

Research Article

Maintenance Optimization Based on Three-Stage Failure Process under Performance-Based Contracting

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As a new form of support contract, performance-based contracting has been extensively applied in both public and private sectors. However, maintenance policies under performance-based contracting have not gotten enough attention. In this paper, a preventive maintenance optimization model based on three-stage failure process for a single-component system is investigated with an objective of maximizing the profit and improving system performance at a lower cost under performance-based contracting. Different from conventional optimization models, the step revenue function is used to correlate profit with availability and cost. Then, a maintenance optimization model is proposed to maximize profit by optimizing the inspection interval. Moreover, the customers' upper limit of funds is considered when we use the revenue function, which has rarely been considered in past studies. Finally, a case study on the cold water pumps along with comparison of linear and step revenue function and sensitivity analysis is provided to illustrate the applicability and effectiveness of our proposed approach.

1. Introduction

As the global economy continues to be more service-based, the role of support and maintenance has become increasingly important. This trend is particularly prominent in capital-intensive industries, such as defence, aviation, and energy [1]. Users of equipment need to spend a lot of money, labour, and materials in annual operation and sustainment cost. For example, the US military needs \$63 billion and 678,000 DOD personnel for support and maintenance. In wind power industry, the sustainment and maintenance market had been expected to reach \$10.6 billion in 2016 and continue to increase at a rate of 16.6% [2].

Traditionally, support and maintenance services are implemented under material-based contracts (MBC) [3]. Under MBC, customers pay service providers based on the materials and labour consumed each time. Because it is easier to implement, MBC have been widely used for supporting equipment operation and maintenance in private and public sectors. However, MBC will lead to a lack of innovation on the supply side because their revenue comes from the components, services, and consumables they sold.

As the equipment ages and becomes obsolete, the suppliers may gain more revenues, and the resulting costs will be borne by the customers [4–8].

One approach that specifically addresses this challenge is “performance-based contracting” (PBC). PBC as a new form of support contract is different from MBC, the buyers attend more to the desired outcome (as the value expected from the solution), rather than the individual components of a solution (e.g., a machine and related services), whereas the suppliers' compensation is tied to successfully achieving this outcome [9].

PBC originated from the Performance-Based Logistics (PBL) of the US military. Performance-Based Logistics is the purchase of support as an integrated, affordable performance package, which is designed to optimize system readiness and meet performance goals through long-term support arrangements with clear lines of authority and responsibility [10]. Its essence is to purchase performance outcomes, rather than individual components or repair services. PBL mentioned has the same meaning as PBC in this paper.

In the past years, successful PBC cases are frequently reported in the US military sectors. For example, the US

navy has witnessed the equipment availability improvement, from 67% under traditional material contracts to 85% for the F/A-18 aircraft under PBC and from 62% to 94% for the Aegis cruiser [11]. In not only the military, but also PBC is widely used in other industries, such as space freight transport (e.g., SpaceX Company), pay per use of aero engines (e.g., the often-cited Rolls Royce aero engine support case), full-service offerings for machinery and equipment (cp. the discussion of outsourced manufacturing), or full-service fleet and mobility management (helicopter to forklift trucks) [12].

With the application of PBC, research on PBC is gradually increasing. However, most of the existing research on PBC focused on performance metrics, frameworks, risks, and incentives. Doerr et al. [13] discussed how an excess of measurement could make administration of comprehensive PBL contracts more costly, while the difficulty with defining and measuring some logistics services make consumer sovereignty difficult to establish. Sols et al. [14] proposed a framework for formulating more efficient and effective PBL contractual agreements and identified the main topics or aspects of a successful PBL initiative. Ng and Nudurupati [15] identified the challenges and risks of implementing PBCs, which include the complexity and unpredictability of costs, the dependability on customer in delivering the service, and the cultural change from traditional setting. Datta and Roy [16] investigated the performance implications of different incentive mechanisms and risk-sharing arrangements for contracting for availability type of performance-based contracts. To the best of our knowledge, maintenance strategies under PBCs are barely addressed.

In this paper, a preventive maintenance optimization model based on three-stage failure process under PBC is proposed. Traditionally, the component state before failure is usually deemed as binary, e.g., normal or defective. This is an approximation, but still applicable to some practical situations where inspections may only reveal whether the component is working normally or not. Then, Christer [17] proposed a delay time concept that considers the failure process as a two-stage process from new to an initial point that a defect can be first identified by an inspection and then from that point to failure if the defect is unattended. Contrasted with many other maintenance models reported, a number of successful case studies have been reported with actual applications in industry using the delay time concept and associated models. For a recent review of the delay time inspection models, see Wang [18]. However, Wang et al. [19] observed that a three-color scheme in industries was used to quantify the plant state before failure into green (normal), yellow (need attention), and red (need immediate attention). Motivated by this observation, Wang [20] extended the delay time concept into a three-stage failure process where the traditionally defined delay time is divided into another two stages corresponding to a minor and a severe defective stage. This implies that the plant item can be in one of the four states, namely, normal, minorly defective, severely defective, and failed at any one time. The durations of the normal, minorly defective, and severely defective states constitute a three-stage failure process. Based on the three-stage failure

process, many researchers have developed models to optimize preventive maintenance [21–23]. Nevertheless, we do not find the work related to maintenance optimization based on three-stage failure process under PBC reported.

Based on the three-stage failure process, we present a model for preventive maintenance optimization of a single-component system subject to a dominant failure mode under PBC. The main objective is to determine the optimal maintenance policy for maximizing the profit and improving system performance at a lower cost. In contrast with existing maintenance models, a step revenue function considering funds limit is used to correlate the availability with the supplier's revenue and profit. Adopting profit as the optimization objective under PBC, the optimal maintenance policy is determined to maximize the service provider's expected profit rate. To illustrate the impact of PBC, we compare the proposed policy with the cost minimization policy. Finally, the difference between the step revenue function and the linear revenue function is compared to illustrate the advantages of the step revenue function.

The rest of the paper is organized as follows. Section 2 discusses the related work. Section 3 provides the problem description, modelling assumption, and notations. Section 4 develops a three-stage preventive maintenance model under PBC and provides the solution algorithm. Section 5 presents a case study on the cold-water pumps in a soft drink company to validate the applicability of the proposed model. The conclusion and future research directions are presented in Section 6.

2. Related Work

In the past two decades, research interests on PBC have gradually increased. In this section, we mainly review the papers related to the application of PBC to maintenance support.

In Table 1, we have reviewed 18 literature related to the application of PBC to maintenance support. These works are sequenced with the publishing time; we summarize their application domains: revenue functions, theory based, and funds limit.

With respect to the application domains, we divide it into spares inventory and maintenance optimization. Then, we note that 12 of the reviewed papers [24–33, 36, 38] focused on the spares inventory. Spare parts are common inventory stock items, which are needed to maintain equipment and the cost of spare parts takes a large share of product lifecycle cost [42]. Therefore, in order to obtain the optimal stock keeping units that maximize a company's profit subject to a collection of performance constraints, PBC is applied and optimization models are developed. By contrast, only 6 of the reviewed papers [34, 35, 37, 39–41] concern maintenance optimization under PBC.

Regarding revenue functions, there are mainly three types, namely, linear, exponential, and step. 10 of the reviewed papers did not use revenue function, while the other 8 reviewed papers used different types of revenue function. Among them, linear and exponential are mainly used to solve a number of practical problems. Wang et al.

TABLE 1: Overview of application domains, revenue functions, funds limit, and theory based, by the main reviewed contributions.

Ref.	Application domain	Type of revenue function	Consider the funds limit?	Theory based
Kim et al. [24]	Spares inventory	—	—	Principal-agent model
Nowicki et al. [25]	Spares inventory	Linear exponential step	No	Metric
Mirzahasseinian and Piplani [26]	Spares inventory	—	—	Metric; Queueing theory; Markov model
Jin and Tian [27]	Spares inventory and reliability	—	—	Reliability optimization; inventory management
Jin and Wang [28]	Spares inventory and reliability	—	—	Reliability optimization; inventory management
Mirzahasseinian and Piplani [29]	Spares inventory and reliability	—	—	Metric
Kang et al. [30]	Spares inventory and reliability	—	—	Simulation
Jin et al. [31]	Spares inventory and reliability	—	—	Game theoretical; Metric
Patriarca et al. [32]	Spares inventory	—	—	Metric
Mirzahasseinian et al. [33]	Spares inventory and reliability	—	—	Reliability optimization; inventory management
Xiang et al. [34]	Maintenance policy	Linear	No	Stochastic degradation process; CBM
Qiu et al. [35]	Maintenance policy	Linear	No	Virtual age model
Hur et al. [36]	Spares inventory	—	No	Markov chain
Yang et al. [37]	Maintenance policy	Linear	No	Two-stage preventive maintenance model
Patra et al. [38]	Spares inventory and reliability	Nonlinear	No	Principal-agent model; Time-series model
Wang et al. [39]	Maintenance policy	Linear; Exponential	No	Time-series model
Wang et al. [40]	Maintenance policy	Linear	No	Degradation-threshold-shock model
Li et al. [41]	Maintenance policy	Exponential	No	Time-series model
This paper	Maintenance policy	Step; Linear	Yes	Three-stage preventive maintenance model

mainly investigated the trade-off between the wind turbine's availability and maintenance under PBC [40]. Linear and exponential revenue functions were both used in their research, and it showed that the linear revenue function is more effective than the exponential revenue function in their case. Li et al. proposed a multicomponent maintenance policy for shipborne antenna to maximize the operating revenue under the requirement of the antenna's availability [41]. They hold that the exponential revenue function, compared with the linear revenue function, can show the influence of system performance more clearly. Nowicki et al. proposed the step revenue function in 2008 [25]. Nevertheless, we found there is no succeeding study using step revenue function.

With respect to the theory based on maintenance optimization, we can see from Table 1 that PBC has applied stochastic degradation model [34], virtual age model [35], two-stage preventive maintenance model [37], time-series model [39, 41], degradation-threshold-shock model [40], and so on. According to Jonge and Scarf [43], there are very rich maintenance optimization models, which were classified according to asset (e.g., system, unit, component, and part), deterioration state spaces (e.g., two states, three states,

discrete state space, and continuous state space), and the maintenance actions carried out on these assets (e.g., preventive and corrective maintenance, replacement, repair, inspection, and condition monitoring). We then can conclude that PBC has only been applied in some of them, and maintenance optimization under PBC needs to be further investigated.

In addition, the annual budget on maintenance support in either private or public sector is usually limited. A very important point in the meaning of PBC is "affordable," which cannot be ignored in the implementation of PBC. It means that PBC is usually under limited funds. Therefore, it is necessary to consider funds limit in the study of maintenance optimization. From Table 1, we note that none of the existing studies considers the funds limit when using revenue functions.

Based on the above discussions, we can conclude the following:

- (1) Only a few of the existing studies have applied PBC in maintenance optimization. Still, there are urgent needs to investigate the problem of maintenance optimization under PBC.

- (2) There are all kinds of maintenance optimization models. PBC have only been applied in very few of them.
- (3) When applying PBC, most papers used linear and exponential revenue function instead of step one. They need to be further extended to step revenue function. Moreover, the existing studies do not consider the funds limit.

This paper attempts to investigate a preventive maintenance optimization model based on three-stage failure process under PBC considering complex situations, including the use of step revenue function and the funds limit.

3. Problem Description and Assumptions

3.1. Problem Description. This study considers a single-component system subject to a single failure mode [44, 45]. Such system could be a battery or a pump. There exist three possible states of the system, i.e., normal, minorly defective, and severely defective before failure, respectively [20]. It is known that for most systems, inspections are part of planned preventive maintenance. We usually take periodic inspections to check the working status of the system. If the system is in a minorly defective state, we should pay attention and take some responses; or in a severely defective state, we need to take immediate response. Failure will be observed immediately and repaired or replaced. Generally, inspections cost money and are critical to system performance (e.g., availability and mission completion rate). Thus, how often to inspect the system or the determination of inspection intervals is one of the key decisions of a maintenance manager, and then inspection interval is usually deemed as the decision variable for system maintenance optimization. Traditionally, the optimization objective of maintenance optimization models is to minimize the long-term expected cost per unit time; therefore, cost is the main concern when determining the inspection interval. In this paper, we deal with a preventive maintenance optimization model based on three-stage failure process under PBC. Different from the existing maintenance optimization models based on three-stage failure process, PBC motivates service providers to implement effective maintenance policies in order to boost profits and improve system performance at a lower cost [27]. For this reason, how to find the optimal inspection interval to achieve the objectives of PBC is what we need to study.

3.2. Model Assumptions. The modelling assumptions used in this paper are as follows:

- (1) The system is single component and subject to a single failure mode.
- (2) The failure process is divided into three stages, namely, normal, minorly defective stage, and severely defective stage. These three stages are independent.
- (3) An inspection of interval, t , is carried out to check which stage the system is in. Checks are

instantaneous and perfect, but the cost of checks cannot be ignored.

- (4) If the system is found to be in a minorly defective stage, the system may not be necessarily replaced immediately. We will shorten the subsequent inspection intervals to be half of the current interval.
- (5) If the system is found to be in the severely defective stage, it is always replaced immediately.
- (6) Failure can be observed immediately, and replacement is carried out at the time of the failure.
- (7) Replacement is regarded as renewing the system. There are two possible renewals (e.g., a failure renewal and an inspection renewal) when a severely defective stage is found.

The notations used in this paper are listed in Table 2.

4. Maintenance Optimization Modelling under Performance-Based Contracting

4.1. Optimization Model of Performance-Based Contracting. The purpose of this section is to calculate and optimize the expected profit rate of the system under PBC. PBC is a typical profit centered policy, which considers the performance and operation cost of the product. In this paper, the performance is measured by the expected average availability, $A(t)$, because most products in defense and aviation need higher system stability and longer operation time, and the operation cost is measured by the expected cost rate per unit time, $EC(t)$.

Before formulating the profit function, we need to express the relationship between the profit and availability with the help of revenue function. In previous studies, the form of the revenue function is usually linear as follows:

$$ER(t) = \begin{cases} 0, & A(t) < A_0, \\ a + b(A(t) - A_0), & A(t) \geq A_0, \end{cases} \quad (1)$$

where a denotes the fixed part of the revenue, $b(A(t) - A_0)$ represents the performance-based incentive, and A_0 is the benchmark availability requirement that the service provider needs to achieve.

The form of linear revenue function is relatively simple, which clearly expresses the relationship between availability and revenue, and only needs to set fewer parameters, so it is easy to implement. However, the lack of adjustable parameters also makes customers do not have enough approaches to bargain with suppliers when signing contracts. Meanwhile, it may lead to the effect of PBC incentive being not obvious.

Effective PBC contains performance incentives and disincentives linked to the support requirements of the customers. Incentives and disincentives should be included in the PBC to achieve the target level of performance [10]. For this reason, this paper adopts a form of revenue function, step revenue function, which is rarely used before. The concept of step revenue function is to further divide the availability, better motivate the behavior of suppliers, and

TABLE 2: Notations used in this paper.

Notations	Descriptions
X_n	Random variable representing the duration of the n th stage of the system, $n = 1, 2, 3$
f_{X_n}	Probability density function of X_n , $n = 1, 2, 3$
F_{X_n}	Cumulative distribution function of X_n , $n = 1, 2, 3$
t	Inspection interval
T_p	Time to preventive replacement
T_f	Time to failure
C_{ins}	Cost per inspection
C_p	Cost per preventive renewal
C_f	Cost per failure renewal
D_p	Downtime per preventive renewal
D_f	Downtime per failure renewal
EC	Expected cost in a renewal cycle
T_{down}	Expected downtime in a renewal cycle
T_{up}	Expected uptime in a renewal cycle
EC(t)	Expected cost rate per unit time
A(t)	Expected average availability
ER(t)	Expected revenue rate per unit time
EP(t)	Expected profit rate per unit time
R_{max}	The upper limit of revenue

give higher incentive for good performance. Different from the linear revenue function, step revenue function contains more intervals and the degree of motivate will increase with availability. The general form of the step revenue function is expressed as

$$ER(t) = \begin{cases} 0, & 0 \leq A(t) < A_0, \\ a_1, & A_0 \leq A(t) < A_1, \\ a_1 + b_1(A(t) - A_1), & A_1 \leq A(t) < A_2, \\ \vdots & \vdots \\ a_{i-1} + b_{i-1}(A(t) - A_{i-1}), & A_{i-1} \leq A(t) < A_i, \\ a_i + b_i(A(t) - A_i), & A_i \leq A(t) \leq 1, \end{cases} \quad (2)$$

where a_i denotes the fixed revenue which is often thought of as guaranteed money, once the system's availability within the corresponding range, and $a_i = a_{i-1} + b_{i-1}(A_i - A_{i-1})$ when $i \geq 2$. $b_i(A(t) - A_i)$ represents the performance-based incentive. In order to motivate the better performance, b_i is usually more than b_{i-1} . A_i is the selected piecewise point.

In order to simplify the modelling and facilitate the subsequent case study, we consider the situation, where the step revenue function consists of four bands (i.e., $i = 2$), in the following discussion. Then, the specific step revenue function when $i = 2$ is shown in Figure 1.

The step revenue function when $i = 2$ can be expressed as follows:

$$ER(t) = \begin{cases} 0, & 0 \leq A(t) < A_0, \\ a_1, & A_0 \leq A(t) < A_1, \\ a_1 + b_1(A(t) - A_1), & A_1 \leq A(t) < A_2, \\ a_2 + b_2(A(t) - A_2), & A_2 \leq A(t) \leq 1, \end{cases} \quad (3)$$

where A_0 denotes the minimum acceptable availability of the system; suppliers could not obtain any revenue when $A(t)$

$< A_0$. A_1 denotes the starting value of the reward, and A_2 represents the starting value of high rewards. If $A_0 \leq A(t) < A_1$, suppliers can only obtain the fixed revenue a_1 . If $A_1 \leq A(t) < A_2$, suppliers will gain reward $b_1(A(t) - A_1)$ on the basis of a_1 . If $A(t) \geq A_2$, suppliers can get reward $b_2(A(t) - A_2)$ on the basis of a_2 . In addition, the upper limit of funds, R_{max} , is considered because the revenue could not increase without limit.

Let EP(t) represent the expected profit rate per unit time to the service provider. Considering the expected cost rate and the expected revenue rate, the maintenance optimization model with the objective of maximizing the expected profit rate is formulated as

$$\begin{aligned} \max EP(t) &= ER(t) - EC(t) \\ t^* &= \arg \max\{EP(t)\} \\ \text{s.t. } 0 &< t \leq t_{max}, \end{aligned} \quad (4)$$

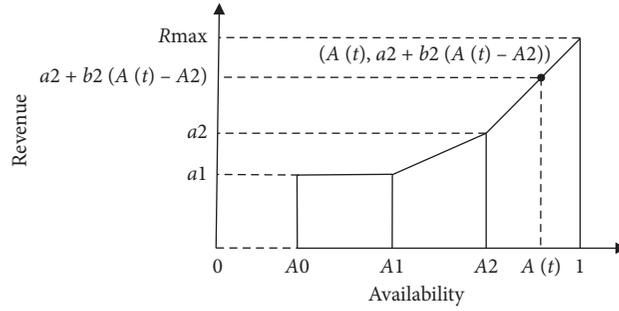
where EC(t) is the expected cost rate per unit time and t_{max} is the maximum inspection interval.

For comparison purposes, we propose a benchmark model with the objective of minimizing cost:

$$\begin{aligned} \min EC(t) \\ t^* &= \arg \min\{EC(t)\} \\ \text{s.t. } 0 &< t \leq t_{max}. \end{aligned} \quad (5)$$

4.2. Calculation of Renewals Probability. In order to obtain A(t) and EC(t) in the expected profit rate function, we need to calculate the probabilities of failure renewal and inspection renewal first.

In this paper, we assume the system is renewed at failure or the time that a severe defect state was first found prior to failure. However, if a minor defective state is identified, the subsequent inspection intervals are halved.

FIGURE 1: Step revenue function when $i = 2$.

4.2.1. *Probability of a Failure Renewal.* This is the probability that failure renewal occurs within an inspection interval. There are two possible scenarios here:

- (a) The system failed in $((k-1)t, kt)$ before any defect was found. Figure 2 shows the illustration of this scenario.

Since the inspection is perfect, the minor and the severe defective must occur within the same interval to cause the failure. In this case, the probability of failure can be expressed as

$$\begin{aligned}
 P((k-1)t < T_f < kt) &= P((k-1)t < X_1 < kt, 0 < X_2 < kt - X_1, 0 < X_3 < kt - X_1 - X_2) \\
 &= \int_{(k-1)t}^{kt} f_{X_1}(x) \int_0^{kt-x} f_{X_2}(y) \int_0^{kt-x-y} f_{X_3}(z) dz dy dx \\
 &= \int_{(k-1)t}^{kt} f_{X_1}(x) \int_0^{kt-x} f_{X_2}(y) F_{X_3}(kt - x - y) dy dx,
 \end{aligned} \tag{6}$$

where $k = 1, \dots$

The probability density function of failure at T_f , $T_f \in ((k-1)t + z, (k-1)t + z + dz)$ and $z \in (0, t)$, can be derived from equation (6) as follows:

$$P \frac{((k-1)t + z < T_f < (k-1)t + z + dz)}{dz} = \int_{(k-1)t}^{(k-1)t+z} f_{X_1}(x) \int_0^{(k-1)t+z-x} f_{X_2}(y) f_{X_3}((k-1)t + z - x - y) dy dx, \tag{7}$$

where $k = 1, \dots$

Similar to equation (6), we have

- (b) The system failed in $(kt + (i-1)t/2, kt + it/2)$ after a minor defect was first found at kt . Figure 3 shows the illustration of this scenario.

$$\begin{aligned}
 P\left(\frac{kt + (i-1)t}{2} < T_f < \frac{kt + it}{2}\right) &= P\left((k-1)t < X_1 < kt, \frac{kt + (i-1)t}{2} - X_1 < X_2 < \frac{kt + it}{2} - X_1, 0 < X_3 < \frac{kt + it}{2} - X_1 - X_2\right) \\
 &= \int_{(k-1)t}^{kt} f_{X_1}(x) \int_{kt+(i-1)t/2-x}^{kt+it/2-x} f_{X_2}(y) F_{X_3}\left(\frac{kt + it}{2} - x - y\right) dy dx,
 \end{aligned} \tag{8}$$

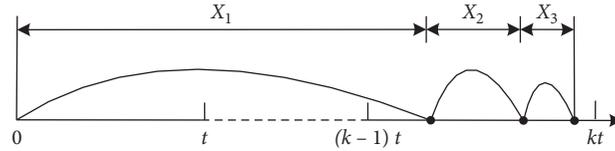


FIGURE 2: A schematic illustration of a failure renewal in $((k-1)t, kt)$.

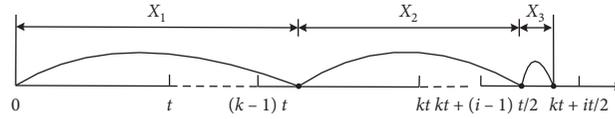


FIGURE 3: A schematic illustration of a failure renewal in $(kt + (i-1)t/2, kt + it/2)$.

where $k = 1, \dots, i = 1, \dots$

The probability density function of failure at T_f , $T_f \in (kt + (i-1)t/2 + z, kt + (i-1)t/2 + z + dz)$, $z \in (0, t/2)$ is following the same way we did for equation (7), given by

$$P \frac{((kt + (i-1)t/2) + z < T_f < (kt + (i-1)t/2) + z + dz)}{dz} = \int_{(k-1)t}^{kt} f_{X_1}(x) \int_{kt+(i-1)t/2-x}^{kt+(i-1)t/2+z-x} f_{X_2}(y) f_{X_3}\left(\frac{kt + (i-1)t}{2} + z - x - y\right) dy dx, \tag{9}$$

where $k = 1, \dots, i = 1, \dots$

Next, we formulate the probability of an inspection renewal.

4.2.2. *Probability of an Inspection Renewal.* There are also two possible scenarios here:

- (a) The system was renewed at kt before any minor defect was found, but a severe defect was found at kt . Figure 4 shows the illustration of this scenario. This is opposite to equation (6), and the severe defective stage must be longer than $kt - x - y$; that is,

$$P(T_p = kt) = P((k-1)t < X_1 < kt, 0 < X_2 < kt - X_1, X_3 > kt - X_1 - X_2) = \int_{(k-1)t}^{kt} f_{X_1}(x) \int_0^{kt-x} f_{X_2}(y) (1 - F_{X_3}(kt - x - y)) dy dx, \tag{10}$$

where $k = 1, \dots$

- (b) The system was renewed at $kt + it/2$ after a minor defect was first found at kt and a severe defect at

$kt + it/2$. The illustration of this scenario is shown in Figure 5. Similar to equation (10) but the severely defective stage must be longer than $kt + it/2 - x - y$, so that

$$P\left(T_p = \frac{kt + it}{2}\right) = P\left((k-1)t < X_1 < kt, \frac{kt + (i-1)t}{2} - X_1 < X_2 < \frac{kt + it}{2} - X_1, X_3 > \frac{kt + it}{2} - X_1 - X_2\right) = \int_{(k-1)t}^{kt} f_{X_1}(x) \int_{kt+(i-1)t/2-x}^{kt+it/2-x} f_{X_2}(y) \left(1 - F_{X_3}\left(\frac{kt + it}{2} - x - y\right)\right) dy dx, \tag{11}$$

where $k = 1, \dots, i = 1, \dots$

4.3. *Calculation of Availability and Cost.* According to the probability of failure and inspection renewal, and

considering all inspection costs and renewal costs, we derive the expected uptime, expected downtime, and expected cost in a renewal cycle. Then, the expected average availability and expected cost rate per unit time are calculated.

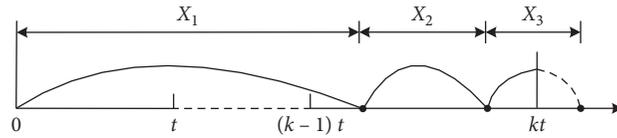


FIGURE 4: A schematic illustration of an inspection renewal at kt .

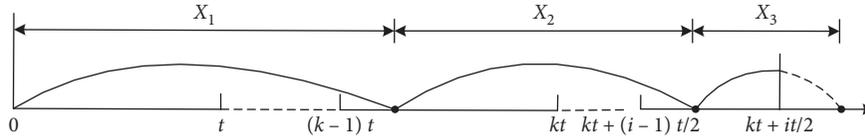


FIGURE 5: A schematic illustration of an inspection renewal at $kt + it/2$.

4.3.1. *Expected Average Availability.* Availability is the most used performance metric in maintenance support services. Generally, the average availability is defined as follows:

$$\begin{aligned} \text{availability} &= \frac{\text{expected uptime in a renewal cycle}}{\text{expected renewal cycle length}} \\ &= \frac{\text{expected uptime in a renewal cycle}}{\text{expected uptime in a renewal cycle} + \text{expected downtime in a renewal cycle}}. \end{aligned} \tag{12}$$

Let T_{up} /denote the expected uptime in a renewal cycle and T_{down} denote the expected downtime in a renewal cycle. Then, the expected average availability can be expressed as

$$A(t) = \frac{T_{\text{up}}}{T_{\text{up}} + T_{\text{down}}}. \tag{13}$$

In previous maintenance researches, downtime is usually ignored when calculating the expected renewal cycle length.

However, downtime must be considered in this paper because we need to calculate the average availability. Therefore, we refer to the expression of uptime and give the expression of downtime.

The expected renewal uptime in a renewal cycle is given by

$$\begin{aligned} T_{\text{up}} &= \sum_{k=1}^{\infty} \left(\int_0^t ((k-1)t + z) \cdot P((k-1)t + z < T_f < (k-1)t + z + dz) \right. \\ &\quad + \int_0^{t/2} \sum_{i=1}^{\infty} \frac{kt + (i-1)t}{2} + z) \\ &\quad \cdot P\left(\frac{kt + (i-1)t}{2} + z < T_f < \frac{kt + (i-1)t}{2} + z + dz\right) \\ &\quad \left. + ktP(T_p = kt) \left(+ \sum_{i=1}^{\infty} \left(\frac{kt + it}{2}\right) PT_p = \frac{kt + it}{2} \right), \right. \end{aligned} \tag{14}$$

and the expected downtime in a renewal cycle is given by

$$T_{\text{down}} = \sum_{k=1}^{\infty} \left(D_f P((k-1)t < T_f < kt) + \sum_{i=1}^{\infty} D_f P\left(\frac{kt + (i-1)t}{2} < T_f < \frac{kt + it}{2}\right) + D_p P(T_p = kt) + \sum_{i=1}^{\infty} D_p P\left(T_p = \frac{kt + it}{2}\right) \right). \quad (15)$$

The component terms for various probabilities in equations (14) and (15) can be calculated using equations (6)–(11).

4.3.2. Expected Cost Rate. According to the classical renewal theorem in [46], the expected cost rate per unit time can be expressed as

$$EC(t) = \frac{EC}{T_{\text{up}} + T_{\text{down}}}, \quad (16)$$

where T_{up} and T_{down} have been derived before; the expected cost in a renewal cycle, EC, is given by

$$\begin{aligned} EC = & \sum_{k=1}^{\infty} ((C_f + kC_{\text{ins}}) \cdot P((k-1)t < T_f < kt)) \\ & + \sum_{i=1}^{\infty} (C_f + (k+i)C_{\text{ins}}) \cdot P\left(kt + \frac{(i-1)t}{2} < T_f < \frac{kt + it}{2}\right) \\ & + (C_p + kC_{\text{ins}})P(T_p = kt) \\ & + \sum_{i=1}^{\infty} (C_p + (k+i)C_{\text{ins}}) \cdot P\left(T_p = \frac{kt + it}{2}\right). \end{aligned} \quad (17)$$

4.4. Solution Algorithm. From the equations deduced above, it is noted that the model is very difficult to be solved directly because of its recurrence relation. Hence, the thought of numerical calculation was adopted, the discrete algorithm was introduced [47], and the calculation process for our proposed model is given in Figure 6:

Step 1. Input the initial parameters of distribution, maintenance, and PBC.

Step 2. Set the maximum inspection interval, t_{max} , and the step size, s . According to the demand of the actual situation and accuracy, choose the reasonable t_{max} and s .

Step 3. Initialize the inspection interval, t . t is the only variable in our proposed model; other values can be solved by adjusting t in the model.

Step 4. Calculate EC, T_{up} , and T_{down} based on equations (6)–(11), and then calculate $A(t)$ and $EC(t)$ based on equations (13) and (16).

Step 5. According to equation (3), judge which situation $A(t)$ is in and calculate $ER(t)$ based on equation (3). If $0 \leq A(t) < A_0$, $ER(t) = 0$; if $A_0 \leq A(t) < A_1$, $ER(t) = a_1$; if $A_1 \leq A(t) < A_2$, $ER(t) = a_1 + b_1(A(t) - A_1)$; and if $A_2 \leq A(t) \leq 1$, $ER(t) = a_2 + b_2(A(t) - A_2)$. Then, calculate $EP(t)$ based on equation (4).

Step 6. Record each value and determine whether to exit the loop. When $t \leq t_{\text{max}}$, $t = t + s$, it should go to Step 4; otherwise, compare and output optimal inspection interval, t^* , according to the optimization objectives.

5. Case Study

In this section, we investigate a previous case study on the maintenance plans of the cold water pumps used in a soft drink company [19]. There is a limited buffer of water stored in tanks between the pumps and the production lines. Failure of the pump, and hence the water supply to production, will result in costs of lost production. Therefore, it is of utmost importance that the pump is well maintained. According to the actual operating conditions of the pump, we treat pump as four categories ranging from machine acceptable to inspect/repair immediately. They are as follows: (1) the pump is operating normally. (2) The pump is operating and shows signs of deterioration. It is advisable to take some preventive action at the next planned maintenance. (3) The pump is operating, but requires immediate attention. (4) The pump has failed.

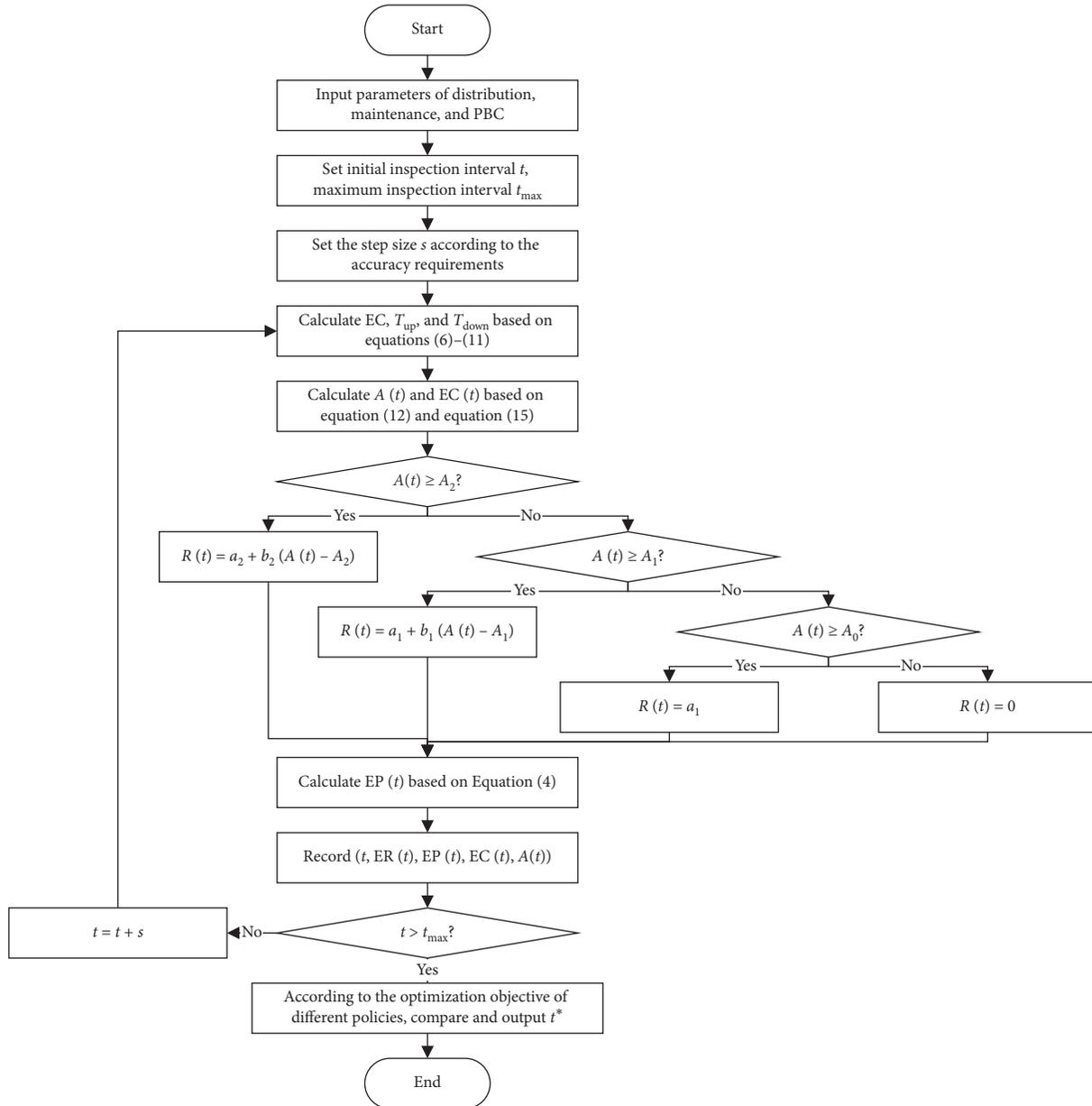


FIGURE 6: Discrete algorithm for the model solution.

TABLE 3: The distribution parameters.

$f_{X1}(x)$		$f_{X2}(y)$		$f_{X3}(z)$	
α_1	β_1	α_2	β_2	α_3	β_3
45.45	1.7	10.2	3.37	5.56	5.81

TABLE 4: The maintenance parameters.

C_{ins}	C_p	C_f	D_p	D_f
100	1000	3000	12	24
		6000		36
		12000		48

The values of several model parameters used in the analysis are adopted from existing literature [20, 21] with slight modifications. Others are assumptions based on typical and plausible values. From the literature in reliability, the time to failure of a component is popularly described as the Weibull distribution; see [48, 49]. So, we assume the three stages follow three Weibull distributions, and the probability density function of the n th ($n = 1, 2, 3$) stage is

$$X_n \sim \text{Weibull}(\alpha_n, \beta_n),$$

$$f_{X_n}(x) = \frac{\beta_n}{\alpha_n} \left(\frac{x}{\alpha_n}\right)^{\beta_n-1} e^{-(x/\alpha_n)^{\beta_n}}, \quad (18)$$

where α_n and β_n are scale parameter and shape parameter, respectively. Their values are shown in Table 3.

Then, we set maintenance parameters. Fixing the cost per inspection, cost per preventive renewal, and downtime per preventive renewal, different levels of cost per failure renewal and downtime per failure renewal are selected for the purpose of sensitivity analysis. The specific values are shown in Table 4.

5.1. Comparison of the Proposed Policy and Conventional Policy. In this section, we compare and analyze the difference between the conventional policy, referring to cost minimization, and the policy proposed in this paper, referring to profit maximization. Before that, the parameters of PBC are needed to be set in advance. It is worth noting that, different from the previous literature, the upper limit of revenue is set based on the actual situation in the implementation of PBC. This consideration is because the supplier's revenue comes from the funds paid by the customer. The funds are limited by the customer's budget, usually within a certain range, and cannot increase without limit, which is rarely considered in the previous studies.

After communicating with the staff of the acquisition sector and combining the optimization results of the cost minimization optimization policy, the parameters of PBC are set as follows: $R_{\max} = 150$, $A_0 = 0.98$, $A_1 = 0.985$, $A_2 = 0.99$, $a_1 = 50$, $a_2 = 80$, $b_1 = 6000$, and $b_2 = 7000$. It can be expressed as

$$ER(t) = \begin{cases} 0, & 0 \leq A(t) < 0.98, \\ 50, & 0.98 \leq A(t) < 0.985, \\ 50 + 6000(A(t) - 0.985), & 0.985 \leq A(t) < 0.99, \\ 80 + 7000(A(t) - 0.99), & 0.99 \leq A(t) \leq 1. \end{cases} \quad (19)$$

Figure 7 indicates the step revenue function defined in equation (19).

Based on the existing models and equation, we use MATLAB to solve the optimal inspection interval between the conventional policy and the policy proposed under different C_f and D_f results of the examples are summarized in Table 5.

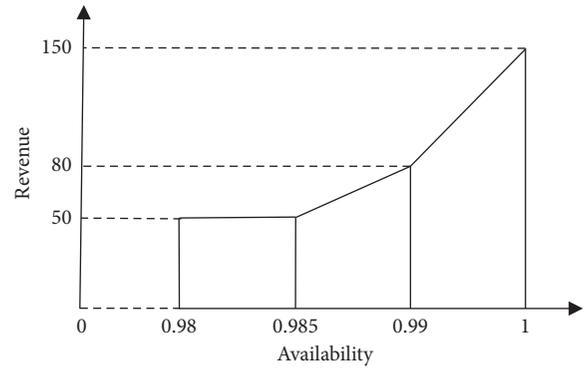


FIGURE 7: Step revenue function with specific values.

To compare the proposed policy with the conventional policy, three criteria are used, namely, cost, availability, and profit. The optimization results are given in Table 5; it shows that compared with the traditional cost minimization policy, the profit maximization policy under PBC can bring higher profits to the supplier. We also note that the improvement in profit rates is per unit time; as time goes by, the total profit will be greater. In addition, it can be observed that under other conditions unchanged, our proposed policy can not only achieve higher profits, but also a higher level of availability. For instance, when $C_{\text{ins}} = 100$, $C_p = 1000$, $C_f = 3000$, $D_p = 12$, and $D_f = 48$, comparing with the conventional policy, the proposed policy has only increased its cost by 5.73%, but its profit has increased by 15.1%, and its availability has also increased by 0.13%. Finally, we can also observe that the more serious the consequences of the failure, that is, the larger C_f and D_f are, the less the supplier's profit is.

5.2. Comparison of Linear and Step Revenue Functions. In order to illustrate the advantages of the step revenue function used in this paper, we have verified this through an example. First, we keep the cost and downtime parameters unchanged, namely, $C_{\text{ins}} = 100$, $C_p = 1000$, $C_f = 6000$, $D_p = 12$, and $D_f = 36$. Then, let $A_0 = 0.98$, $A_1 = 0.985$, $A_2 = 0.99$, and $a_1 = 50$ in the step revenue function, and the upper limit of revenue, R_{\max} , is still 150. It is noticed that when other parameters are fixed and a_2 changes, b_1 and b_2 will change with a_2 . Therefore, several sets of different step revenue functions are tested by changing the a_2 . Next, we keep corresponding variables of the linear revenue function the same as the step revenue function, namely, $R_{\max} = 150$, $A_0 = 0.98$, $a = 50$, and $b = 5000$. The comparison results are shown in Table 6.

From Table 6, it can be observed that, compared with the linear return function, the step return function has lower revenue rate and lower profit rate, but higher availability. It has been already known that the supplier's revenue comes from the funds paid by customers, so using the step revenue function is a more ideal solution for customers. Customers have reduced the cost of payment while gaining higher availability. On the other hand, in the

TABLE 5: The optimization results under conventional policy and proposed policy.

C_{ins}	C_p	C_f	D_p	D_f	Conventional policy			Proposed policy				
					t	EC(t)	A(t)	EP(t)	t	EC(t)	A(t)	EP(t)
100	1000	3000	12	24	9.7	33.00	0.989888	46.32	8.4	33.65	0.990162	47.49
				36	9.7	32.98	0.989275	42.66	7.7	34.53	0.990068	45.95
				48	9.7	32.96	0.988662	39.01	7.5	34.85	0.989959	44.90
100	1000	6000	12	24	8.3	35.35	0.990176	45.88	7.8	35.51	0.990232	46.11
				36	8.3	35.34	0.989912	44.14	7.4	35.86	0.990124	45.01
				48	8.3	35.33	0.989649	42.56	7.1	36.26	0.990061	44.16
100	1000	12000	12	24	7.3	37.53	0.990267	44.34	7.2	37.54	0.990272	44.36
				36	7.3	37.52	0.99014	43.46	6.9	37.71	0.990192	43.64
				48	7.3	37.52	0.990014	42.58	6.7	37.91	0.990135	43.03

TABLE 6: Comparison results of linear revenue function and step revenue function.

	Linear				Step						
	$b = 5000$	$a_2 = 78$	$b_1 = 5600$ $b_2 = 7200$	$a_2 = 79$	$b_1 = 5800$ $b_2 = 7100$	$a_2 = 80$	$b_1 = 6000$ $b_2 = 7000$	$a_2 = 81$	$b_1 = 6200$ $b_2 = 6900$	$a_2 = 82$	$b_1 = 6400$ $b_2 = 6800$
t	7.6						7.4				
EC(t)	35.66						35.86				
A(t)	0.990088						0.990124				
ER(t)	100.44		78.90		79.88		80.87		81.86		82.85
EP(t)	64.78		43.03		44.02		45.01		45.99		46.98

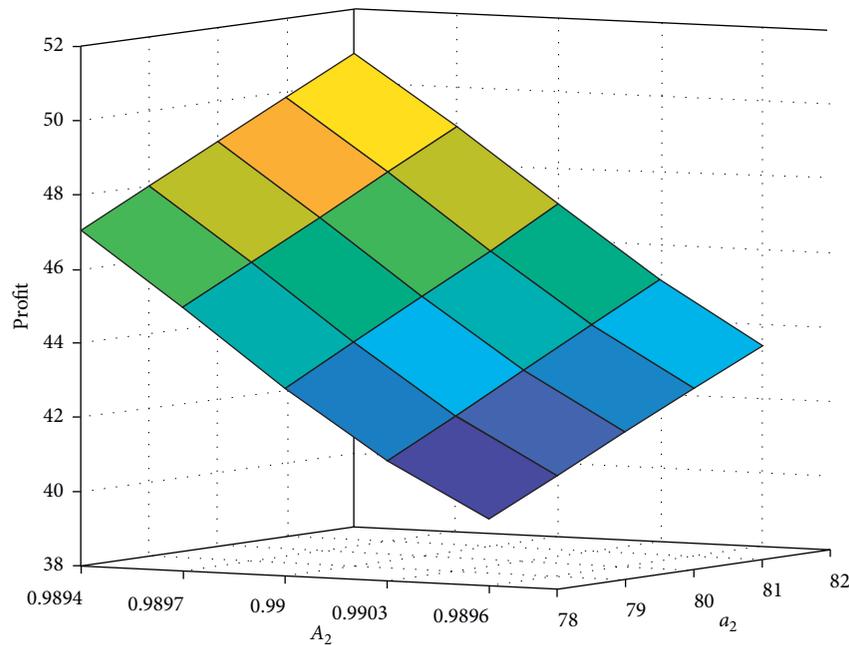


FIGURE 8: Sensitivity analysis of A_2 on the maximum profit.

case of using a step revenue function, it is also possible to change the revenue and profit by changing a_2 . As a_2 increases, revenue and profit are increasing. When suppliers and customer sign performance-based contracts, they can negotiate and adjust the parameters to get a result that both parties are satisfied.

5.3. Sensitivity Analysis. To further verify the performance and applicability of our proposed maintenance policy, sensitivity analysis is carried out on a critical parameter, A_2 .

Firstly, we keep the other parameters unchanged, namely, $C_{ins} = 100$, $C_p = 1000$, $C_f = 6000$, $D_p = 12$, $D_f = 36$, $A_0 = 0.98$, $A_1 = 0.985$, $a_1 = 50$, and $R_{max} = 150$. Then, it can be

observed that under different a_2 , changes in A_2 cause changes in profit, and notice that the value of b_1 and b_2 will change with A_2 . The results are shown in Figure 8.

According to Figure 8, it can be known, under different a_2 , when A_2 is lower, the profit is higher. This is because when $(A(t) - A_2)$ is bigger, the reward revenue, $b_2(A(t) - A_2)$, is greater, which will make suppliers get more revenue. As A_2 becomes higher, $(A(t) - A_2)$ becomes less, and the reward revenue, $b_2(A(t) - A_2)$, and profit will decrease. In addition, the results again verify the results obtained in the previous section, that is, as a_2 increase, profits also increase.

6. Conclusion and Future Research

This paper puts forward a three-stage preventive maintenance model for single-component system under PBC. This is the first paper to apply PBC to three-stage preventive maintenance optimization. Different from traditional maintenance optimization models, profit is usually used as an optimization objective under PBC. Based on which, step revenue function is used to link availability and cost to the profit, and the profit is maximized by optimizing the inspection interval. Besides, funds limit is taken into account in step revenue function, which is more accordant with the actual situation. Our work further expands the application of PBC in maintenance optimization.

Several interesting research directions remain to be explored in the future. Firstly, this paper only considers the situation of single component; it is also worth considering multicomponent cases, e.g., series-parallel system, cold standby system, and k -out-of- n system. Moreover, with respect to maintenance policies, imperfect inspections and imperfect repairs rather than perfect inspections and perfect repairs may be more practical in system maintenance. For instance, an inspection may be false positive or even cause system failure, and the improvement effect of a repair may be random. These two directions mentioned above can be considered in future research, which is of both theoretical and practical interest to address maintenance optimization under PBC.

Data Availability

All the data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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