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Research Article

Compensation Method for Inclination Errors in Measurement Results of Tooth Surface of Spiral Bevel Gear

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The manufacturing error of spiral bevel gear tooth surface has a great influence on transmission efficiency and gear life. The error of the gear tooth surface needs to be measured accurately and fed back to the machine tool to adjust the parameters. When measuring the spiral bevel gear using a gear measuring machine, combined with the measurement theory of the tooth flank of spiral bevel gear, this paper proposed a method to compensate the inclination error in the measurement result precisely. Based on the iterative search method, a precision matching method for the theoretical and the measured tooth surface of the spiral bevel gear was designed to calculate the compensation results. The experimental results show that the inclination errors included in tooth surface measurement results reduced from more than 3 μm to less than 0.5 μm, and more than 70% of the errors are compensated by the proposed method. The accuracy of the measurement results improved significantly after compensation, and furthermore, it can provide a more accurate basis for the adjustment of machine tool parameters in the manufacturing process.

1. Introduction

Spiral bevel gear is widely used in the field of mechanical transmissions, such as automobile, aviation, wind turbines, and so on. When manufacturing the gear by machine tools, the manufacturing error affects the accuracy of the gear tooth surface and then directly affects the performance of gear transmission efficiency, noise, lifetime, and reliability [1–4]. In the closed-loop manufacturing system, in order to feedback and adjust the machine tool parameters, the gear tooth surface requires to improve its measuring accuracy. At present, the coordinate measurement method is one of the most widely used methods to measure the tooth surface error of the spiral bevel gear [5–7].

When measuring the tooth surface of spiral bevel gear by gear measuring machine (GMM), the manufacturing and assembly error of the gear is inevitable. The inclination error caused by the nonparallel between the gear axis and the vertical rotation axis of GMM will affect the measurement results of spiral bevel gear [8, 9]. Therefore, to improve the measurement accuracy of the spiral bevel gear tooth surface, first of all, the measured tooth surface of the gear and its theoretical tooth surface must be accurately matched to eliminate the inclination error in the measurement result of the gear tooth surface [10].

Many works have been done on the measurement error and the compensation method of spiral bevel gear tooth surface. In order to predict the relative errors of alignment between spiral bevel gears, Fuentes et al. proposed a procedure of determination of the relative spatial position of spiral bevel gear supporting shafts during torque transmission [11–13]. The misalignment problem for the spiral bevel and hypoid gears usually exists in the actual manufacturing and transmission, and Ding et al. presented an automatic data-driven operation and optimization to determine the uncertain misalignment [14]. Considering residual tooth flank form error, Shao et al. presented an accurate systematic CMM measurement method to prescribe and data-driven control the tooth flank form error and get a flexible compensation of the error [15]. Fu et al. proposed a new online detection method of flatness of spiral bevel gears, and the data of flatness is evaluated by the least square method after error separation [16]. When manufacturing the spiral bevel gears, Shih and You developed an on-machine measurement system on five-axis machines using a quasi-3D probe and verified by comparing the evaluation results with the same gear measured on a dedicated GMM [17]. By using the virtual conjugate reference tooth surface, Li et al. presented a novel scanning measurement method to measure the tooth flank form of hypoid gears [18, 19]. Wang et al. established the on-machine measuring and the data processing method for spiral bevel gears, and the method was validated by comparing simulation results with the actual measuring result [20, 21]. Simon researched the influence of misalignments of the mating members and tooth errors on mesh performances of the spiral bevel gear and found that the misalignments and the tooth spacing error could increase the angular position error of the driven gear [22]. By considering spiral bevel gear geometry, Shunmugam reported a method of determining the normal deviation from the theoretical surface of the bevel gear and then gave the validation of the method [23]. Peng et al. researched the effects of eccentricity error on the gear dynamic responses and proposed a simple method using translational kinematic transmission error modification to reduce the computational time [24]. Using the truncated singular value decomposition and the L curve method, Chen and Yan solved the identification equation of the tooth surface deviation and compared it with the least square method to verify the validation of this method [25]. Xie et al. presented a coordinate measuring method for the variable ratio noncircular bevel gear by coordinate measuring machine [26]. Combined with the theoretical gear and coordinate measuring machine, Cao and Deng provided a method for measuring the errors between the actual surface and the theoretical surface of the bevel gear and then developed the corrective machine setting [27]. Zhou et al. investigated a higher-accuracy fitting method for the form error tooth flank, and the universal machine tool settings are exploited for the identification of the real tooth flank form error [28]. Mo and Zhang gained the corresponding digitized true tooth surface by changing the machining adjustment parameters, which can lay a solid basis for the subsequent finite element analysis of gear contact and transmission error analysis [29]. However, few works have been focused on the measurement error of the spiral bevel gear caused by the inclination error, so it is necessary to research the compensation method for the inclination errors.

Combined with the measurement theory of the tooth flank of spiral bevel gear, this paper proposed a precision matching method for the theoretical tooth surface and the measured tooth surface of the spiral bevel gear, and then compensated the inclination error in the measurement result precisely. The experimental results show that the accuracy of the measurement results improved significantly after compensation.

2. Establishment of the Measuring Coordinate System

The design basis of spiral bevel gear is the apex of the pitch cone of the bevel gear, and its design coordinate system is set as $\{O: X, Y, Z\}$. When measuring spiral bevel gear with GMM, the inner end of the gear was fixed upward, and the outer end was fixed downward on the platform, and the measurement basis is the rotation axis of the gear and the outer end surface of the gear.

As shown in Figure 1, the measuring coordinate system of spiral bevel gear $\{O_m: X_m, Y_m, Z_m\}$ is set to coincide with the coordinate system $\{O: X, Y, Z\}$. According to the above coordinate systems, the position of the theoretical gear pitch cone apex in the Z-axis direction can be determined by the Z-axis coordinate value of the mounting platform and the mounting distance of the gear, and the coordinate relationship between the measured pitch cone apex and the theoretical pitch cone apex can be solved.

3. Precision Compensation Method for the Measurement Errors of Gear Tooth Surface

When measuring the tooth surface of spiral bevel gear by GMM, the theoretical data point cloud and the corresponding unit normal vector of the gear tooth surface measuring points should be input GMM first. According to the coordinate value of the theoretical data point cloud, the GMM measures the actual coordinates of the points on the tooth surface and finally obtains the actual measurement error. In order to compensate the inclination error of the rotating axis of the gear, matching the measured tooth surface with the theoretical tooth surface of the gear accurately is the first step.

The spatial relative positions of two curved surfaces can be represented by six relative position parameters of the corresponding points P and P_1: $\{\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z\}$. When matching the two curved surfaces, the above six parameters should be calculated first, and then one of the two surfaces translated and rotated according to the value of the six parameters to match the other surface in spatial. This paper mainly focuses on the translation deviations Δx and Δy along the X and Y axis, and the angle deviations $\Delta\theta_x$ and $\Delta\theta_y$ around the X and Y axis, and finally, find a compensation method for the above four parameters.

Assuming that the point on the measured tooth surface is recorded as point P_m (x_m, y_m, z_m), and the corresponding point on the theoretical tooth surface is named as point P_t (x_t, y_t, z_t). After the surface matching process, the measured tooth surface moves to a new position and the corresponding point of P_m is recorded as point P_c (x_c, y_c, z_c). The distance from the point P_m to the theoretical tooth surface is Δd, and the intersection between the normal vector of the theoretical tooth surface that passing through the point P_c and the theoretical tooth surface is point P_s (x_s, y_s, z_s). The objective function of the matching method is shown in the following:

FIGURE 1: Schematic diagram of measuring coordinate system of spiral bevel gear.

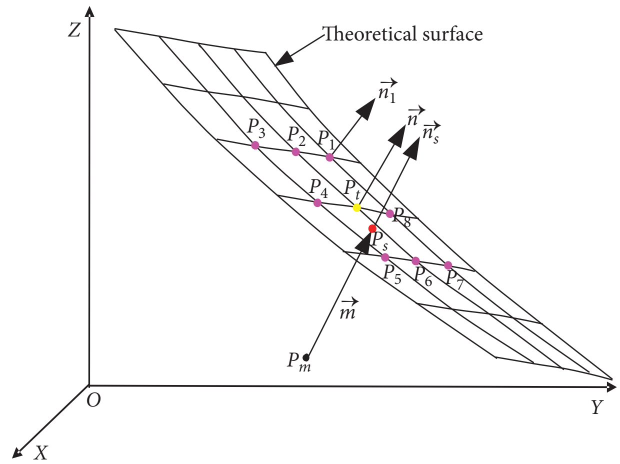

FIGURE 2: The search region for searching the point P_s.

$$\min F(\Delta x, \Delta y, \Delta\theta_x, \Delta\theta_y) = \min \sum_{i=1}^{m}\sum_{j=1}^{n}\left|\left(\vec{P}_s - \vec{P}_m\right)\right|^2 = \min \sum_{i=1}^{m}\sum_{j=1}^{n}\left|\Delta d_{ij}\right|^2. \quad (1)$$

Due to the inclination error, the position relationship between the measured tooth surface and the theoretical tooth surface is uncertain. Therefore, the position of the point P_s corresponding to the point P_m needs to be obtained by the search calculation method. As the error of the actual tooth surface is generally small, the position of the point P_s is considered nearby the point P_t, as shown in Figure 2.

If the corresponding normal vector $\vec{n}_s(n_{sx}, n_{sy}, n_{sz})$ of point P_s passes through the point P_m, the point P_s is the closest point from the point P_m to the theoretical tooth surface, and the cosine value of the angle between the vectors $\vec{m}$ and $\vec{n}_s$ is 1. In order to find the position of the nearest point P_s by searching, eight measured points P_i (x_i, y_i, z_i) $(i = 1\sim8)$ around the point P_t and their corresponding unit normal vectors $\vec{n}_i(n_{xi}, n_{yi}, n_{zi})$ are selected to form the search region, and the objective function is established as in equation (2). The position of the point P_s could be searched by iteratively calculating and comparing the value of the function f of the point P_i.

$$\min f(x_s, y_s, z_s) = \min\left(1 - \cos\langle\vec{m}, \vec{n}_s\rangle\right). \quad (2)$$

This paper researches the single direction matching method with the example of tooth surface matched by moving along the X-axis and rotating around the X-axis.

3.1. Matching Method for the Measured and Its Theoretical Tooth Surface in Single Direction. The iterative search calculation diagram is shown in Figure 3. The point P_m is taken as the origin of the coordinate system, the moving distance and rotating angle are taken as the X-axis and Y-axis, and the optimal matching point P_c of the two surfaces is searched by the iteration method in this figure.

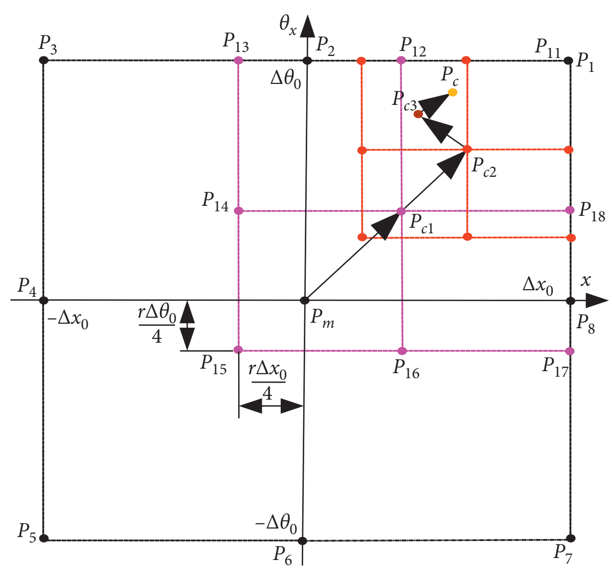

FIGURE 3: The iterative search calculation diagram.

First, set the initial search step Δx_0 and $\Delta\theta_0$ to ensure that the initial search region could enclose the point P_c. Set the matching parameters $\Delta x = 0$, and $\Delta\theta_x = 0$. The specific process is as follows:

(1) Move the point P_m with a distance Δx_0 along with the positive and negative directions of the X-axis, a distance $\Delta\theta_0$ along with the positive and negative directions of the θ_x axis, respectively. Together with the point P_m, nine new matched points are obtained and named $P_0\sim P_8$. According to equation (1), the objective function value F_i $(i = 0 - 8)$ could be calculated. Compare the value F_i $(i = 0 - 8)$, and the new search region is judged according to the minimum value. For example, if F_1 is the minimum value, the new search region is the first quadrant shown in Figure 3.

(2) The new search direction is determined according to the new search region, and the step size is reduced to

half of the original step, that is $\Delta x_0/2^k$ and $\Delta\theta_{x0}/2^k$. If the point P_c is in the first quadrant, the coefficients corresponding to the search direction are recorded as $\alpha_1 = 1$ and $\beta_1 = 1$. If the point P_c is in the second quadrant, the coefficients is recorded as $\alpha_1 = -1$ and $\beta_1 = 1$, and so on. In order to ensure that the optimization point is always in the search region, when a new search process is carried out, the search region is expanded with a distance of $r\Delta x_0/2^{k+1}$ and $r\Delta\theta_{x0}/2^{k+1}$, where r is the ratio. Generally, the value of r is 0.5. The new step size can be calculated by the following:

$$\begin{cases} \Delta x_1 = \Delta x_0 \cdot \left(2^{-k} + 2^{-(k+1)}r\right), \\ \Delta\theta_{x1} = \Delta\theta_{x0} \cdot \left(2^{-k} + 2^{-(k+1)}r\right). \end{cases} \quad (3)$$

(3) Along the search direction, move the point P_m with a distance $\Delta x_1'$ and $\Delta\theta_{x1}'$ to get a new matched point P_{c1}. The distance could be calculated by the following:

$$\begin{cases} \Delta x_1' = \alpha_1 \cdot \Delta x_0 \cdot \left(2^{-k} - 2^{-(k+1)}r\right), \\ \Delta\theta_{x1}' = \beta_1 \cdot \Delta\theta_{x0} \cdot \left(2^{-k} - 2^{-(k+1)}r\right). \end{cases} \quad (4)$$

Take the point P_{c1} as the center, and Δx_1 and $\Delta\theta_{x1}$ as the step size, repeat steps (1) and (2) to get the new search region, the search direction coefficients α_2 and β_2, the new step size Δx_2 and $\Delta\theta_{x2}$, and the moving distance $\Delta x_2'$ and $\Delta\theta_{x2}'$.

(4) After the k-th iteration, the moving distance of the matched point is calculated. Move the point according to the moving distance $\Delta x_k'$ and $\Delta\theta_{xk}'$ to get the new matched point P_{ck}, and then repeat step (1).

(5) After the k-th iteration, if the steps Δx_k along the X-axis and $\Delta\theta_{xk}$ around the X-axis are less than ε, the iteration search results meet the accuracy requirements, and the iteration ended.

After the iteration, the matched results of the two tooth surfaces could be obtained, and the compensation value for the tooth surface measurement errors of the spiral bevel gear could be calculated by the following:

$$\begin{cases} \Delta x' = \sum_{1}^{k} \alpha_k \cdot \Delta x_0 \cdot \left(2^{-k} - 2^{-(k+1)}r\right), \\ \Delta\theta_x' = \sum_{1}^{k} \beta_k \cdot \Delta\theta_{x0} \cdot \left(2^{-k} - 2^{-(k+1)}r\right). \end{cases} \quad (5)$$

The specific iteration process is shown in Figure 4.

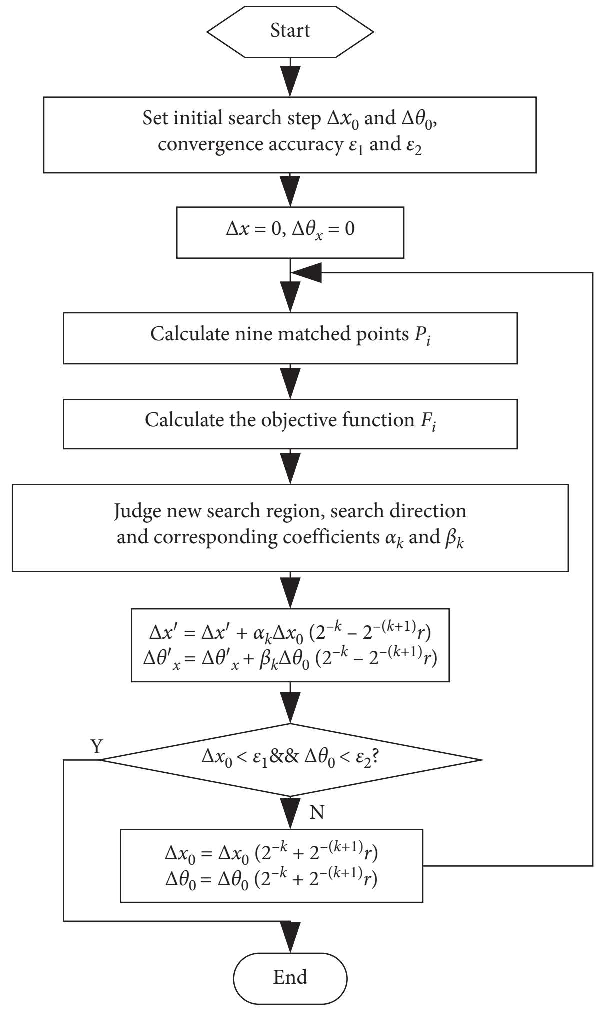

Figure 4: Flow chart of the iteration process.

3.2. Matching Method for the Measured and Its Theoretical Tooth Surface in Multiple Directions. In this iteration process, four parameters Δx, Δy, $\Delta\theta_x$ and $\Delta\theta_y$ should be calculated. In order to reduce the calculation work, the initial iteration (coarse matching) is carried out first, and then the accurate iteration (precision matching) in two directions is carried out. The specific process is shown in Figure 5.

The coarse matching process is as follows: According to the matching method for a single direction, match the tooth surface along and around X-axis for n times and then match the tooth surface along and around Y-axis for n times. Judge whether the tooth surface deviation is less than the end criterion of the iteration. If the deviation is less than the criterion, skip the cycle and turn it into the accurate iteration process; otherwise, continue the iteration.

The accurate iteration process is as follows: first, match the tooth surface along and around X-axis for 1 time according to the matching method for a single direction, and then match the surface along and around Y-axis for 1 time. Judge whether the deviation meets the end criterion of the iteration. If it does not meet the criterion, return and continue the iteration until the criterion is met, and then calculate the matching results of the tooth surface according to the matching method.

Finally, the compensation value of the tooth surface measurement errors of the spiral bevel gear in two directions could be calculated by equation (5).

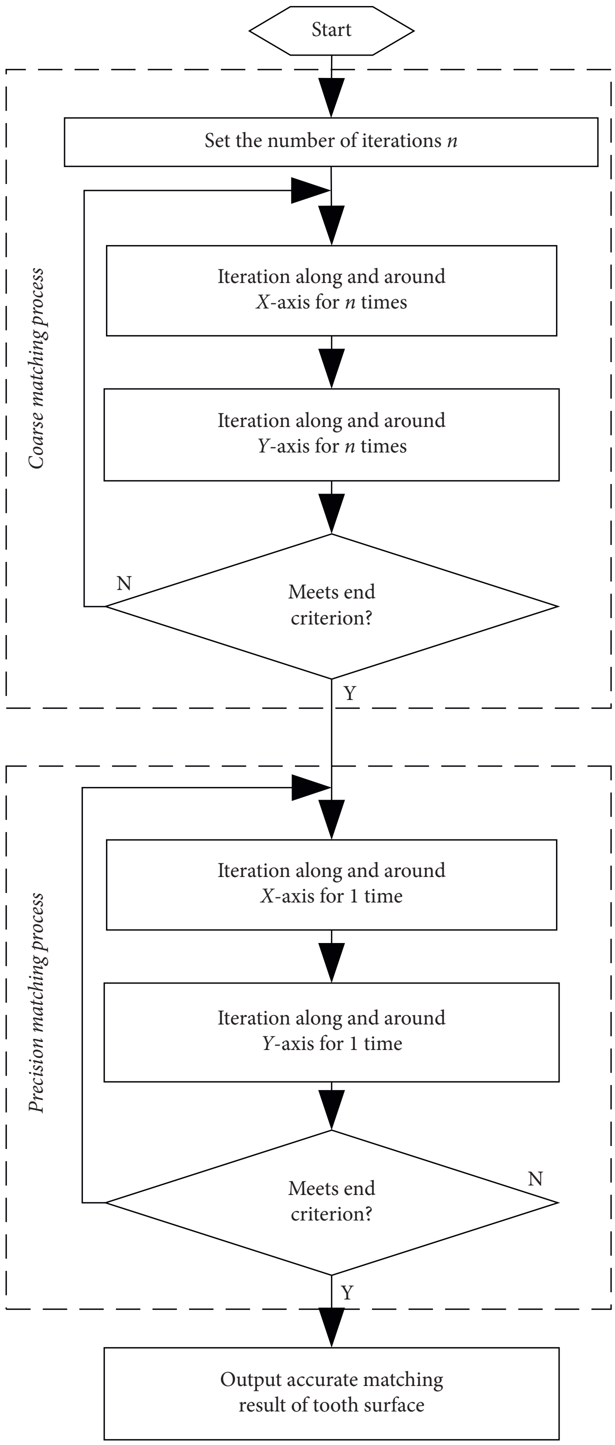

FIGURE 5: Flow chart of a precision matching method for multiple directions.

4. Experiment Results and Analysis of the Compensation Method

4.1. Basic Data of the Measured Gear. The parameters of the measured spiral bevel gear are shown in Table 1.

The theoretical points of the tooth surface are selected along three profile lines named PL7, PL15, and PL23, and one tooth trace line named TL5, as shown in Figure 6. Each line has 113 theoretical points. The theoretical data both for the convex and the concave surfaces contain the coordinates and unit normal vectors of these theoretical points. According to these theoretical data, actual measurement experiments are done and the measured tooth surface are the convex and concave surfaces of Tooth 1, Tooth 15, Tooth 29, and Tooth 43 of the gear. In the actual measurement experiment, some assembly errors are added artificially.

TABLE 1: The parameters of the measured spiral bevel gear.

Type	Gear
No. of teeth	56
Pitch diameter (mm)	72.800
Pressure angle (°)	20.000
Shaft angle (°)	90.000
Pitch angle (°)	75.967
Face angle (°)	76.567
Root angle (°)	73.867
Mounting distance (mm)	22.350

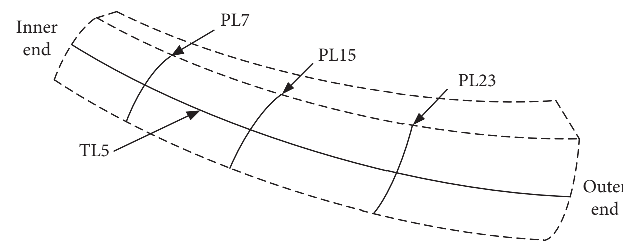

FIGURE 6: The theoretical points of the tooth surface of the spiral bevel gear.

According to the compensation method proposed in the paper, the point cloud of the measured tooth surface could be matched with the theoretical tooth surface, and the compensation values could also be calculated by equation (5). Compared with the errors of the measured value, the accuracy of the compensation could be calculated.

4.2. The Compensation Results and Analysis. Figure 7 shows the comparisons of the measurement errors of the measured points along the tooth trace line both on the concave and convex tooth surface of Tooth 1 of the gear before and after compensation. Figures 7(a) and 7(b) are the compensation results of the convex tooth surface in the X direction and Y direction, respectively, and Figures 7(c) and 7(d) are the compensation results of the concave tooth surface in the X direction and Y direction, respectively.

After matching and compensation between the measured tooth surface and the theoretical tooth surface, the average measurement errors of the tooth surface along the X-axis and Y-axis of all measured tooth surfaces are shown in Tables 2 and 3, and the corresponding figures of the average errors of four teeth of the gear along the X-axis and Y-axis before and after compensation are shown in Figures 8 and 9, respectively.

From Tables 2 and 3, Figures 8 and 9, it can be easily seen that, before the compensation process, the average errors of

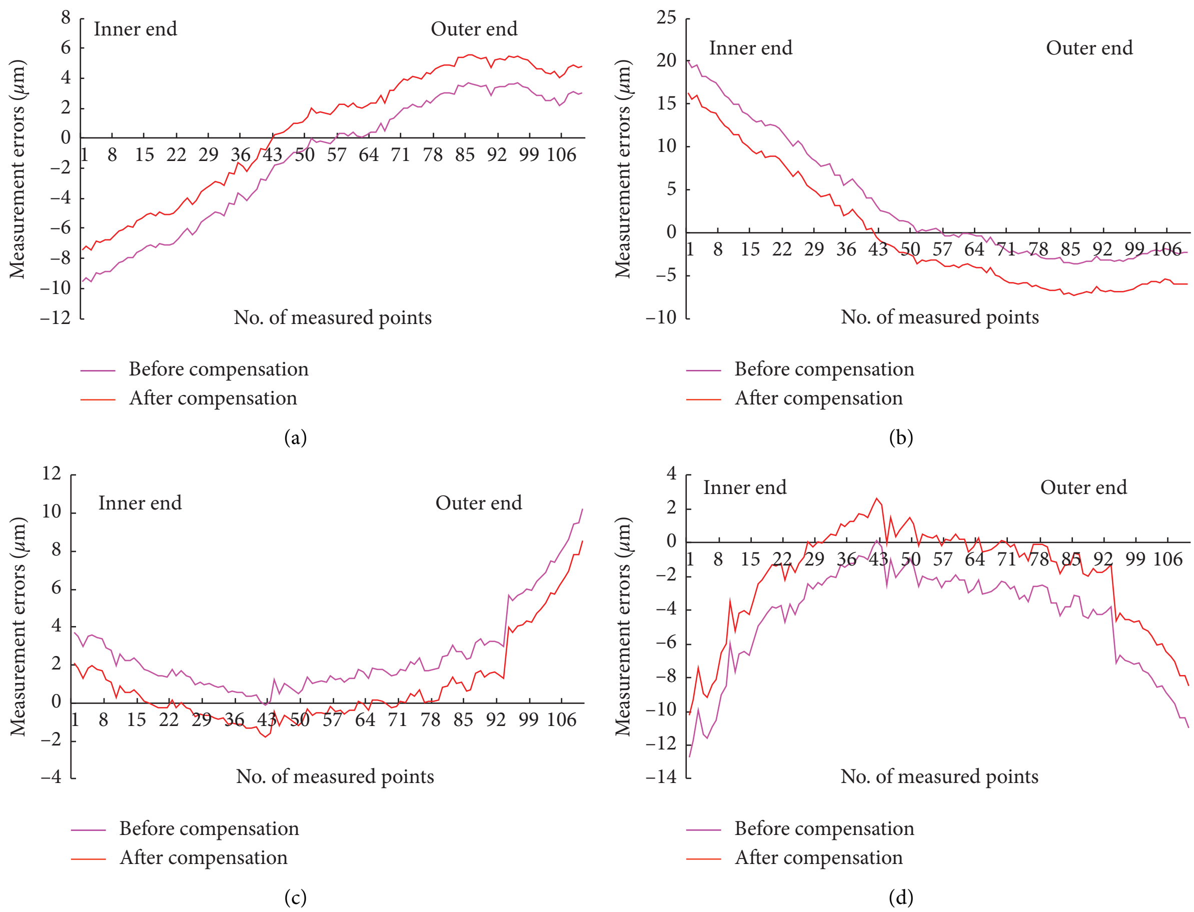

Figure 7: Comparisons of the measurement errors of the tooth trace line of tooth 1. (a) The convex flank along X direction. (b) The convex flank along Y direction. (c) The concave flank along X direction. (d) The concave flank along Y direction.

Table 2: Average errors of four tooth flanks before and after compensation (X direction).

No. of tooth surface		Errors before compensation (μm)	Compensation value (μm)	Errors after compensation (μm)	Compensation effects (%)
Convex surface	Tooth 1	−1.73	1.85	0.12	93.1
	Tooth 15	−0.011	0.012	0.001	88.5
	Tooth 29	−2.03	2.48	0.45	77.8
	Tooth 43	−1. 65	1.98	0.33	79.6
Concave surface	Tooth 1	1.72	−1.67	0.05	97.1
	Tooth 15	3.15	−3.62	−0.47	85.1
	Tooth 29	1.34	−1.14	0.20	85.4
	Tooth 43	1.69	−1.84	−0.15	90.9

Table 3: Average errors of four tooth flanks before and after compensation (*Y* direction).

No. of tooth surface		Errors before compensation (μm)	Compensation value (μm)	Errors after compensation (μm)	Compensation effects (%)
Convex surface	Tooth 1	3.27	−3.61	−0.34	89.6
	Tooth 15	1.68	−1.20	0.48	71.3
	Tooth 29	3.46	−3.40	0.06	98.4
	Tooth 43	2.64	−2.41	0.23	91.4
Concave surface	Tooth 1	−2.85	2.67	−0.18	93.5
	Tooth 15	−4.38	4.01	−0.37	91.5
	Tooth 29	−1.86	1.626	−0.24	87.2
	Tooth 43	−3.72	3.75	0.03	99.0

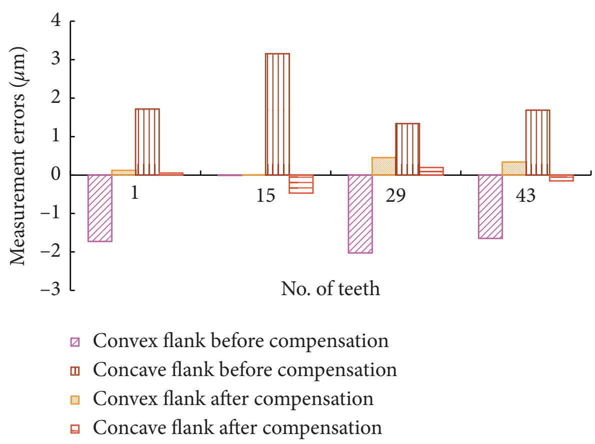

Figure 8: Average errors of all tooth flanks before and after compensation (*X* direction).

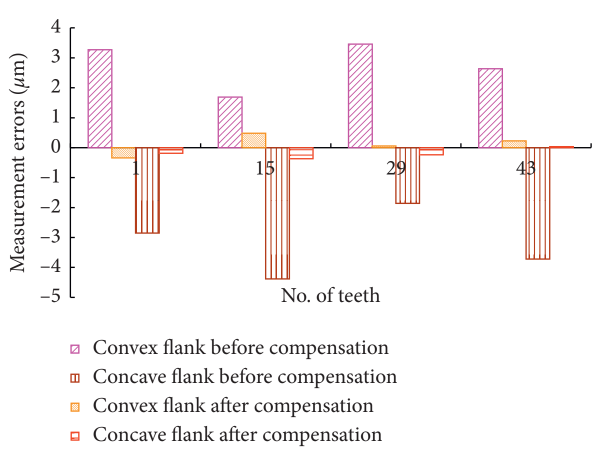

Figure 9: Average errors of all tooth flanks before and after compensation (*Y* direction).

some tooth flank are more than 3 μm. After matching and the compensation process, the average errors of the measurement results of all tooth surfaces are less than 0.5 μm, and more than 70% of the measurement errors are compensated by the proposed method.

5. Conclusions

In order to improve the accuracy of the measurement results of the spiral bevel gear tooth surface, this paper presents a precision compensation method for the inclination error of the gear tooth surface. Based on the iterative search method, this paper established an objective function to match the theoretical tooth surface and actual tooth surface along and around *X*-axis and *Y*-axis accurately and then compensated the measurement errors of the tooth surface according to the matching method.

The experimental results show that the errors included in the tooth surface measurement results along and around the *X*-axis, *Y*-axis are reduced from more than 3 μm to less than 0.5 μm, and more than 70% of the errors are compensated by the proposed compensation method. The results show that the measurement errors in the above directions can be eliminated obviously, and this method can provide a more accurate basis for the adjustment of machine tool parameters in the manufacturing process.

Data Availability

The processed data cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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