Research Article

Robust Fixed-Time Synchronization for Coupled Delayed Neural Networks with Discontinuous Activations Subject to a Quadratic Polynomial Growth

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In this paper, we focus on the robust fixed-time synchronization for discontinuous neural networks (NNs) with delays and hybrid couplings under uncertain disturbances, where the growth of discontinuous activation functions is governed by a quadratic polynomial. New state-feedback controllers, which include integral terms and discontinuous factors, are designed. By Lyapunov–Krasovskii functional method and inequality analysis technique, some sufficient criteria, which ensure that networks can realize the robust fixed-time synchronization, are addressed in terms of linear matrix inequalities (LMIs). Moreover, the upper bound of the settling time, which is independent on the initial values, can be determined to any desired values in advance by the configuration of parameters in the proposed control law. Finally, two examples are provided to illustrate the validity of the theoretical results.

1. Introduction

In recently years, coupled neural networks (CNNs), as a special sort of complex dynamic networks, have attracted the widespread attention from a lot of scholars due to its potential applications in parallel computation, multiagent cooperative control, cryptography, nuclear magnetic resonance instrument, and other aspects [1–5]. Particularly, the synchronization with respect to CNNs has been extensively studied in many science fields [6–11] and the references therein.

Time delay often arises in the transmission of signals in CNNs [12, 13]. In [13], Shao and Zhang considered the delay-dependent stabilization for CNNs with two additive input delays. In [14], He et al. investigated the pinning synchronization for CNNs with hybrid couplings and delays by an adoptive approach. In [15], Wang and Huang discussed the pinning synchronization of delayed CNNs with reaction-diffusion effects.

Recently, many works are devoted to the synchronization behaviors of NNs with discontinuous activations. For example, in [16], by applying the state-feedback control strategies, the global finite-time synchronization conditions are addressed for delayed NNs with discontinuous activations. In [17], the authors discussed the global synchronization for NNs with time-varying delays and discontinuous right-hand side. The exponential synchronization for discontinuous NNs with delays has been considered in [18]. In [19], the discrete nonfragile control strategy was designed to achieve the synchronization for fractional-order NNs by adjusting the coupling gain.

In engineering applications, the synchronization is required to be achieved in a finite time [20, 21]. In [22], the authors pointed out that the finite-time control can demonstrate better disturbance and robustness rejection properties. Therefore, it is meaningful to investigate the global synchronization in finite time for CNNs [23–27].

However, it should be noted that the upper bound of the settling time greatly depends on the initial states of CNNs under the finite-time synchronization. In generality, it is difficult to obtain the initial values in the real world. This shows that there exists a constraint to practical applications of CNNs [28]. To solve this problem, the fixed-time synchronization control strategy is considered in [28], where the settling time is independent on the initial conditions of CNNs. At present, as a kind of efficient control strategies, the fixed-time synchronization control has received tremendous attention from many researchers. The global fixed-time synchronization was discussed for semi-Markovian jumping neural networks with time-varying delays and discontinuous activation functions in [29]. The authors discussed the global fixed-time stability of dynamical nonlinear systems and realized the global fixed-time synchronization for CNNs with discontinuous activations in [30]. In [31], the nonsingular fixed-time consensus tracking was considered for second-order multiagent networks. Then, Zheng et al. discussed the fixed-time synchronization of memristive fuzzy BAM cellular neural networks with time-varying delays based on feedback controllers in [32]. Some criterions were obtained for the fixed-time synchronization of Cohen–Grossberg CNNs with and without parameter uncertainties in [33]. By fixed-time controllers, the global synchronization of delayed hybrid CNNs is studied in [34]. In [35, 36], Zhu et al. discussed the synchronization in fixed time with hybrid couplings and delays for semi-Markovian switching complex NNs, and the upper bound of settling time could be determined under the designed controller. The authors considered the global synchronization in fixed time for CNNs with delays and discontinuous/continuous activations and proposed two discontinuous control protocols in [37]. Event-triggered synchronization in fixed time was investigated for semi-Markov switching dynamical complex networks with multiple weights and discontinuous nonlinearity in [38]. It is worth pointing out, to the best of authors' knowledge, there are few results on the global synchronization for hybrid CNNs with discontinuous activations subject to a nonlinear function growth.

Inspired by the above discussion, in this paper, we focus on the global robust fixed-time synchronization for hybrid CNNs with delays and discontinuous activations in the presence of disturbances. Under the designed controllers with integral terms and discontinuous factors, the global synchronization conditions in fixed time are addressed in the terms of LMIs. Moreover, the upper bound of the settling time is estimated explicitly. The primary contributions of this paper are listed as follows:

- (1) The neuron activation functions are modeled to be discontinuous and subject to a quadratic polynomial growth.
- (2) On the basis of the configuration of parameters in the designed control law, the upper bound of the settling

time can be determined to any desired values in advance.

- (3) The robust synchronization conditions in fixed time are achieved in the form of LMIs.

The rest of this paper is organized as follows. In Section 2, some preliminaries and CNNs model are provided. In Section 3, the state-feedback discontinuous controllers are designed, and the global fixed-time synchronization conditions are addressed in the form of LMIs. In Section 4, two numerical simulations verifying the theoretic findings are presented. Conclusion is received in Section 5.

Notation. R refers to the set of real numbers. R^n represents the n -dimensional Euclidean space, and $R^{n \times n}$ denotes the set of all $n \times n$ real matrices. Given $A \in R^{n \times n}$, $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) stands for the maximal (minimal) eigenvalues of A . $A > 0$ ($A < 0$) represents A is a positive (negative) definite matrix. Set $\eta = (\eta_1, \eta_2, \dots, \eta_n)^T \in R^n$, where the superscript T is the transpose operator. Define $\|\eta\|_q = (\sum_{i=1}^n |\eta_i|^q)^{(1/q)}$, $|\eta|^\alpha = (|\eta_1|^\alpha, |\eta_2|^\alpha, \dots, |\eta_n|^\alpha)^T$, and $\text{sig}^\alpha(\eta) = (|\eta_1|^\alpha \text{sign}(\eta_1), (|\eta_2|^\alpha \text{sign}(\eta_2), \dots, |\eta_n|^\alpha \text{sign}(\eta_n))^T$, where α and q are positive constants and $|\cdot|$ denotes the absolute real value. For a set $\Phi \subset \mathbb{R}^n$, $\bar{\text{co}}[\Phi]$ represents the closure of the convex hull of Φ . Let A, B, C , and D be matrices with appropriate dimensions. $\otimes$ denotes the Kronecker product.

2. Preliminaries and System Description

Consider an array of hybrid CNNs described by

$$\begin{aligned} \dot{x}_i(t) = & -Qx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)) + c_1 \sum_{j=1}^N d_{ij}x_j(t) \\ & + c_2 \sum_{j=1}^N h_{ij}x_j(t-\tau) + J(t) + u_i(t) + \Delta_i(t, x_i(t)), \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, N$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T$ is the state of the i th network at time t ; $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T$, $Q = \text{diag}(q_1, \dots, q_n)$, $q_i > 0$; $A = (a_{lr})_{n \times n}$ represents the connection weight matrix; $B = (b_{lr})_{n \times n}$ denotes the delayed connection matrix; c_1 and c_2 are couple strength; $f(x_i(t)) = (f_1(x_{i1}(t)), \dots, f_n(x_{in}(t)))^T$ intends neuron activation function; $\tau > 0$ is delay. $u_i(t)$ stands for the control input; $J(t) = (J_1(t), J_2(t), \dots, J_n(t))^T$ is an external input; and $D = (d_{ij})_{N \times N}$ and $H = (h_{ij})_{N \times N}$ indicate coupling configuration matrix and delayed coupling configuration matrix, respectively. If there exists an edge from node i to j , then $d_{ij} = d_{ji} > 0$; otherwise, $d_{ij} = d_{ji} = 0$ ($i \neq j$). Laplacian matrix $L = (l_{ij})_{N \times N}$ of a graph corresponding to D is given by $l_{ii} = \sum_{j=1, j \neq i}^N d_{ij}$, $l_{ij} = -d_{ij}$, $i \neq j$. Δ_i expresses the uncertain disturbance.

In system (1), $f_i(\cdot)$ is made to satisfy the following assumptions:

- (A1) $f_i(\cdot)$ is continuous except on a countable set of isolate points ρ_k^i , and $f_i(\cdot)$ has at most a limited number of discontinuous points on any compact

interval of R ; in addition, at the discontinuous points ρ_k^i , the finite right limit $f_i(\rho_k^{i+})$ and left limit $f_i(\rho_k^{i-})$ exist.

(A2) Let $\widehat{D} \in R^n$ be a domain containing the origin. There exist positive real constants ν_l, ω_l , and g_l for each $l = 1, 2, \dots, n, i = 1, 2, \dots, N, \forall t \geq 0$, such that

$$\sup |\gamma_{il}(t) - \tilde{\gamma}_l(t)| \leq g_l |x_{il}(t) - y_l(t)|^2 + \nu_l |x_{il}(t) - y_l(t)| + \omega_l, \quad (2)$$

holds for $\forall x_{il}(t), y_l(t) \in \widehat{D}$, where $\tilde{\gamma}_l(t) \in \overline{co}[f_l(y_l(t))], \gamma_{il} \in \overline{co}[f_l(x_{il}(t))]$.

Based on A1, it follows that $\overline{co}[f(x_i)] = (\overline{co}[f_1(x_{i1})], \overline{co}[f_2(x_{i2})], \dots, \overline{co}[f_n(x_{in})])^T$, $\overline{co}[f_n(x_{in})]^T$, where $\overline{co}[f_i(x_{il})] = [\min\{f_i(x_{il}^-), f_i(x_{il}^+)\}, \max\{f_i(x_{il}^-), f_i(x_{il}^+)\}]$.

Under A1, system (1) is a functional differential equation with discontinuous right-hand side [38]. In this paper, analogous to [39, 40], we use the definition of Filippov solutions for system (1).

Definition 1 (see [41]). $x_i(t)$ is solution of system (1) in Filippov sense, if the following holds:

- (i) $x_i(t)$ is continuous on $[-\tau, T]$, and $x_i(t)$ is absolutely continuous on $[0, T]$.
- (ii) for a.e. $t \in [0, T]$.

$$\begin{aligned} \dot{x}_i(t) \in F(x_i) = & -Qx_i(t) + A\overline{co}[f(x_i(t))] \\ & + B\overline{co}[f(x_i(t-\tau))] + c_1 \sum_{j=1}^N d_{ij}x_j(t) \\ & + c_2 \sum_{j=1}^N h_{ij}x_j(t-\tau) + J(t) \\ & + \overline{co}[u_i(t)] + \Delta_i(t, x_i(t)). \end{aligned} \quad (3)$$

Noting that set-valued map $F(x_i)$ has a nonempty, compact, and convex value and is upper semicontinuous, so it is measurable. By measurable selection theorem [41], there exists measurable function $\gamma_i(t) = (\gamma_{i1}(t), \dots, \gamma_{in}(t))^T: [-\tau, T] \rightarrow R^n, \gamma_i \in \overline{co}[f(x_i(t))]$ and $\xi_i(t) = (\xi_{i1}(t), \xi_{i2}(t), \dots, \xi_{in}(t))^T: [-\tau, T] \rightarrow R^n, \xi_i \in \overline{co}[u_i(t)]$ such that, for a.e. $t \in [0, T]$,

$$\begin{aligned} \dot{x}_i(t) = & -Qx_i(t) + A\gamma_i(t) + B\gamma_i(t-\tau) + J(t) + \xi_i(t) \\ & + c_1 \sum_{j=1}^N d_{ij}x_j(t) \\ & + c_2 \sum_{j=1}^N h_{ij}x_j(t-\tau) + \Delta_i(t, x_i(t)). \end{aligned} \quad (4)$$

Definition 2 (IVP [42]). For any $\varphi_i = (\varphi_{i1}, \varphi_{i2}, \dots, \varphi_{iN})^T: [-\tau, 0] \rightarrow R^N$ and any measurable selection $\phi_i = (\phi_{i1}, \phi_{i2}, \dots, \phi_{iN})^T: [-\tau, 0] \rightarrow R^n$, where $\phi_i(s) \in \overline{co}[f_i(\phi_i(s))]$ for a.e. $s \in [-\tau, 0]$. Absolute continuous function $x_i(t) = x_i$

(t, φ, ϕ) associated with measurable function $\gamma_i(t) \in \overline{co}[f(x_i(t))]$ is said to be solution of IVP of system (1) on $[0, T]$ with initial value $(\varphi_i(s), \phi_i(s))$ if

$$\begin{cases} \dot{x}_i(t) = -Qx_i(t) + A\gamma_i(t) + B\gamma_i(t-\tau) + c_1 \sum_{j=1}^N d_{ij}x_j(t) \\ + c_2 \sum_{j=1}^N h_{ij}x_j(t-\tau) + J(t) + \xi_i(t) + \Delta_i(t, x_i(t)), \\ \text{for a.e. } t \in [0, T] \\ x_i(s) = \tilde{\varphi}_i(s), \quad \forall s \in [-\tau, 0], \\ \gamma_i(s) = \tilde{\phi}_i(s), \quad \text{for a.e. } s \in [-\tau, 0]. \end{cases} \quad (5)$$

Consider the following isolated neural network:

$$\dot{y}(t) = -Qy(t) + Af(y(t)) + Bf(y(t-\tau)) + J(t). \quad (6)$$

Analogous to Definition 2, the IVP associated with system (6) is obtained as follows:

Definition 3 (IVP [42]). For any $\psi = (\psi_1, \psi_2, \dots, \psi_N)^T: [-\tau, 0] \rightarrow R^n$ and any measurable selection $\omega = (\omega_1, \omega_2, \dots, \omega_N)^T: [-\tau, 0] \rightarrow R^n$, where $\omega_i(s) \in \overline{co}[f_i(\psi_i(s))]$ for a.e. $s \in [-\tau, 0]$. Absolute continuous function $y(t) = y(t, \psi, \omega)$ associated with measurable function $\tilde{\gamma}(t) \in \overline{co}[f(y(t))]$ is said to be solution of IVP of system (6) on $[0, T]$ with initial value $(\psi(s), \omega(s))$ if

$$\begin{cases} \dot{y}(t) = -Qy(t) + A\tilde{\gamma}(t) + B\tilde{\gamma}(t-\tau) + J(t), \\ \text{for a.e. } t \in [0, T], \\ y(s) = \tilde{\psi}(s), \quad \forall s \in [-\tau, 0], \\ \tilde{\gamma}(s) = \omega(s), \quad \text{for a.e. } s \in [-\tau, 0]. \end{cases} \quad (7)$$

In this paper, our objective is to design new feedback controllers to realize the robust fixed-time synchronization between CNNs (1) and isolated network (6).

Set synchronization error $e_i(t) = x_i(t) - y(t)$. Then,

$$\begin{aligned} \dot{e}_i(t) = & -Qe_i(t) + A\tilde{f}(e_i(t)) + B\tilde{f}(e_i(t-\tau)) + c_1 \sum_{j=1}^N d_{ij}e_j(t) \\ & + c_2 \sum_{j=1}^N h_{ij}e_j(t-\tau) + u_i(t) + \Delta_i(t, x_i(t)), \end{aligned} \quad (8)$$

where $\tilde{f}_i(t) = f(x_i(t)) - f(y(t))$ and $\tilde{f}_i(t-\tau) = f(x_i(t-\tau)) - f(y(t-\tau))$.

According to Definitions 2 and 3, IVP of error system can be written as

$$\begin{cases} \dot{e}_i(t) = -Qe_i(t) + A\hat{\gamma}_i(t) + B\hat{\gamma}_i(t - \tau) + \xi_i(t) + c_1 \sum_{j=1}^N d_{ij}e_j(t) \\ + c_2 \sum_{j=1}^N h_{ij}e_j(t - \tau) + \Delta_i(t, x_i(t)), \\ \text{for a.e. } t \in [0, T], \\ e_i(s) = \bar{e}_i(s), \quad \forall s \in [-\tau, 0], \\ \hat{\gamma}_i(s) = \bar{\eta}_i(s), \quad \text{for a.e. } s \in [-\tau, 0], \end{cases} \quad (9)$$

where $\hat{\gamma}_i(t) = \gamma_i(t) - \bar{\gamma}(t) \in \overline{co}[f(x_i(t))] - \overline{co}[f(y(t))]$
 $\in \tilde{f}(e_i(t))$, $\bar{e}_i(s) = \bar{\varphi}_i(s) - \bar{\psi}(s)$, and $\bar{\eta}_i(s) = \bar{\phi}_i(s) - \omega(s)$.

In order to derive the robust synchronization results of CNNs (1), for the terms $\Delta_i(t, x)$, we make the following assumption:

(A3) The uncertain disturbances $\Delta_i(t, x)$ are bounded by

$$|\Delta_i(t, x)| \leq \Delta_{\max}, \quad (10)$$

where $\Delta_{\max}$ is a known nonnegative constant.

Definition 4. Under the designed controller $u_i(t)$, if there exists time function $T(\bar{e}_i, \bar{\eta}_i) > 0$ such that $\lim_{t \rightarrow T(\bar{e}_i, \bar{\eta}_i)} \|e_i(t)\| = 0$ and $\|e_i(t)\| \equiv 0$, $t \geq T(\bar{e}_i, \bar{\eta}_i)$, then system (1) is said to be globally robust finite-time synchronized with system (6). Moreover, if there exists scalar $T_{\max} > 0$ such that $T(\bar{e}_i, \bar{\eta}_i) \leq T_{\max}$, then system (1) is said to be globally robust fixed-time synchronized with system (6). $T(\bar{e}_i, \bar{\eta}_i)$ is called as the settling time function, and $T_{\max}$ is the upper bound of the settling time function.

Definition 5 (see [43]). For function $\mathcal{V}(x_i): R^n \rightarrow R$, if

- (i) $\mathcal{V}(x_i)$ is regular in R^n ,
- (ii) for $x_i = 0$, $\mathcal{V}(0) = 0$, and for $x_i \neq 0$, $\mathcal{V}(x_i) > 0$,
- (iii) $\mathcal{V}(x_i) \rightarrow +\infty$ as $\|x_i\| \rightarrow +\infty$,

then $\mathcal{V}(x_i)$ is called as C -regular.

Lemma 1 (chain rule [44]). If function $\mathcal{V}(x_i)$ is C -regular and $x_i(t)$ is absolutely continuous on $[0, +\infty)$, then $\mathcal{V}(x_i(t))$ is differentiable for $x_i(t) \in [0, +\infty)$ and

$$\frac{d}{dt} \mathcal{V}(x_i(t)) = v^T x_i(t), \quad \forall v \in \partial \mathcal{V}(x_i), \quad (11)$$

where $\partial \mathcal{V}(x_i)$ is the Clarke generalized gradient of $\mathcal{V}(x_i)$ at x .

Lemma 2 (see [44]). Suppose that $\mathcal{V}: R^n \rightarrow R$ is C -regular and $x_i(t): [0, +\infty) \rightarrow R^n$ is absolutely continuous. Let $\hat{v}_i(t) = \mathcal{V}(x_i(t))$. If there exist a continuous function $\varsigma: [0, +\infty) \rightarrow R$ with $\varsigma(\cdot) > 0$, $\gamma \in (0, +\infty)$, such that

$$\dot{\hat{v}}_i(t) \leq -\varsigma(\hat{v}_i(t)), \quad \text{for a.e. } t > 0,$$

$$\int_0^{\hat{v}_i(0)} \frac{1}{\varsigma(\varrho)} d\varrho = t_1 < +\infty, \quad (12)$$

then $\lim_{t \rightarrow t_1} x_i(t) = 0$ and $x_i(t) = 0$, $t \geq t_1$. Especially, if for all $\varrho \in (0, +\infty)$ and $\varsigma(\varrho) = M\varrho^\mu$, where $0 < \mu < 1$ and $M > 0$, then t_1 can be calculated by

$$t_1 = \frac{\hat{v}_i^{1-\mu}(0)}{M(1-\mu)}. \quad (13)$$

Lemma 3 (see [30]). Suppose that $\mathcal{V}: R^n \rightarrow R$ is a C -regular function and $e_i(t): [0, +\infty) \rightarrow R^n$ is solution with initial value $(\bar{e}_i, \bar{\eta}_i)$ of error system (9). If there exist constants $\bar{a} > 0$, $\bar{b} > 0$, $\bar{\delta} > 0$, $\bar{k} > 0$, and $\bar{\delta}\bar{k} > 1$, such that

$$\frac{d}{dt} \mathcal{V}(e_i(t)) \leq -\left(\bar{a}\bar{\mathcal{V}}^{\bar{\delta}}(e_i(t)) + \bar{b}\right)^{\bar{k}}, \quad e_i(t) \in R^n \setminus \{0\}, \text{ for a.e. } t > 0, \quad (14)$$

then the upper bound settling time $T(\bar{e}_i, \bar{\eta}_i)$ is estimated by

$$T(\bar{e}_i, \bar{\eta}_i) \leq T_{\max} \triangleq \frac{1}{\bar{b}^{\bar{k}}} \left(\frac{\bar{a}}{\bar{b}}\right)^{(1/\bar{\delta})} \left(1 + \frac{1}{\bar{\delta}\bar{k} - 1}\right). \quad (15)$$

Lemma 4 (see [45, 46]). Let $\delta_1, \delta_2, \dots, \delta_n \geq 0$, $q > 1$, and $p \in (0, 1]$. And then,

$$\begin{aligned} \sum_{i=1}^n \delta_i^q &\geq n^{1-q} \left(\sum_{i=1}^n \delta_i\right)^q, \\ \sum_{i=1}^n \delta_i^p &\geq \left(\sum_{i=1}^n \delta_i\right)^p. \end{aligned} \quad (16)$$

Lemma 5 (see [47, 48]). Let $\zeta_1, \zeta_2, \dots, \zeta_n \geq 0$, $0 < p < q$. Then,

$$\left(\sum_{i=1}^n \zeta_i^p\right)^{(1/p)} \geq \left(\sum_{i=1}^n \zeta_i^q\right)^{(1/q)}. \quad (17)$$

3. Main Results

Set $\hat{a} = \max_{l,r \in [1,n]} |a_{lr}|$, $\hat{b} = \max_{l,r \in [1,n]} |b_{rl}|$, $\hat{\nu} = \max_{r \in [1,n]} \nu_r$, and $\hat{\omega} = \max_{r \in [1,n]} \omega_r$. Define $\tilde{L} = (\tilde{l}_{ij})_{N \times N}$ and $\tilde{D} = (\tilde{d}_{ij})_{N \times N}$, $i, j = 1, 2, \dots, N$, where

$$\begin{aligned} \tilde{d}_{ij} &= \begin{cases} 2\hat{a}_{ij}^{(2/3)}, & i \neq j, \\ 0, & i = j, \end{cases} \\ \tilde{l}_{ij} &= \begin{cases} -\tilde{d}_{ij}, & i \neq j, \\ \sum_{j=1, j \neq i}^N \tilde{d}_{ij}, & i = j, \end{cases} \end{aligned} \quad (18)$$

$$\tilde{\Lambda} = \text{diag}\left\{(2\tilde{\beta}_1)^{(2/3)}, (2\tilde{\beta}_2)^{(2/3)}, \dots, (2\tilde{\beta}_N)^{(2/3)}\right\}. \quad (19)$$

The controller $u_i(t)$ is designed as follows:

$$\begin{aligned} u_i(t) = & -\Gamma \|e_i(t-\tau)\|_1^2 \text{sign}(e_i(t)) \\ & + \beta \sum_{j=1}^N d_{ij} \text{sig}^2(e_j(t) - e_i(t)) - \beta \tilde{\beta} \text{sig}^2(e_i(t)) \\ & - a \text{sign}(e_i(t)) - K \frac{e_i(t)}{\|e_i(t)\|^2} \\ & + \hat{K} \sum_{l=1}^n \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \frac{e_i(t)}{\|e_i(t)\|^2}, \end{aligned} \quad (20)$$

where $\Gamma > 0$, $\beta > 0$, $\text{sign}(e_i(t)) = \text{diag}\{\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{im}(t))\}$, $a > 0$, $K > 0$, $\hat{K} > 0$, $\beta = \text{diag}\{\beta_1, \dots, \beta_N\}$. There exists a path between network i and (6), if and only if, $\tilde{\beta}_i > 0$.

Note that controller (20) is discontinuous, we have

$$\begin{aligned} \overline{\text{co}}[u_i(t)] = & -\Gamma \|e_i(t-\tau)\|_1^2 \overline{\text{co}}[\text{sign}(e_i(t))] \\ & + \beta \sum_{j=1}^N d_{ij} \overline{\text{co}}[\text{sig}^2(e_j(t) - e_i(t))] \\ & - \beta \tilde{\beta} \overline{\text{co}}[\text{sig}^2(e_i(t))] - a \overline{\text{co}}[\text{sign}(e_i(t))] \\ & - K \frac{e_i(t)}{\|e_i(t)\|^2} \\ & + \hat{K} \sum_{l=1}^n \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \frac{e_i(t)}{\|e_i(t)\|^2}, \end{aligned} \quad (21)$$

where $e_i(t) > 0$, $\overline{\text{co}}[\text{sign}(e_i(t))] = 1$; $e_i(t) = 0$, $\overline{\text{co}}[\text{sign}(e_i(t))] = [-1, 1]$; $e_i(t) < 0$, $\overline{\text{co}}[\text{sign}(e_i(t))] = -1$.

Let $\xi_i(t) \in \overline{\text{co}}[u_i(t)]$, then there exists $\text{Sign}(e_i(t)) \in \overline{\text{co}}[\text{sign}(e_i(t))]$, such that

$$\begin{aligned} \xi_i(t) = & -\Gamma \|e_i(t-\tau)\|_1^2 \text{Sign}(e_i(t)) \\ & + \beta \sum_{j=1}^N d_{ij} |e_j(t) - e_i(t)|^2 \text{Sign}(e_j(t) - e_i(t)) \\ & - \beta \tilde{\beta} |e_i(t)|^2 \text{Sign}(e_i(t)) \\ & + \hat{K} \sum_{l=1}^n \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \frac{e_i(t)}{\|e_i(t)\|^2} \\ & - a \text{Sign}(e_i(t)) - K \frac{e_i(t)}{\|e_i(t)\|^2}. \end{aligned} \quad (22)$$

Theorem 1. Suppose that (A_1) , (A_2) , and (A_3) are satisfied, and the coupling interaction topology is undirected and connected. If the following conditions

$$\Theta = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} < 0, \quad (23)$$

$$a - \Delta_{\max} - n\hat{\omega}\hat{b} - n\hat{\omega}\hat{a} > 0, \quad (24)$$

$$\Gamma - \hat{g}\hat{b} > 0, \quad (25)$$

$$\chi_2 - 2n\hat{g}\hat{a} > 0, \quad (26)$$

hold, where $\Phi_{11} = -2(I_N \otimes Q) + (2n\hat{\omega}\hat{a} + 1)(I_{nN}) - 2c_1(L \otimes I_n) + c_2(H \otimes I_n)(H \otimes I_n)^T$, $\Phi_{12} = \Phi_{21}^T = \hat{\gamma}\hat{b} \otimes I_n$, $\Phi_{21} = \hat{\gamma}\hat{b} \otimes I_n$, $\Phi_{22} = (c_2 - 1) \otimes I_n$, $\chi_2 = (\lambda_{\min}(\hat{L} + \hat{\Lambda}))^{(3/2)} \beta \tilde{n}^{(-1/2)}$, $\hat{K} = \chi_2 - 2n\hat{g}\hat{a}$, $\tilde{n} = nN^2$, then CNNs (1) are globally robust fixed-time synchronized with system (6) under the designed controller (20). And the upper bound $T_{\max}$ of settling time is estimated by

$$T_{\max} = \frac{3}{2K} \left(\frac{\hat{K}}{2K} \right)^{(2/3)}. \quad (27)$$

Proof. Construct the Lyapunov–Krasovskii functional

$$\mathcal{V}(t) = \sum_{i=1}^N \sum_{l=1}^n \int_{t-\tau}^t e_{il}^2(s) ds + \sum_{i=1}^N e_i^T(t) e_i(t). \quad (28)$$

Calculating the derivative of $\mathcal{V}(t)$ at time t along the trajectories of error system (9), it follows that

$$\begin{aligned} \dot{\mathcal{V}}(t) = & \sum_{i=1}^N 2e_i^T(t) \dot{e}_i(t) - \sum_{i=1}^N e_i^T(t-\tau) e_i(t-\tau) + \sum_{i=1}^N e_i^T(t) e_i(t) \\ = & \sum_{i=1}^N 2e_i^T(t) (-Qe_i(t) + A\tilde{f}(e_i(t)) + B\tilde{f}(e_i(t-\tau))) \\ & + c_1 \sum_{j=1}^N d_{ij} e_j(t) + \Delta_i(t, x_i(t)) + c_2 \sum_{j=1}^N h_{ij} e_j(t-\tau) \\ & + u_i(t) + \sum_{i=1}^N e_i^T(t) e_i(t) - \sum_{i=1}^N e_i^T(t-\tau) e_i(t-\tau) \\ = & 2 \sum_{i=1}^N e_i^T(t) A\tilde{\gamma}_i(t) - 2 \sum_{i=1}^N e_i^T(t) Qe_i(t) \\ & + 2 \sum_{i=1}^N e_i^T(t) B\tilde{\gamma}_i(t-\tau) + 2 \sum_{i=1}^N e_i^T(t) \Delta_i(t, x_i(t)) \\ & + 2c_2 \sum_{i=1}^N \sum_{j=1}^N h_{ij} e_i^T(t) (e_j(t-\tau) - e_i(t-\tau)) \\ & + 2c_1 \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) e_j(t) + 2 \sum_{i=1}^N e_i^T(t) u_i(t) \\ & - \sum_{i=1}^N e_i^T(t-\tau) e_i(t-\tau) + \sum_{i=1}^N e_i^T(t) e_i(t). \end{aligned} \quad (29)$$

Substituting (22) into (31), we can obtain

$$\begin{aligned}
\dot{\mathcal{V}}(t) = & -2 \sum_{i=1}^N e_i^T(t) Q e_i(t) + 2 \sum_{i=1}^N e_i^T(t) A \hat{\gamma}_i(t) + \sum_{i=1}^N e_i^T(t) e_i(t) \\
& + 2 \sum_{i=1}^N e_i^T(t) B \hat{\gamma}_i(t - \tau) + 2c_1 \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) (e_j(t) - e_i(t)) \\
& + 2c_2 \sum_{i=1}^N \sum_{j=1}^N h_{ij} e_i^T(t) (e_j(t - \tau) - e_i(t - \tau)) \\
& - 2 \sum_{i=1}^N e_i^T(t) \Gamma \|e_i(t - \tau)\|_1^2 \text{Sign}(e_i(t)) - \sum_{i=1}^N e_i^T(t - \tau) e_i(t - \tau) \\
& - 2a \sum_{i=1}^N e_i^T(t) \text{Sign}(e_i(t)) + 2 \sum_{i=1}^N e_i^T(t) \\
& \cdot \beta \sum_{j=1}^N d_{ij} |e_j(t) - e_i(t)|^2 \cdot \text{Sign}(e_j(t) - e_i(t)) \\
& + 2 \sum_{i=1}^N e_i^T(t) \Delta_i(t, x_i(t)) - 2\beta\tilde{\beta} \sum_{i=1}^N e_i^T(t) |e_i(t)|^2 \text{Sign}(e_i(t)) \\
& - 2K \sum_{i=1}^N e_i^T(t) \frac{e_i(t)}{\|e_i(t)\|^2} + 2\tilde{K} \sum_{i=1}^N e_i^T(t) \\
& \cdot \sum_{l=1}^n \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \frac{e_i(t)}{\|e_i(t)\|^2}.
\end{aligned} \tag{30}$$

Due to $d_{ij} = d_{ji}$, it follows that

$$\begin{aligned}
& \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) (e_j(t) - e_i(t)) \\
& = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) (e_j(t) - e_i(t)) \\
& \quad + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ji} e_j^T(t) (e_i(t) - e_j(t)) \\
& = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) (e_j(t) - e_i(t)) \\
& \quad - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_j^T(t) (e_j(t) - e_i(t)) \\
& = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ij} (e_j(t) - e_i(t))^T (e_j(t) - e_i(t)) \\
& = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} (e_{jl}(t) - e_{il}(t))^2 \\
& = -e^T(t) (L \otimes I_n) e(t).
\end{aligned} \tag{31}$$

It is easy to derive that

$$\begin{aligned}
& 2c_2 \sum_{i=1}^N \sum_{j=1}^N h_{ij} e_i^T(t) (e_j(t - \tau) - e_i(t - \tau)) \\
& = 2c_2 e^T(t) (H \otimes I_n) e(t - \tau) \\
& \leq c_2 e^T(t) (H \otimes I_n) (H \otimes I_n)^T e(t) + c_2 e^T(t - \tau) e(t - \tau).
\end{aligned} \tag{32}$$

Similar to (31), one obtains

$$\begin{aligned}
& \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) |e_j(t) - e_i(t)|^2 \text{Sign}(e_j(t) - e_i(t)) \\
& = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ij} e_i^T(t) |e_j(t) - e_i(t)|^2 \text{Sign}(e_j(t) - e_i(t)) \\
& \quad + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ji} e_j^T(t) |e_j(t) - e_i(t)|^2 \text{Sign}(e_i(t) - e_j(t)) \\
& = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ji} e_i^T(t) |e_j(t) - e_i(t)|^2 \text{Sign}(e_j(t) - e_i(t)) \\
& \quad - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N d_{ji} e_j^T(t) |e_j(t) - e_i(t)|^2 \text{Sign}(e_j(t) - e_i(t)) \\
& = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} |e_{jl}(t) - e_{il}(t)|^3.
\end{aligned} \tag{33}$$

By means of Assumptions (A₁), (A₂), and (A₃), we get

$$\begin{aligned}
\sum_{i=1}^N e_i^T(t) A \hat{\gamma}_i(t) & = \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n e_{il}(t) a_{lr} \tilde{\gamma}_{lr}(t) \\
& \leq \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n |e_{il}(t)| \|a_{lr}\| |\tilde{\gamma}_{lr}(t)| \\
& \leq \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n |e_{il}(t)| |a_{lr}| (|g_r| |e_{ir}(t)|^2 + \nu_r |e_{ir}(t)| + \omega_r) \\
& \leq n\hat{\gamma}\hat{a} \sum_{i=1}^N e_i^T(t) e_i(t) + n\hat{g}\hat{a} \sum_{i=1}^N e_i^T(t) e_i^2(t) + n\hat{\omega}\hat{a} \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)|.
\end{aligned} \tag{34}$$

By Assumption (A₂), one has

$$\begin{aligned}
& \sum_{i=1}^N e_i^T(t) B \hat{\gamma}_i(t-\tau) \\
&= \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n e_{il}(t) b_{lr} \tilde{\gamma}_{ir}(t-\tau) \\
&\leq \hat{b} \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n |e_{il}(t)| (\tilde{\gamma}_{ir}(t-\tau)) \\
&\leq \hat{b} \sum_{i=1}^N \sum_{r=1}^n \sum_{l=1}^n |e_{il}(t)| (|g_r| e_{ir}(t-\tau)^2 + \nu_r |e_{ir}(t-\tau)| + \omega_r) \\
&\leq \hat{g} \hat{b} \sum_{i=1}^N |e_i(t)|^T |e_i^2(t-\tau)| + \hat{\nu} \hat{b} \sum_{i=1}^N |e_i(t)|^T |e_i(t-\tau)| \\
&\quad + n \hat{\omega} \hat{b} \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)|.
\end{aligned} \tag{35}$$

Under Assumption (A₃), combining (30) with (35), it follows that

$$\begin{aligned}
\dot{\mathcal{V}}(t) &\leq -2e^T(t) (I_N \otimes Q) e(t) \\
&\quad + 2n\hat{\alpha} \sum_{i=1}^N e_i^T(t) e_i(t) 2n\hat{g}\hat{a} \sum_{i=1}^N e_i^T(t) e_i^2(t) \\
&\quad + 2n\hat{\omega}\hat{a} \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| + 2n\hat{\omega}\hat{b} \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| \\
&\quad + 2\hat{\nu}\hat{b} \sum_{i=1}^N \sum_{l=1}^n |e_i^T(t)| |e_i(t-\tau)| \\
&\quad - 2c_1 e^T(t) (L \otimes I_n) e(t) + 2\hat{g}\hat{b} \sum_{i=1}^N \|e_i(t)\|_1 \|e_i(t-\tau)\|_1^2 \\
&\quad + c_2 e^T(t) (H \otimes I_n) (H \otimes I_n)^T e(t) + c_2 e^T(t-\tau) e(t-\tau) \\
&\quad - 2 \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \chi_2 \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \frac{e_i(t)}{\|e_i(t)\|^2} \\
&\quad + 2\Delta_{\max} \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| \\
&\quad - 2\beta \sum_{i=1}^N \tilde{\beta}_i \sum_{l=1}^n |e_{il}(t)|^3 - 2 \sum_{i=1}^N e_i^T(t) K \frac{e_i(t)}{\|e_i(t)\|} \\
&\quad - \sum_{i=1}^N e_i^T(t-\tau) e_i(t-\tau) \\
&\quad + \sum_{i=1}^N e_i^T(t) e_i(t) - 2\Gamma \sum_{i=1}^N \|e_i^T(t)\|_1 \|e_i(t-\tau)\|_1^2 \\
&\quad - \beta \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} |e_{jl}(t) - e_{il}(t)|^3 - 2a \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| \\
&\quad + 2n\hat{g}\hat{a} \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)}.
\end{aligned} \tag{36}$$

The inequality above can be rewritten as

$$\begin{aligned}
\dot{\mathcal{V}}(t) &\leq \eta \Theta \eta^T - (-2n\hat{\omega}\hat{a} - 2n\hat{\omega}\hat{b} + 2a - 2\Delta_{\max}) \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| \\
&\quad + 2n\hat{g}\hat{a} \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \\
&\quad + 2n\hat{g}\hat{a} \sum_{i=1}^N e_i^T(t) e_i^2(t) \\
&\quad - (-2\hat{g}\hat{b} + 2\Gamma) \sum_{i=1}^N \|e_i(t)\| \|e_i(t-\tau)\|^2 \\
&\quad - 2\chi_2 \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \\
&\quad - \beta \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} |e_{jl}(t) - e_{il}(t)|^3 \\
&\quad - 2\beta \sum_{i=1}^N \tilde{\beta}_i \sum_{l=1}^n |e_{il}(t)|^3 - 2K,
\end{aligned} \tag{37}$$

where $\eta = [e_i(t), e_i(t-\tau)]^T$.

Together (24)–(26) and (37), one has

$$\begin{aligned}
\dot{\mathcal{V}}(t) &\leq 2n\hat{g}\hat{a} \sum_{i=1}^N e_i^T(t) e_i^2(t) \\
&\quad + 2n\hat{g}\hat{a} \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)} \\
&\quad - \beta \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} |e_{jl}(t) - e_{il}(t)|^3 \\
&\quad - 2\beta \sum_{i=1}^N \tilde{\beta}_i \sum_{l=1}^n |e_{il}(t)|^3 - 2K \\
&\quad - 2\chi_2 \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(3/2)}.
\end{aligned} \tag{38}$$

Using Lemma 4 yields

$$\sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| \geq \left(\sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)|^2 \right)^{(1/2)} = (e^T(t) e(t))^{(1/2)}. \tag{39}$$

By Lemma 3 and (18), one has

$$\begin{aligned}
& -\beta \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n d_{ij} |e_{jl}(t) - e_{il}(t)|^3 - 2\beta \sum_{i=1}^N \tilde{\beta}_i \sum_{l=1}^n |e_{il}(t)|^3 \\
& \leq -\beta \bar{n}^{-(1/2)} \left(\sum_{l=1}^n \left(\sum_{i=1}^N \sum_{j=1}^N d_{ij}^{(2/3)} |e_{jl}(t) - e_{il}(t)|^2 + \sum_{i=1}^N (2\tilde{\beta}_i)^{(2/3)} |e_{il}(t)|^2 \right) \right)^{(3/2)} \\
& = -\beta \bar{n}^{-(1/2)} \left(\sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^n \frac{1}{2} \tilde{d}_{ij} (e_{jl}(t) - e_{il}(t))^2 + \sum_{l=1}^n \sum_{i=1}^N (2\tilde{\beta}_i)^{(2/3)} |e_{il}(t)|^2 \right)^{(3/2)} \\
& = -\beta \bar{n}^{-(1/2)} \left(e^T(t) (\tilde{L} \otimes I) e(t) + e^T(t) (\tilde{\Lambda} \otimes I) e(t) \right)^{(3/2)} = -\beta \bar{n}^{-(1/2)} \left(e^T(t) ((\tilde{\Lambda} + \tilde{L}) \otimes I) e(t) \right)^{(3/2)},
\end{aligned} \tag{40}$$

where m is the real number nodes of $\tilde{\beta}_i$, $\bar{n} = n(N(N-1) + m)$, and $m_{\max} = N$. Then, $\bar{n} = n(N(N-1) + m) = nN^2 \geq \bar{n}$. Combining (39) with (40), we have

$$\dot{\mathcal{V}}(t) \leq -(-2n\hat{g}\hat{a} + \beta \bar{n}^{-(1/2)} (\lambda_{\min}(\tilde{\Lambda} + \tilde{L}) \otimes I)^{(3/2)}) \mathcal{V}^{(3/2)}(t) - 2K. \tag{41}$$

Noting that $\tilde{\Lambda} + \tilde{L}$ is positive definite, we can get

$$\dot{\mathcal{V}}(t) \leq -(\hat{K} \mathcal{V}^{(3/2)}(t) + 2K), \tag{42}$$

where $\hat{K} = -2n\hat{g}\hat{a} + \beta \bar{n}^{-(1/2)} (\lambda_{\min}(\tilde{\Lambda} + \tilde{L}) \otimes I)^{(3/2)} > 0$.

By Lemma 3, we can conclude that error system (9) is globally robust fixed-time stable. This shows that system (1) can achieve the global robust fixed-time synchronization with system (6) under the controller (20). The upper bound $T_{\max}$ of settling time is estimated by $T_{\max} = (3/2K)(\hat{K}/2K)^{(2/3)}$. The proof is completed. $\square$

Remark 1. In [30, 33–36], the global fixed-time synchronization issues were considered for delayed CNNs with discontinuous activations, where activation function $f(\cdot)$ is

subject to linear growth. However, in Theorem 1, discontinuous activations $f(\cdot)$ are nonlinear growth and subject to a quadratic polynomial function. In addition, in [37], the Lyapunov function $\mathcal{V}(t) = \sum_{i=1}^N e_i^T(t) e_i(t)$ is used to achieve the global fixed-time synchronization conditions. In this paper, the integral item $\sum_{i=1}^N \sum_{l=1}^n \int_{t-\tau}^t e_{il}^2(s) ds$ is introduced in Lyapunov functional (28). Obviously, compared with the above works in [30, 33–36], the result in this paper is more general.

Remark 2. It should be pointed out that the upper bound of the settling time, $T_{\max} = (3/2K)(\hat{K}/2K)^{(2/3)}$, is independent on initial conditions. In addition, it is easy to see that, on the basis of the configuration for parameters K in the proposed control law, the upper bound $T_{\max}$ can be determined in advance to any desired values.

In the designed controller (20), the integral item $\sum_{l=1}^n (\int_{t-\tau}^t e_{il}^2(s) ds)^{(3/2)}$ is used, which may bring difficulties in implement. We remove this integral item and design the following controller:

$$\begin{aligned}
u_i(t) = & -\Gamma \|e_i(t-\tau)\|_1 \text{sign}(e_i(t)) + \beta \sum_{j=1}^N d_{ij} \text{sig}^2(e_j(t) - e_i(t)) - \beta \tilde{\beta} \text{sig}^2(e_i(t)) \\
& - a \text{sign}(e_i(t)) - K \frac{e_i(t)}{\|e_i(t)\|^2} - \Gamma_2 \|e_i(t-\tau)\|_1 \text{sign}(e_i(t)).
\end{aligned} \tag{43}$$

Applying controller (43), we can obtain the following result.

Corollary 1. Suppose that Assumptions (A_1) , (A_2) , and (A_3) are satisfied and couple interaction topology is connected and undirected. If the following conditions

$$\begin{aligned}
\Theta_1 = & \begin{pmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{pmatrix} < 0, \\
a - \Delta_{\max} - n\hat{\omega}\hat{b} - n\hat{\omega}\hat{a} & > 0, \\
\Gamma - \hat{g}\hat{b} & > 0,
\end{aligned} \tag{44}$$

hold, where $\Psi_{11} = -2(I_N \otimes Q) + 2n\hat{w}\hat{a}(I_{nN}) - 2c_1(L \otimes I_n) + c_2(H \otimes I_n)(H \otimes I_n)^T + (I_N \otimes I_n)\Psi_{12} = \Phi_{21} = n\hat{w}\hat{b}I_{nN}$, $\Psi_{21} = n\hat{w}\hat{b} \otimes I_n$, $\Psi_{22} = c_2I_{nN}$, $\chi_2 = (\lambda_{\min}(\tilde{L} + \tilde{\Lambda}))^{(3/2)}\beta\tilde{n}^{(-1/2)}$, $\tilde{n} = nN^2$, then CNNs (1) are globally robust fixed-time synchronized with system (6) under controller (43). Also, the settling time is estimated by

$$T_{\max} = \frac{3}{2\Gamma_2} \left(\frac{\chi_2}{2\Gamma_2} \right)^{(2/3)}. \quad (45)$$

Proof. Consider $\mathcal{V}(t) = \sum_{i=1}^N e_i^T(t)e_i(t)$ and follow the similar proof process of Theorem 1, and Corollary 1 can easily be proved, hence omitted here.

Next, we developed the global robust finite-time synchronization conditions for CNNs (1). Design the following controller:

$$\begin{aligned} u_i(t) = & -\Gamma \|e_i(t-\tau)\|_1^2 \text{sign}(e_i(t)) - \Pi \|e_i(t)\|_1^2 \text{sign}(e_i(t)) \\ & - a \text{sign}(e_i(t)) + \hat{K} \sum_{l=1}^n \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(1/2)} \frac{e_i(t)}{\|e_i(t)\|^2}, \end{aligned} \quad (46)$$

where $\Gamma > 0$, $\Pi > 0$, $a > 0$, $\hat{K} > 0$, and $\text{sign}(e_i(t)) = \text{diag}\{\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{in}(t))\}$. $\square$

Theorem 2. Suppose that Assumptions (A_1) , (A_2) , and (A_3) are satisfied and couple interaction topology is connected and undirected. If the following conditions

$$\Theta_2 = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} < 0, \quad (47)$$

$$\Pi - n\hat{g}\hat{a} > 0, \quad (48)$$

$$2\Gamma - 2\hat{g}\hat{b} > 0, \quad (49)$$

hold, where $\Phi_{11} = -2(I_N \otimes Q) + (2n\hat{w}\hat{a} + 1)I_{nN} - 2c_1(L \otimes I_n) + c_2(H \otimes I_n)(H \otimes I_n)^T$, $\Phi_{12} = \Phi_{21} = n\hat{w}\hat{b} \otimes I_n$, $\Phi_{22} = (c_2 - 1)I_{nN}$, $\chi_1 = -n\hat{w}\hat{a} - n\hat{w}\hat{b} + a - \Delta_{\max}$, $\mathcal{V}(0) = \sum_{i=1}^N \|e_i(0)\|_1$, then CNNs (1) are globally robust finite-time synchronized under the controller (46). And the setting time is estimated by

$$t_1(\tilde{\varphi}_i(s), \tilde{\varphi}_i(s)) = \frac{2V^{(1/2)}(0)}{\chi_1}. \quad (50)$$

Proof. Consider Lyapunov–Krasovskii functional (28). Analogous as the proof of Theorem 1, we can get

$$\begin{aligned} \dot{\mathcal{V}}(t) \leq & -(-2n\hat{w}\hat{a} - 2n\hat{w}\hat{b} + 2a - 2\Delta_{\max}) \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)| + \eta \Theta_2 \eta^T \\ & + 2\chi_1 \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(1/2)} \\ & - (\Pi - 2n\hat{g}\hat{a}) \sum_{i=1}^N |e_i^T(t)| |e_i(t)|^2 \\ & - (-2\hat{g}\hat{b} + 2\Gamma) \sum_{i=1}^N \|e_i(t)\| \|e_i(t-\tau)\|^2, \end{aligned} \quad (51)$$

where $\eta = [e_i(t), e_i(t-\tau)]^T$.

By (47)–(49) and (51), it follows that

$$\begin{aligned} \dot{\mathcal{V}}(t) \leq & 2\chi_1 \sum_{i=1}^N \sum_{l=1}^n e_i^T(t) \left(\int_{t-\tau}^t e_{il}^2(s) ds \right)^{(1/2)} \\ & - (-2n\hat{w}\hat{a} - 2n\hat{w}\hat{b} + 2a - 2\Delta_{\max}) \sum_{i=1}^N \sum_{l=1}^n |e_{il}(t)|. \end{aligned} \quad (52)$$

Furthermore, we have

$$\dot{\mathcal{V}}(t) \leq \chi_1 \mathcal{V}^{(1/2)}(t). \quad (53)$$

Applying Lemma 2, we can conclude that error system (9) is globally robust finite-time stable. This shows that system (1) can achieve the global robust finite-time synchronization with system (6) under the controller (46). The settling time is estimated by $t_1(\tilde{\varphi}_i(s), \tilde{\varphi}_i(s)) = ((2\mathcal{V}^{(1/2)}(0))/\chi_1)$. The proof is completed. $\square$

4. Numerical Examples

Example 1. Consider an array CNNs (1) with five NNs, in which the dynamical equation of each network is described by

$$\begin{aligned} \dot{x}_{il}(t) = & -Qx_{il}(t) + Af(x_{il}(t)) + Bf(x_{il}(t-\tau)) + \Delta_{il}(t, x_i(t)) \\ & + c_1 \sum_{j=1}^5 d_{ij}(x_{jl}(t) - x_{il}(t)) \\ & + c_2 \sum_{j=1}^5 h_{ij}(x_{jl}(t-\tau) - x_{il}(t-\tau)) \\ & + J(t) + u_i(t), \quad l = 1, 2, i = 1, \dots, 5. \end{aligned} \quad (54)$$

The corresponding isolated network (6) is given by

$$\dot{y}(t) = -Qy(t) + Af(y(t)) + Bf(y(t-\tau)) + J(t), \quad (55)$$

FIGURE 1: State trajectories of variables $x_{i1}(t)$ and $y_1(t)$, $i = 1, \dots, 5$.

where $A = \begin{pmatrix} 0.12 & 0.3 \\ 0.44 & 0.5 \end{pmatrix}$, $B = \begin{pmatrix} 0.2 & 0.3 \\ -0.36 & -0.06 \end{pmatrix}$, and

$$Q = \begin{pmatrix} 5.51 & 0 \\ 0 & 6.91 \end{pmatrix}. \quad \text{Take } D = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 & 0 \\ -1 & 0 & -1 & 3 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{pmatrix}, \quad \text{and } H = \begin{pmatrix} 1 & -1 & 0 & -1 & -2 \\ -1 & 1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 1 \\ -1 & -1 & 0 & 1 & -1 \\ -2 & 0 & -1 & -1 & 1 \end{pmatrix}.$$

The initial values are selected as $\tilde{\varphi}_1(t) = (2t^2 + 1, 2\cos 2t + 2)^T$, $\tilde{\varphi}_2(t) = (-2t^2 + 1, 2\sin 2t + 3)^T$, $\tilde{\varphi}_3(t) = (0.2\sin 2t + 1, 0.1e^{3t} + e^t)^T$, $\tilde{\varphi}_4(t) = (2\sin 2t - 0.5, e^t)^T$, and $\tilde{\varphi}_5(t) = (2\cos 2t + 1, 0.1e^{3t} + e^t)^T$. The initial condition of the corresponding isolated network is $\tilde{\psi}(t) = (2\sin 2t, 2\sin t)^T$. Take activation function f as $f(x_{il}(t)) = 0.001x_{il}^2(t) + 0.05\text{sign}(x_{il}(t))$, $i = 1, 2, \dots, 5$, where $x_{il}(t) \in \hat{D}$, $\hat{D} = [-10, 10] \times [-10, 10]$, then $\hat{g} = 0.001$, $\hat{\nu} = 0$, $\hat{\omega} = 0.45$.

$$\Delta(x_i(t)) = (0.7 \sin(x_{i1}(t)), 0.6 \sin(x_{i2}(t)))^T, \quad (56)$$

$$i = 1, 2, \dots, 5, \Delta_{\max} = \sqrt{0.85}.$$

Set $\tau = 1$, $c_1 = 0.4$, $c_2 = 1$, $a = 4.15$, $\tilde{\beta} = \text{diag}\{3.14, 5.19, 0, 0, 0\}$, $K = 0.25$, $\beta = 1.412$, and $\Gamma = 5.863$. By simple calculation, $a - \Delta_{\max} - n\hat{\omega}\hat{b} - n\hat{\omega}\hat{a} = 2.544 > 0$, $\Gamma - \hat{g}\hat{b} = 5.862 > 0$, $\chi_2 = \lambda_{\min}(\tilde{L} + \tilde{\Lambda})^{(3/2)}\beta\tilde{n}^{(-1/2)} = 0.636$, and $\hat{K} = \chi_2 - 2n\hat{g}\hat{a} = 0.6344 > 0$.

Then, we calculate $T_{\max} = 7.032s$. Moreover, it is easy to verify that condition (23) in Theorem 1 is satisfied. As shown in Figures 1–3, the simulation results agree well with the theoretical analysis. Specifically, Figures 1 and 2 depict the state variables of system (54) and (55). Figure 3 shows the

evolutions of e_{ij} , $i, j = 1, 2, \dots, 5$. From Figure 3, it is easy to view that systems (54) and (55) can achieve the global robust synchronization in fixed time $T_{\max} = 1.2s$ under controller (20).

Example 2. Consider an array delayed CNNs (1) with three NNs, in which the dynamical equation of each network described by

$$\begin{aligned} \dot{x}_{il}(t) = & -Qx_{il}(t) + Af(x_{il}(t)) + Bf(x_{il}(t - \tau)) + \Delta_{il}(t, x_i(t)) \\ & + c_1 \sum_{j=1}^3 d_{ij}(x_{jl}(t) - x_{il}(t)) \\ & + c_2 \sum_{j=1}^3 h_{ij}(x_{jl}(t - \tau) - x_{il}(t - \tau)) \\ & + J(t) + u_i(t), \quad l = 1, 2, 3, i = 1, 2, 3. \end{aligned} \quad (57)$$

The corresponding isolated network (6) is given by

$$\dot{y}(t) = -Qy(t) + Af(y(t)) + Bf(y(t - \tau)) + J(t), \quad (58)$$

where $A = \begin{pmatrix} 0.6 & 0.11 & -0.1 \\ 0.4 & 0.65 & 1 \\ 1 & 0.7 & 0.4 \end{pmatrix}$, $B = \begin{pmatrix} 0.2 & 0.3 & -0.19 \\ -0.26 & 0.06 & 0.03 \\ 0.3 & 0.1 & 0.1 \end{pmatrix}$, and $Q = \begin{pmatrix} 5.1 & 0 & 0 \\ 0 & 2.2 & 0 \\ 0 & 0 & 2.01 \end{pmatrix}$. Take $D = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $L = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$, and $H = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$. The initial values are selected as $\tilde{\varphi}_1(t) = (2t^2 + 1, 2.1\cos 2t + 2, 2.5t^2 + 1)^T$, $\tilde{\varphi}_2(t) = (-2t^2 + 1, 2.5t + 3, 2\sin t + 1)^T$, and $\tilde{\varphi}_3(t) = (0.2\sin 2t + 1, 0.1e^{3t} + e^t, e^{3t})^T$. The trivial point $y(t) = 0$ is taken as the objective trajectory.

FIGURE 2: State trajectories of variables $x_{i2}(t)$ and $y_2(t), i = 1, \dots, 5$.

FIGURE 3: Time evolutions of $e_{ij}, i, j = 1, \dots, 5$ under the controller (20).

We take the activation functions f as $f(x_{il}(t)) = 0.01x_{il}^2(t) + 0.3\text{sign}(x_{il}(t))$, $i = 1, 2, 3$, where $x_{il}(t) \in \hat{D}$, $\hat{D} = [-10, 10] \times [-10, 10]$, then $\hat{g} = 0.01, \hat{\nu} = 0, \hat{\omega} = 0.5$.

Let $\Delta(x_i(t)) = (0.2 \sin(x_{i1}(t)), 0.3 \sin(x_{i2}(t)), 0.4 \sin(x_{i3}(t)))^T, i = 1, 2, 3$, $\Delta_{\max} = \sqrt{0.29}$. Set $\tau = 1$, $c_1 = 0.04$, $c_2 = 1.8$, $a = 3.01$, $\Pi = 0.48$, $\hat{K} = 1.121$, and $\Gamma = 0.42$. By simple computation, $\Pi - n\hat{g}\hat{a} = 0.45 > 0$, and $\Gamma - \hat{g}\hat{b} = 0.414 > 0$.

From (50), $\chi_1 = -n\hat{\omega}\hat{a} - n\hat{\omega}\hat{b} + a - \Delta_{\max} = 0.522$. Then, we calculate $t_1(\bar{\varphi}_i(s), \bar{\phi}_i(s)) = 21.979s$. Moreover, it is easy to verify that condition (50) in Theorem 2 also satisfied. As shown in Figure 4, the simulation results agree well with the theoretical analysis. Figure 4 shows the evolutions of $e_{ij}, i, j = 1, 2, 3$, and it is obvious to view that systems (57) and

FIGURE 4: Time response of synchronization errors $\|e_i\|, i = 1, 2, 3$ under the controller (46).

(58) can achieve THE global robust finite-time synchronization before $T = 3.8s$ under controller (46).

5. Conclusion

In this paper, the global robust synchronization in fixed time and global synchronization in finite time have been investigated for a class of hybrid coupled delayed NNs with discontinuous activation functions. Under the designed discontinuous feedback controller, the global synchronization conditions have been presented in the forms of LMIs, and the settling time, which is independent on initial conditions, has been also evaluated. Compared with the existing works, where the neuron activations are supposed to be linear growth, the results proposed in this paper are more general.

It is worth noting that the designed feedback controllers contain the sign function and the integral term, which may bring the chatter in actual implement, indicating that the designed control schemes have certain limitation.

Future work will be focused on how to remove the chatter of the designed controller and to extend the results here obtained for stochastic sampled-date synchronization control for CNNs with delays and discontinuous activations.

Data Availability

The underlying data supporting the results of our study can be found in the original paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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