Research Article

Consensus of the Distributed Multiagent System with the Framework of the Small-World Network

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This paper investigates the consensus problem of the distributed multiagent system (MAS) with the small-world framework. A distributed consensus protocol is provided for the node-to-node communication. According to an error between every two neighbor agents, several consensus criteria among the agents are obtained firstly. Then, consensus criteria are obtained via the diameter of the graph of the MAS. Finally, based on the small-world framework, the consensus criteria are obtained; also, the relations among the consensus, the diameter of the path in the small-world framework, and the errors of agents are disclosed. Finally, one numerical example shows the reliability of the proposed methods.

1. Introduction

In recent years, the coordination problem of the MAS has been drawing considerable attention due to its wide range of application domains [1, 2], such as swarming [3, 4], clustering [5, 6], and flocking [7, 8]. Consensus refers to the fact that the state of each agent converges to a common value.

To solve the consensus problem, the main task is to design the distributed protocol for each agent, and all agents achieve the consensus via the local communications. Hence, the well-designed protocol and optimized topology are usually considered when we investigate the problem.

For different circumstances, we need an effective consensus protocol to ensure the system keeps the coordinative behave, and there are many related results about it, such as the first-order MAS [9–12], the second-order MAS [13, 14], the high-order MAS [15, 16], and the fractional-order MAS [17–21]. Combined with weights and inequalities, sufficient conditions for consistency between the leader node and the following node are studied for the first-order MAS in [9]. H. J. LeBlanc and X. Koutsoukos studied the standards of the first-order MAS and the higher-order MAS consistency [10]. Based on the conditions of spanning trees and strong connectivity in the orientation diagram, the timing consistency of the first-order nonlinear MAS is studied in [12],

and the main conditions for MAS consistency are obtained: in the leader-following case, the topology should have a generated tree; in the absence of a reader node, the MAS topology is strongly connected. Based on the eigenvalues of the matrix, the controllability of the second-order MAS is studied in [13], and some equivalent conditions for the second-order MAS controllability are obtained. In [14], the tracking problem of the second-order nonlinear MAS with time-variable and multiple leader nodes is studied, and the proposed control protocol can effectively deal with the problems such as the disturbance and the unknown input of the reader node. Using self-triggering control and dynamic output feedback control methods, the consensus of the high-order MAS is studied, and the obtained results can improve the communication efficiency of the MAS [15].

In many applications, the topology of the MAS should be optimized; hence, the fixed topology and switching topology are considered frequently. For example, in [20], based on the switching topology, quasi-consistency of the MAS with competition and collaboration is studied, and the results show that, as long as the collaboration time is long enough, the system can achieve the consensus. In addition, with the Lipschitz conditions, the adaptive consensus of the MAS under fixed topology is studied in [21]. Additionally, state estimation problems with Markovian switching are

considered in [22, 23], and the fixed-time coordination problems are considered in [24–26].

Furthermore, by optimizing the topology of the MAS, the consensus problems of the MAS are further studied. Considering that the power system is affected by actuator saturation, the event trigger and self-trigger strategy are proposed, and the obtained results can save the resources of the network effectively [27].

Notice that most of the existed results about the consensus problems of the MAS are based on the fixed node set, and when the consensus is considered, the states of all agents are taken into account. In fact, in some applications, it is difficult to obtain all the information of the MAS, such as the BA model of scale-free network (SFN) [28], and the newly added node is connected to the related original node with a certain probability. Therefore, by comparing the error between every two nodes on a route of the communication graph, the related problems have been considered. Based on the balance between the in-degree and the out-degree, the consensus problem has been studied in [29]. In the noise environment, the consensus problem of the MAS under the SFN structure is studied in [30], and the results show that noise has limited influence on system consistency; in addition, under the SFN topology, it shows that the system has strong robustness. According to an adaptive consensus protocol, the adaptive consensus problem of the MAS with the SFN structure is studied in [31], and several conditions of consensus are obtained.

In the last decade, the properties of small-world networks have been studied in [32–34]. A small-world network refers to an ensemble of networks in which the mean geodesic (i.e., shortest-path) distance between nodes increases sufficiently slowly as a function of the number of nodes in the network [35]. Based on the framework of the small-world network, many problems of different research fields have been solved, such as the efficiency issue of computing and maintaining the eccentricity distribution on a large dynamic network [36], the viral-style information diffusion [37], and the visual analytics [38]. Furthermore, the coordination problem has been considered in the small-world network. In [39], the statistical properties of the consensus and synchronization of the small-world networks are studied, and the results show that the Cheeger constant, which is a major determinant to measure the convergence rate of the consensus and synchronization of the small-world networks, is investigated.

In fact, there is another way to study the consensus problem in the small-world framework. That is, by

comparing the error between every two agents in the small-world framework, the related consensus protocol is designed, and less error will be influenced, whereas all errors should be considered in the traditional results. Motivated by the idea, in this paper, we investigate the consensus problem in the small-world framework.

The remainder of this article is organized as the following. Section 2 introduces some notations and concepts. Section 3 proposes the consensus problem. In Section 4, several consensus criteria are obtained for the MAS. In Section 5, a numerical example shows the reliability of the proposed methods. In Section 6, we get the conclusions.

2. Preliminaries

Let $\mathcal{V} = \{v_i | i \in N\}$ be an agent set and $\mathcal{E}$ be the edge set, where N is the natural number set.

Let $\mathcal{A} = (a_{ij})_{N \times N}$ be a weighted symmetric adjacent matrix, where a_{ij} is the weight between nodes v_i and v_j , $a_{ij} = a_{ji}$, and a_{ij} takes values of 1 or 0. If $a_{ij} = 1$, then it shows that there exists information flow between agents v_i and v_j ; or else, $a_{ij} = 0$.

Let $\mathcal{E} = \{(i, j) | a_{ij} = 1, i \neq j\}$ be the edge set, where (i, j) refers to an edge between v_i and v_j . N_i is the neighbor set of v_i , where $N_i = \{j | (v_i, v_j) \in \mathcal{E}\}$. Then, the undirected weighted graph of a multiagent system (MAS) can be shown by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$.

$\forall v_i \in \mathcal{V}$, let $x_i(t) \in R^n$ be the state of agent v_i , where $t \in R$, and the consensus of the MAS can be interpreted as follows.

Definition 1 (see [31]). $\forall v_i, v_j \in \mathcal{V}$, if

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \quad (1)$$

then the system achieves the consensus, where $\|x_i(t) - x_j(t)\|$ is the norm of function $x_i(t) - x_j(t)$.

3. Problem Statement

Consider a MAS with the node set $\mathcal{V}$; the state of each node satisfies

$$\dot{x}_i'(t) = u_i(t), \quad i \in N, \quad (2)$$

where $u_i(t)$ is the consensus protocol of node v_i .

If $u_i(t)$ is implemented as

$$u_i(t) = \sum_{j \in N_i, \|x_j(t)\| > 0} \frac{a_{ij}}{d_i} \left[\frac{c_i(t) \|x_i(t)\| x_j(t)}{\|x_j(t)\|} - x_i(t) \right] - \sum_{j \in N_i, \|x_j(t)\| = 0} \frac{a_{ij}}{d_i} x_i(t), \quad v_i \in \mathcal{V}, \quad (3)$$

where $d_i = \sum_{j \in N_i} a_{ij}$ and $c_i(t)$ is a function, then we have the following system:

$$x'_i(t) = \sum_{j \in N_i, \|x_j(t)\| > 0} \frac{a_{ij}}{d_i} \left[\frac{c_i(t) \|x_i(t)\| \|x_j(t)\|}{\|x_j(t)\|} - x_i(t) \right] - \sum_{j \in N_i, \|x_j(t)\| = 0} \frac{a_{ij}}{d_i} x_i(t), \quad v_i \in \mathcal{V}, \quad (4)$$

or in a compact form,

$$X'(t) = [A(t) - I]X(t), \quad (5)$$

where $X(t) = [x_1^T(t), x_2^T(t), \dots]^T$, $I = \text{diag}\{1, 1, \dots, 1, \dots\}$, and

$$A(t) = \begin{bmatrix} b_{11}(t) & b_{12}(t) & b_{13}(t) & \dots \\ b_{21}(t) & b_{22}(t) & b_{23}(t) & \dots \\ \vdots & & \ddots & \end{bmatrix},$$

$$d_i^1 = \sum_{j \in N_i, \|x_j(t)\| > 0} a_{ij},$$

$$d_i^2 = \sum_{j \in N_i, \|x_j(t)\| = 0} a_{ij}, \quad (6)$$

$$d_i = d_i^1 + d_i^2,$$

$$b_{ij}(t) = \begin{cases} \frac{1}{d_i^1} \frac{\|x_i(t)\|}{\|x_j(t)\|}, & \|x_j(t)\| \neq 0 \text{ and } i \neq j, \\ 0, & i = j. \end{cases}$$

In the following sections, we are going to discuss the consensus of system (4), and we need the following assumption.

Assumption 1. We assume that $\forall v_i, v_q \in \mathcal{V}$, there exist two positive functions $c_i(t)$ and ε_{iq} such that if $\|x_i(t)\| \neq \|x_q(t)\|$, then

$$1 > \varepsilon_{iq} > c_i(t) \frac{\|x_i(t)\|^2 - 2\|x_i(t)\|\|x_q(t)\|\cos \theta_{iq} + \|x_q(t)\|^2}{(\|x_i(t)\| - \|x_q(t)\|)^2}, \quad (7)$$

where

$$\cos \theta_{iq} = \frac{x_q(t)x_i(t)}{\|x_q(t)\|\|x_i(t)\|},$$

$$c_i(t) = \begin{cases} \min_{v_i \in N_j \in \mathcal{V}} \frac{(\|x_i(t)\| - \|x_j(t)\|)^2}{(\|x_i(t)\|^2 - 2\|x_i(t)\|\|x_j(t)\|\cos \theta_{ij} + \|x_j(t)\|^2)}, & x_i(t) \neq x_j(t), \\ 1, & x_i(t) = x_j(t). \end{cases} \quad (8)$$

Remark 1. If $\|x_i(t)\| = \|x_q(t)\| = 0$, then the error between $v_i(t)$ and $v_j(t)$ is zero, and they achieve the consensus. If one of $\|x_i(t)\|$ and $\|x_q(t)\|$ is zero, then their angle of intersection is an arbitrary angle, and we define $\cos \theta_{iq} = 1$.

It shows that the real networks have the following property; for any two nodes in a network, the distance between them is relatively small, while, at the same time, the level of transitivity or clustering is relatively high [40]. This property, which is shared by many real-world networks, is called the small-world phenomenon. In the following sections, based on the small-world framework of

graph $\mathcal{G}$, the consensus problems of system (5) will be studied.

4. The Main Results

In this section, the consensus problem of the MAS is studied. Firstly, we propose the state error between every two neighbor agents. Then, in the connected topology, the state error between every two agents is provided via the route which associates with two agents. Finally, according to the diameter of a graph, consensus criteria are obtained;

meanwhile, as a special case, consensus criteria are obtained for the small-world framework.

Theorem 1. *If $q \in N_i$, then under protocol (5),*

$$\lim_{t \rightarrow \infty} [x_i(t) - x_q(t)] = 0, \quad (9)$$

where $\cos \theta_{iq} = ((x_q(t)x_i(t))/(\|x_q(t)\|\|x_i(t)\|))$.

Proof. First, we estimate the error between $x_i(t)$ and $x_i(t)$ under the condition $\|x_i(t)\| \cdot \|x_i(t)\| \neq 0$. Since

$$x'_i(t) = \sum_{j \in N_i, \|x_j(t)\| > 0} \frac{a_{ij}}{d_i} \left[\frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - x_i(t) \right] - \sum_{j \in N_i, \|x_j(t)\| = 0} \frac{a_{ij}}{d_i} x_i(t), \quad (10)$$

$$x'_q(t) = \sum_{v \in N_q, \|x_v(t)\| > 0} \frac{a_{qv}}{d_q} \left[\frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|} - x_q(t) \right] - \sum_{v \in N_q, \|x_v(t)\| = 0} \frac{a_{qv}}{d_i} x_q(t),$$

it holds that

$$\begin{aligned} x'_i(t) - x'_q(t) &= \sum_{j \in N_i} \frac{1}{d_i} \left[\frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - x_i(t) \right] - \sum_{v \in N_q} \frac{1}{d_q} \left[\frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|} - x_q(t) \right], \\ &= -[x_i(t) - x_q(t)] + \sum_{j \in N_i} \frac{1}{d_i} \frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - \sum_{v \in N_q} \frac{1}{d_q} \frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|}. \end{aligned} \quad (11)$$

For convenience, denote

$$\delta_{iq}(t) = x_i(t) - x_q(t),$$

$$f_{iq}(t) = \sum_{j \in N_i} \frac{1}{d_i} \frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - \sum_{v \in N_q} \frac{1}{d_q} \frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|}. \quad (12)$$

We have

$$\begin{aligned} \|f_{iq}(t)\| &\leq \left\| \sum_{j \in N_i} \frac{1}{d_i} \frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - \sum_{v \in N_q} \frac{1}{d_q} \frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|} \right\|, \\ &= \left\| \sum_{j \in N_i, v \in N_q} \left[\frac{1}{d_q d_i} \frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - \frac{1}{d_q d_i} \frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|} \right] \right\|, \\ &\leq \sum_{j \in N_i, v \in N_q} \frac{1}{d_q d_i} \left\| \frac{c_i(t)\|x_i(t)\|x_j(t)}{\|x_j(t)\|} - \frac{c_q(t)\|x_q(t)\|x_v(t)}{\|x_v(t)\|} \right\|. \end{aligned} \quad (13)$$

Let $c(t) = \max\{c_i(t), c_q(t)\}$; it follows that

$$\begin{aligned}
\|f_{iq}(t)\| &\leq \sum_{j \in N_i, v \in N_q} \frac{c(t)}{d_q d_i} \left(\sqrt{\frac{\|x_i(t)\| \|x_j(t)\|}{\|x_j(t)\|} - \frac{\|x_q(t)\| \|x_v(t)\|}{\|x_v(t)\|}} \right)^2 \\
&\leq \sum_{j \in N_i, v \in N_q} \frac{c_i(t)}{d_q d_i} \sqrt{\|x_i(t)\|^2 - 2 \frac{\|x_i(t)\| \|x_j(t)\|}{\|x_j(t)\|} \frac{\|x_q(t)\| \|x_v(t)\|}{\|x_v(t)\|} + \|x_q(t)\|^2} \\
&= \sum_{j \in N_i, v \in N_q} \frac{1}{d_q d_i} \sqrt{c_i(t)^2 \left[\|x_i(t)\|^2 - 2 \|x_i(t)\| \|x_q(t)\| \cos \theta_{jv} + \|x_q(t)\|^2 \right]}.
\end{aligned} \tag{14}$$

From Assumption 1, it holds that

$$\frac{\varepsilon_{iq} - c(t)^2}{\varepsilon_{iq} - c(t)^2 \cos \theta_{iq}} > \frac{2 \|x_i(t)\| \|x_q(t)\|}{\|x_i(t)\|^2 + \|x_q(t)\|^2}, \tag{15}$$

and it follows that

$$\varepsilon_{iq} \left(\|x_i(t)\|^2 + 2 \|x_i(t)\| \|x_q(t)\| + \|x_q(t)\|^2 \right) > c_i(t)^2 \left(\|x_i(t)\|^2 + 2 \|x_i(t)\| \|x_q(t)\| \cos \theta_{iq} + \|x_q(t)\|^2 \right). \tag{16}$$

Hence, we have

$$\begin{aligned}
\|f_{iq}(t)\| &< \sum_{j \in N_i, v \in N_q} \frac{c}{d_q d_i} \sqrt{c_i(t)^2 \left[\|x_i(t)\|^2 - 2 \|x_i(t)\| \|x_q(t)\| + \|x_q(t)\|^2 \right]}, \\
&\leq \varepsilon_{iq} \sum_{j \in N_i, v \in N_q} \frac{1}{d_q d_i} \left[\|x_i(t)\| - \|x_q(t)\| \right] \\
&\leq \varepsilon_{iq} \sum_{j \in N_i, v \in N_q} \frac{1}{d_q d_i} \|x_i(t) - x_q(t)\|, \\
&\leq \varepsilon_{iq} \|x_i(t) - x_q(t)\|.
\end{aligned} \tag{17}$$

Let the Lyapunov candidate be $V_{iq}(t) = (1/2) \delta_{iq}(t)^T \delta_{iq}(t)$; then, we get

$$\begin{aligned}
V_{iq}'(t) &= \frac{1}{2} \left[\delta_{iq}'(t)^T \delta_{iq}(t) + \delta_{iq}(t)^T \delta_{iq}'(t) \right], \\
&\leq \frac{1}{2} \left[-\delta_{iq}(t) + f_{iq}(t) \right]^T \delta_{iq}(t) + \delta_{iq}(t)^T \left[-\delta_{iq}(t) + f_{iq}(t) \right], \\
&\leq -\delta_{iq}(t)^T \delta_{iq}(t) + \frac{1}{2} \left[f_{iq}(t) \right]^T \delta_{iq}(t) + \delta_{iq}(t)^T f_{iq}(t), \\
&\leq -\delta_{iq}(t)^T \delta_{iq}(t) + \varepsilon_{iq} \|x_i(t) - x_q(t)\| \|\delta_{iq}(t)\|, \\
&= -\|\delta_{iq}(t)\|^2 + \varepsilon_{iq} \|\delta_{iq}(t)\|^2, \\
&= 2(-1 + \varepsilon_{iq}) V_{iq}(t), \\
&< 0,
\end{aligned} \tag{18}$$

and $\lim_{t \rightarrow \infty} \delta_{iq}(t) = 0$, namely, $\lim_{t \rightarrow \infty} [x_i(t) - x_q(t)] = 0$.

If $\|x_i(t)\| \cdot \|x_q(t)\| = 0$, then at least one of $\|x_i(t)\|$ and $\|x_q(t)\|$ is 0. Without loss of generality, we suppose

$\|x_q(t)\| = 0$, namely, $x_q(t)$ is the zero vector, and we can get the same result. This completes the proof.

Theorem 2. If $\mathcal{G}$ is connected, then under protocol (5), system (4) achieves the consensus.

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_q(t)\| \leq \lim_{t \rightarrow \infty} \|x_i(t) - x_{l_1}(t)\| + \lim_{t \rightarrow \infty} \|x_{l_1}(t) - x_{l_2}(t)\| + \cdots + \lim_{t \rightarrow \infty} \|x_{l_n}(t) - x_q(t)\| = 0, \quad (19)$$

and on the contrary, $\lim_{t \rightarrow \infty} \|x_i(t) - x_q(t)\| \geq 0$; then, we have $\lim_{t \rightarrow \infty} \|x_i(t) - x_q(t)\| = 0$, and this completes the proof. $\square$

Remark 2. The consensus criterion of Theorem 2 is a rough result; it has not disclosed the relations among the scale of the network, the bound of tolerate errors, and the diameter of the graph G . In the following results, the relations will be discussed.

Assumption 2. For each diameter $P = v_{i_1} v_{i_2} \dots v_{i_s} \dots v_{i_{n+1}}$ of graph $\mathcal{G}$, we assume that there exist two functions $V_{iM}(t)$ and $V_{im}(t)$ such that

$$V_{iM} \geq V_{iq}(t) \geq V_{im}, \quad (20)$$

Proof. If $q \in N_i$, then $\lim_{t \rightarrow \infty} [x_i(t) - x_q(t)] = 0$ has been proved in Theorem 1.

Since $\mathcal{G}$ is connected, if $q \notin N_i$, then there exists a path $v_i v_{l_1} v_{l_2} \dots v_{l_n} v_q$ between v_i and v_q such that

where $V_{iq}(t) = (1/2)\delta_{iq}^T \delta_{iq}(t)$, $v_q \in N_i$, $i, q \in \{v_{i_1}, v_{i_2}, \dots, v_{i_s}, \dots, v_{i_n}\}$, and V_{iM} and V_{im} are non-negative constants.

Theorem 3. If $P = v_{i_1} v_{i_2} \dots v_{i_{L+1}}$ is the diameter of graph $\mathcal{G}$ and G is connected, then under the control of protocol (5), system (4) achieves the consensus if

$$\sum_{l=1}^L \varepsilon_{i_l i_{l+1}} < 1, \quad (21)$$

where L is the length of path $i_1 i_2 \dots i_L i_q$ in the small-world network of the MAS.

Proof. Let $V_{iq}(t) = (1/2)\delta_{i_1 i_2}^T(t)\delta_{i_1 i_2}(t) + (1/2)\delta_{i_2 i_3}^T(t)\delta_{i_2 i_3}(t) + \cdots + (1/2)\delta_{i_L i_{L+1}}^T(t)\delta_{i_L i_{L+1}}(t)$; then, it holds that

$$\begin{aligned} V_{i_l i_{l+1}}'(t) &= \frac{1}{2} \left[\delta_{i_1 i_2}^T(t)\delta_{i_1 i_2}(t) + \delta_{i_1 i_2}^T(t)\delta_{i_1 i_2}'(t) + \cdots + \delta_{i_L i_{L+1}}^T(t)\delta_{i_L i_{L+1}}(t) + \delta_{i_L i_{L+1}}^T(t)\delta_{i_L i_{L+1}}'(t) \right], \\ &\leq \frac{1}{2} \left[[-\delta_{i_1 i_2}(t) + f_{i_1 i_2}(t)]^T \delta_{i_1 i_2}(t) + \cdots + \delta_{i_L i_{L+1}}(t)^T [-\delta_{i_L i_{L+1}}(t) + f_{i_L i_{L+1}}(t)], \right. \\ &\quad \left. + \delta_{i_1 i_2}^T(t) [-\delta_{i_1 i_2}(t) + f_{i_1 i_2}(t)] + \cdots + [-\delta_{i_n i_{L+1}}(t) + f_{i_n i_{L+1}}(t)]^T \delta_{i_n i_{L+1}}(t) \right], \\ &\leq -\delta_{i_1 i_2}(t)^T \delta_{i_1 i_2}(t) + [f_{i_1 i_2}(t)]^T \delta_{i_1 i_2}(t) + \cdots + \delta_{i_n i_{L+1}}(t)^T f_{i_n i_{L+1}}(t), \\ &\leq -\delta_{iq}(t)^T \delta_{iq}(t) + \varepsilon_{i_1 i_2} \|x_{i_1}(t) - x_{i_2}(t)\| \|\delta_{i_1 i_2}(t)\| + \cdots + \varepsilon_{iq} \|x_{i_n}(t) - x_{i_{L+1}}(t)\| \|\delta_{i_n i_{L+1}}(t)\|, \\ &\leq -\delta_{iq}(t)^T \delta_{iq}(t) + \varepsilon_{iq} [\varepsilon_{i_1 i_2} \delta_{i_1 i_2}(t)^T \delta_{i_1 i_2}(t) + \cdots + \varepsilon_{i_L i_{L+1}} \delta_{i_L i_{L+1}}(t)^T \delta_{i_L i_{L+1}}(t)], \\ &= 2[-V_{iq} + V_{iq}] = 0, \end{aligned} \quad (22)$$

and it follows that $\forall v_i, v_q \in \mathcal{V}$, $\lim_{t \rightarrow \infty} \|x_i(t) - x_q(t)\| = 0$. This completes the proof. $\square$

Corollary 1. If $P = v_{i_1} v_{i_2} \dots v_{i_{L+1}}$ is the diameter of a small-world graph $\mathcal{G}$, G is connected, and $V_{im}(t) = m > 0$ and $V_{iM}(t) = M > 0$, then under the control of protocol (5), system (4) achieves the consensus if $\forall v_i, v_q \in \mathcal{V}$,

$$0 < \varepsilon_{iq} < \frac{m}{LM}. \quad (23)$$

Proof. According to the proof of Theorem 3, it holds that $V_{iq}'(t) \leq 2[-V_{im} + \varepsilon_{iq} LV_{iM}] < 0$.

Hence, system (4) achieves consensus, and this completes the proof.

In this section, the consensus problem of the MAS has been studied. In the next section, a numerical example shows the reliability of the obtained methods. $\square$

FIGURE 1: The topology of MAS (4).

FIGURE 2: All agents of MAS (4) achieve the consensus.

FIGURE 3: Function of $c(t)$.

5. A Numerical Example

Example 1. Consider a connected MAS (4) with seven agents, where $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$, and the topology is shown in Figure 1. Suppose $x_i(t) \in R$, let

$$\varepsilon_{i,q} = \frac{1}{4} c_i(t)^2 \frac{\|x_i(t)\|^2 - 2\|x_i(t)\|\|x_q(t)\|\cos \theta_{iq} + \|x_q(t)\|^2}{(\|x_i(t)\| - \|x_q(t)\|)^2}, \quad (24)$$

where $i, q = 1, 2, \dots, 7$, and according to Corollary 1, under the control of protocol (3), MAS (5) achieves the consensus. The simulation result is shown in Figures 2 and 3, where $c(t) = \max\{c_i(t) | i = 1, 2, \dots, 7\}$.

6. Conclusion

In this paper, the consensus problem of the MAS has been investigated via the node-to-node communications. A kind of distributed protocol has been designed firstly. Then, we have obtained a system with scale-free topology. By the theoretical analysis, the results have shown that based on the protocols, all agents can achieve the consensus; meanwhile, a criterion on the small-world framework has been obtained. Finally, the numerical example has shown the reliability of the proposed methods.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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