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Research Article

Finite Horizon Robust Nonlinear Model Predictive Control for Wheeled Mobile Robots

Phuong Nam Dao,[1] **Hong Quang Nguyen**,[2] **Thanh Long Nguyen**,[1,3] **and Xuan Sinh Mai**[1]

[1]*School of Electrical Engineering, Hanoi University of Science and Technology, Hanoi, Vietnam*
[2]*Thai Nguyen University of Technology, No. 666 3/2 Street, Thai Nguyen, Vietnam*
[3]*Hung Yen University of Technology and Education, Hung Yen, Vietnam*

Correspondence should be addressed to Hong Quang Nguyen; quang.nguyenhong@tnut.edu.vn

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The control of mobile robotic systems with input constraints is still a remarkable problem for many applications. This paper studies the model predictive control-based kinematic control scheme after implementing the decoupling technique of wheeled mobile robots (WMRs). This method enables us to obtain the easier optimization problem with fixed initial state. The finite horizon in cost function of model predictive control (MPC) algorithm requires the appropriate terminal controller as well as the equivalent terminal region. The stability of MPC is determined by feasible control sequence. Finally, offline simulation results validate that the computation load is significantly reduced and also validate trajectory tracking control effectiveness of our proposed control scheme.

1. Introduction

Mobile vehicle systems have a broad application prospective in many important fields (military, industry, and hospital task) and are attractive to researchers throughout the world following several main directions including interaction between chaotic systems and mobile robots [1, 2], multirobot coordination [3], trajectory tracking control, and motion-force control problem. The conventional nonlinear control technique has been mentioned in many works by researchers [4]. The classical sliding mode control (SMC) was considered in the work of [4] based on the computation the control signals in each stages consisting of converging to sliding surface and moving on it. Furthermore, the SMC technique has been implemented for different systems such as bilateral teleoperators, under the consideration of order reduction [5]. Because of the nonholonomic constraints and under-actuated description, it is worth noting that a robotic system can be classified into two subsystems. It leads us to consider a cascade controller and stabilization problem [5–9]. Under the influence of unknown wheel slip, this separation technique can be still considered for robotic systems to develop the back-stepping controller achieving the whole system stability [8]. Moreover, the authors in [7] still employ the back-stepping technique for output feedback problem as well as for tractor-trailer systems in [9].

During the last three decades, the motion-force control design of robotic systems under the effect of underactuated mechanical systems and nonholonomic constraints has been considered as a remarkable challenge. The fact is that the above separation method only depends on the trajectory tracking problem due to the constraint coefficient elimination after the transformation. Therefore, the motion/force controller for robotics has been proposed in [10] after obtaining the equivalent map. On the other hand, the challenge of actuator saturation has been handled by classical nonlinear controllers [6]. However, in general nonlinear systems, it is necessary to consider the model predictive control (MPC) solution because of the advantage in handling the actuator saturation.

Optimal control solution has the remarkable way that can solve the above constraint problems by considering the constraint-based optimization [11–14], and model predictive control (MPC) is also one of the most effective solutions to handle the constraint in single systems [15, 16] as well as multiagent systems [17]. In the direction of optimal control, finding the explicit solution of Ricatti equation and partial differential HJB (Hamilton–Jacobi–Bellman) equation in general nonlinear systems [11] is necessary. Thanks to the neural network approximation effectiveness, the novel online adaptive dynamic programming (ADP) algorithm which enables to adjust simultaneously both actor and critic terms [11] is proposed. The training technique of the critic neural network (NN) was implemented by the modified Levenberg–Marquardt method to minimize the square residual error. Furthermore, the weight convergence and stability problem were shown by the need of persistence of excitation (PE) condition [11]. Considering the extension of this work, a model-free adaptive reinforcement learning has been proposed via the special cost function without the knowledge of the system dynamics [12]. Moreover, the nonlinear systems were acted by online adaptive reinforcement learning with completely unknown dynamics after implementing the data collection and Kronecker product.

In recent years, the model predictive control (MPC) has been considered as an effective approach to deal with state/input constraints along with guaranteeing many control objectives under choosing the equivalent cost functions [18, 19]. The main difference between MPC and conventional nonlinear control approaches is that we give out the control sequence by solving optimization problem at each sampling time [20, 21]. Moreover, the tracking problem as well as stability effectiveness of closed systems should be mentioned in MPC algorithm after satisfying the optimization problem. A number of two directions are considered in MPC solution to ensure the stability effectiveness of closed systems, including finite horizon [15, 22–24] and infinite horizon cost function [25, 26]. In order to satisfy the feasibility problem in the MPC control system, the linear matrix inequalities (LMIs) were mentioned in the case of infinite horizon cost function [25, 26], and the appropriate terminal cost function, equivalent terminal controller, terminal region were found in the case of finite horizon [15, 22–24]. This method was also considered for active car suspension and linear parameter varying (LPV) systems in the description of discrete time systems by using LMIs technique without finding the terminal controller in the work of [16, 27], respectively.

Furthermore, a different approach to consider the stability of MPC control systems was also investigated by using Lyapunov-based MPC solution [28]. In [22–24], the authors considered the way to handle the disturbance based on nominal systems, which are obtained by eliminating the disturbance, and this technique was extended by an additional disturbance observer [29]. Furthermore, the extension of this method is considered via a cascade controller with different terminal controller [15]. Similar to the work in [22–24, 30], the robust MPC was applied for manipulators based on optimization problem with which the initial state belongs to the region obtained from disturbance influence [31]. The linearization technique was applied for the advantage of obtaining easier optimization problem in manipulator [32] and in mobile robots [33]. Extending the work in [22–24], the event-based model predictive control for tracking of a nonholonomic mobile robot was investigated in [34]. Moreover, the motion-force control objective can be considered after the work in [35] employing nonlinear MPC for chained form systems. In [36], the nonlinear MPC was investigated for autonomous underwater vehicles (AUVs), not only the kinematic subsystem but also dynamic model. Inspired by the above contents and consideration of MPC problem for robotic systems, the work focuses on the MPC-based kinematic control for mobile robots with main contributions listed as follows:

(1) In comparison with the previous papers [15, 22–24, 30], a MPC-based kinematic control scheme with an easier optimization problem considering fixed initial state and eliminating the predicted model in MPC law
(2) The strict proof concerning the new terminal controller as well as the terminal region is given based on the Lyapunov stability theory

The remainder of our paper is organized as follows. The robot mathematical model and problem statements are given in Section 2. The model predictive-based kinematic tracking control design is presented in Section 3. Subsequently, the simulation results are shown in Section 4. Finally, the conclusions are determined in Section 5.

2. Robot Mathematical Model and Problem Statements

In [8, 9], we can establish the Lagrange function after obtaining the kinematic and dynamic energy. Based on Lagrange dynamic equation, the mathematical model of WMRs can be represented in Figure 1 as mentioned in [8, 9]:

$$\begin{cases} M(\eta)\ddot{\eta} + C(\eta,\dot{\eta})\dot{\eta} + B(\eta)\left(F(\dot{\eta}) + \tau_d\right) = B(\eta)\tau + J^T(\eta)\lambda, \\ J(\eta)\dot{\eta} = 0, \end{cases} \tag{1}$$

where $\eta = [x, y, \theta]^T$ is the vector of joint variables. Let us describe in detail about this vector: three variables, namely, x, y, and θ are sequentially denoted by the position coordinates, the orientation angle of the mobile robot with respect to the axis. All the parameters as well as variables are described in Table 1. Moreover, $M(\eta)$ is the symmetric and positive-definite inertia matrix; $C(\eta,\dot{\eta})$ denotes the centripetal and Coriolis matrix; $F(\dot{\eta})$ and τ_d represent the friction term and bounded external disturbances, respectively; $B(\eta)$ denotes the input transformation matrix; $J^T(\eta)$ is the associated constraint matrix; λ is the Lagrange coefficient; and the mobile robot is acted by the input vector $\tau = [\tau_l, \tau_r]^T$ including left and right torques. The nonholonomic description of general mobile robotic systems (1) leads us to obtain many appropriate control designs based on the existence of solutions $S^T(\eta)$ satisfying the equation

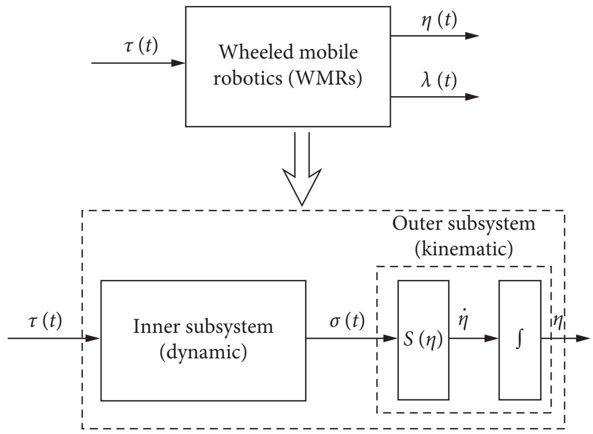

Figure 1: The model of wheeled mobile robotics.

Table 1: Parameters and physical variables of WMRs.

Parameter	Meaning	Unit
b	Half of distance between two wheels	m
r	The wheel's radius	m
r	The wheel's radius	m
m_r	The weight of platform	Kg
I_{rz}	Inertia moment of platform with respect to z-axis	kgm^2
I_{wz}	Inertia moment of wheels with respect to z-axis	kgm^2
I_{wy}	Inertia moment of wheels with respect to wheel axis	kgm^2
x	Projection of the coordinate with respect to x-axis	m
y	Projection of the coordinate with respect to y-axis	m
θ	Heading angle of mobile robots with respect to x-axis	rad
θ_r, θ_l	Angular displacement of each wheel	rad
v	WMR's velocity	m/s

$S^T(\eta)$. $J^T(\eta) = 0$. Consequently, there exists a vector σ such that $\dot{\eta}(t) = S(\eta)\sigma$. Therefore, taking the time derivative of this equation, we obtain that

$$\ddot{\eta} = \dot{S}(\eta)\sigma + S(\eta)\dot{\sigma}. \quad (2)$$

In the case of WMRs, the vector σ was determined as $\sigma = [\vartheta, \omega]^T$, where ϑ and ω are the linear and angular velocities, respectively. It is worth noting that the state variables can be represented by $[\eta^T, \sigma^T]^T = [x, y, \theta, \vartheta, \omega]^T$. By multiplying on both sides of mathematical model (1), we achieve the two connected subsystems of WMRs as follows:

$$\begin{cases} \dot{\eta} = S(\eta)\sigma, \\ D(\eta)S(\eta)\dot{\eta} + C_1(\eta, \dot{\eta})\eta = B(\eta)\tau. \end{cases} \quad (3)$$

Remark 1. It can be verified that the above decoupling technique (3) has the following advantage. The dynamic part (inner subsystem) is a fully actuated subsystem being influenced by unknown parameters and unmodeled disturbances. Meanwhile, the kinematic part, namely, the outer subsystem is the only underactuated system without any uncertain terms.

Remark 2. The decoupling technique was the starting idea of robust adaptive control design in mobile robotic systems [9]. However, it is worth noting that this technique does not guarantee implementing motion/force control objective due to elimination of constraint force factor being the Lagrangian constraint coefficient (3). In order to implement the motion/force control problem, the chain form-based decoupling techniques were mentioned in [25].

The control objective is to find the control signal satisfying the trajectory tracking control problem based on the model predictive control (MPC) algorithm with the advantages not only the optimization problem but also the closed loop stability. Additionally, the purpose of this article is also to reduce the complexity of MPC law by decoupling the WMRs model.

3. Model Predictive-Based Kinematic Tracking Control Design

In this section, we first design the tracking controller using the MPC technique and then analyze the feasibility and closed loop stability of the proposed MPC algorithm. According to (3), in the case of WMRs, the kinematic model can be represented as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \quad (4)$$

Because of the existence of trigonometric factor in (4), according to the desired trajectory $[x_r, y_r, \theta_r]^T$ and equivalent $[v_r, \omega_r]^T$, the tracking error model of kinematic subsystem should be transformed as follows:

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix}. \quad (5)$$

Therefore, the tracking error model's dynamic of the kinematic subsystem can be obtained as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t} p_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} \omega y_e - v + v_r \cos(\theta_e) \\ -\omega x_e + v_r \sin(\theta_e) \\ \omega_r - \omega \end{bmatrix}, \quad (6)$$

where the modified control inputs:

$$u_e = \begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix} = \begin{bmatrix} v_r \cos(\theta_e) - v \\ \omega_r - \omega \end{bmatrix}. \quad (7)$$

Remark 3. The difference in description method in comparison with [15] is described by choosing the equivalent coordinate. In our work, the local coordinate frame of the WMR uses the geometrical center. Meanwhile, the mass center of WMR was chosen for the local coordinate frame in [15]. It leads us to find the appropriate terminal region, terminal controller. Furthermore, it is obviously different from the existing control methods for WMRs in [4, 6–10]. The proposed robust MPC is first structured to obtain the optimization purpose being the extension of classical trajectory tracking control.

Thanks to the decoupling technique above, the disturbance has been eliminated in the kinematic subsystem (6) and then leads us to present the MPC algorithm with modified optimization problem considering the fixed starting point as follows:

$$\underset{u(\tau|t_k)}{\arg\min} J(p_e, u_e) = \int_{t_k}^{t_k+T} L(p_e(\tau|t_k), u_e(\tau|t_k))\mathrm{d}\tau + g(p_e(t_k+T|t_k)), \quad (8)$$

subject to

$$\begin{aligned}
&\dot{p}_e(\tau|t_k) = f(x_e(\tau|t_k), u_e(\tau|t_k)), \quad \tau \in [t_k, t_k+T],\\
&p_e(t_k|t_k) = p_e(t_k),\\
&u(\tau|t_k) \in U\\
&\quad = \left\{[v,\omega]^T : 0 \le v \le v_{\max}, -\omega_{\max} \le \omega \le \omega_{\max}\right\},\\
&\quad \tau \in [t_k, t_k+T],\\
&p_e(t_k+T) \in \Omega,
\end{aligned} \quad (9)$$

where

$$\begin{aligned}
L(p_e(\tau|t_k), u_e(\tau|t_k)) &= \|p_e(\tau|t_k)\|_Q^2 + \|u_e(\tau|t_k)\|_P^2,\\
g(p_e(t_k+T|t_k)) &= \frac{1}{2}\|p_e(t_k+T|t_k)\|^2,\\
Q &= \mathrm{diag}\{q_1, q_2, q_3\},\\
P &= \mathrm{diag}\{p_1, p_2\},\\
T &= N\delta; \quad N \in, N > 0,
\end{aligned} \quad (10)$$

and Ω is the terminal region, and then it will be defined and found later.

Remark 4. Thanks to the decoupling technique, the kinematic model belongs to certain systems. Therefore, the optimization problem will be considered with the fixed initial state, as described in the following algorithm.

Remark 5. Unlike that the classical MPC needs to employ the predictive model, this above optimization problem can solve directly based on the kinematic model without external disturbances and uncertainties. The modified optimization problem is solved online at each step by consideration of the optimal control problem without implementing the computation of predicted model. It should be noted that, by combining with the consideration of a fixed initial point in optimization problem, it is able to obtain simplicity in implementing the MPC law. Furthermore, the unification of optimization and tracking problem is solved in the Theorem 1.

The following theorem shows the tracking problem of closed systems using the modified optimization-based algorithm (Algorithm 1).

Theorem 1. *Consider the outer loop error system (6) with the above kinematic tracking algorithm. Then, the closed control system is ISS.*

In order to obtain the proof of Theorem 1, several definitions and lemmas are considered as follows.

Definition 1. Consider the tracking error model (6), the terminal region Ω, and the equivalent terminal controller $u^L(.)$ are described that if $p_e(t_k+T|t_k) \in \Omega$, then the closed system with this terminal controller satisfies, for any $\tau \in (t_k+T, t_{k+1}+T]$:

$$\begin{cases}
p_e(\tau|t_k) \in \Omega;\\
u(\tau|t_k) \in U;\\
\dfrac{\mathrm{d}g(p_e(\tau|t_k))}{\mathrm{d}\tau} + L(p_e(\tau|t_k), u_e(\tau|t_k)) \le 0.
\end{cases} \quad (11)$$

Lemma 1 enables to find the equivalent terminal region for a terminal controller. It leads us to determine the stability of closed system by using the intermediate estimation.

Lemma 1. *For the outer loop error system (6), the following set*

$$\Omega = \left\{\begin{array}{c}
p_e: |x_e| \ge |y_e|, \quad y_e\theta_e < 0,\\
\dfrac{v_r\cos\theta_e - v_{\max}}{k_1} \le x_e \le \dfrac{v_r\cos\theta_e + v_{\max}}{k_1}\\
-\dfrac{(\omega_{\max}+\omega_r)}{k_2} \le \theta_e \le \dfrac{(\omega_{\max}-\omega_r)}{k_2}
\end{array}\right\}, \quad (12)$$

is a terminal region for the equivalent terminal controller:

$$u^L(t|t_k) = \begin{bmatrix} -k_1x_e + v_r\cos(\theta_e)\\ k_2\theta_e \end{bmatrix}, \quad (13)$$

for any $\tau \in [t_k+T, t_{k+1}+T)$, *where* k_1 *and* k_2 *satisfy* $k_1 - q_1 - p_1k_1^2 > q_2, k_2 - q_3 - p_2k_2^2 > 0$.

Proof. It can be seen that the terminal controller satisfies $u^L \in U$ if $p_e(t_k|t_k) \in \Omega$.

Taking the time derivative of $p_e(t_k|t_k) \in \Omega$ with respect to τ, we have

$$\begin{aligned}
\frac{\mathrm{d}g(p_e(\tau|t_k))}{\mathrm{d}\tau} &= x_e\dot{x}_e + y_e\dot{y}_e + \theta_e\dot{\theta}_e\\
&= -k_1x_e^2 - k_2\theta_e^2 + y_ev_r\sin(\theta_e).
\end{aligned} \quad (14)$$

(1) At time t_k, implement the measurement of actual state
(2) Solve the modified optimization problem to obtain the controller $u_e(t)$
(3) Apply the result during the sampling time interval $[t_k, t_k + 1)$
(4) Update the time instant $t_k \longrightarrow t_{k+1}$

Algorithm 1: MPC-based kinematic tracking algorithm.

According to (12) and (14), we imply $(\mathrm{d}g(p_e(\tau \mid t_k))/\mathrm{d}\tau) < 0$. Therefore, the first condition in (11) is satisfied. Furthermore, based on $p_e(\tau \mid t_k) \in \Omega$, we obtain the result as follows:

$$\begin{aligned}\frac{\mathrm{d}g(\tilde{p}_e(\tau \mid t_k))}{\mathrm{d}\tau} &+ L(p_e(\tau \mid t_k), u_e(\tau \mid t_k)) \\ &= x_e\dot{x}_e + y_e\dot{y}_e + \theta_e\dot{\theta}_e + q_1x_e^2 + q_2y_e^2 \\ &\quad + q_3\theta_e^2 + p_1u_{1e}^2 + p_2u_{2e}^2 \\ &= -(k_1 - q_1 - p_1k_1^2)x_e^2 - (k_2 - q_3 - p_2k_2^2)\theta_e^2 \\ &\quad + y_ev_r\sin(\theta_e) + q_2y_e^2 < 0.\end{aligned} \tag{15}$$

It is obvious that the third condition of (11) is satisfied, which completes Lemma 1. □

Proof. First, it is necessary to consider the feasibility problem of Algorithm 1. Assuming that there exists a feasible solution, an optimal solution $u^*(t_k)$ is obtained at the sampling instant time t_k. Implementing the application of this control sequence $u^*(t_k)$ to (6), the state trajectory is driven into the terminal region Ω, by means that $p_e^*(t_k + T \mid t_k) \in \Omega$. Furthermore, $p_e(t_{k+1} \mid t_k) = p_e^*(t_{k+1})$ is a feasible initial state for the modified optimization problem. Therefore, a feasible control sequence is considered as the intermediate control sequence for estimation in Step 2, which can be established as follows:

$$u(\tau \mid t_{k+1}) = \begin{cases} u^*(\tau \mid t_k), & \tau \in [t_{k+1}, t_k + T), \\ u^L(\tau \mid t_k), & \tau \in [t_k + T, t_{k+1} + T). \end{cases} \tag{16}$$

Secondly, in order to prove the stability of closed system, we choose the Lyapunov function using optimal cost function as $V(t_k) = J(p_e^*(t_k), u_e^*(t_k)), k = \overline{1,\infty}$.

Considering the deviation of the two Lyapunov candidate functions at time t_k and $t_k + 1$,

$$\begin{aligned}\Delta V &= V(t_{k+1}) - V(t_k) \\ &= J(p_e^*(t_{k+1}), u_e^*(t_{k+1})) - J(p_e^*(t_k), u_e^*(t_k)) \\ &\le J(p_e(t_{k+1}), u_e(t_{k+1})) - J(p_e^*(t_k), u_e^*(t_k)) \\ &= -\int_{t_{k+T}}^{t_{k+1}} \left(\|p_e^*(t \mid t_k)\|_Q^2 + \|u_e^*(t \mid t_k)\|_P^2\right)\mathrm{d}t \\ &\quad + \int_t^{t_{k+1}+T} \left(\|p_e(t \mid t_{k1})\|_Q^2 + \|u_e(t \mid t_{k1})\|_P^2\right)\mathrm{d}t \\ &\quad + \frac{1}{2}\|p_e(t_{k+1} + T \mid t_{k+1})\|^2 - \frac{1}{2}\|p_e^*(t_k + T \mid t_k)\|^2.\end{aligned} \tag{17}$$

According to (11), integrating from $(t_k + T)$ to $(t_{k+1} + T)$, we imply that

$$\begin{aligned}&\frac{1}{2}\|p_e(t_{k+1} + T \mid t_{k+1})\|^2 - \frac{1}{2}\|p_e^*(t_k + T \mid t_k)\|^2 \\ &+ \int_t^{t_{k+1}+T} \left(\|p_e(t \mid t_{k+1})\|_Q^2 + \|u_e(t \mid t_{k+1})\|_P^2\right)\mathrm{d}t \le 0.\end{aligned} \tag{18}$$

According to (17) and (18), the following holds:

$$\begin{aligned}\Delta V &= V(t_{k+1}) - V(t_k) \\ &\le -\int_{t_{k+T}}^{t_{k+1}} \left(\|p_e^*(t \mid t_k)\|_Q^2 + \|u_e^*(t \mid t_k)\|_P^2\right)\mathrm{d}t \\ &\le -\int_{t_{k+T}}^{t_{k+1}} \left(\|p_e^*(t \mid t_k)\|_Q^2\right)\mathrm{d}t.\end{aligned} \tag{19}$$

Furthermore, we obtain $V(\infty) - V(0) \le -\int_0^\infty (\|p_e^*(t)\|_Q^2)\mathrm{d}t$.

Because $V(\infty) \ge 0$, there exists the finite integral $\int_0^\infty (\|p_e^*(t)\|_Q^2)\mathrm{d}t$. Moreover, according to (19), a finite limitation $\lim_{k \longrightarrow \infty} V(t_k) = \phi < \infty$ exists. It is obvious that

$$\begin{aligned}0 &\le \lim_{k \longrightarrow \infty} \int_{t_k}^{t_{k+1}} \|p_e^*(t)\|_Q^2 \mathrm{d}\tau \\ &\le \lim_{k \longrightarrow \infty} \int_{t_k}^{t_{k+1}} \left(\|p_e^*(t)\|_Q^2 + \|u_e^*(t \mid t_k)\|_P^2\right)\mathrm{d}t \\ &\le \lim_{k \longrightarrow \infty} (V(t_{k+1}) - V(t_k)) \\ &= \phi - \phi = 0.\end{aligned} \tag{20}$$

Therefore, we conclude that $\lim_{k \longrightarrow \infty} \int_{t_k}^{t_{k+1}} \|p_e^*(\tau)\|_Q^2 \mathrm{d}\tau = 0 \Longrightarrow \lim_{t \longrightarrow \infty} \|p_e^*(t)\|_Q^2 = 0$. □

Remark 6. Unlike the proposed solutions in [15, 22–24], the terminal controller and equivalent terminal region in our work are based on the local coordinate frame using the geometrical center.

4. Simulation Results

The offline simulation is implemented by Casadi tool for modified optimization problem with fixed initial state. The trajectory of WMR to be tracked as follows: the desired trajectory $x_r = 0.8\cos(0.5t)$, $y_r = 0.8\sin(0.5t)$ and parameters of the controller are set to be $q_1 = q_2 = q_3 = 0.5$, $r_1 = r_2 = 0.2$, $k_1 = 2$, and $k_2 = 1$. Algorithm 1 is used for the kinematic model of WMR to obtain the tracking trajectory as described in Figure 2. Furthermore, we also present the response of velocities and the joint variable's error in Figures 3 and 4, respectively. The good behaviours in Figures 2–4 validate the

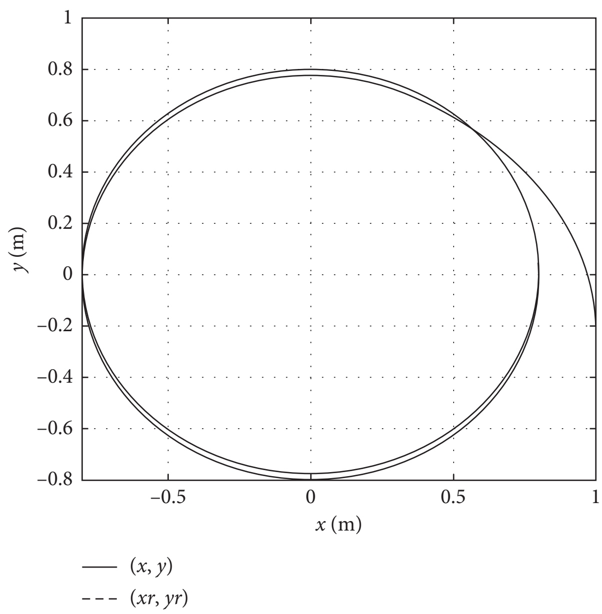

FIGURE 2: The trajectory response.

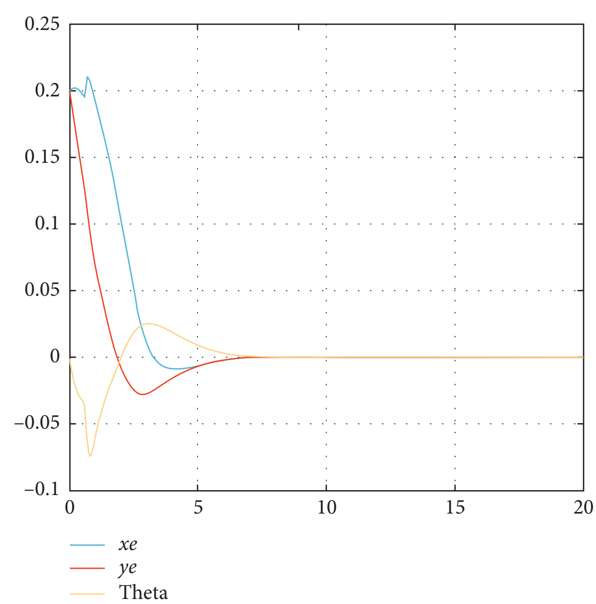

FIGURE 3: The joint variable's response.

high effectiveness of the proposed solution in paper. Furthermore, the computational complexity is easier than the previous works in [15, 22–24] because of the advantage of the fixed initial state in the modified optimization problem.

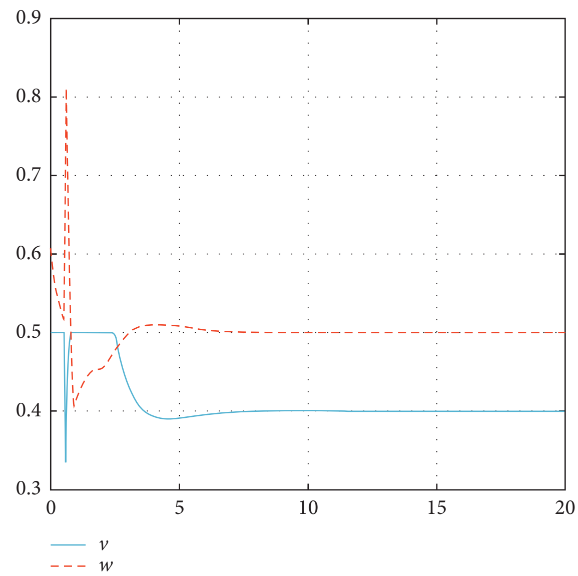

FIGURE 4: The control signals.

5. Conclusions

In this paper, the MPC-based kinematic control scheme has been developed for WMRs with input constraint and disturbances. The work was implemented under the advantage of decoupling technique of the WMR model based on the nonholonomic property in WMRs. The MPC algorithm, based on modified optimization problem with fixed initial state and eliminating the computation of predicted model, enables us to obtain easier computational complexity. The unification of optimization and tracking problem is handled by considering the feasibility and feasible region, appropriate Lyapunov function candidate. Using the Casadi tool in MATLAB software, simulation results and theoretical analysis demonstrated the effectiveness of the proposed solution. Future work of this research encompasses experimental validation of mobile robotic systems.

Data Availability

The multiple datasets supporting this study are cited in references.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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