Research Article

Improved Results on Delay-Dependent Robust H_∞ Control of Uncertain Neutral Systems with Mixed Time-Varying Delays

Jingsha Zhang , Yongke Li , Xiaolin Ma , Zhilong Lin , and Changlong Wang

Department of UAV Engineering, Army Engineering University, Shijiazhuang Campus, Shijiazhuang 05003, Hebei, China

Correspondence should be addressed to Yongke Li; dyp001@126.com and Changlong Wang; wang-oec@126.com

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In this paper, the problem of the delay-dependent robust H_∞ control for a class of uncertain neutral systems with mixed time-varying delays is studied. Firstly, a robust delay-dependent asymptotic stability criterion is shown by linear matrix inequalities (LMIs) after introducing a new Lyapunov–Krasovskii functional (LKF). The LKF including vital terms is expected to obtain results of less conservatism by employing the technique of various efficient convex optimization algorithms and free matrices. Then, based on the obtained criterion, analyses for uncertain systems and H_∞ controller design are presented. Moreover, on the analysis of the state-feedback controller, different from the traditional method which multiplies the matrix inequality left and right by some matrix and its transpose, respectively, we can obtain the state-feedback gain directly by calculating the LMIs through the toolbox of MATLAB in this paper. Finally, the feasibility and validity of the method are illustrated by examples.

1. Introduction

Strictly speaking, some degrees of time delays are frequently encountered in almost all practical engineering systems such as [1–3], in which the existence of the delays may lower the performances of these engineering control systems and even result in the instability of such control systems. Then, it is an essential requirement to investigate the stability for systems with time delays. And quantities of results on stability for time-delay systems in actual applications have been obtained in the near past [4–6]. The stability analyses of the systems can be classified into delay-dependent ones and delay-independent ones. The stability analyses criteria in delay-dependent systems are always less conservative than the delay-independent ones. In addition, the stability criteria are developed by applying the LKF methodology. Otherwise, for some time-delay systems in practice such as air-feed system [7] and power converters system [8], the controllers are designed to make the systems more practical by employing advanced methods for the stability analyses. Though the time delay can be constant or varying, the former is a special case of the latter which is always more complex and typical. Thus,

the stabilization problem of dynamical systems with time-varying delays has been taken into consideration.

On the other hand, there are also many typical practical models of time-delay neutral dynamical systems such as partial element equivalent circuit model [9] and population genetics and community ecology [10], which are described by neutral functional differential equations containing time delays not only in the state but also in its derivative. Obviously, studying neutral systems with mixed delays (NSMDs) is more challenging than the system with a delay only in the state. Results about stability analysis for NSMD have been obtained in recent years. In the following, some classical methods are shown for NSMD. In [11], delay-dependent conditions were improved via utilizing the LMI method. In [12], by applying a candidate LKF, less conservative criteria were proposed. Besides, to reduce conservatism, an approach of piece-wise delay was expressed in [13]. Advanced integral inequalities were used in [14] to get the delay-dependent stability conditions, which were less conservative. NSMD with uncertainties were observed, and then the robust stability analyses were discussed in [15]. As we can see in [16], compared with an inequality which might not cause a less conservative result, a tighter inequality based

on a suitable LKF could get the advantage we want. From [11–16], we can see that the methods of LKF and integral terms are the mainstream ones. By constructing an LKF with inequality terms and applying the LMI method, sufficient conditions are obtained.

As known, uncertainties can lead to poor performance in many time-delay systems. Therefore, much attention has been paid to the analysis of stability and stabilization for systems with parametric uncertainties. And robust control can be used for the analysis and design of the systems including uncertainties which are complex to handle in dynamical models. The problem of robust stabilization for a class of uncertain neutral time-delay dynamical systems with the uncertain matrix coefficients of both the delayed state and derivative of the delayed state is considered so that we can ensure the discussed systems stable under specific conditions. In addition, to overcome the external disturbances of actual operations in engineering models such as in [17, 18] and make them stable artificially, H_∞ control is also an important issue. In [17], robust H_∞ control was designed for PFC rectifiers. For the cooperative driving system with external disturbances and communication delays in the vicinity of traffic signals, robust H_∞ control was studied in [18]. As presented in [19], the issue on robust H_∞ control was discussed. Besides, H_∞ control of neutral systems has been expressed in many papers up to now. In [20], the problem of robust H_∞ control with state feedback was solved in the uncertain neutral system. For an uncertain neutral system in [21], H_∞ performance for Markovian uncertain systems via the sliding mode control method was considered. Moreover, delay-dependent H_∞ filtering conditions were proposed for uncertain neutral stochastic systems with Markovian switching and mode-dependent time delays [22]. By applying a H_∞ state-feedback controller in [23], a neutral-type time-delay system was studied. And in another paper [24], the H_∞ feedback controller was used to obtain the results.

Taking all the discussed elements into consideration, a representative system named the teleoperation system was introduced in [25]. The teleoperation system is composed of a master manipulated by a human operator and a slave operating in a remote environment through a communication channel. Time-varying delays existed in the long-range or flexible communication channel [26]; thus, the problem of stability of teleoperation systems for time-varying delays by neutral LMI techniques was discussed in [25]. Besides, due to that no real system is pure linear, time-varying model uncertainties considered as perturbations existed in real implementation of bilateral teleoperation [27], and then the study on robust control design for teleoperation systems was done in [28]. Moreover, the stability criteria above were all given in the form of LMIs, which can be used to compute the maximum values of delays. And in order to obtain the controller parameters directly by solving LMIs, state-feedback control law was found to stabilize the system with guaranteeing performance in [29]. Above all, in our paper, both time-varying delays and uncertainties are considered in the neutral model to analysis the robust control with

state-feedback controller by the technique of LMIs. Different from the general delay system such as [16, 23], this paper also considers delays in the neutral terms. And unlike the delays in [11–15, 20, 24], the delays shown in both discrete and distributed forms in this article are all varying and different instead of constant or the same delay function. Then, the model derived from the actual application is more complex but more representative. Moreover, appropriate Lyapunov function is constructed to deal with the complicated model terms technically and get the control gain directly without applying the traditional method in [30].

Hence, motivated by the discussions above, the problem of delay-dependent robust H_∞ control of uncertain neutral systems with mixed time-varying delays is explored in this article. After introducing an improved LKF and using the LMI method, a sufficient condition for the asymptotic stability of the system is established. Analyses on uncertainties and H_∞ control are made then. Numerical examples are given to verify the results finally.

The main contributions of this paper are summered as follows:

- (1) In the LKF newly constructed, all terms are taken into account to obtain a larger delay bound.
- (2) The methods including not only free-weighting matrices but also efficient convex optimization algorithms are applied technically to get results of less conservatism.
- (3) Sufficient conditions are given all by LMIs, and the solutions can be solved by MATLAB Toolbox directly.
- (4) Though similar papers concerning the delay-dependent robust control of uncertain neutral systems with mixed time-varying delays have already been many, there are some differences between the current submission and latest works. To the extent of the authors' knowledge, none of the existing works including all the terms of robust control, neutral systems, uncertainties, and mixed time-varying delays derived from actual engineering problems. Besides, the Lyapunov functional proposed in this article can get better conservatism with larger delay bounds when compared with these in [31–33], which is shown in Example 3. Finally, different from the traditional method which multiplies the matrix inequality left and right by some matrix and its transpose, respectively, such as in [30], the controller gain illustrated in Theorem 4 of this paper can be obtained more easily and simply.

1.1. Notation. The notations throughout this paper are fairly standard. A^{-1} and A^T denote, respectively, the matrix inverse and transpose of A . $\mathcal{R}^n$ and $\mathcal{R}^{n \times m}$ are the n -dimensional Euclidean space and the set of $n \times m$ real matrices. $B > C$ means that $B - C$ is positive definite. I is a unit matrix with compatible dimensions. $\begin{bmatrix} M & W \\ * & N \end{bmatrix}$ denotes $\begin{bmatrix} M & W \\ W^T & N \end{bmatrix}$. Let

$\|f\|_2 = \sqrt{\int_0^\infty \|f\|_2^2 dt}$, $f(t) \in L_2[0, \infty]$. $\|f\|$ expresses the Euclidean norm of function $f(t)$. $L_2[0, \infty]$ refers to the space of square integral vectors.

2. Description of the Problem

Consider the NSMD with uncertainties:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_1(t)x(t-d(t)) + A_2(t)\dot{x}(t-h(t)) + A_3(t) \int_{t-d(t)}^t x(s)ds + A_4(t) \int_{t-h(t)}^t x(s)ds \\ + Bu(t) + B_w w(t), \\ z(t) = C(t)x(t) + C_1(t)x(t-d(t)) + D(t)w(t), \\ x(t) = \varphi(t), \dot{x}(t) = \phi(t), t \in [-H, 0], \end{cases} \quad (1)$$

with $x(t) \in \mathfrak{R}^n$, $u(t) \in \mathfrak{R}^m$, $w(t) \in \mathfrak{R}^p$, and $z(t) \in \mathfrak{R}^q$ as the state vector, the control input, the disturbance which belongs to $L_2[0, \infty]$, and the controlled output, respectively;

$\varphi(t)$ and $\phi(t)$ are given continuous initial condition functions. Continuous functions $d(t)$ and $h(t)$ are time-varying delays which satisfy

$$0 \leq d(t) \leq d_M, \dot{d}(t) \leq u_1 < 1, 0 \leq h(t) \leq h_M, \dot{h}(t) \leq u_2 < 1, H = \max\{d_M, h_M\}, \quad (2)$$

where d_M, h_M, u_1, u_2 are constants. $A(t), A_1(t), A_2(t), A_3(t), A_4(t), C(t), C_1(t)$ and $D(t)$ are matrices with uncertainties satisfying

$$\begin{aligned} A(t) &= A + \Delta A(t), \\ A_1(t) &= A_1 + \Delta A_1(t), \\ A_2(t) &= A_2 + \Delta A_2(t), \\ A_3(t) &= A_3 + \Delta A_3(t), \\ A_4(t) &= A_4 + \Delta A_4(t), \\ C(t) &= C + \Delta C(t), \\ C_1(t) &= C_1 + \Delta C_1(t), \\ D(t) &= D + \Delta D(t), \end{aligned} \quad (3)$$

where $A, A_1, A_2, A_3, A_4, C, C_1$ and D are known constant matrices with suitable dimensions. The time-varying unknown matrices $\Delta A(t), \Delta A_1(t), \Delta A_2(t), \Delta A_3(t), \Delta A_4(t), \Delta C(t), \Delta C_1(t)$, and $\Delta D(t)$ satisfy the following form:

$$\begin{aligned} &[\Delta A(t) \ \Delta A_1(t) \ \Delta A_2(t) \ \Delta A_3(t) \ \Delta A_4(t) \ \Delta C(t) \ \Delta C_1(t) \ \Delta D(t)] \\ &= MF(t)[N_A \ N_{A1} \ N_{A2} \ N_{A3} \ N_{A4} \ N_C \ N_{C1} \ N_D], \end{aligned} \quad (4)$$

where $M, N_A, N_{A1}, N_{A2}, N_{A3}, N_{A4}, N_C, N_{C1}$ and N_D are known constant matrices with appropriate dimensions and

$F(t)$ as an unknown matrix function satisfies $F^T(t)F(t) \leq I$ for any $t \geq 0$.

For system (1), the nominal form is given as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-d(t)) + A_2\dot{x}(t-h(t)) + A_3 \int_{t-d(t)}^t x(s)ds + A_4 \int_{t-h(t)}^t x(s)ds \\ + Bu(t) + B_w w(t), \\ z(t) = Cx(t) + C_1x(t-d(t)) + Dw(t), \\ x(t) = \varphi(t), \dot{x}(t) = \phi(t), t \in [-H, 0]. \end{cases} \quad (5)$$

For system (5), a state-feedback controller is designed by

$$u(t) = Kx(t), \quad (6)$$

where $K \in \mathfrak{R}^{m \times n}$ is a constant matrix to be designed.

Combining (1) with (3) and the controller expression (6), we can get the closed-loop system of (1):

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t) + BK)x(t) + (A_1 + \Delta A_1(t))x(t - d(t)) + (A_2 + \Delta A_2(t))\dot{x}(t - h(t)) \\ \quad + (A_3 + \Delta A_3(t)) \int_{t-d(t)}^t x(s)ds + (A_4 + \Delta A_4(t)) \int_{t-h(t)}^t x(s)ds + B_w w(t), \\ z(t) = (C + \Delta C(t))x(t) + (C_1 + \Delta C_1(t))x(t - d(t)) + (D + \Delta D(t))w(t), \\ x(t) = \phi(t), \dot{x}(t) = \phi(t), t \in [-H, 0]. \end{cases} \quad (7)$$

Remark 1. The mixed time-varying delays, respectively, distribute in the state and the derivative of the state, and the system could not always be stable with all values of delays. Then, we must find the bound of each time-varying delay, the larger the boundary and the better conservative system indicating that the broader time scope can be got to keep the system stable.

Remark 2. The two kinds of both discrete and distributed delays exist in the actual systems. And the large delay always causes the instability of the system. And either of the large delay can cause the instability of the system. Thus, considering the bounds of the discrete and distributed delays is a must.

Lemma 1. Schur complement [34]: given matrices X, Y, Z with $Y > 0$, then we have

$$\begin{bmatrix} X & Z \\ * & -Y \end{bmatrix} < 0 \Leftrightarrow X + ZY^{-1}Z^T < 0. \quad (8)$$

Lemma 2. Jensen's inequality [35]: for any scalar $\eta > 0$ and any constant matrix $M \in \mathfrak{R}^{n \times n}$, $M = M^T > 0$, the following inequality holds

$$\left[\int_0^\eta w(s)ds \right]^T M \left[\int_0^\eta w(s)ds \right] \leq \eta \int_0^\eta w^T(s)Mw(s)ds. \quad (9)$$

Lemma 3. Cauchy matrix inequality [36]: let $N \in \mathfrak{R}^{n \times n}$ be a positive definite matrix, then

$$-2y^T x \leq x^T N x + y^T N^{-1} y, \quad \forall x, y \in \mathfrak{R}^n. \quad (10)$$

Lemma 4 (see [37]). For any constant matrix $Q \in \mathfrak{R}^{n \times n}$, $Q = Q^T > 0$, and free-weighting matrix V , we have

$$-\int_{t-d(t)}^t \dot{x}^T(s)Q\dot{x}(s)ds \leq d(t)\xi^T(t)VQ^{-1}V^T\xi(t) + 2\xi^T(t)V(x(t) - x(t - d(t))), \quad (11)$$

where

$$\begin{aligned} \xi^T(t) = & \left[x^T(t)x^T(t - d(t)) \left(\int_{t-d(t)}^t x(s)ds \right)^T x^T(t - d_M)x^T(t - h(t)) \left(\int_{t-h(t)}^t x(s)ds \right)^T \right. \\ & \left. \cdot x^T(t - h_M)\dot{x}^T(t)\dot{x}^T(t - d(t))\dot{x}^T(t - h(t)) \right]. \end{aligned} \quad (12)$$

Lemma 5 (see [38]). Given matrices D, E with suitable dimensions and $F(t)$ satisfying $F^T(t)F(t) \leq I$, then for any constant $\varepsilon > 0$, the inequality holds

$$DF(t)E + E^T F^T(t)D^T \leq \varepsilon DD^T + \varepsilon^{-1}E^T E. \quad (13)$$

3. Main Results

In this section, a new sufficient condition criterion of asymptotic stability for system (14) is presented by using the LKF method and LMI technique:

$$\begin{cases} \dot{x}(t) - A_2 \dot{x}(t-h(t)) = Ax(t) + A_1 x(t-d(t)) + A_3 \int_{t-d(t)}^t x(s)ds + A_4 \int_{t-h(t)}^t x(s)ds, \\ x(t) = \varphi(t), \dot{x}(t) = \phi(t), t \in [-H, 0]. \end{cases} \quad (14)$$

Theorem 1. For the given constants d_M, u_1, h_M , and u_2 , system (14) is asymptotically stable if there exist positive-definite matrices $Q_1, Q_2, Q_a, Q_b, Z_i, (i = 1, 2, 3, 4, 5, 6, 7, 8), R_j, (j = 1, 2, 3, 4), S_k, (k = 1, 2), P_1$ and

matrices with appropriate dimensions $P_l (l = 2, \dots, 9), U_\alpha, V_\alpha, G_\alpha, H_\alpha, L_\alpha, J_\alpha (\alpha = 1, 2), W$ satisfy the following LMI:

$$\begin{aligned} \Omega &= \Omega_1 + \Omega_2 + \Omega_2^T, \Omega_1 = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{12}^T, \\ \varphi_{11} &= P_2^T A + A^T P_2 + Q_1 + Q_2 + Q_{a11} + Q_{b11} + d_M(Z_5 + Z_6) + h_M(Z_7 + Z_8) + h_1 U_1 + h_1 U_1^T + h_2 V_1 + h_2 V_1^T + d_M^2 R_3 + h_M^2 R_4, \\ \varphi_{12} &= P_2^T A_1 + A^T P_3 - h_1 U_1 + h_1 U_2^T, \varphi_{13} = P_2^T A_3 + A^T P_4, \varphi_{14} = A^T P_5, \varphi_{15} = A^T P_6 - h_2 V_1 + h_2 V_2^T, \\ \varphi_{16} &= P_2^T A_4 + A^T P_7, \varphi_{17} = A^T P_8, \varphi_{18} = P_1 - P_2^T + A^T P_9 + Q_{a12} + Q_{b12}, \\ \varphi_{22} &= P_3^T A_1 + A_1^T P_3 - h_1 Q_{a11} - h_1 U_2 - h_1 U_2^T, \varphi_{23} = P_3^T A_3 + A_1^T P_4, \varphi_{24} = A_1^T P_5, \varphi_{25} = A_1^T P_6, \\ \varphi_{26} &= P_3^T A_4 + A_1^T P_7, \varphi_{27} = A_1^T P_8, \varphi_{28} = -P_3^T + A_1^T P_9, \varphi_{33} = P_4^T A_3 + A_3^T P_4 - h_1 R_3, \varphi_{34} = A_3^T P_5, \\ \varphi_{35} &= A_3^T P_6, \varphi_{36} = P_4^T A_4 + A_3^T P_7, \varphi_{37} = A_3^T P_8, \varphi_{38} = -P_4^T + A_3^T P_9, \varphi_{44} = -Q_1, \varphi_{46} = P_5^T A_4, \\ \varphi_{48} &= -P_5^T, \varphi_{55} = -h_2 Q_{b11} - h_2 V_2 - h_2 V_2^T, \varphi_{56} = P_6^T A_4, \varphi_{58} = -P_6^T, \varphi_{66} = P_7^T A_4 + A_4^T P_7 - h_2 R_4, \\ \varphi_{67} &= A_4^T P_8, \varphi_{68} = -P_7^T + A_4^T P_9, \varphi_{77} = -Q_2, \varphi_{78} = -P_8^T, \\ \varphi_{88} &= -P_9 - P_9^T + Q_{a22} + Q_{b22} + d_M(Z_1 + Z_2) + h_M(Z_3 + Z_4) + d_M R_1 + h_M R_2 + d_M^2 S_1 + h_M^2 S_2, \\ \Omega_{11} &= \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14} & \varphi_{15} & \varphi_{16} & \varphi_{17} & \varphi_{18} & 0 & P_2^T A_2 \\ * & \varphi_{22} & \varphi_{23} & \varphi_{24} & \varphi_{25} & \varphi_{26} & \varphi_{27} & \varphi_{28} & -h_1 Q_{a12} & P_3^T A_2 \\ * & * & \varphi_{33} & \varphi_{34} & \varphi_{35} & \varphi_{36} & \varphi_{37} & \varphi_{38} & 0 & P_4^T A_2 \\ * & * & * & \varphi_{44} & 0 & \varphi_{46} & 0 & \varphi_{48} & 0 & P_5^T A_2 \\ * & * & * & * & \varphi_{55} & \varphi_{56} & 0 & \varphi_{58} & 0 & P_6^T A_2 - h_{12} Q_{b12} \\ * & * & * & * & * & \varphi_{66} & \varphi_{67} & \varphi_{68} & 0 & P_7^T A_2 \\ * & * & * & * & * & * & \varphi_{77} & \varphi_{78} & 0 & P_8^T A_2 \\ * & * & * & * & * & * & * & \varphi_{88} & 0 & P_9^T A_2 \\ * & * & * & * & * & * & * & * & -h_1 Q_{a22} & 0 \\ * & * & * & * & * & * & * & * & * & -h_{12} Q_{b22} \end{bmatrix}, \\ \Omega_{12} &= \begin{bmatrix} h_1 d_M U_1 & h_2 h_M V_1 \\ h_1 d_M U_2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & h_2 h_M V_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ \Omega_{22} &= \begin{bmatrix} -h_1 d_M R_1 & 0 \\ * & -h_2 h_M R_2 \end{bmatrix}, h_1 = 1 - u_1, h_2 = 1 - u_2, \\ \Omega_2 &= \begin{bmatrix} G_1 + L_1 + G_2 + L_2 + 2d_M J_1 + 2h_M J_2 & H_1 - G_1 & 0 & -H_1 - L_1 & H_2 - G_2 & 0 & -H_2 - L_2 \\ 0 & 0 & 0 & W[-A & -A_1 & -A_3 & 0 & 0 & -A_4 & 0 & I & 0 & -A_2] \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
G_1 &= \begin{bmatrix} G_{01} \\ G_{02} \\ \vdots \\ G_{10} \end{bmatrix}, H_1 = \begin{bmatrix} H_{01} \\ H_{02} \\ \vdots \\ H_{10} \end{bmatrix}, L_1 = \begin{bmatrix} L_{01} \\ L_{02} \\ \vdots \\ L_{10} \end{bmatrix}, J_1 = \begin{bmatrix} J_{01} \\ J_{02} \\ \vdots \\ J_{10} \end{bmatrix}, \\
G_2 &= \begin{bmatrix} G_{11} \\ G_{12} \\ \vdots \\ G_{20} \end{bmatrix}, H_2 = \begin{bmatrix} H_{11} \\ H_{12} \\ \vdots \\ H_{20} \end{bmatrix}, L_2 = \begin{bmatrix} L_{11} \\ L_{12} \\ \vdots \\ L_{20} \end{bmatrix}, J_2 = \begin{bmatrix} J_{11} \\ J_{12} \\ \vdots \\ J_{20} \end{bmatrix}, W = \begin{bmatrix} W_{01} \\ W_{02} \\ \vdots \\ W_{10} \end{bmatrix}, \\
\Xi &= \begin{bmatrix} \Omega & d_M G_1 & d_M H_1 & d_M L_1 & h_M G_2 & h_M H_2 & h_M L_2 & 2d_M J_1 & d_M J_1 & 2h_M J_2 & h_M J_2 & d_M^2 J_1 & h_M^2 J_2 \\ * & -d_M Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -d_M Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -d_M Z_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -h_M Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -h_M Z_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -h_M Z_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -2d_M Z_5 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -d_M Z_6 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -2h_M Z_7 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -h_M Z_8 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -d_M^2 S_1 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -h_M^2 S_2 \end{bmatrix} < 0. \tag{15}
\end{aligned}$$

Proof. Choose an LKF candidate as

where

$$V(x_t) = \sum_{i=1}^7 V_i(x_t), \tag{16}$$

$$V_1(x_t) = \zeta^T(t) E P \zeta(t),$$

$$V_2(x_t) = \int_{t-d_M}^t x^T(s) Q_1 x(s) ds + \int_{t-h_M}^t x^T(s) Q_2 x(s) ds,$$

$$V_3(x_t) = \int_{t-d(t)}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T Q_a \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds + \int_{t-h(t)}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T Q_b \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds$$

$$\begin{aligned}
V_4(x_t) &= \int_{-d_M}^0 du \int_{t+u}^t \dot{x}^T(s) (Z_1 + Z_2) \dot{x}(s) ds + \int_{-h_M}^0 du \int_{t+u}^t \dot{x}^T(s) (Z_3 + Z_4) \dot{x}(s) ds \\
&\quad + \int_{-d_M}^0 du \int_{t+u}^t x^T(s) (Z_5 + Z_6) x(s) ds + \int_{-h_M}^0 du \int_{t+u}^t x^T(s) (Z_7 + Z_8) x(s) ds,
\end{aligned} \tag{17}$$

$$V_5(x_t) = \int_{t-d(t)}^t [s - (t - d(t))] \dot{x}^T(s) R_1 \dot{x}(s) ds + \int_{t-h(t)}^t [s - (t - h(t))] \dot{x}^T(s) R_2 \dot{x}(s) ds,$$

$$V_6(x_t) = d_M \int_{t-d(t)}^t [s - (t - d(t))] x^T(s) R_3 x(s) ds + h_M \int_{t-h(t)}^t [s - (t - h(t))] x^T(s) R_4 x(s) ds,$$

$$V_7(x_t) = 2 \int_{-d_M}^0 du \int_u^t \dot{x}^T(s) S_1 \dot{x}(s) ds + 2 \int_{-h_M}^0 du \int_u^t \dot{x}^T(s) S_2 \dot{x}(s) ds,$$

with

$$\begin{aligned}
 E &= \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
 P &= \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 \end{bmatrix}, \\
 \zeta(t) &= \begin{bmatrix} x^T(t)x^T(t-d(t)) \left(\int_{t-d(t)}^t x(s)ds \right)^T x^T(t-d_M)x^T(t-h(t)) \left(\int_{t-h(t)}^t x(s)ds \right)^T x^T(t-h_M)\dot{x}^T(t) \end{bmatrix}^T.
 \end{aligned} \tag{18}$$

The time derivative of $V(x_t)$ is given by

$$\dot{V}_1(x_t) = 2\dot{\zeta}^T(t)EP\dot{\zeta}(t) = 2\dot{\zeta}^T(t) \begin{bmatrix} P_1\dot{x}(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 2\dot{\zeta}^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} \tag{19}$$

$$= 2\dot{\zeta}^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ -\dot{x}(t) + Ax(t) + A_1x(t-d(t)) + A_2\dot{x}(t-h(t)) + A_3 \int_{t-d(t)}^t x(s)ds + A_4 \int_{t-h(t)}^t x(s)ds \end{bmatrix},$$

$$\dot{V}_2(x_t) = x^T(t)Q_1x(t) - x^T(t-d_M)Q_1x(t-d_M) + x^T(t)Q_2x(t) - x^T(t-h_M)Q_2x(t-h_M), \tag{20}$$

$$\begin{aligned}
 \dot{V}_3(x_t) &= \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T Q_a \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - (1-\dot{d}(t)) \begin{bmatrix} x(t-d(t)) \\ \dot{x}(t-d(t)) \end{bmatrix}^T Q_a \begin{bmatrix} x(t-d(t)) \\ \dot{x}(t-d(t)) \end{bmatrix} \\
 &+ \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T Q_b \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - (1-\dot{h}(t)) \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T Q_b \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix} \\
 &\leq \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{a11} + Q_{b11} & Q_{a12} + Q_{b12} \\ * & Q_{a22} + Q_{b22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \\
 &- (1-u_1) \begin{bmatrix} x(t-d(t)) \\ \dot{x}(t-d(t)) \end{bmatrix}^T \begin{bmatrix} Q_{a11} & Q_{a12} \\ * & Q_{a22} \end{bmatrix} \begin{bmatrix} x(t-d(t)) \\ \dot{x}(t-d(t)) \end{bmatrix} \\
 &- (1-u_2) \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T \begin{bmatrix} Q_{b11} & Q_{b12} \\ * & Q_{b22} \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix},
 \end{aligned} \tag{21}$$

$$\begin{aligned}
 \dot{V}_4(x_t) &= d_M \dot{x}^T(t)(Z_1 + Z_2)\dot{x}(t) - \int_{t-d(t)}^t \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d_M}^{t-d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d_M}^t \dot{x}^T(s)Z_2\dot{x}(s)ds \\
 &+ h_M \dot{x}^T(t)(Z_3 + Z_4)\dot{x}(t) - \int_{t-h(t)}^t \dot{x}^T(s)Z_3\dot{x}(s)ds - \int_{t-h_M}^{t-h(t)} \dot{x}^T(s)Z_3\dot{x}(s)ds - \int_{t-h_M}^t \dot{x}^T(s)Z_4\dot{x}(s)ds \\
 &+ d_M x^T(t)(Z_5 + Z_6)x(t) - \int_{t-d(t)}^t x^T(s)Z_5x(s)ds - \int_{t-d_M}^{t-d(t)} x^T(s)Z_5x(s)ds - \int_{t-d_M}^t x^T(s)Z_6x(s)ds \\
 &+ h_M x^T(t)(Z_7 + Z_8)x(t) - \int_{t-h(t)}^t x^T(s)Z_7x(s)ds - \int_{t-h_M}^{t-h(t)} x^T(s)Z_7x(s)ds - \int_{t-h_M}^t x^T(s)Z_8x(s)ds,
 \end{aligned} \tag{22}$$

$$\begin{aligned}
\dot{V}_5(x_t) &= t\dot{x}^T(t)R_1\dot{x}(t) - (1 - \dot{d}(t))(t - d(t))\dot{x}^T(t - d(t))R_1\dot{x}(t - d(t)) \\
&\quad - (1 - \dot{d}(t)) \int_{t-d(t)}^t \dot{x}^T(s)R_1\dot{x}(s)ds - (t - d(t))[\dot{x}^T(t)R_1\dot{x}(t) - (1 - \dot{d}(t))\dot{x}^T(t - d(t))R_1\dot{x}(t - d(t))] \\
&\quad + t\dot{x}^T(t)R_2\dot{x}(t) - (1 - \dot{h}(t))(t - h(t))\dot{x}^T(t - h(t))R_2\dot{x}(t - h(t)) \\
&\quad - (1 - \dot{h}(t)) \int_{t-h(t)}^t \dot{x}^T(s)R_2\dot{x}(s)ds - (t - h(t))[\dot{x}^T(t)R_2\dot{x}(t) - (1 - \dot{h}(t))\dot{x}^T(t - h(t))R_2\dot{x}(t - h(t))] \\
&= d(t)\dot{x}^T(t)R_1\dot{x}(t) - (1 - \dot{d}(t)) \int_{t-d(t)}^t \dot{x}^T(s)R_1\dot{x}(s)ds + h(t)\dot{x}^T(t)R_2\dot{x}(t) \\
&\quad - (1 - \dot{h}(t)) \int_{t-h(t)}^t \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\leq d(t)\dot{x}^T(t)R_1\dot{x}(t) - (1 - u_1) \int_{t-d(t)}^t \dot{x}^T(s)R_1\dot{x}(s)ds + h(t)\dot{x}^T(t)R_2\dot{x}(t) \\
&\quad - (1 - u_2) \int_{t-h(t)}^t \dot{x}^T(s)R_2\dot{x}(s)ds.
\end{aligned} \tag{23}$$

By applying Lemma 4,

$$\begin{aligned}
\dot{V}_5(x_t) &\leq d_M\dot{x}^T(t)R_1\dot{x}(t) + (1 - u_1)d_M\xi^T(t)UR_1^{-1}U^T\xi(t) + 2(1 - u_1)\xi^T(t)U[x(t) - x(t - d(t))] \\
&\quad + h_M\dot{x}^T(t)R_2\dot{x}(t) + (1 - u_2)h_M\xi^T(t)VR_2^{-1}V^T\xi(t) + 2(1 - u_2)\xi^T(t)V[x(t) - x(t - h(t))], \\
U &= [U_1^T \ U_2^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, \\
V &= [V_1^T \ 0 \ 0 \ 0 \ V_1^T \ 0 \ 0 \ 0 \ 0 \ 0]^T.
\end{aligned} \tag{24}$$

Similarly,

$$\begin{aligned}
\dot{V}_6(x_t) &= d(t)d_Mx^T(t)R_3x(t) - (1 - \dot{d}(t))d_M \int_{t-d(t)}^t x^T(s)R_3x(s)ds \\
&\quad + h(t)h_Mx^T(t)R_4x(t) - (1 - \dot{h}(t))h_M \int_{t-h(t)}^t x^T(s)R_4x(s)ds \\
&\leq d_M^2x^T(t)R_3x(t) - (1 - \dot{d}(t))d(t) \int_{t-d(t)}^t x^T(s)R_3x(s)ds \\
&\quad + h_M^2x^T(t)R_4x(t) - (1 - \dot{h}(t))h(t) \int_{t-h(t)}^t x^T(s)R_4x(s)ds.
\end{aligned} \tag{25}$$

By using Lemma 2, we can get

$$\begin{aligned}
\dot{V}_6(x_t) &\leq d_M^2 \dot{x}^T(t) R_3 x(t) + \left(\int_{t-d(t)}^t x(s) ds \right)^T [-(1-u_1)R_3] \left(\int_{t-d(t)}^t x(s) ds \right) \\
&\quad + h_M^2 \dot{x}^T(t) R_4 x(t) + \left(\int_{t-h(t)}^t x(s) ds \right)^T [-(1-u_2)R_4] \left(\int_{t-h(t)}^t x(s) ds \right), \\
\dot{V}_7(x_t) &= 2 \int_{-d_M}^0 du \int_u^0 [\dot{x}^T(t) S_1 \dot{x}(t) - \dot{x}^T(t+\nu) S_1 \dot{x}(t+\nu)] d\nu \\
&\quad + 2 \int_{-h_M}^0 du \int_u^0 [\dot{x}^T(t) S_2 \dot{x}(t) - \dot{x}^T(t+\nu) S_2 \dot{x}(t+\nu)] d\nu \\
&= d_M^2 \dot{x}^T(t) S_1 \dot{x}(t) - 2 \int_{-d_M}^0 du \int_{t+u}^t \dot{x}^T(s) S_1 \dot{x}(s) ds + h_M^2 \dot{x}^T(t) S_2 \dot{x}(t) \\
&\quad - 2 \int_{-h_M}^0 du \int_{t+u}^t \dot{x}^T(s) S_2 \dot{x}(s) ds.
\end{aligned} \tag{26}$$

Based on the Newton–Leibniz formula, we have

$$\begin{aligned}
2\xi^T(t)G_1 \left[x(t) - x(t-d(t)) - \int_{t-d(t)}^t \dot{x}(s) ds \right] &= 0, 2\xi^T(t)H_1 \left[x(t-d(t)) - x(t-d_M) - \int_{t-d_M}^{t-d(t)} \dot{x}(s) ds \right] = 0, \\
2\xi^T(t)L_1 \left[x(t) - x(t-d_M) - \int_{t-d_M}^t \dot{x}(s) ds \right] &= 0, 2\xi^T(t)G_2 \left[x(t) - x(t-h(t)) - \int_{t-h(t)}^t \dot{x}(s) ds \right] = 0, \\
2\xi^T(t)H_2 \left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s) ds \right] &= 0, 2\xi^T(t)L_2 \left[x(t) - x(t-h_M) - \int_{t-h_M}^t \dot{x}(s) ds \right] = 0, \\
2\xi^T(t)J_1 \left[2d_M x(t) - \int_{t-d(t)}^t x(s) ds - \int_{t-d_M}^{t-d(t)} x(s) ds - \int_{t-d_M}^t x(s) ds - 2 \int_{-d_M}^0 du \int_{t+u}^t \dot{x}(s) ds \right] &= 0, \\
2\xi^T(t)J_2 \left[2h_M x(t) - \int_{t-h(t)}^t x(s) ds - \int_{t-h_M}^{t-h(t)} x(s) ds - \int_{t-h_M}^t x(s) ds - 2 \int_{-h_M}^0 du \int_{t+u}^t \dot{x}(s) ds \right] &= 0.
\end{aligned} \tag{27}$$

According to Lemmas 2 and 3, we can get

$$\begin{aligned}
2\xi^T(t)G_1 \int_{t-d(t)}^t \dot{x}(s) ds &= 2\xi^T(t)G_1 \left(- \int_{t-d(t)}^t \dot{x}(s) ds \right) = 2\sqrt{d_M} \xi^T(t)G_1 \times \frac{1}{\sqrt{d_M}} \left(- \int_{t-d(t)}^t \dot{x}(s) ds \right) \\
&\leq d_M \xi^T(t)G_1 Z_1^{-1} G_1^T \xi(t) + \frac{1}{d_M} \left(\int_{t-d(t)}^t \dot{x}(s) ds \right)^T Z_1 \left(\int_{t-d(t)}^t \dot{x}(s) ds \right) \\
&\leq d_M \xi^T(t)G_1 Z_1^{-1} G_1^T \xi(t) + \int_{t-d(t)}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds.
\end{aligned} \tag{28}$$

Then, we can obtain

$$\begin{aligned}
& -2\xi^T(t)G_1 \int_{t-d(t)}^t \dot{x}(s)ds \leq d_M \xi^T(t)G_1 Z_1^{-1} G_1^T \xi(t) + \int_{t-d(t)}^t \dot{x}^T(s)Z_1 \dot{x}(s)ds, \\
& -2\xi^T(t)H_1 \int_{t-d_M}^{t-d(t)} \dot{x}(s)ds \leq d_M \xi^T(t)H_1 Z_1^{-1} H_1^T \xi(t) + \int_{t-d_M}^{t-d(t)} \dot{x}^T(s)Z_1 \dot{x}(s)ds, \\
& -2\xi^T(t)L_1 \int_{t-d_M}^t \dot{x}(s)ds \leq d_M \xi^T(t)L_1 Z_2^{-1} L_1^T \xi(t) + \int_{t-d_M}^t \dot{x}^T(s)Z_2 \dot{x}(s)ds, \\
& -2\xi^T(t)G_2 \int_{t-h(t)}^t \dot{x}(s)ds \leq h_M \xi^T(t)G_2 Z_3^{-1} G_2^T \xi(t) + \int_{t-h(t)}^t \dot{x}^T(s)Z_3 \dot{x}(s)ds, \\
& -2\xi^T(t)H_2 \int_{t-h_M}^{t-h(t)} \dot{x}(s)ds \leq h_M \xi^T(t)H_2 Z_3^{-1} H_2^T \xi(t) + \int_{t-h_M}^{t-h(t)} \dot{x}^T(s)Z_3 \dot{x}(s)ds, \\
& -2\xi^T(t)L_2 \int_{t-h_M}^t \dot{x}(s)ds \leq h_M \xi^T(t)L_2 Z_4^{-1} L_2^T \xi(t) + \int_{t-h_M}^t \dot{x}^T(s)Z_4 \dot{x}(s)ds, \\
& -2\xi^T(t)J_1 \int_{t-d(t)}^t x(s)ds \leq d_M \xi^T(t)J_1 Z_5^{-1} J_1^T \xi(t) + \int_{t-d(t)}^t x^T(s)Z_5 x(s)ds, \\
& -2\xi^T(t)J_1 \int_{t-d_M}^{t-d(t)} x(s)ds \leq d_M \xi^T(t)J_1 Z_5^{-1} J_1^T \xi(t) + \int_{t-d_M}^{t-d(t)} x^T(s)Z_5 x(s)ds, \\
& -2\xi^T(t)J_1 \int_{t-d_M}^t x(s)ds \leq d_M \xi^T(t)J_1 Z_6^{-1} J_1^T \xi(t) + \int_{t-d_M}^t x^T(s)Z_6 x(s)ds, \\
& -2\xi^T(t)J_2 \int_{t-h(t)}^t x(s)ds \leq h_M \xi^T(t)J_2 Z_7^{-1} J_2^T \xi(t) + \int_{t-h(t)}^t x^T(s)Z_7 x(s)ds, \\
& -2\xi^T(t)J_2 \int_{t-h_M}^{t-h(t)} x(s)ds \leq h_M \xi^T(t)J_2 Z_7^{-1} J_2^T \xi(t) + \int_{t-h_M}^{t-h(t)} x^T(s)Z_7 x(s)ds, \\
& -2\xi^T(t)J_2 \int_{t-h_M}^t x(s)ds \leq h_M \xi^T(t)J_2 Z_8^{-1} J_2^T \xi(t) + \int_{t-h_M}^t x^T(s)Z_8 x(s)ds, \\
& -4\xi^T(t)J_1 \int_{-d_M}^0 du \int_{t+u}^t \dot{x}(s)ds \leq d_M^2 \xi^T(t)J_1 S_1^{-1} J_1^T \xi(t) + 2 \int_{-d_M}^0 du \int_{t+u}^t \dot{x}^T(s)S_1 \dot{x}(s)ds, \\
& -4\xi^T(t)J_2 \int_{-h_M}^0 du \int_{t+u}^t \dot{x}(s)ds \leq h_M^2 \xi^T(t)J_2 S_2^{-1} J_2^T \xi(t) + 2 \int_{-h_M}^0 du \int_{t+u}^t \dot{x}^T(s)S_2 \dot{x}(s)ds.
\end{aligned} \tag{29}$$

From system (14), it is easy to get

$$2\xi^T(t)W \left[\dot{x}(t) - Ax(t) - A_1 x(t-d(t)) - A_2 \dot{x}(t-h(t)) - A_3 \int_{t-d(t)}^t x(s)ds - A_4 \int_{t-h(t)}^t x(s)ds \right] = 0. \tag{30}$$

Combing formulas (19) to (30), we have

$$\begin{aligned}
\dot{V}(x_t) \leq & \xi^T(t) \left(\Omega + d_M G_1 Z_1^{-1} G_1^T + d_M H_1 Z_1^{-1} H_1^T + d_M L_1 Z_2^{-1} L_1^T + h_M G_2 Z_3^{-1} G_2^T + h_M H_2 Z_3^{-1} H_2^T \right. \\
& + h_M L_2 Z_4^{-1} L_2^T + 2d_M J_1 Z_5^{-1} J_1^T + d_M J_1 Z_6^{-1} J_1^T + 2h_M J_2 Z_7^{-1} J_2^T + h_M J_2 Z_8^{-1} J_2^T \\
& \left. + d_M^2 J_1 S_1^{-1} J_1^T + h_M^2 J_2 S_2^{-1} J_2^T \right) \xi(t).
\end{aligned} \tag{31}$$

By Lemma 1, we can obtain $\dot{V}(x_t) \leq \xi^T(t) \Xi \xi(t)$. Obviously, system (14) is ensured to be asymptotically stable. $\square$

Remark 3. In the field of analysis of delay-dependent stability, the main purpose is to find the maximum delay

bounds for guaranteeing stability. In the discussed LKF $V(x_t)$, the three terms $\zeta^T(t)EP\zeta(t)$, $\int_{-d_M}^0 du \int_u^0 dv \int_{t+v}^t \dot{x}^T(s)S_1\dot{x}(s)ds$, and $\int_{-h_M}^0 du \int_u^0 dv \int_{t+v}^t \dot{x}^T(s)S_2\dot{x}(s)ds$ play an important role in reducing the conservatism of our results. More specifically, by taking the states $\int_{t-d_M}^t x(s)ds$, $\int_{t-h_M}^t x(s)ds$, $\int_{t-d(t)}^t x(s)ds$, $\int_{t-h(t)}^t x(s)ds$, $\int_{-d_M}^0 du \int_{t+u}^t \dot{x}(s)ds$, $\int_{-h_M}^0 du \int_{t+u}^t \dot{x}(s)ds$, $\int_{-d_M}^0 du \int_{t+u}^t x(s)ds$, $\int_{-h_M}^0 du \int_{t+u}^t x(s)ds$ as variables, more information can be sufficiently utilized on state variables, which can yield less conservatism. And as mentioned in [37], in order to derive an upper bound of $\dot{V}_5(x_t)$ in each intervals by Lemma 4, two different free-weighting matrices U and V are introduced at each interval. In addition, the integral terms in $V_6(x_t)$ devote to a less result with Jensen's inequality presented.

Remark 4. The LMI problem illustrated in Theorem 1 is a convex problem in relation to solution variables. And various convex optimization algorithms can solve the problem. In this paper, Matlab LMI Toolbox, as a useful software, for solving convex optimization problem, is used in which the interior-point algorithm was implemented.

Remark 5. The proposed criteria in Theorem 1 may give larger stability regions in compassion with some of the existing criteria. This claim is due to the following characteristics of the proposed stability analysis scheme: in [16], the relationship between the augmented LKF and the tightness of inequalities was explored. It has been shown that a tighter inequality does not always lead to a less conservative criterion. Nevertheless, a suitable augmented LKF and a tighter inequality, both of which are indispensable, can effectively reduce conservatism. In this paper, an augmented LKF combined with more cross terms can be well dealt with Lemma 4 technically. In addition, double- and triple-integral terms help to further increase the stability region.

Remark 6. (i) The system modeled in (1) is more representative than the ones in [11, 16, 36, 37]. For example, if $\Delta A(t) = \Delta A_1(t) = \Delta A_2(t) = A_3(t) = A_4(t) = 0$, the neutral items are linear, then system (1) here can reduce to the neutral model without uncertainties in [11]. Besides, when $A_2(t) = A_3(t) = A_4(t) = 0$, the system becomes the general uncertain delay time model in [16, 37]. If $A(t) = A$, $A_1(t) = A_1$, $A_2(t) = A_3(t) = A_4(t) = 0$, then the general delay time systems without uncertainties in [16, 36] are got. And if $d(t) = d$, $h(t) = h$, the varying delay terms are turned into constant delay ones. (ii) In the system of this article, all the coefficient matrices of the states and the derivative are functional matrices with uncertainties, different from the known constant matrices or few uncertain matrices in the existing literature. In the majority of the papers including all the cited papers in this paper, uncertainties are neglected more or less. However, in this article, uncertainties are all considered in the states and the derivative vector. (iii) Most articles about the content of neutral time-delay systems are concerned with the stability problem such as [9, 11–15], and our article not only solves the stability problem but also contains the control strategies.

Remark 7. If $A_3 = 0$ and $A_4 = 0$, we can get a special case of system (14):

$$\begin{cases} \dot{x}(t) - A_2\dot{x}(t-h(t)) = Ax(t) + A_1x(t-d(t)), \\ x(t) = \varphi(t), \dot{x}(t) = \phi(t), t \in [-H, 0]. \end{cases} \quad (32)$$

By applying Theorem 1, solutions of system (32) and the upper delay bounds comparing with other results are shown in Example 1.

Theorem 2. For a prescribed scalar $\gamma > 0$ and given constants d_M, u_1, h_M, u_2 , system (5) with $u(t) = 0$ is robustly asymptotically stable with H_∞ performance γ if there exist positive-definite matrices Q_1, Q_2, Q_a, Q_b, Z_i , ($i = 1, 2, 3, 4, 5, 6, 7, 8$), R_j , ($j = 1, 2, 3, 4$), S_k , ($k = 1, 2$), P_l , and matrices with appropriate dimensions P_l ($l = 2, \dots, 9$), $\bar{U}_\alpha, \bar{V}_\alpha, \bar{G}_\alpha, \bar{H}_\alpha, \bar{L}_\alpha, \bar{J}_\alpha$, ($\alpha = 1, 2$), $\bar{W}$ satisfying the LMI:

$$\begin{aligned} \bar{\Omega} &= \bar{\Omega}_1 + \bar{\Omega}_2 + \bar{\Omega}_2^T, \bar{\Omega}_1 = \bar{\Omega}_{11} - \bar{\Omega}_{12}\bar{\Omega}_{22}^{-1}\bar{\Omega}_{12}^T, \\ \bar{\Omega}_{11} &= \begin{bmatrix} \Omega_{11} & 0 \\ * & -\gamma^2 I \end{bmatrix}, \bar{\Omega}_{12} = \begin{bmatrix} \Omega_{12} \\ 0 \end{bmatrix}, \bar{\Omega}_{22} = \Omega_{22}, \bar{E} = \begin{bmatrix} C & C_1 & \underbrace{0 \cdots 0}_8 & D \end{bmatrix}^T, \\ \bar{\Omega}_2 &= \begin{bmatrix} \bar{G}_1 + \bar{L}_1 + \bar{G}_2 + \bar{L}_2 + 2d_M\bar{J}_1 + 2h_M\bar{J}_2 & \bar{H}_1 - \bar{G}_1 & 0 & -\bar{H}_1 - \bar{L}_1 & \bar{H}_2 - \bar{G}_2 & 0 & -\bar{H}_2 - \bar{L}_2 \\ 0 & 0 & 0 & 0 + \bar{W}[-A & -A_1 & -A_3 & 0 & 0 & -A_4 & 0 & I & 0 & -A_2 & -B_w] \end{bmatrix}, \\ \bar{G}_1 &= \begin{bmatrix} \bar{G}_{01} \\ \bar{G}_{02} \\ \vdots \\ \bar{G}_{11} \end{bmatrix}, \bar{H}_1 = \begin{bmatrix} \bar{H}_{01} \\ \bar{H}_{02} \\ \vdots \\ \bar{H}_{11} \end{bmatrix}, \bar{L}_1 = \begin{bmatrix} \bar{L}_{01} \\ \bar{L}_{02} \\ \vdots \\ \bar{L}_{11} \end{bmatrix}, \bar{J}_1 = \begin{bmatrix} \bar{J}_{01} \\ \bar{J}_{02} \\ \vdots \\ \bar{J}_{11} \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\bar{G}_2 &= \begin{bmatrix} \bar{G}_{11} \\ \bar{G}_{12} \\ \vdots \\ \bar{G}_{21} \end{bmatrix}, \bar{H}_2 = \begin{bmatrix} \bar{H}_{11} \\ \bar{H}_{12} \\ \vdots \\ \bar{H}_{21} \end{bmatrix}, \bar{L}_2 = \begin{bmatrix} \bar{L}_{11} \\ \bar{L}_{12} \\ \vdots \\ \bar{L}_{21} \end{bmatrix}, \bar{J}_2 = \begin{bmatrix} \bar{J}_{11} \\ \bar{J}_{12} \\ \vdots \\ \bar{J}_{21} \end{bmatrix}, \bar{W} = \begin{bmatrix} \bar{W}_{01} \\ \bar{W}_{02} \\ \vdots \\ \bar{W}_{11} \end{bmatrix}, \\
\bar{\Xi} &= \begin{bmatrix} \bar{\Omega} & d_M \bar{G}_1 & d_M \bar{H}_1 & d_M \bar{L}_1 & h_M \bar{G}_2 & h_M \bar{H}_2 & h_M \bar{L}_2 \\ * & -d_M Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -d_M Z_1 & 0 & 0 & 0 & 0 \\ * & * & * & -d_M Z_2 & 0 & 0 & 0 \\ * & * & * & * & -h_M Z_3 & 0 & 0 \\ * & * & * & * & * & -h_M Z_3 & 0 \\ * & * & * & * & * & * & -h_M Z_4 \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{bmatrix} \\
&\quad \begin{bmatrix} 2d_M \bar{J}_1 & d_M \bar{J}_1 & 2h_M \bar{J}_2 & h_M \bar{J}_2 & d_M^2 \bar{J}_1 & h_M^2 \bar{J}_2 & \bar{E} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2d_M Z_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -d_M Z_6 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2h_M Z_7 & 0 & 0 & 0 & 0 \\ * & * & * & -h_M Z_8 & 0 & 0 & 0 \\ * & * & * & * & -d_M^2 S_1 & 0 & 0 \\ * & * & * & * & * & -h_M^2 S_2 & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (33)
\end{aligned}$$

where $\Omega_{11}, \Omega_{12}, \Omega_{22}$ are the same terms as in Theorem 1.

Proof. Introduce the following H_∞ performance index under the zero initial condition:

For any nonzero disturbance $w(t) \in L_2[0, \infty]$, we can see

$$J_\infty = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t)] dt. \quad (34)$$

$$\begin{aligned}
J_\infty &= \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}] dt - V \leq \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}] dt \\
&= \int_0^\infty \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix}^T \bar{\Xi} \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix} dt. \quad (35)
\end{aligned}$$

Based on the method which is similar to the one utilized in Theorem 1, we can conclude $J_\infty < 0$, which means $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$. That is to say, system (5) with $u(t) = 0$ is robustly asymptotically stable with H_∞ performance γ . $\square$

Remark 8. To overcome the external disturbances in actual operations of engineering models and make them stable artificially, considering the problem of robust H_∞ control is a must. The study [18] has shown the artificial applications of robust H_∞ control under external disturbances.

Remark 9. As a special case of system (5), the following time-varying system is considered:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-d(t)) + B_w w(t), \\ z(t) = Cx(t) + C_1x(t-d(t)), \\ x(t) = \varphi(t), t \in [-d_M, 0]. \end{cases} \quad (36)$$

Let $A_2 = 0, A_3 = 0, A_4 = 0$, then the stability analysis can be got for system (36) by Theorem 2, and the conservatism of this method is shown in Example 2.

Theorem 3. For a prescribed scalar $\gamma > 0$ and given constants d_M, u_1, h_M, u_2 , system (1) with $u(t) = 0$ is robustly asymptotically stable with H_∞ performance γ if there exist positive-definite matrices $Q_1, Q_2, Q_a, Q_b, Z_i (i = 1, 2, 3, 4, 5, 6, 7, 8), R_j (j = 1, 2, 3, 4), S_k (k = 1, 2), P_1$, and matrices with appropriate dimensions $P_l (l = 2, \dots, 9), \bar{U}_\alpha, \bar{V}_\alpha, \bar{G}_\alpha, \bar{H}_\alpha, \bar{L}_\alpha, \bar{J}_\alpha (\alpha = 1, 2), \bar{W}$, and $\lambda > 0$ satisfying the following LMI:

$$\bar{\Xi} = \begin{bmatrix} \bar{\Xi} & \Lambda_m & \lambda \Lambda_n \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0, \quad (37)$$

where

$$\begin{aligned} \Lambda_m &= \begin{bmatrix} \Lambda_{m1} M & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ 0 & \underbrace{0 \dots 0}_{12} & & & M \end{bmatrix}, \\ \Lambda_n^T &= \begin{bmatrix} \Lambda_{n1} & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \\ \Lambda_{n2} & \underbrace{0 \dots 0}_{12} & & & 0 \end{bmatrix}, \\ \Lambda_{m1} &= -\bar{W} + [P_2 \ P_3 \ P_4 \ P_5 \ P_6 \ P_7 \ P_8 \ P_9 \ 0 \ 0 \ 0]^T, \\ \Lambda_{n1} &= [N_A \ N_{A1} \ N_{A3} \ 0 \ 0 \ N_{A4} \ 0 \ 0 \ 0 \ N_{A2} \ 0], \\ \Lambda_{n2} &= \begin{bmatrix} N_C \ N_{C1} & \underbrace{0 \dots 0}_8 & N_D \end{bmatrix}, \end{aligned} \quad (38)$$

with the same terms in Theorem 2.

Proof. $A, A_1, A_2, A_3, A_4, C, C_1, D$ in Theorem 2 are substituted by $A + \Delta A(t), A_1 + \Delta A_1(t), A_2 + \Delta A_2(t), A_3 + \Delta A_3(t), A_4 + \Delta A_4(t), C + \Delta C(t), C_1 + \Delta C_1(t), D + \Delta D(t)$, respectively, and then we can obtain

$$\begin{aligned} &\bar{\Xi} + \Lambda_m \bar{F}(t) \Lambda_n^T + \Lambda_n \bar{F}(t)^T \Lambda_m^T < 0, \\ \bar{F}(t) &= \begin{bmatrix} F(t) & 0 & \dots & 0 & 0 \\ 0 & F(t) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & F(t) & 0 \\ 0 & \underbrace{0 \dots 0}_{12} & & & F(t) \end{bmatrix}. \end{aligned} \quad (39)$$

According to Lemma 5, there exists $\lambda > 0$ satisfying

$$\bar{\Xi} + \lambda^{-1} \Lambda_m \Lambda_m^T + \lambda \Lambda_n \Lambda_n^T < 0. \quad (40)$$

Applying Lemma 1, obviously, (40) is equal to (37) and the theorem is proved. $\square$

Remark 10. The uncertain terms are solved in Theorem 3, which makes the system less complex to deal with.

Remark 11. The method applied with parameters in Example 3 is proved to be less conservative.

In the following theorem, we will put our focus on the synthesis of the controller for system (1) and the goal is to

construct a state-feedback control law (6). On the basis of the results of the previous section, closed-loop system (7) is analyzed. The following Theorem 4 gives an LMI-based computational procedure to determine a state-feedback controller.

Theorem 4. For given constants $d_M, u_1, h_M, u_2, \gamma > 0$ and a small enough scalar $\varepsilon > 0$, system (1) is robustly asymptotically stable with performance γ if there exist positive-definite matrices $Q_1, Q_2, Q_a, Q_b, Z_i (i = 1, 2, 3, 4, 5, 6, 7, 8), R_j (j = 1, 2, 3, 4), S_k (k = 1, 2), P_1$, and matrices with appropriate dimensions $P_l (l = 2, \dots, 9), \bar{U}_\alpha, \bar{V}_\alpha, \bar{G}_\alpha, \bar{H}_\alpha, \bar{L}_\alpha, \bar{J}_\alpha (\alpha = 1, 2), \bar{W}, K$, and $\lambda > 0$ satisfying the following LMI:

$$\begin{aligned} \hat{\Xi} &= \begin{bmatrix} \bar{\Xi} & \Lambda_m & \lambda \Lambda_n & \Lambda_k (\Lambda_B + \varepsilon I) \\ * & -\lambda I & 0 & 0 \\ * & * & -\lambda I & 0 \\ * & * & * & -(\Lambda_B + \varepsilon I)^T \end{bmatrix} < 0, \\ \Lambda_k &= \begin{bmatrix} \Lambda_{k1} & \underbrace{0 \ \cdots \ 0}_{13} \\ 0 & 0 \ \cdots \ 0 \\ \vdots & \vdots \ \ddots \ \vdots \\ 0 & 0 \ \cdots \ 0 \end{bmatrix}, \\ \Lambda_B &= \begin{bmatrix} \Lambda_{B1} & \underbrace{0 \ \cdots \ 0}_{13} \\ 0 & 0 \ \cdots \ 0 \\ \vdots & \vdots \ \ddots \ \vdots \\ 0 & 0 \ \cdots \ 0 \end{bmatrix}, \\ \Lambda_{k1} &= \begin{bmatrix} -\bar{W}_{01} + P_2^T & -K^T \\ -\bar{W}_{02} + P_3^T & 0 \\ -\bar{W}_{03} + P_4^T & 0 \\ -\bar{W}_{04} + P_5^T & 0 \\ -\bar{W}_{05} + P_6^T & 0 \\ -\bar{W}_{06} + P_7^T & 0 \\ -\bar{W}_{07} + P_8^T & 0 \\ -\bar{W}_{08} + P_9^T & 0 \\ -\bar{W}_{09} & 0 \\ -\bar{W}_{10} & 0 \\ -\bar{W}_{11} & 0 \end{bmatrix}, \\ \Lambda_{B1} &= \begin{bmatrix} 0 & -B \\ -B^T & 0 \end{bmatrix}, \end{aligned} \quad (41)$$

with the same terms in Theorem 3.

Proof. Substituting A in Theorem 3 by $A + BK$, it is easy to get

$$\begin{aligned} \bar{\Xi} + \lambda^{-1} \Lambda_m \Lambda_m^T + \lambda \Lambda_n \Lambda_n^T + \Lambda_k \Lambda_B \Lambda_k^T &< 0, \\ \Lambda_k \Lambda_B \Lambda_k^T &= \begin{bmatrix} \Lambda_{k1} \Lambda_{B1} \Lambda_{k1}^T & \underbrace{0 \cdots 0}_{13} \\ 0 & 0 \cdots 0 \\ \vdots & \vdots \ddots \vdots \\ 0 & 0 \cdots 0 \end{bmatrix}, \\ \Lambda_{k1} \Lambda_{B1} \Lambda_{k1}^T &= \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{18} & -K^T B^T \bar{W}_{09}^T & -K^T B^T \bar{W}_{10}^T & -K^T B^T \bar{W}_{11}^T \\ \psi_{21} & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \psi_{81} & 0 & \cdots & 0 & 0 & 0 & 0 \\ -\bar{W}_{09} BK & 0 & \cdots & 0 & 0 & 0 & 0 \\ -\bar{W}_{10} BK & 0 & \cdots & 0 & 0 & 0 & 0 \\ -\bar{W}_{11} BK & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned} \quad (42)$$

where

$$\begin{aligned} \psi_{11} &= K^T B^T (-\bar{W}_{01}^T + P_2) + (-\bar{W}_{01} + P_2^T) BK, \\ \psi_{12} &= K^T B^T (-\bar{W}_{02}^T + P_3), \psi_{18} = K^T B^T (-\bar{W}_{08}^T + P_9), \\ \psi_{21} &= (-\bar{W}_{02} + P_3^T) BK, \psi_{81} = (-\bar{W}_{08} + P_9^T) BK. \end{aligned} \quad (43)$$

There must be a small enough scalar $\varepsilon > 0$ subject to

$$\bar{\Xi} + \lambda^{-1} \Lambda_m \Lambda_m^T + \lambda \Lambda_n \Lambda_n^T + \Lambda_k \Lambda_B \Lambda_k^T + \varepsilon \Lambda_k \Lambda_k^T < 0, \quad (44)$$

which equals to

$$\bar{\Xi} + \lambda^{-1} \Lambda_m \Lambda_m^T + \lambda \Lambda_n \Lambda_n^T + \Lambda_k (\Lambda_B + \varepsilon I) \Lambda_k^T < 0. \quad (45)$$

Then, we can obtain (41). Thus, the theorem is proved. $\square$

Remark 12. To get the controller gain in Theorem 4, the method utilized in this paper is faster and easier than the traditional method which multiplies the matrix inequality left and right by some matrix and its transpose, respectively.

Remark 13. The feasibility of the method mentioned in Theorem 4 is shown in Example 4.

Remark 14. In this paper, we focus on the improvement of the robust stability analysis of H_∞ control for delay-dependent uncertain neutral systems with mixed time-varying delays which shows in the following aspects. First, based on a new LKF, robust stabilization criteria for time-varying delays are established. Second, by introducing free-weighting matrices and using the technique of convex combination, sufficient

conditions are derived in terms of LIMs. And it is convenient for us to get all the solutions by MATLAB toolbox directly.

Remark 15. In order to guarantee the solutions of the discussed system asymptotically stable, we develop a continuous state-feedback controller scheme with a rather simple structure to make a closed loop of the system. And when the parameters are given, we can know the value of the controller gain whether to be feasible or not by MATLAB toolbox directly. It is easy to know that the larger control gain is, the less conservative of Theorem 4 is. However, a larger control gain means higher control cost. Therefore, we need to choose the control gain as small as possible on the basis of satisfying the inequalities in Theorem 4.

4. Numerical Examples

In this section, some examples are presented to demonstrate that the proposed methods are feasible and effective.

Example 1. Consider neutral-type system (32) with parameters

$$\begin{aligned} A &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}. \end{aligned} \quad (46)$$

Case I: for $c = 0.1, u_1 = 0.9, u_2 = 0, d_M = 1, h_M = 1$, by LMI Toolbox, the main solutions are

$$\begin{aligned}
P_1 &= \begin{bmatrix} 4.4021 & -0.0581 \\ -0.0581 & 4.4021 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.3829 & -0.0092 \\ -0.0092 & 0.3829 \end{bmatrix}, Q_2 = \begin{bmatrix} 0.3829 & -0.0092 \\ -0.0092 & 0.3829 \end{bmatrix}, \\
Q_a &= \begin{bmatrix} 0.4348 & -0.0000 & 0.2490 & -0.0475 \\ -0.0000 & 0.4348 & -0.0475 & 0.2490 \\ 0.2490 & -0.0475 & 0.2624 & 0.0080 \\ -0.0475 & 0.2490 & 0.0080 & 0.2624 \end{bmatrix}, Q_b = \begin{bmatrix} 0.4348 & -0.0000 & 0.0856 & 0.0012 \\ -0.0000 & 0.4348 & 0.0012 & 0.0856 \\ 0.0856 & 0.0012 & 0.2920 & 0.0057 \\ 0.0012 & 0.0856 & 0.0057 & 0.2920 \end{bmatrix}, \\
Z_1 &= \begin{bmatrix} 0.0508 & 0.0014 \\ 0.0014 & 0.0508 \end{bmatrix}, Z_2 = \begin{bmatrix} 0.0420 & 0.0016 \\ 0.0016 & 0.0420 \end{bmatrix}, Z_3 = \begin{bmatrix} 0.2695 & 0.0056 \\ 0.0056 & 0.2695 \end{bmatrix}, Z_4 = \begin{bmatrix} 0.2268 & 0.0059 \\ 0.0059 & 0.2268 \end{bmatrix}, \\
Z_5 &= \begin{bmatrix} 0.0709 & 0.0033 \\ 0.0033 & 0.0709 \end{bmatrix}, Z_6 = \begin{bmatrix} 0.0342 & 0.0025 \\ 0.0025 & 0.0342 \end{bmatrix}, Z_7 = \begin{bmatrix} 0.6812 & 0.0056 \\ 0.0056 & 0.6812 \end{bmatrix}, Z_8 = \begin{bmatrix} 0.7719 & 0.0061 \\ 0.0061 & 0.7719 \end{bmatrix}, \\
R_1 &= \begin{bmatrix} 0.0393 & 0.0018 \\ 0.0018 & 0.0393 \end{bmatrix}, R_2 = \begin{bmatrix} 0.1542 & 0.0054 \\ 0.0054 & 0.1542 \end{bmatrix}, R_3 = \begin{bmatrix} 0.0507 & -0.0093 \\ -0.0093 & 0.0507 \end{bmatrix}, R_4 = \begin{bmatrix} 0.3829 & -0.0092 \\ -0.0092 & 0.3829 \end{bmatrix}, \\
S_1 &= \begin{bmatrix} 0.0049 & 0.0002 \\ 0.0002 & 0.0049 \end{bmatrix}, S_2 = \begin{bmatrix} 0.2268 & 0.0059 \\ 0.0059 & 0.2268 \end{bmatrix}, U_1 = \begin{bmatrix} 24.6682 & -1.3527 \\ -1.3527 & 24.6682 \end{bmatrix}, U_2 = \begin{bmatrix} 2.6363 & -0.0556 \\ -0.0556 & 2.6363 \end{bmatrix}, \\
V_1 &= \begin{bmatrix} -0.2166 & -0.0261 \\ -0.0261 & -0.2166 \end{bmatrix}, V_2 = \begin{bmatrix} -0.0086 & -0.0015 \\ -0.0015 & -0.0086 \end{bmatrix}.
\end{aligned} \tag{47}$$

Because of the large number and big dimensions, the solution to $P_l (l = 2, \dots, 9), U_\alpha, V_\alpha, G_\alpha, H_\alpha, L_\alpha, J_\alpha (\alpha = 1, 2)$ and W is omitted here.

The results of the state response of the neutral system with parameters in case I are, respectively, depicted in Figure 1.

Case II: by taking parameter $c = 0.1$, the admissible upper bounds of $d(t)$ for different u_2 are listed in Table 1 by Theorem 1. From the comparison, we can see that the methods proposed in this paper are less conservative.

As we can see in Table 1, the results of delay bound to guarantee stability with two different u_2 were presented in [12, 31, 39] and the best results were in [31]. However, by applying Theorem 1, we can obtain larger values than [31].

Remark 16. The example above has shown that our obtained methods not only can solve the normal stability problem as a special case proposed in this paper but also can get results which are better than some existing results.

Example 2. Consider time-varying delay system (36) with matrix parameters

$$\begin{aligned}
A &= \begin{bmatrix} -0.6238 & -1.0132 \\ 2.0116 & -0.2106 \end{bmatrix}, \\
A_1 &= \begin{bmatrix} -0.5011 & 0.7871 \\ -0.3002 & -0.5231 \end{bmatrix}, \\
B_w &= \begin{bmatrix} -0.4326 & 0.1253 \\ 1.6656 & 0.2877 \end{bmatrix}, \\
C &= \begin{bmatrix} 0.2134 & -0.0191 \\ 0.1119 & -0.1665 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.0816 & 0.1290 \\ 0.0712 & 0.0669 \end{bmatrix}.
\end{aligned} \tag{48}$$

By applying the conditions of Theorem 2, Table 2 reports the maximum upper bound d_M for different γ with $u_1 = 0, u_2 = 0.1, h_M = 0.1$, and simulation results are conducted to show the differences between some typical different parameters in Figure 2.

In Table 2, comparison of maximum allowable delay bound d_M for different values of γ is listed. With the increase of γ , the maximum allowable delay bound becomes larger. And it also shows that Theorem 2 provides improved delay bounds.

Example 3. Consider the uncertain neutral system

FIGURE 1: State responses with some typical parameters of Example 1.

TABLE 1: Example 1: comparison of maximum allowable delay bound d_M .

u_2	0	0.6
[39]	1.8842	1.6904
[12]	3.7024	3.3150
[31]	4.4239	3.9563
Theorem 1	6.0317	5.5131;

TABLE 2: Example 2: comparison of maximum allowable delay bound d_M for given γ .

γ	2.0	2.5	3.0	3.5	4.0	4.5
[40]	0.4203	0.4779	0.5146	0.5401	0.5589	0.5733
[30]	0.4734	0.5237	0.5545	0.5754	0.5904	0.6018
Theorem 2	0.6013	0.6302	0.6425	0.6562	0.6635	0.6700;

FIGURE 2: State responses with some typical parameters of Example 2.

TABLE 3: Example 3: comparison of maximum allowable upper bound $d_M = h_M$ for $u_1 = 0.1, u_2 = 0.1$ of different values of c .

c	0	0.1	0.2	0.3	0.4	0.5
[41]	0.9242	0.7386	0.5599	0.4123	0.2935	0.1914
[42]	0.9755	0.7805	0.6020	0.4560	0.3112	0.1997
[11]	1.0720	0.9208	0.7516	0.5991	0.4625	0.3402
[39]	1.1660	0.9621	0.7781	0.6164	0.4727	0.3467
[43]	1.1726	0.9665	0.7815	0.6181	0.4744	0.3476
[44]	1.2723	1.0360	0.8302	0.6512	0.4958	0.3607
[45]	1.3209	1.0777	0.8701	0.6827	0.5123	0.3596
[33]	1.3418	1.0841	0.8585	0.6664	0.5204	0.3649
[31]	1.3519	1.1127	0.8972	0.7055	0.5366	0.3882
[32]	2.2165	1.7187	1.3022	0.9579	0.6764	0.4488
Theorem 3	2.7612	2.0351	1.5294	1.0763	0.7205	0.4711;

$$\begin{cases} \dot{x}(t) - A_2 \dot{x}(t - h(t)) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - d(t)), \\ x(t) = \varphi(t), \dot{x}(t) = \phi(t), t \in [-H, 0], \end{cases} \quad (49)$$

where

$$\begin{aligned} A &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \\ M &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ N_A &= \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, \\ N_{A1} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ |c| &< 1. \end{aligned} \quad (50)$$

For $u_1 = 0.1, u_2 = 0.1$ with different values of $|c|$ and $u_2 = 0.1, c = 0.1$ with different values of u_1 , Tables 3 and 4 give out the related comparative results, respectively. From messages of the tables, it is easy to get the conclusion that the maximum allowable delay bounds increase along with the decrease of $|c|$ or u_1 . Moreover, we can see that the results obtained in Theorem 3 are less conservative than those in [11, 31–33, 39, 42, 43, 45, 46], which illustrates the effectiveness of the methods we proposed.

Example 4. Let us consider system (1) with the parameters

$$\begin{aligned} A &= \begin{bmatrix} -0.05 & -0.5 \\ -0.01 & -0.9 \end{bmatrix}, A_1 = \begin{bmatrix} 1.8 & 0.3 \\ -1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 1.5 & 0.1 \\ -0.1 & -0.1 \end{bmatrix}, A_3 = \begin{bmatrix} -0.1 & -0.05 \\ -0.1 & -0.1 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} -0.1 & -0.01 \\ -0.1 & -0.01 \end{bmatrix}, B = \begin{bmatrix} -0.1 \\ -0.5 \end{bmatrix}, B_w = \begin{bmatrix} 0.01 & -0.1 \\ 0.1 & -0.01 \end{bmatrix}, C = \begin{bmatrix} 0.1 & -0.5 \\ -0.01 & -0.05 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.1 & -0.5 \\ -0.1 & -0.05 \end{bmatrix}, D = \begin{bmatrix} -0.3 & 0.01 \\ 0.05 & -0.15 \end{bmatrix}, M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, N_A = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.15 \end{bmatrix}, \\ N_{A1} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.14 \end{bmatrix}, N_{A2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.13 \end{bmatrix}, N_{A3} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.12 \end{bmatrix}, N_{A4} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.12 \end{bmatrix}, \\ N_C &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.11 \end{bmatrix}, N_{C1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, N_D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.09 \end{bmatrix}. \end{aligned} \quad (51)$$

Let $\gamma = 0.935, d_M = 3, h_M = 0.1, u_1 = 0.2, u_2 = 0.3, \varepsilon = 0.51, F(t) = \sin t$, according to the strict LIMs of

Theorem 4 by LMI toolbox in MATLAB, we can obtain the state-feedback gain:

TABLE 4: Example 3: comparison of maximum allowable delay bound $d_M = h_M$ for $u_2 = 0.1, c = 0.1$ with different values of u_1 .

u_1	0.1	0.3	0.5	0.7	0.9
[42]	0.7712	0.6895	0.5780	0.4219	0.1704
[11]	0.9109	0.8537	0.7980	0.7472	0.7199
[39]	0.9623	0.9075	0.8503	0.7891	0.7740
[46]	0.9668	0.9202	0.8876	0.8660	0.8537
[43]	0.9685	0.9598	0.9589	0.9584	0.9581
[45]	1.0657	0.9426	0.9132	0.8943	0.8943
[31]	1.1004	1.0266	0.9858	0.9708	0.9701
[33]	1.1026	1.0086	0.9751	0.9720	0.9704
[32]	1.7187	1.6135	1.5120	1.4249	1.3901
Theorem 3	2.1038	1.8977	1.6603	1.5302	1.4498;

$$K = [1.8148 \ 0.7237], \quad (52)$$

and the value $\gamma_{\text{opt}} = 0.3601$.

5. Conclusion

In this work, the robust H_∞ stability and stabilization for the delay-dependent uncertain neutral system with mixed time-varying delays have been studied in detail. Based on an LKF newly constructed and methods of free-weighting matrices and various efficient convex optimization algorithms, several new robust H_∞ stability and stabilization criteria are expressed in terms of LMIs. Especially, a desired state-feedback controller gain can be obtained directly by MATLAB, and it can lead to a less conservative result. Finally, numerical examples are shown to illustrate the superiority and feasibility of the proposed method.

The weakness of the obtained results is that terms in the constructed Lyapunov function should be handled technically to make the system stable, and then appropriate analysis techniques and methods must be provided. Thus, dealing with a bit too many terms is a challenge. And another weakness is that the nonlinear disturbances have not been contained in our modeled systems. Thus, dealing with nonlinear disturbances in the neutral-type system is in our future work. Besides, the stability problem of neutral-type neural networks with time-varying delay has been more popular recently. Hence, we will also focus on the stability content of different types of delayed neutral-type neural networks and the neural networks with time delays: (i) fixed-time stabilization of neutral-type inertial neural networks with discrete and distributed time-varying delays; (ii) stability criteria Hopfield neural networks of neutral-type with multiple delays.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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