_A}, \overline{f_A})$ and $\overline{B} = (\overline{t_B}, \overline{f_B})$ are defined by the following:

$$\overline{V} = V$$

$$\overline{t_A}(a) = t_A(a), \overline{f_A}(a) = f_A(a), \forall a \in V$$

$$\overline{t_B}(ab) = \begin{cases} 0, & \text{if } t_B(ab) > 0, \\ \min(t_A(a), t_A(b)), & \text{if } t_B(ab) = 0. \end{cases}$$

$$\overline{f_B}(ab) = \begin{cases} 0, & \text{if } f_B(ab) > 0, \\ \max(f_A(a), f_A(b)), & \text{if } f_B(ab) = 0. \end{cases}$$

*Definition 5* (see [31]). A vague path in a VG $\xi = (A, B)$ is a sequence of distinct nodes $b_1, b_2, \ldots, b_n$ *so that either* $t_B(b_k b_{k+1}) > 0$ or $f_B(b_k b_{k+1}) > 0$, $\forall\, 1 \leq k \leq n-1$. It was shown by $P_n$.

*Definition 6* An edge $ab$ of a VG $\xi$ is called an effective edge if $t_B(ab) = t_A(a) \wedge t_A(b)$ and $f_B(ab) = f_A(a) \vee f_A(b)$. Otherwise, it is called a noneffective edge.

*Definition 7* (see [31]). An edge $ab$ in a VG $\xi$ is called a strong edge if $t_B(ab) \geq t_B^{\infty}(ab)$ and $f_B(ab) \leq f_B^{\infty}(ab)$.

*Definition 8* (see [31]). Let $\xi = (A, B)$ be a VG. Let $a, b \in V$, then $a$ dominates $b$ in $\xi$, if there exists a strong edge between $a$ and $b$.

*Definition 9* (see [32]). The cartesian product of two VGs $\xi_1 = (A_1, B_1)$ and $\xi_2 = (A_2, B_2)$ of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, denoted by $\xi_1 \times \xi_2 = (A_1 \times A_2, B_1 \times B_2)$, is defined as follows:

$$\text{(i)}\begin{cases}\left(t_{A_1}\times t_{A_2}\right)(a,b)=\min\left\{t_{A_1}(a),t_{A_2}(b)\right\},\\ \left(f_{A_1}\times f_{A_2}\right)(a,b)=\max\left\{f_{A_1}(a),f_{A_2}(b)\right\},\end{cases}\quad \text{for all } (a,b)\in V,$$

$$\text{(ii)}\begin{cases}\left(t_{B_1}\times t_{B_2}\right)((a,a_1)(a,b_1))=\min\left\{t_{A_1}(a),t_{B_2}(a_1b_1)\right\},\\ \left(f_{B_1}\times f_{B_2}\right)((a,a_1)(a,b_1))=\max\left\{f_{A_1}(a),f_{B_2}(a_1b_1)\right\},\end{cases}\quad \text{for all } a\in V_1, a_1b_1\in E_2, \tag{1}$$

$$\text{(iii)}\begin{cases}\left(t_{B_1}\times t_{B_2}\right)((a,a_1)(b,a_1))=\min\left\{t_{B_1}(ab),t_{A_2}(a_1)\right\},\\ \left(f_{B_1}\times f_{B_2}\right)((a,a_1)(b,a_1))=\max\left\{f_{B_1}(ab),f_{A_2}(a_1)\right\},\end{cases}\quad \text{for all } a_1\in V_2, ab\in E_1.$$

*Definition 10* (see [35]). A node $a$ in a VG $\xi$ is said to be an isolated node if $t_B(ab)=0$ and $f_B(ab)=0$, for all $b\in V$ and $a\neq b$. That is, $\mathcal{N}(a)=\varnothing$.

*Definition 11* (see [31]). $S\subseteq V$ is called a *DS* in $\xi$ if, $\forall a\in V-S$, $\exists b\in S$, so that $a$ dominates $b$.

*Definition 12* (see [31]). Let $\xi=(A,B)$ be a VG. The vertex cardinality of $S\subseteq V$ is defined as

$$|S|=\left|\sum_{a\in S}\frac{1+t_A(a)-f_A(a)}{2}\right|. \tag{2}$$

Notations are shown in Table 1.

## 3. Certain Notions of Domination in Vague Graphs

*Definition 13.* Let $\xi=(A,B)$ be a VG and let $k\geq 1$ be an integer. A subset $S\subseteq V$ is called a K-DS of $\xi$ if, for each node $a\in V-S$, there exists an $a-b$ vague path which includes at least $k$ effective edges for $b\in S$. The K-DN of $\xi$, denoted by $\gamma_k(\xi)$, is described as the minimum cardinality among all K-DS in $\xi$.

*Definition 14.* Let $\xi=(A,B)$ be a VG and let $k\geq 1$ be an integer. A subset $S\subseteq V$ is called a T-KDS of $\xi$ if, for each node $a\in V$, $\exists$ an $a-b$ vague path that includes at least $k$ effective edges for $b\in S$. The T-KDN of $\xi$, demonstrated by $\gamma_{tk}(\xi)$, is described as the minimum cardinality between all T-KDS in $\xi$.

*Example 1.* Consider an example of a T-2DS *of* VG $\xi$ shown in Figure 1.

It is clear from Figure 1 that $S=\{a,f\}$ is a minimal T-2DS of VG $\xi$. The T-2DN of $\xi$ is $\gamma_k(\xi)=0.8$.

*Definition 15.* Let $\xi$ be a VG. A set $S\subseteq V$ is called an RDS of $\xi$ if each node in $V-S$ dominates a node in $S$ and also a node in $V-S$. The RDN of $\xi$, demonstrated by $\gamma_r(\xi)$, is described as the minimum cardinality of an RDS in $\xi$.

*Example 2.* Consider a VG $\xi=(A,B)$ as shown in Figure 2. It is obvious that $S=\{c,d\}$ is an RDS of $\xi$. The RDN of $\xi$ is $\gamma_r(\xi)=0.75$.

*Definition 16.* Let $\xi$ be a VG. A set $S\subseteq V$ is called GRDS of $\xi$ if it is an RDS of both $\xi$ and $\overline{\xi}$. The GRDN of $\xi$, denoted by $\gamma_{gr}(\xi)$, is described as the minimum cardinality of a GRDS in $\xi$.

*Example 3.* Consider $\xi$ and $\overline{\xi}$ as shown in Figures 3 and 4. It is easy to see that $S_1=\{a,b\}$ and $S_2=\{c,d\}$ are GRDSs of $\xi$. The GRDN of $\xi$ is $\gamma_{gr}(\xi)=0.9$.

**Theorem 1.** *Suppose that $\xi=(A,B)$ is a CVG; then $\gamma_r(\xi)=|V|$.*

*Proof.* Assume that $\xi=(A,B)$ is a CVG. Then, $t_B(ab)=\min(t_A(a),t_A(b))$ and $f_B(ab)=\max(f_A(a),f_A(b))$. Let $S$ be the MI-RDS of $\xi$. Then, each node in $V-S$ dominates a node in $S$ and also a node in $V-S$. Hence, each node dominates all other nodes. So, $\gamma_r(\xi)=|V|$. □

*Definition 17.* Let $\xi=(A,B)$ be a VG. A subset $S\subseteq V$ is called a PDS of $\xi$ if, for each node $b\in V-S$, there exists exactly one node $a\in S$ so that $a$ dominates $b$.

*Definition 18.* We say that a PDS $S$ is an MI-PDS if, for every $a\in S$, the set $S-\{a\}$ is not a PDS in $\xi$. The minimum cardinality among all MI-PDSs is called the PDN of $\xi$ and it was shown by $\gamma_{\text{pif}}(\xi)$ or simply $\gamma_{\text{pif}}$.

*Example 4.* Consider a VG $\xi=(A,B)$ as shown in Figure 5. It is obvious that $S=\{a,d\}$ is an MI-PDS. The PDN of $\xi$ is $\gamma_{\text{pif}}=0.9$.

**Theorem 2.** *Every DS in CVG $\xi=(A,B)$ is a PDS.*

*Proof.* Let $S$ be an MI-DS of a VG $\xi$. Since $\xi$ is complete, each edge in $\xi$ is one effective edge and each node $b\in V-S$ is neighbor to exactly one node $a\in S$. Hence, each DS in $\xi$ is a PDS. □

*Definition 19.* The strong product of two VGs $\xi_1=(A_1,B_1)$ and $\xi_2=(A_2,B_2)$ of the graphs $G_1^*=(V_1,E_1)$ and $G_2^*=(V_2,E_2)$, where $V_1\cap V_2=\varnothing$, denoted by $G_1\times G_2=(A_1\times A_2,B_1\times B_2)$, is defined as follows:

Table 1: Some basic notations.

| Notation | Meaning |
| --- | --- |
| FG | Fuzzy graph |
| VS | Vague set |
| $\xi$ | Vague graph |
| DS | Dominating set |
| RDS | Restrained dominating set |
| IDS | Independent dominating set |
| IDN | Independent dominating number |
| IR-DN | Irredundance dominating number |
| K-DS | K-dominating set |
| PDS | Perfect dominating set |
| PDN | Perfect dominating number |
| K-DN | K-dominating number |
| T-KDN | Total K-dominating number |
| MI-PDS | Minimal perfect dominating set |
| T-KDS | Total K-dominating set |
| RDN | Restrained dominating number |
| GRDN | Global restrained dominating number |
| CVG | Complete vague graph |
| EDS | Equitable dominating set |
| GRDS | Global restrained dominating set |

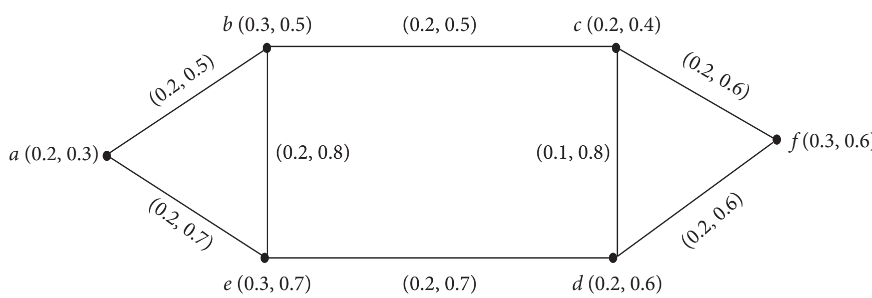


Figure 1: Total 2-dominating set of $\xi$.

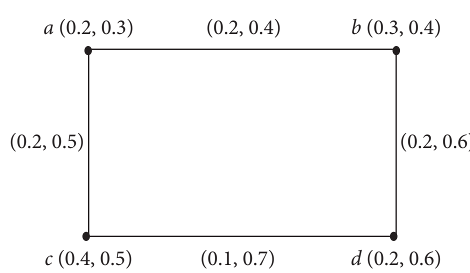


Figure 2: RDS of $\xi$.

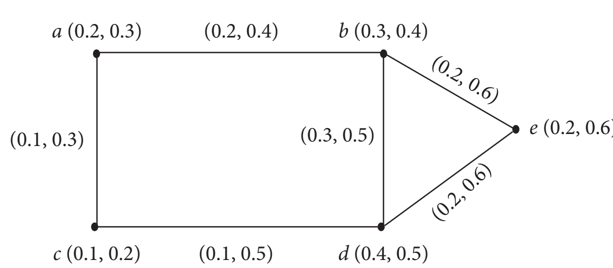


Figure 3: Global RD set of $\xi$.

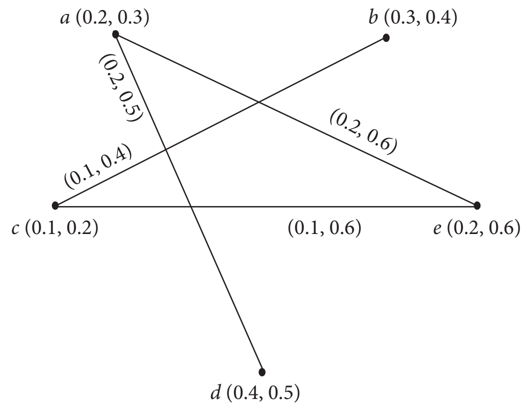


FIGURE 4: $\overline{\xi}$.

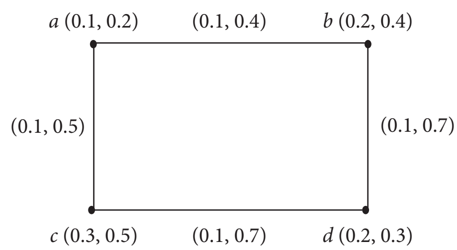


FIGURE 5: VG $\xi$.

$$\begin{aligned}
&\text{(i)} \begin{cases} \left(t_{A_1} \boxtimes t_{A_2}\right)(a,b) = \min\left\{t_{A_1}(a), t_{A_2}(b)\right\}, \\ \left(f_{A_1} \boxtimes f_{A_2}\right)(a,b) = \max\left\{f_{A_1}(a), f_{A_2}(b)\right\}, \end{cases} \quad \forall (a,b) \in V, \\
&\text{(ii)} \begin{cases} \left(t_{B_1} \boxtimes t_{B_2}\right)((a,a_1)(a,b_1)) = \min\left\{t_{A_1}(a), t_{B_2}(a_1b_1)\right\}, \\ \left(f_{B_1} \boxtimes f_{B_2}\right)((a,a_1)(a,b_1)) = \max\left\{f_{A_1}(a), f_{B_2}(a_1b_1)\right\}, \end{cases} \quad \text{for all } a \in V_1 \text{ and } a_1b_1 \in E_2, \\
&\text{(iii)} \begin{cases} \left(t_{B_1} \boxtimes t_{B_2}\right)((a,a_1)(b,a_1)) = \min\left\{t_{B_1}(ab), t_{A_2}(a_1)\right\}, \\ \left(f_{B_1} \boxtimes f_{B_2}\right)((a,a_1)(b,a_1)) = \max\left\{f_{B_1}(ab), f_{A_2}(a_1)\right\}, \end{cases} \quad \forall a_1 \in V_2 \text{ and } ab \in E_1, \\
&\text{(iv)} \begin{cases} \left(t_{B_1} \boxtimes t_{B_2}\right)((a,a_1)(b,b_1)) = \min\left\{t_{B_1}(ab), t_{B_2}(a_1b_1)\right\}, \\ \left(f_{B_1} \boxtimes f_{B_2}\right)((a,a_1)(b,b_1)) = \max\left\{f_{B_1}(ab), f_{B_2}(a_1b_1)\right\}, \end{cases} \quad \forall ab \in E_1 \text{ and } a_1b_1 \in E_2.
\end{aligned} \tag{3}$$

**Theorem 3.** *Let $\xi_1 = (A_1, B_1)$ and $\xi_2 = (A_2, B_2)$ be two VGs with $V_1 \cap V_2 = \varnothing$. The strong product $\xi = \xi_1 \times \xi_2$ remains connected even after removal of all noneffective edges in it.*

*Proof.* Assume that $\xi = \xi_1 \times \xi_2$ is a strong product of two VGs $\xi_1$ and $\xi_2$. Let $e = ((a,a_1)(b,b_1))$ be a noneffective edge in $\xi$; that is, $t_B((a,a_1)(b,b_1)) < \min\{t_A(a,a_1), t_A(b,b_1)\}$, and $f_B((a,a_1)(b,b_1)) > \max\{f_A(a,a_1), f_A(b,b_1)\}$. Let $\xi' = \xi - e$, and suppose that $\xi'$ is disconnected. The edge $e$ disconnects the graph into more than one component. Hence, there is no path among $(a,a_1)$ and $(b,b_1)$ except the edge $e = ((a,a_1),(b,b_1))$ in $\xi'$. This implies that $t_B((a,a_1)(b,b_1)) = \{t_A(a,a_1), t_A(b,b_1)\}$ and $f_B((a,a_1)(b,b_1)) = \{f_A(a,a_1), f_A(b,b_1)\}$, which is a contradiction. So, $\xi'$ is connected. □

*Remark 1.* The strong product $\xi_1 \boxtimes \xi_2$ of two connected vague graphs is a connected vague graph.

**Theorem 4.** *If a node $a$ dominates a node $b$ in $\xi_1$ and a node $a_1$ dominates a node $b_1$ in $\xi_2$, then the node $ab$ does not dominate the node $a_1b_1$ in $\xi_1 \times \xi_2$.*

*Proof.* Suppose that $a$ dominates $b$ in $\xi_1$. Then $\exists$ an effective edge $ab$ in $\xi_1$, i.e., $t_B(ab) = \min\{t_A(a), t_A(b)\}$ and $f_B(ab) = \max\{f_A(a), f_A(b)\}$. Similarly, assume that a node $a_1$ dominates a node $b_1$ in $\xi_2$, so that $t_B(a_1b_1) = \min\{t_A(a_1), t_A(b_1)\}$ and $f_B(a_1b_1) = \max\{f_A(a_1), f_A(b_1)\}$. Now, by definition of Cartesian product, there does not exist any edge among the nodes $(a,a_1)$ and $(b,b_1)$ in $\xi_1 \times \xi_2$; i.e., $t_B((a,a_1)(b,b_1)) = 0$ and $f_B((a,a_1)(b,b_1)) = 0$. Therefore, $(a,a_1)$ does not dominate $(b,b_1)$ in $\xi_1 \times \xi_2$. □

*Definition 20.* The direct product of two VGs $\xi_1 = (A_1, B_1)$ and $\xi_2 = (A_2, B_2)$ of the graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, where $V_1 \cap V_2 = \varnothing$, denoted by $\xi_1 \otimes \xi_2 = (A_1 \otimes A_2, B_1 \otimes B_2)$, is defined as follows:

$$\begin{aligned}
&\text{(i)} \begin{cases} \left(t_{A_1} \otimes t_{A_2}\right)(a,b) = \min\left\{t_{A_1}(a), t_{A_2}(b)\right\}, \\ \left(f_{A_1} \otimes f_{A_2}\right)(a,b) = \max\left\{f_{A_1}(a), f_{A_2}(b)\right\}, \quad \forall (a,b) \in V, \end{cases} \\
&\text{(ii)} \begin{cases} \left(t_{B_1} \otimes t_{B_2}\right)((a,a_1)(b,b_1)) = \min\left\{t_{B_1}(ab), t_{B_2}(a_1b_1)\right\}, \\ \left(f_{B_1} \otimes f_{B_2}\right)((a,a_1)(b,b_1)) = \max\left\{f_{B_1}(ab), f_{B_2}(a_1b_1)\right\}, \quad \forall\, ab \in E_1 \text{ and } a_1b_1 \in E_2. \end{cases}
\end{aligned} \tag{4}$$

*Note 1.* If a node $a$ dominates a node $b$ in $\xi_1$ and a node $a_1$ dominates a node $b$ in $\xi_2$, then the node $(a, a_1)$ dominates the node $(b, b_1)$ in $\xi_1 \otimes \xi_2$. It is shown in Figure 6.

*Example 5.* Consider a VG as in Figure 6. It is obvious that the node $a$ dominates $b$ in $\xi_1$ and $e$ dominates $f$ in $\xi_2$. Likewise, the node $(a, e)$ dominates the node $(b, f)$ in $\xi_1 \times \xi_2$.

*Definition 21.* Let $\xi = (A, B)$ be a VG. A subset $S \subseteq V$ is called an EDS of $\xi$ if, for each node $b \in V - S$, $\exists$ a node $a \in S$ so that $ab \in E$, $|\deg^t(a) - \deg^t(b)| \le 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$, $|\deg^f(a) - \deg^f(b)| \ge 1$, and $f_B(ab) = \max\{f_A(a), f_A(b)\}$. The EDN of $\xi$, denoted by $\gamma_e(\xi)$, is defined as the minimum cardinality of an EDS $S$.

*Example 6.* Consider a VG $\xi = (A, B)$, as shown in Figure 7.

It is obvious that the MI-EDS of a VG $\xi$ is $S = \{a\}$. The EDN is $\gamma_e(\xi) = 0.45$. Note that an EDS $S$ is called an MI-EDS of $\xi$, if, for each node $b \in S$, the set $S - \{b\}$ is not an EDS.

**Theorem 5.** *Let $\xi_1$ and $\xi_2$ be VGs on nonempty sets $V_1$ and $V_2$, respectively. Then,*

$$\gamma_e(\xi_1 \otimes \xi_2) \le \min\left(\left|S_1 \times V_1\right|, \left|S_2 \times V_2\right|\right). \tag{5}$$

*Proof.* Assume that $S_1$ and $S_2$ are EDSs of minimum cardinality of $\xi_1$ and $\xi_2$, respectively. Then, for each node $b \in V - S_1$, $\exists$ a node $a \in S_1$ so that $|\deg^t(b) - \deg^t(a)| \le 1$, $t_B(ab) = \min\{t_A(a), t_A(b)\}$, $|\deg^f(b) - \deg^f(a)| \ge 1$, and $f_B(ab) = \max\{f_A(a), f_A(b)\}$. Similarly, for each node $c \in V - S_2$, $\exists$ a node $d \in S_2$ so that $|\deg^t(d) - \deg^t(c)| \le 1$, $t_B(c\,d) = \min\{t_A(c), t_A(d)\}$, $|\deg^f(d) - \deg^f(c)| \ge 1$, and $f_B(c\,d) = \max\{f_A(c), f_A(d)\}$. That is, $a$ dominates $b$ in $\xi_1$ and $d$ dominates $c$ in $\xi_2$. Therefore, by Note 1, the node $(a, d)$ dominates the node $(b, c)$ in $\xi_1 \otimes \xi_2$. Thus, $|\deg^t(a,d) - \deg^t(b,c)| \le 1$, $t_B((a,d)(b,c)) = \min\{t_A(a,d), t_A(b,c)\}$, $|\deg^f(a,d) - \deg^f(b,c)| \ge 1$, and $f_B((a,d)(b,c)) = \max\{f_A(a,d), f_A(b,c)\}$. So, $\gamma_e(\xi_1 \otimes \xi_2) \le \min(|S_1 \times V_1|, |S_2 \times V_2|)$. □

**Theorem 6.** *Let $S_1$ and $S_2$ be K-DSs of connected VGs $\xi_1 = (A_1, B_1)$ and $\xi_2 = (A_2, B_2)$, respectively; then (1) $\xi_1 \times \xi_2$ is connected. (2) If $S_1$ is a connected K-DS of $\xi_1$, then $S_1 \times V_2$ is a connected K-DS of $\xi_1 \times \xi_2$. (3) If $S_2$ is a connected K-DS of $\xi_2$, then $V_1 \times S_2$ is a connected K-DS of $\xi_1 \times \xi_2$.*

*Proof.* To prove that $\xi_1 \times \xi_2$ is connected, consider any two arbitrary distinct nodes $(a, d)$ and $(b, c)$ of $V_1 \times V_2$. Then, by definition of Cartesian product, $\exists$ a path between these two nodes in the following cases:

(1) If $a = b$, then, since $\xi_2$ is a connected VG, $\exists$ a path $P$: $d, d_1, d_2, \dots, c$ so that $t_B(ef) > 0$ and $f_B(ef) > 0$ for any two nodes $e, f$ of vague path $P$. Hence, $t_B((a,d)(a,c)) = \min\left\{t_{A_1}(a), t_{B_2}(cd)\right\}$ and $f_B((a,d)(a,c)) = \max\left\{f_{A_1}(a), f_{B_2}(cd)\right\}$. So, $P\prime$: $(a, d)$ $(a, d_1)$, $(a, d_2), \dots, (a, c)$ is the vague path between $(a, d)$, $(b, c)$ in $\xi_1 \times \xi_2$.

(2) If $d = c$, then, since $\xi_1$ is a connected VG, $\exists$ a vague path $Q = a, a_1, a_2, \dots, b$ so that $t_{B_1}(ab) > 0$ and $f_{B_1}(ab) > 0$ for any two nodes $e, f$ of vague path Q. Hence, $t_B((a,d)(b,d)) = \min\left\{t_{B_1}(a,b), t_{A_2}(d)\right\}$, and $t_B((a,d), \dots, (b,d))$ is the vague path between $(a, d)$, $(b, c)$ in $\xi_1 \times \xi_2$.

(3) If $a \ne b$ and $c \ne d$, then, by case 1, $\exists$ a vague path between the nodes $(a, d)$ and $(a, c)$ in $\xi_1 \times \xi_2$. Likewise, by case 2, $\exists$ a vague path between the nodes $(a, d)$ and $(b, d)$ in $\xi_1 \times \xi_2$. Hence, the union of these two disjoint vague paths is a vague path between the nodes $(a, d)$ and $(b, c)$ in $\xi_1 \times \xi_2$. Now, if $S_1$ and $S_2$ are K-DSs of $\xi_1$ and $\xi_2$, respectively, then $\gamma_k(\xi_1 \times \xi_2) = \min\{|V_1 \times S_2|, |S_1 \times V_2|\}$. So, $V_1 \times S_2$ and $S_1 \times V_2$ are K-DSs of $\xi_1 \times \xi_2$ and the connectivity can be proved similarly. □

**Theorem 7.** *Let $\xi_1$ and $\xi_2$ be VGs on nonempty sets $V_1$ and $V_2$, respectively. Let $S_1$ and $S_2$ be K-DSs of $\xi_1$ and $\xi_2$; then $S_1 \times V_2$ is an independent K-DS of $\xi_1 \times \xi_2$ if and only if $S_1$ is k-independent and (i) $t_{B_1}(ab) < t_{A_2}(d)$ and $f_{B_1}(ab) > f_{A_2}(d)$, for $a, b \in S_1$ and $d \in V_2$; (ii) $t_{B_2}(c\,d) < t_{A_1}(a)$ and $f_{B_2}(c\,d) > f_{A_1}(a)$, for $a \in S_1$ and $c, d \in V_2$; and (iii) $t_{B_2}(c\,d) < \min\left\{t_{A_2}(d), t_{A_2}(c)\right\}$ and $f_{B_2}(c\,d) > \max\left\{f_{A_2}(d), f_{A_2}(c)\right\}$, for $c, d \in V_2$.*

*Proof.* To prove that each two distinct nodes $(a, d)$, $(b, c)$ in $S_1 \times V_1$ are not neighbor, we consider three conditions. If $a = b$, then

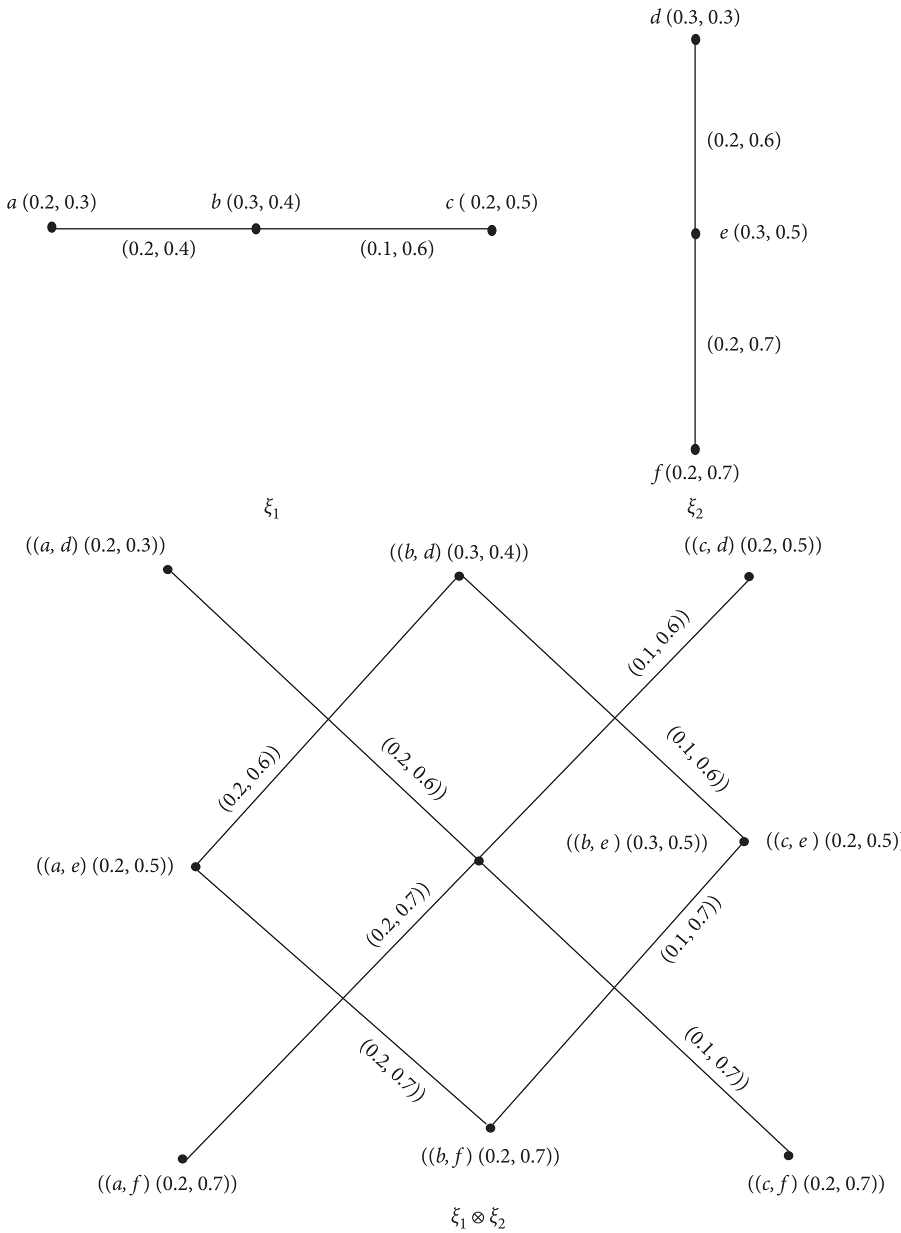


Figure 6: Direct product of $\xi_1$ and $\xi_2$.

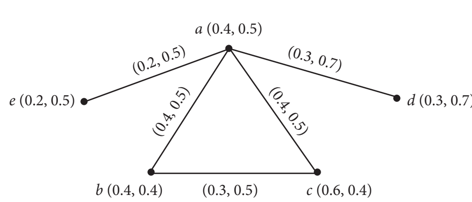


Figure 7: Equitable dominating set of $\xi$.

$$
\begin{aligned}
t_B((a,d)(a,c)) &= \min\{t_{A_1}(a), t_{B_2}(c\ d)\}\\
&< \min\{t_{A_1}(a), \min\{t_{A_2}(c), t_{A_2}(d)\}\}\\
&< \min\{t_A(a,d), t_A(a,c)\},\\
f_B((a,d)(a,c)) &= \max\{f_{A_1}(a), f_{B_2}(c\ d)\}\\
&> \max\{f_{A_1}(a), \max\{f_{A_2}(c), f_{A_2}(d)\}\}\\
&> \max\{f_A(a,d), f_A(a,c)\}.
\end{aligned}
\tag{6}
$$

If $c = d$, the result is obtained by independence of $a, b$ of $S_1$.

If $a \neq b$ and $c \neq d$, then, by definition, we have $t_B((a,d)(b,c)) = 0$ and $f_B((a,d)(b,c)) = 0$. So, $(a,d)$, $(b,c)$ are not neighbors in $G_1 \times G_2$. Conversely, assume that (iii) is false. That is, $\exists$ nodes $b, c \in V_2$, so that $t_{B_2}(bc) = \min\{t_{A_2}(b), t_{A_2}(c)\}$ and $f_{B_2}(bc) = \max\{f_{A_2}(b), f_{A_2}(c)\}$. Let $a \in S_1$; then

$$
\begin{aligned}
t_B((a,b)(a,c)) &= \min\{t_{A_1}(a), t_{B_2}(bc)\}\\
&= \min\{t_{A_1}(a), \min\{t_{A_2}(b), t_{A_2}(c)\}\}\\
&= \min\{t_A(a,b), t_A(a,c)\},\\
f_B((a,b)(a,c)) &= \max\{f_{A_1}(a), f_{B_2}(bc)\}\\
&= \max\{f_{A_1}(a), \max\{f_{A_2}(b), f_{A_2}(c)\}\}\\
&= \max\{f_A(a,b), f_A(a,c)\}.
\end{aligned}
\tag{7}
$$

Hence, $S_1 \times V_2$ is not independent. Therefore, condition (iii) is true, i.e., $t_{B_2}(b,c) < \min\{t_{A_2}(b), t_{A_2}(c)\}$ and $f_{B_2}(b, c) > \max\{f_{A_2}(b), f_{A_2}(c)\}$. □

## 4. Application of Domination in Medical Sciences

One year has passed since the beginning of the coronary heart disease pandemic in the world. During this year, many people have died in all countries and the lives of all people have been affected. During this period, no definitive cure for this disease has been found and many countries, in attempts to develop a corona vaccine to prevent the disease, are highly contagious. China, Russia, India, and the United States are among these countries, and of course Iran has made efforts in this regard. Most vaccines are in the final stages of production and are about to be sacrificed, and many countries have prepurchased several million doses of these vaccines at this stage. Some vaccines are artificially made from antibodies created following disease; and some other viruses have been killed or weakened. The effectiveness of the study population and less side effects are the most important issues in choosing a vaccine. Relations between countries and political issues between them are also factors affecting the type and amount of vaccines purchased. Although it has been said that the whole world should be safe and these vaccines should be given to all countries, the issues mentioned are definitely on the time required to establish comprehensive security in each country will be effective. Therefore, in this paper, we try to discuss the application and importance of domination in the field of medical sciences and discuss the topic in today's world, namely, the corona vaccine. For this purpose, we consider five countries: Iran, China, USA, India, and Russia. In fact, we want to buy the most effective vaccine for Iran, given the effectiveness of the vaccine and the political relations that exist between this country and other countries. In this vague graph, the nodes representing the countries and edges indicate the extent of political relations and friendship between the two countries.

The vertex of China $(0.6, 0.3)$ shows that the efficiency and effectiveness of vaccines in this country are 60%, and, unfortunately, it is as harmful as 30%. The edge Iran-India shows that only 20% on the friendship is established between the two countries and there is 70% of the political conflict and tension between them. The restrained dominating sets (RDSs) for Figure 8 are as follows:

$$
\begin{aligned}
S_1 &= \{a,b\},\\
S_2 &= \{a,d\},\\
S_3 &= \{b,c\},\\
S_4 &= \{b,e\},\\
S_5 &= \{c,d\},\\
S_6 &= \{d,e\},\\
S_7 &= \{a,b,c\},\\
S_8 &= \{a,b,e\},\\
S_9 &= \{b,c,e\},\\
S_{10} &= \{c,d,e\}.
\end{aligned}
\tag{8}
$$

After calculating the cardinality of $S_1, S_2, \ldots, S_{10}$, we obtain

$$
\begin{aligned}
|S_1| &= 1,\\
|S_2| &= 1.05,\\
|S_3| &= 1.3,\\
|S_4| &= 1.1,\\
|S_5| &= 1.35,\\
|S_6| &= 1.15,\\
|S_7| &= 1.65,\\
|S_8| &= 1.45,\\
|S_9| &= 1.75,\\
|S_{10}| &= 1.8.
\end{aligned}
\tag{9}
$$

It is clear that $S_1$ has the smallest size among other RDSs, so we conclude that it can be the best choice because, first, China has the most effective vaccine in terms of susceptibility to the virus, and, second, there is a relatively good friendship between Iran and China. Therefore, governments must provide the necessary facilities for the delivery of efficient and useful vaccines to deprived countries in order to prevent the transmission of this deadly virus to the rest of the people as soon as possible.

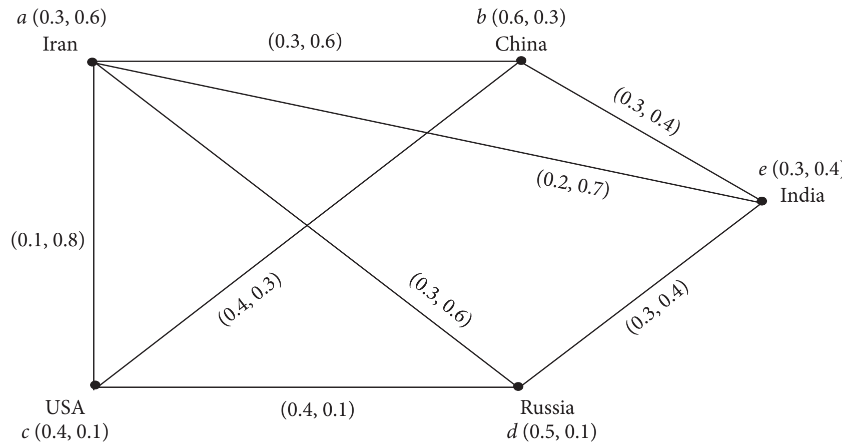


Figure 8: VG $\xi$.

## 5. Conclusion

Domination in FGs theory is one of the most widely discussed topics in other sciences including psychology, computer science, nervous systems, artificial intelligence, and combinations. They have also been utilized in summarizing document and in designing secure systems for electrical grids. Hence, in this paper, we introduced several concepts of domination, such as RDS, PDS, GRDS, EDS, and total K-dominating set in VGs and also investigated their properties by some examples. Finally, we described an application of domination in the field of medical sciences and discussed a topic in today's world, namely, the coronavirus. In our future work, we will introduce vague incidence graphs and study the concepts of connected perfect dominating set, regular perfect dominating set, inverse perfect dominating set, and independent perfect dominating set on vague incidence graph.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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