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Research Article

A New Optimal Guidance Law with Impact Time and Angle Constraints Based on Sequential Convex Programming

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This paper proposed an optimal time-varying proportional navigation guidance law based on sequential convex programming. The guidance law can achieve the desired impact angle and impact time with look angle and lateral acceleration constraints. By treating the multiconstraints' guidance problem as an optimization problem and changing the independent variable to linearize the problem and constraints, the original nonlinear and nonconvex problem is transformed into a series of convex optimization problem so that it can be quickly solved by sequential convex programming. Numerical simulations compared to nonlinear programming and traditional analytical guidance law demonstrate the effectiveness and efficiency of the proposed algorithm. Finally, the proposed guidance law is verified to satisfy different impact time periods and impact angle constraints.

1. Introduction

In modern warfare, the aims of guidance law are not only limited to zero miss distance interception but also required to intercept the target with a certain impact time and impact angle. Impact angle constraint at terminal interception engagement is critical for the homing missile to attack modern warships, tanks, and ballistic missiles, which can increase the effectiveness and lethality of the missile's warhead and escape the limited defense zone of the target. Impact time constraint is important to facilitate a salvo attack against the advanced close-in weapon system (CIWS). Salvo attacks with impact time and impact angle constraints simultaneously have the advantage to effectively destroy the target.

Guidance law considering impact angle and impact time constraints has been widely studied in past decades. Optimal control theory has been implemented to solve optimal guidance law under impact time and angle constraints [1–3]. Kim and Zhao have converted guidance law to a polynomial form with respect to the range-to-go, and three coefficients in the polynomial are designed to control the impact time, impact angle, and zero miss distance [4, 5], respectively. Besides, the errors of impact time and angle also have been selected as sliding surface to constraint terminal impact time and angle by sliding-mode control, whereas not optimal [6–8]. Switched proportional navigation (PN) guidance gain also has been derived to achieve the desired impact time and angle by Harrison [9]. Based on the switched proportional navigation guidance law, Hu [8] proposed a two-stage guidance law by setting a virtual target. In the first stage, nonsingular terminal sliding-mode guidance law is employed to intercept the virtual target with a specific impact angle; then, the proportional navigation guidance law is used to get the desired impact angle. The impact time and the virtual target position are designed by an optimization routine. The optimization process is too complicated to use onboard. To simplify the solving process, He [10] designed a simple decentralized midcourse guidance law for the first stage to get the desired impact time and impact angle for the pure proportional navigation in the second stage. Chen [11] proposed a two-stage impact time constraint guidance law with field-of-view constraint; however, the impact angle is out of control. Kim and Kim [12] proposed a guidance law based on the backstepping method to control impact time with reduced seeker field-of-view constraint.

Besides the previous traditional analytical form of guidance laws, the computational guidance law has been studied in [13–15]. In fields of plant entry [16, 17], spacecraft descent guidance [18, 19], rendezvous and proximity [20],orbit transfer problem [21], powered descent and landing [22], trajectory optimization [23–25], etc., convex programming has been verified that it can solve nonlinear optimization problem efficiently and accurately. Liu [26] has used convex programming to derive a time-varying PN guidance law for impact angle constraint. To control impact time, Jiang [27] has imported the impact time to kinematic equations so that it can be limited as the impact angle. However, the impact time is linked to the range-to-go, which will lead the impact time to be infinite at the end of optimization engagement and cause the problem infeasible. To overcome the inherent infeasible problem, the impact time in this paper is treated as a separated variable and independent with range-to-go. Different discretization strategies are carried out to avoid the infeasible problem. In summary, the main contributions of this paper can be concluded as follows:

(1) The multiconstrained guidance problem is transformed into a convex optimization problem. The differential motion equations, impact angle constraint, look angle constraint, lateral acceleration constraint, and performance index are converted to convex constraints.

(2) The impact time is introduced as an independent differential equation, therefore is decoupled with the range-to-go. The flight time is discretized as equality constraints to avoid the influence of range-to-go so that it contributes to the feasibility of convex programming.

(3) The sequential convex programming is proposed to calculate the optimal time-varying PN guidance law with impact time and impact angle constraints.

This paper is organized as follows. In section 2, the mathematical formulation and original optimization problem are derived. Then, the original optimization problem is converted and solved by sequential convex programming in Section 3, followed by numerical simulations in Section 4. Finally, Section 5 concludes this work.

2. Problem Formulation

The planar-engagement geometry between a missile and a stationary target is shown in Figure 1, where M and T denote the missile and target, respectively. r is the missile and target relative distance,V_M is the missile velocity, a_M is the lateral acceleration perpendicular to the velocity vector, γ, λ, and ε denote the flight path angle, line-of-sight(LOS) angle, and look angle, respectively, and γ_f is the terminal flight path angle. The kinematic equations can be derived as follows [28]:

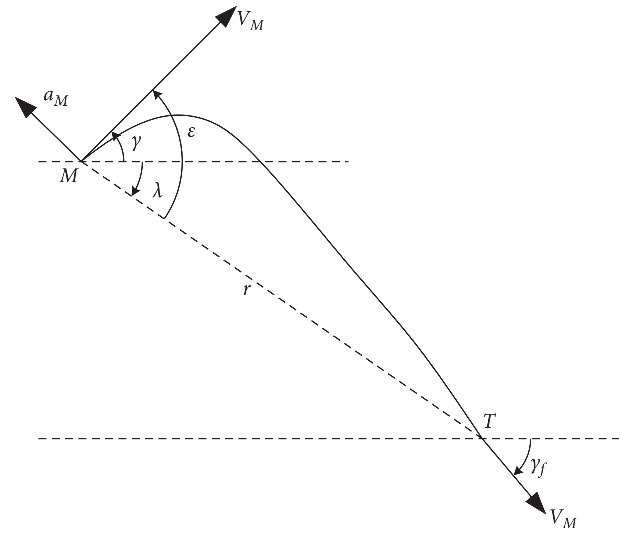


Figure 1: Planar-engagement geometry.

$$\begin{cases} \dot{r} = -V_M \cos \varepsilon, \\ r\dot{\lambda} = -V_M \sin \varepsilon, \\ \dot{\varepsilon} = \dot{\gamma} - \dot{\lambda}, \\ \dot{\gamma} = \dfrac{a_M}{V_M}. \end{cases} \quad (1)$$

Note that γ, λ, and ε are positive if measured counterclockwise from the reference line, otherwise are negative. The proportional navigation calls for the LOS angular rate for calculating the missile lateral acceleration, that is,

$$a_M = NV_M\dot{\lambda}, \quad (2)$$

where in standard PN guidance law, the gain N is constant. Then, the look angle ε can be derived as follows:

$$r\dot{\varepsilon} = -(N-1)V_M \sin \varepsilon. \quad (3)$$

In sum, the kinematic equations of missile under the PN guidance law are given as

$$\begin{cases} \dot{r} = -V_M \cos \varepsilon, \\ r\dot{\lambda} = -V_M \sin \varepsilon, \\ r\dot{\varepsilon} = -(N-1)V_M \sin \varepsilon. \end{cases} \quad (4)$$

In contrast to the standard PN guidance law with a constant PN gain, we design a time-varying PN gain $N(t)$to not only satisfy all the missile physical constraints and trajectory constraints, which will be discussed later, but also minimize the energy consumption. To distinguish between the proposed PN gain and standard PN gain, the PN gain is expressed as

$$u(t) = N(t). \quad (5)$$

A general energy performance index is considered as follows for the energy optimization problem:

$$J = \int_0^{t_f} a_M(t)^2 \mathrm{d}t = \int_0^{t_f} \frac{V_M^4[\sin \varepsilon(t)]^2}{r(t)^2} u(t)^2 \mathrm{d}t. \quad (6)$$

The physical constraints and trajectory constraints must be satisfied in the problem which includes the following:

(1) Kinematics constrains: they are given in kinematic equation (4) with initial conditions $r(0)$, $\lambda(0)$, and $\varepsilon(0)$.

(2) Field of view constraint: to accurately intercept the target, the target must be locked by the seeker during the intercept engagement. Due to the physical limitation of the seeker's field of view, the field of view constraint is described as follows:

$$|\varepsilon(t)| \le \varepsilon_{\max}, \quad (7)$$

where $\varepsilon_{\max} < (\pi/2)$ is an acceptable magnitude of the look angle [29,30].

(3) Acceleration constraint: the maneuverability of missile is limited by the aerodynamic and physical characteristics, an the lateral acceleration is bounded as

$$|a(t)| = \left|\frac{V_M^2 \sin \varepsilon(t) u(t)}{r(t)}\right| \le a_{\max}, \quad (8)$$

where $a_{\max}$ is the maximum lateral acceleration.

(4) Terminal constraints: to impact the target at the specified time and angle, we require $t(t_f) = t_f^*$, $\gamma(t_f) = \gamma_f^*$, and $r(t_f) = 0$. The look angle $\varepsilon(t_f) = 0$ is required to make sure the missile terminal velocity directly points to the target, which is equivalent to $\lambda(t_f) = \gamma_f^*$. Therefore, the terminal constraints are

$$\begin{aligned} t(t_f) &= t_f^*, \\ r(t_f) &= 0, \\ \lambda(t_f) &= \gamma_f^*, \\ \varepsilon(t_f) &= 0. \end{aligned} \quad (9)$$

According to the performance index and constrains, the optimal problem can be stated as Problem O: minimize $J = \int_0^{t_f} ((V_M^4[\sin \varepsilon(t)]^2)/r(t)^2) u(t)^2 \mathrm{d}t$.

Subject to equations (4), (7), (8), and (9), the solution to Problem O will give optimal PN guidance gain $N^*(t) = u^*(t)$; then, the missile lateral acceleration is $a_M(t) = N^*(t) V_M \dot{\lambda}(t)$. Because the precise analytical solution to this nonlinear problem is not available, computational guidance is implemented here to acquire the PN guidance gain. For practical use onboard, the numerical-solution process must be reliable and converges fast. These requirements motivate us to solve the optimization problem using the sequential second-order cone convex programming (SOCP) algorithm [31].

3. Numerical Solution by Sequential SOCP

To transform Problem O into the sequential SOCP problem, the kinematic equations must be discretized as linear equations, and the integrated performance index must be discretized for numerical integration; meanwhile, all non-convex constraints must transform into convex constraints.

First, we convert the nonlinear kinematic equation (4) into linear equations. To begin with, we set the missile and target relative distance r as an independent variable instead of time t. Note that r is monotone decreasing due to the look angle limitation $|\varepsilon| < (\pi/2)$. Equation (4) can be rewritten as [26]

$$\begin{cases} r\lambda' = r\left(\dfrac{\mathrm{d}\lambda}{\mathrm{d}r}\right) = \tan \varepsilon, \\ r\varepsilon' = r\left(\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}r}\right) = (u-1)\tan \varepsilon. \end{cases} \quad (10)$$

For simplicity, we define

$$\sigma = \tan \varepsilon. \quad (11)$$

Then, equation (10) becomes

$$rx' = f(x) + B(x)u, \quad (12)$$

where $x = [\lambda \;\; \sigma]^T$ is the state vector, and

$$\begin{aligned} f(x) &= \begin{bmatrix} \sigma \\ -\sigma\left(1+\sigma^2\right) \end{bmatrix}, \\ B(x) &= \begin{bmatrix} 0 \\ \sigma\left(1+\sigma^2\right) \end{bmatrix}. \end{aligned} \quad (13)$$

Note that this form is still nonlinear, but it is a control-affine system, and it can be easily transformed into a linear system in [32]. The main idea of this method is partly linearized control-affine system based on small-disturbance linearization. Let $[x^{(k)}; u^{(k)}]$be the kth successful solution; then, we convert equation (13) to the following partially linearized system:

$$rx' = A\left(x^{(k)}\right)x + B\left(x^{(k)}\right)u + c\left(x^{(k)}\right), \quad (14)$$

where $x^{(k)} = \left[\lambda^{(k)} \;\; \sigma^{(k)}\right]^T$ and

$$\begin{aligned} A\left(x^{(k)}\right) &= \frac{\mathrm{d}f}{\mathrm{d}x}\left(x^{(k)}\right) = \begin{bmatrix} 0 & 1 \\ 0 & -1-3\left(\sigma^{(k)}\right)^2 \end{bmatrix}, \\ B\left(x^{(k)}\right) &= \begin{bmatrix} 0 \\ \sigma^{(k)} + \left(\sigma^{(k)}\right)^3 \end{bmatrix}, \\ c\left(x^{(k)}\right) &= f\left(x^{(k)}\right) - A\left(x^{(k)}\right)x^{(k)} = \begin{bmatrix} 0 \\ 2\left(\sigma^{(k)}\right)^3 \end{bmatrix}. \end{aligned} \quad (15)$$

Besides, a trust region must be added to maintain the validity of proposed linearization:

$$\left|x - x^{(k)}\right| \leq \delta, \tag{16}$$

where $\delta \in \mathbb{R}^2$ is a constant vector, and the inequality is applied componentwise.

Next, with r as the independent variable, the nonlinear performance index becomes

$$J = \int_{r_0}^{r_f} \frac{V_M^3 \sigma \sin \varepsilon}{r^2} u^2 (-\mathrm{d}r), \tag{17}$$

where $\varepsilon = \tan^{-1} \sigma$. The equivalent form of the performance index can be reformulated as

$$J = \int_{r_0}^{r_f} \eta (-\mathrm{d}r), \tag{18}$$

where η is a slack variable, and

$$\frac{V_M^3 \sigma \sin \varepsilon}{r^2} u^2 \leq \eta. \tag{19}$$

Now, the performance index (18) can be linear discretization. The inequality constraint (19) can be approximated by a "lagging" technique:

$$u^2 \leq w^{(k)} \eta, \tag{20}$$

where $w^{(k)} = (r^2/([V_M^3 \sigma^{(k)} \sin(\varepsilon^{(k)})]))$.

For the nonlinear acceleration constraint in equation (8), we can approximate it with

$$|u| \leq u_{\max}^{(k)}, \tag{21}$$

where $u_{\max}^{(k)} = (r a_{\max}/(V_M^2 \sin \varepsilon^{(k)}))$. The look angle constraint in equation (7) becomes

$$|\sigma| \leq \tan(\varepsilon_{\max}). \tag{22}$$

For the time constraint, through the first equation in equation (4), we can get the derivative of t with respect to r as follows:

$$t' = \frac{1}{V_M \cos \varepsilon}. \tag{23}$$

Substituting equation (11) into equation (23) yields

$$t' = \frac{\sqrt{1 + \left(\sigma^{(k)}\right)^2}}{V_M}. \tag{24}$$

Finally, the terminal constraint becomes

$$\begin{aligned} \lambda\left(r_f\right) &= \gamma_f^*, \\ \sigma\left(r_f\right) &= 0, \\ t\left(r_f\right) &= t_f^*. \end{aligned} \tag{25}$$

Before the numerical solution through sequential convex programming, the kinematic equation (14) and time constraint equation (24) must be discretized by the trapezoidal rule as

$$x_i = x_{i-1} + \frac{e}{2} \left(\dot{x}_{i-1} + \dot{x}_i\right), \tag{26}$$

where e is the discretization step length and $i = 2, 3, \ldots, n$ is the discretization number.

In this problem, the kinematic equation (14) and time constraint equation (24) will be discretized at $n + 1$ uniformly distributed discretized points in $[r_0, r_f]$; then, the step length is $e = ((r_f - r_0)/n)$. Therefore, the kinematic equation (14) can be discretized and rearranged as

$$H_{i-1} x_{i-1} - H_i x_i + G_{i-1} u_{i-1} + G_i u_i = -\frac{e}{2} \left(\frac{c_{i-1}\left(x^{(k)}\right)}{r_{i-1}} + \frac{c_i\left(x^{(k)}\right)}{r_i} \right), \tag{27}$$

where the coefficients are

$$\begin{aligned} H_{i-1} &= I + \frac{e}{2r_{i-1}} A_{i-1}\left(x^{(k)}\right), \\ H_i &= I - \frac{e}{2r_i} A_i\left(x^{(k)}\right), \\ G_{i-1} &= \frac{e}{2r_{i-1}} B_{i-1}\left(x^{(k)}\right), \\ G_i &= \frac{e}{2r_i} B_i\left(x^{(k)}\right). \end{aligned} \tag{28}$$

As the same with equation (14), the time constraints can be discretized as

$$t_i = t_{i-1} + \frac{e}{2} \left(\frac{\sqrt{1 + \left(\sigma^{(k-1)}\right)^2}}{V_M} + \frac{\sqrt{1 + \left(\sigma^{(k)}\right)^2}}{V_M} \right). \tag{29}$$

Besides, the performance index equation (18) is discretized as

$$J = \int_{r_0}^{r_f} \eta (-\mathrm{d}r) = \sum_{i=2}^{n} -\frac{e}{2} \left(\eta_{i-1} + \eta_i\right). \tag{30}$$

In summary, problem O can be approximated as follows: Problem $\mathfrak{R}$: minimize equation (30), subject to equations 20)–(22, (25), (27), and (29).

This problem can be efficiently solved by convex optimization algorithm because it is a SOCP problem, which is mathematically defined in [33].

Because Problem $\mathfrak{R}$is an approximation to Problem O, sequential convex-programming is proposed here to get the solution to approach the exact solution problem O . The procedures are described as follows:

(1) Set $k = 0$. Initialize the states $\lambda(r_0) = \lambda_0$, $\lambda(r_f) = \gamma_f^*$, $\sigma(r_0) = \sigma_0$, $\sigma(t_f) = 0$, $t(r_0) = 0$, and $t(t_f) = t_f^*$. Note that the initial states in the convex optimization algorithm need not satisfy the constraints, so we can generate $x^{(0)}$ by linear interpolating between the initial condition and the final condition.

(2) For $k \geq 1$. Compute the $x^{(k-1)}$-dependent parameters in equations (15), (28), and (29) using $x^{(k-1)}$. Then, Problem $\mathfrak{R}$ can be solved with the initial states $x^{(k-1)}$

and the preceding calculated parameters. Therefore, we can get a solution $\{x^{(k)}; u^{(k)}\}$.

(3) Check the convergence condition:

$$\sup_{r_0 \le r \le r_f} \left| x^{(k)} - x^{(k-1)} \right| \le \xi, \tag{31}$$

where $\xi \in \mathbb{R}^2$ is a prescribed tolerance value for convergency. If the preceding convergence condition is satisfied, go to step 4; otherwise, set $k = k + 1$ and go to step 2.

(4) The solution to Problem O is found to be $x^* = x^{(k)}$ and $u^* = u^{(k)}$.

Remarks. (1) According to equation (27), when discretizing the state equations, the parameters of the last point may be quite small or big to cause problem infeasible due to $r_f \longrightarrow 0$ so that we can use the Euler rule instead of the trapezoidal rule at the last point.

(2) The initial states need not satisfy all the constraints in Problem $\mathfrak{R}$, but different initial states will influence the iteration times to converge in sequential convex programming. The more accurate it is to the trust region, the fewer iteration times it costs.

(3) A theoretical proof of the convergence of the sequential convex programming still needs works due to the presence of various constraints and nonlinear kinematics. Lu and Liu [20] and Wang [34] provided several examples of the convergence of sequential convex programming. The numerical simulations in Section 4 show the convergence and effectiveness of the proposed algorithm.

(4) The proposed method requires no time-to-go information compared to the traditional analytical optimal guidance law. The approximation error of time-to-go degrades the performance index.

The schematic diagram of the proposed method is summarized in Figure 2. The main idea of this approach is to transfer the nonconvex guidance problem to a sequential SOCP problem and use sequential convex programming to solve the sequential SOCP problem until convergence to obtain the optimal time-varying proportional navigation guidance law.

4. Numerical Simulation

In this section, firstly, we will verify the convergence and effectiveness of the proposed sequential convex programming algorithm compared to a nonlinear-programming solver. Next, we will show the super performance of the proposed guidance law to the traditional analytical optimal guidance law. Finally, we will show the effectiveness of the proposed guidance law for different impact times and angles.

To begin with, we set trust region as $\delta = [\,(50\pi \quad /180)\ \tan(50\pi \quad /180)\,]$ in equation (16) and convergence condition as$\xi = [\,(0.1\pi \quad /180)\ \tan(0.1\pi/180)\,]$ in equation (31). The simulations are implemented in MATLAB R2017b on a laptop computer equipped with Intel Core i5–8250 U 1.6 GHz and 8 GB RAM. Convex programming problems are solved using MOSEK [35] in CVX [36].

A vertical interception engagement of a ground-launched missile is considered as follows. The missile and target initial relative distance $r_0 = 12000\,\text{m}$. The missile velocity is a constant $V_M = 300\,(\text{m/s})$. The initial look angle is $\varepsilon_0 = 30^\circ$, and the initial LOS angle is $\lambda_0 = 0$. In addition, the field of view constraint is $\varepsilon_{\max} = 60^\circ$, and lateral acceleration constraint is $a_{\max} = (50\,\text{m/s}^2)$.

4.1. The Effectiveness of Proposed Sequential Convex Programming. In this part, we set the impact time as$t_f^* = 47\,\text{s}$ and impact angle as $\gamma_f^* = -60^\circ$. To validate the optimality and convergence of the proposed algorithm, the independent software GPOPS [37] is selected to directly solve the original Problem O.

The performance index values of the proposed algorithm in each iteration are shown in Figure 3. It indicates that the sequential algorithm converges in 9 steps with interpolation initial states for the problem. More quantitative results of the converging process are shown in Table 1. If looser tolerance is used, fewer steps will be needed for convergence. In each step, it costs about 0.3–0.5 s CPU time. In sum, the algorithm costs 4.21 s to solve Problem $\mathfrak{R}$, while the GPOPS costs 92.31 s to solve the original Problem O. The efficiency of the proposed algorithm illustrates that the proposed algorithm has the potential to use onboard.

Figure 4 presents the PN guidance gains, trajectory profiles, lateral acceleration profiles, LOS angle profiles, look angle profiles, and performance index values obtained by the proposed algorithm and GPOPS, respectively. It can be seen that the guidance gain obtained by the proposed algorithm successfully achieves the desired impact time and impact angle, furthermore, satisfies all the constraints. The very similar solution obtained by GPOPS validates the optimality of the proposed algorithm. In the last several seconds, the obtained PN guidance gain is different from GPOPS, and it may be caused by several potential reasons, such as the linearization and approximation of kinematic dynamics and different discretization strategies. The GPOPS employs the hp-adaptive Radau Pseudospectral method to approximate the kinematics. The main idea of GPOPS is to use orthogonal polynomial to approximate the control variable and states at the Legendre–Gauss–Radau points. The inherent drawback of polynomial approximation is not suitable for the non-smooth problem. The linearization and discretization in this paper are based on the kinematics of the guidance problem. Generally speaking, the proposed method is faster than GPOPS for the guidance optimization problem and needs no initial guesses.

To further demonstrate the performance of the proposed algorithm with different initial states, we take two different initial trajectories to trigger the proposed algorithm. The initial condition 1 is the interpolation trajectory, and the initial condition 2 is the proportional navigation guidance

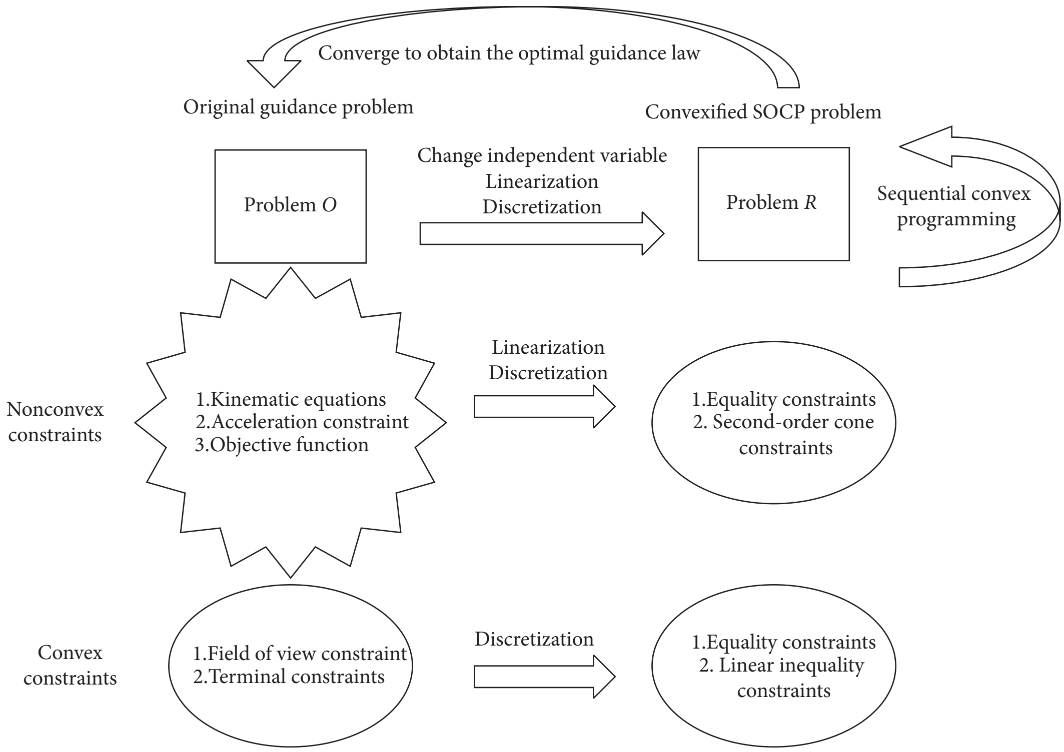


Figure 2: The schematic diagram of the proposed method.

trajectory with guidance gain 3. The initial condition 1 takes 9 steps to converge, while initial condition 2 takes 10. The look angle and LOS angle profiles of these two initial conditions are present in Figure 5. Figure 5(d) indicates that the initial trajectory of the proposed algorithm need not satisfy the constraint. The proposed algorithm relaxes the requirements for the initial trajectory.

4.2. Compared to Traditional Optimal Guidance Law. To show the difference between the proposed guidance law with the traditional analytical guidance law, numerical simulation results are compared with those obtained by a guidance law based on the optimality of error dynamics (GOED) [3]. The analytical form of the guidance law is expressed as follows:

$$
\begin{aligned}
a_M &= 4V_M\dot{\lambda} + \frac{2V_M\left(\lambda-\gamma_f\right)}{t_{\mathrm{go}}} + \frac{30KV_M^2}{rt_{\mathrm{go}}\left(4\varepsilon-\varepsilon_f\right)}\varepsilon_t,\\
t_{\mathrm{go}} &= \frac{r}{V_M}\left(1+\frac{1}{15}\left(\varepsilon^2+\varepsilon_f^2-\frac{1}{2}\varepsilon\varepsilon_f\right)\right),\\
\varepsilon_t &= t_f^* - t_{\mathrm{go}} - t,
\end{aligned}
\tag{32}
$$

where $t_{go}$is the time-to-go, which is estimated by the second equation, and t denotes the current flight time. Note that the gain K is a designed parameter that should be sufficiently large to enable a successful impact time constraint. According to the result in KIM [38], the guidance gain should be set as $K \geq 4$. The performance index of GOED is the same as equation (6). It can be seen from the simulation results in [3] that the bigger the K value is selected, the larger the performance index value is obtained. Therefore, we set $K = 4$ to get the smallest performance index value. The field of view constraint is set as $\varepsilon_{\max} = 42^{\circ}$.The other parameters are the same as the previous case.

The PN guidance gains, trajectory profiles, lateral acceleration profiles, LOS angle profiles, look angle profiles, and performance index values obtained by the proposed algorithm and GOED guidance law are shown in Figure 6, respectively. Obviously, both algorithms can satisfy the impact time and impact angle constraints, which mean that the two methods achieve the desired impact angle at the desired impact time. While the look angle profiles in Figure 6(e) show that the traditional GOED guidance law cannot satisfy the look angle constraint. The proposed algorithm can maintain the look angle will not exceed the maximum look angle constraint. Figure 6(c) presents the required lateral acceleration of GOED is larger than the proposed algorithm at the start of the engagement, which is caused by the error of the time-to-go estimation. The performance index values in Figure 6(f) indicate that the proposed algorithm needs less control energy than the GOED guidance law. To sum up, the proposed algorithm not only satisfies the look angle constraint but also has less energy consumption.

Compared to the traditional guidance law, the proposed optimal guidance law utilizes large computational resources to optimize the total trajectory to satisfy the constraint and increase the performance of guidance law. Therefore, it requires the missile to have sufficient computational ability onboard. The traditional guidance law is easy to realize but

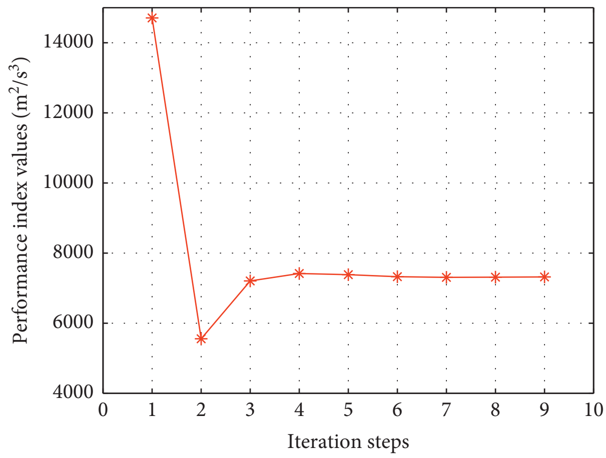


FIGURE 3: Performance index values of each iteration.

TABLE 1: The difference of the states between each iteration.

Iteration step	$\Delta\|\lambda\|$, deg	$\Delta\|\arctan(\sigma)\|$, deg
1	12.602	34.885
2	11.332	13.351
3	1.0875	4.4881
4	0.2051	2.1228
5	0.1811	1.0894
6	0.0730	0.6234
7	0.0368	0.0322
8	0.0202	0.1364
9	0.0066	0.0759

has to sacrifice some performance. With the development of computer science, the proposed computational guidance law will be employed to enhance the performance of guidance law.

4.3. Effectiveness of Different Impact Times and Angles. To verify the effectiveness of the proposed algorithm for different impact times and impact angles, we set the different impact times as $\gamma_f^* = -60^\circ$ and $t_f^* = 44\,\mathrm{s}, 47\,\mathrm{s}, 50\,\mathrm{s}$ and different impact angles as $t_f^* = 47s$ and $\gamma_f^* = -60^\circ, -75^\circ, -90^\circ$. Figure 7 shows the PN guidance gains, trajectory profiles, lateral acceleration profiles, LOS angle profiles, look angle profiles, and performance index values obtained by the proposed algorithm for different impact times. It can be seen from Figure 7(b) that the trajectory needs to be higher for a longer time to intercept the target, which leads to larger lateral acceleration to change the flight path angle and finally requires more control effort. Furthermore, the look angle constraint is satisfied in all cases. Figure 8 shows the PN guidance gains, trajectory profiles, lateral acceleration profiles, LOS angle profiles, look angle profiles, and performance index values obtained by the proposed algorithm for different impact angles. Figure 8(c) illustrates that the larger impact angle requires smaller lateral acceleration at the beginning of the engagement and requires bigger lateral acceleration at the end of the engagement to achieve the desired impact angle. The curvature of trajectory in Figure 8(b) proves that larger impact angle calls for smaller lateral acceleration at the beginning of interception engagement because the curvature of the first half trajectory changes smoothly and larger lateral acceleration at the end because of the fast curvature alters of second-half trajectory. Therefore, it results in the curvature changes of look angle, LOS angle, and performance index value during the interception engagement. In summary, the proposed algorithm can generate different PN guidance gains for achieving different impact times and impact angles under all constraints.

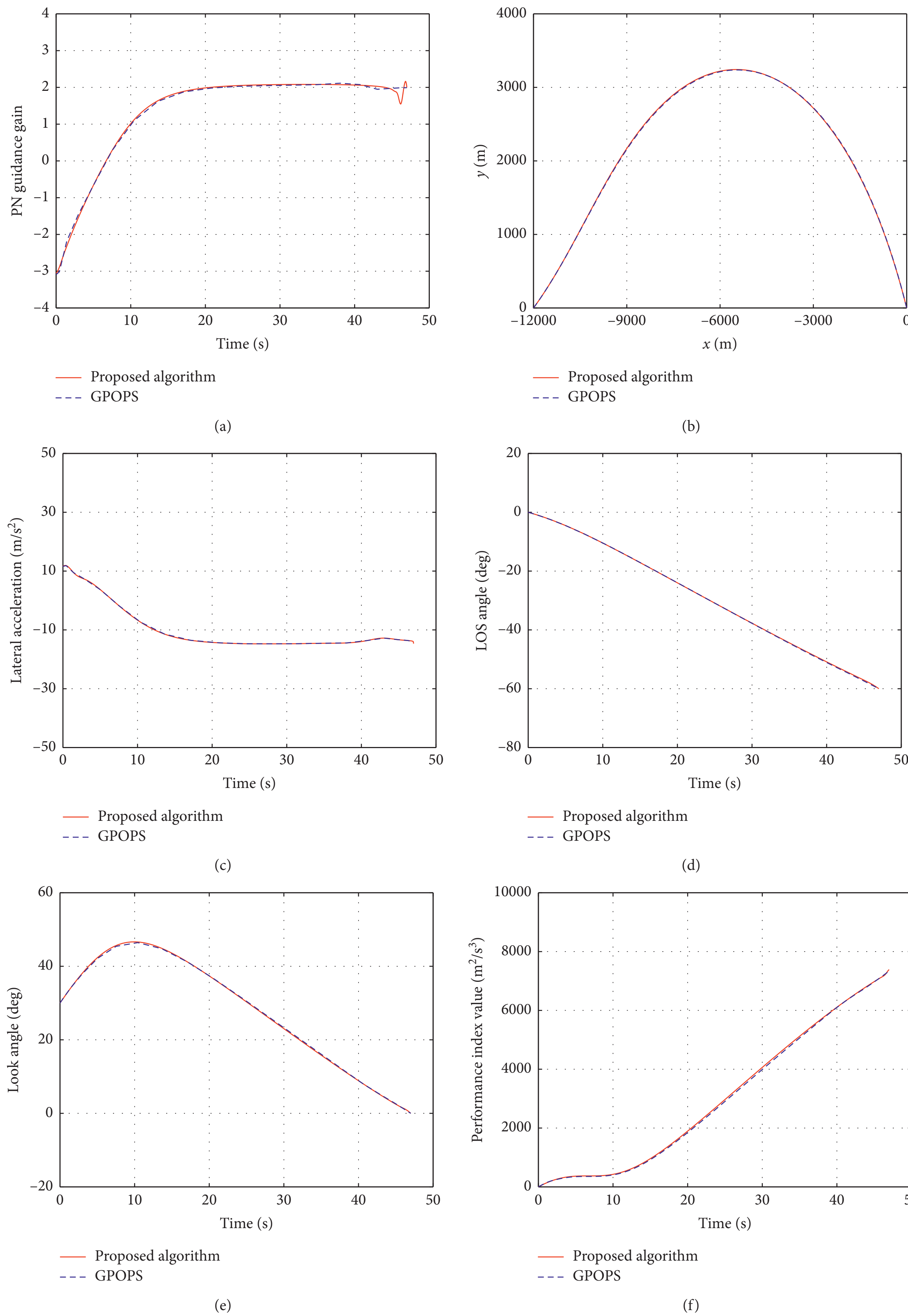


FIGURE 4: (a) PN guidance gains, (b) trajectory, (c) lateral acceleration, (d) LOS angle, (e) look angle, and (f) performance index values obtained by the proposed algorithm and GPOPS.

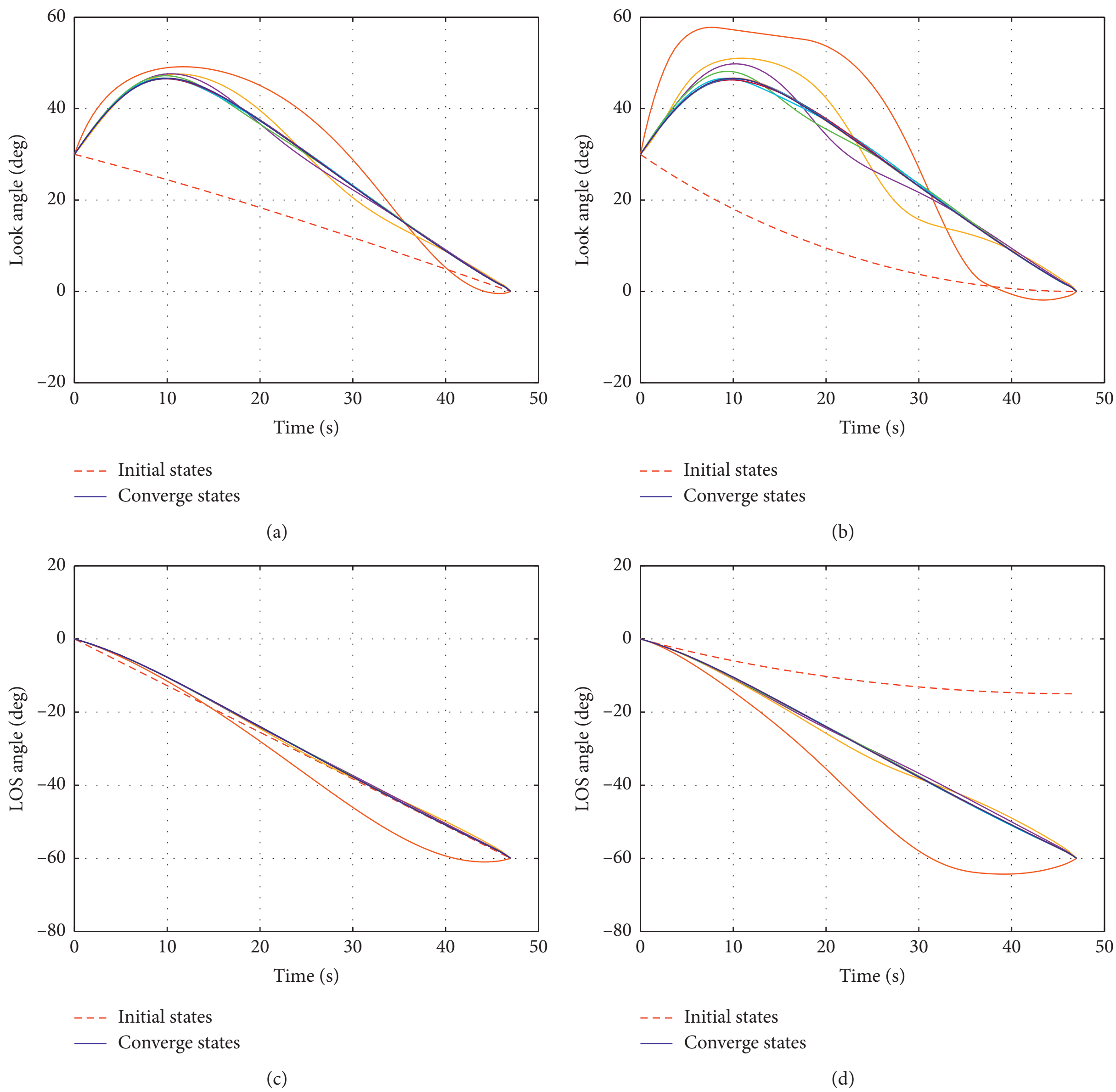


Figure 5: Look angle and LOS angle profiles for different initial conditions. (a) Look angle profiles for initial condition 1. (b) Look angle profiles for initial condition 2. (c) LOS angle profiles for initial condition 1. (d) LOS angle profiles for initial condition 2.

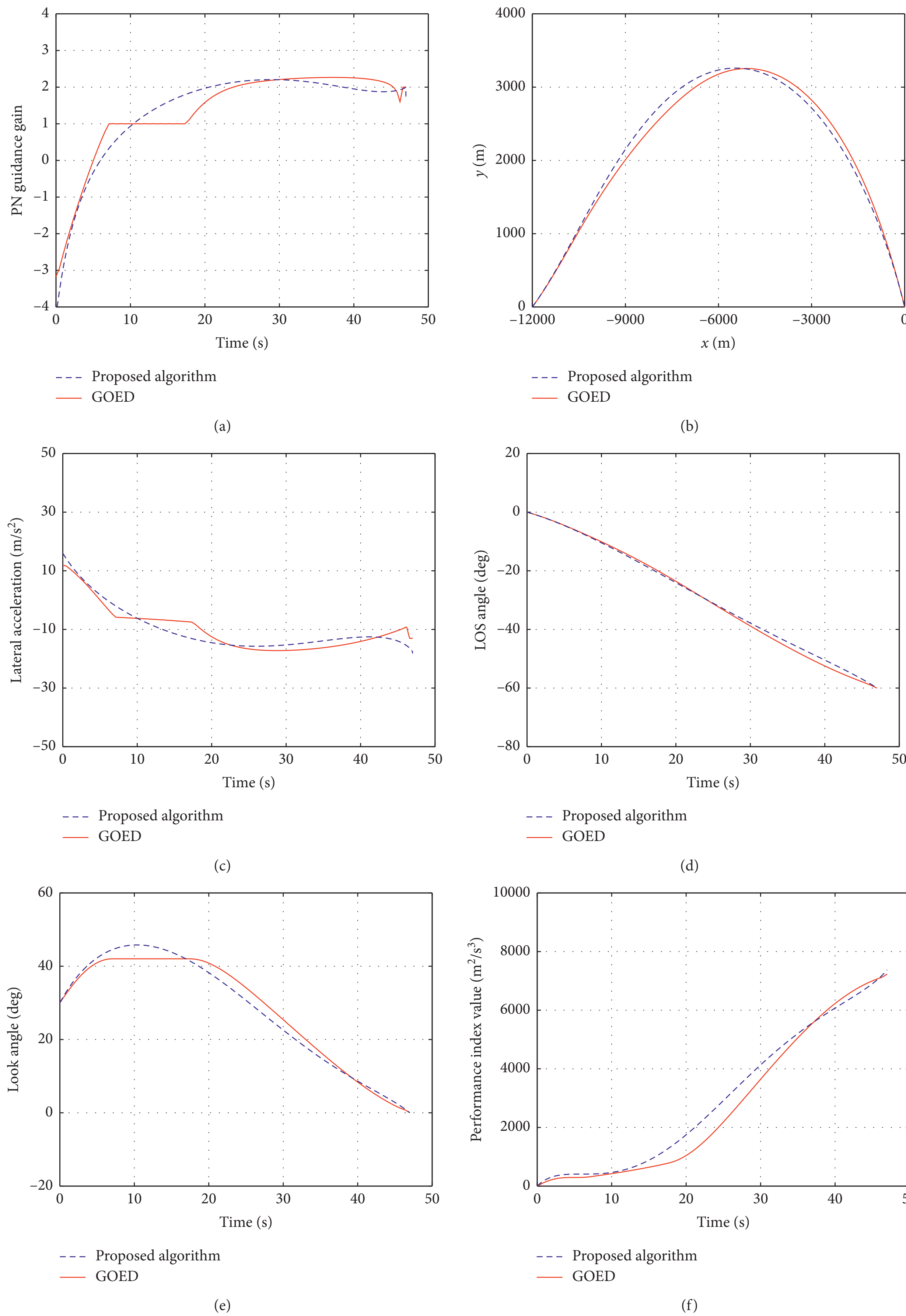
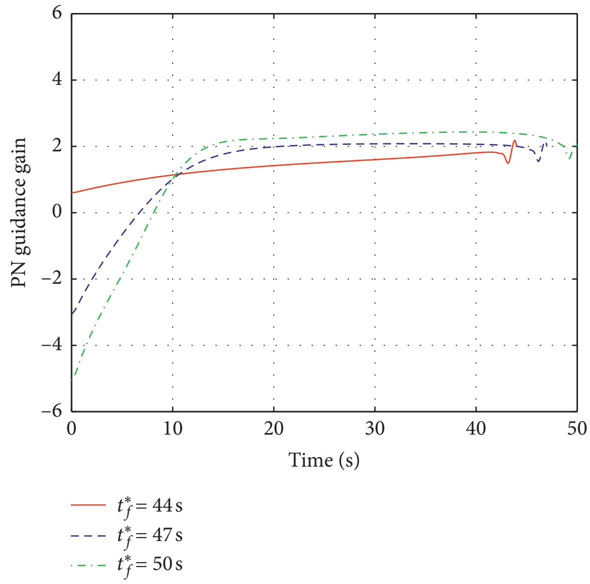
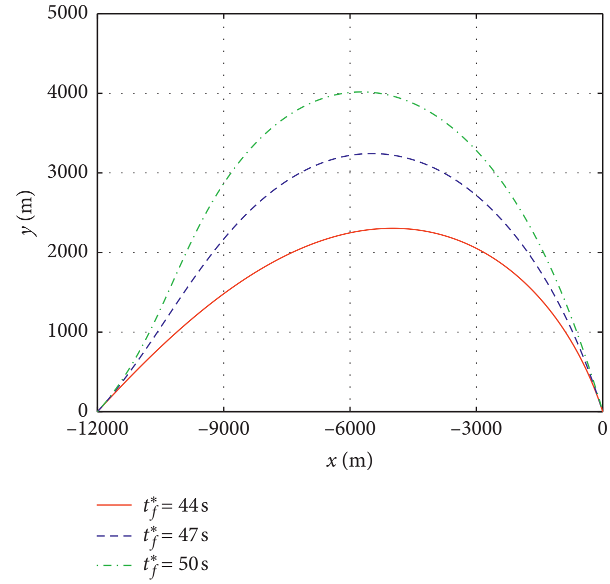


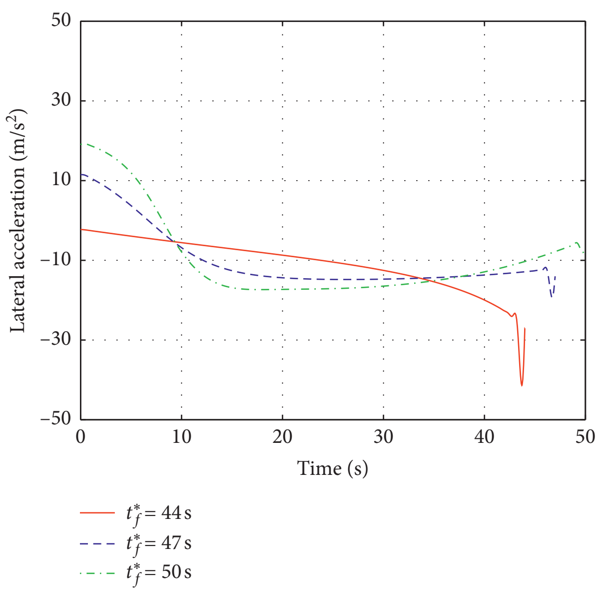
FIGURE 6: (a) PN guidance gains, (b) trajectory, (c) lateral acceleration, (d) LOS angle, (e) look angle, and (f) performance index values obtained by the proposed algorithm and GOED guidance law.



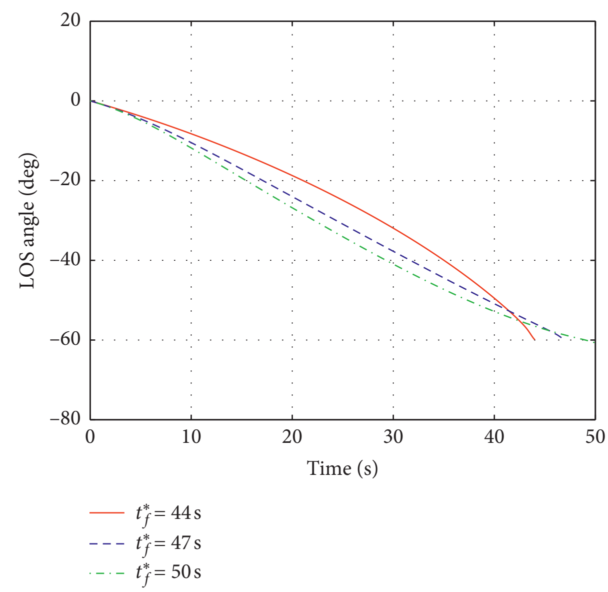
(a)



(b)

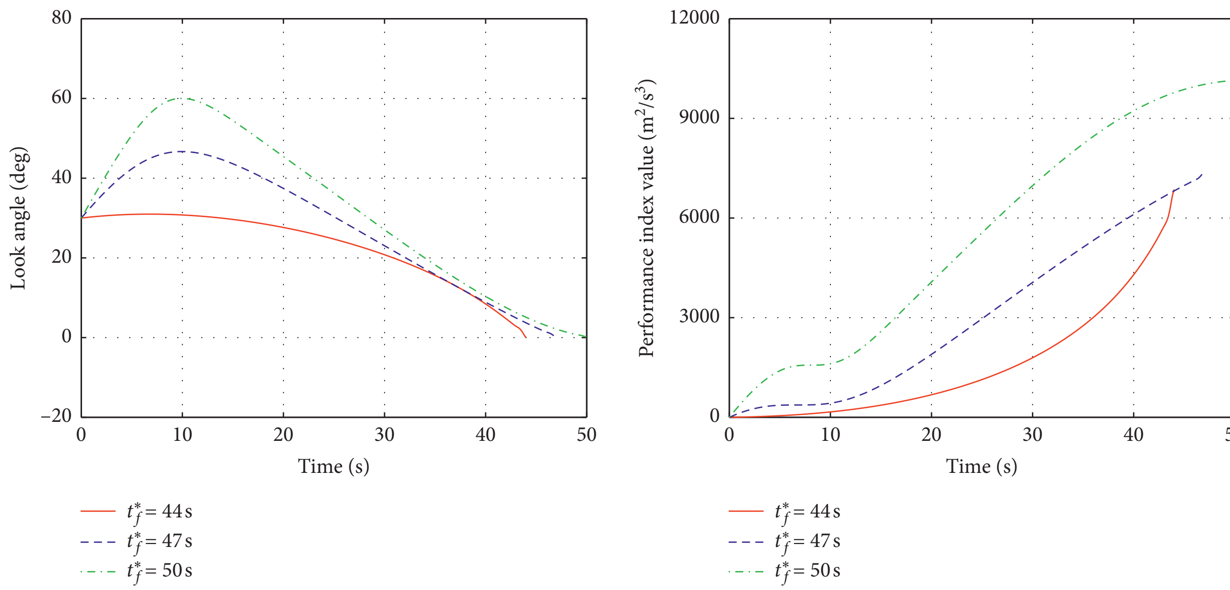


(c)



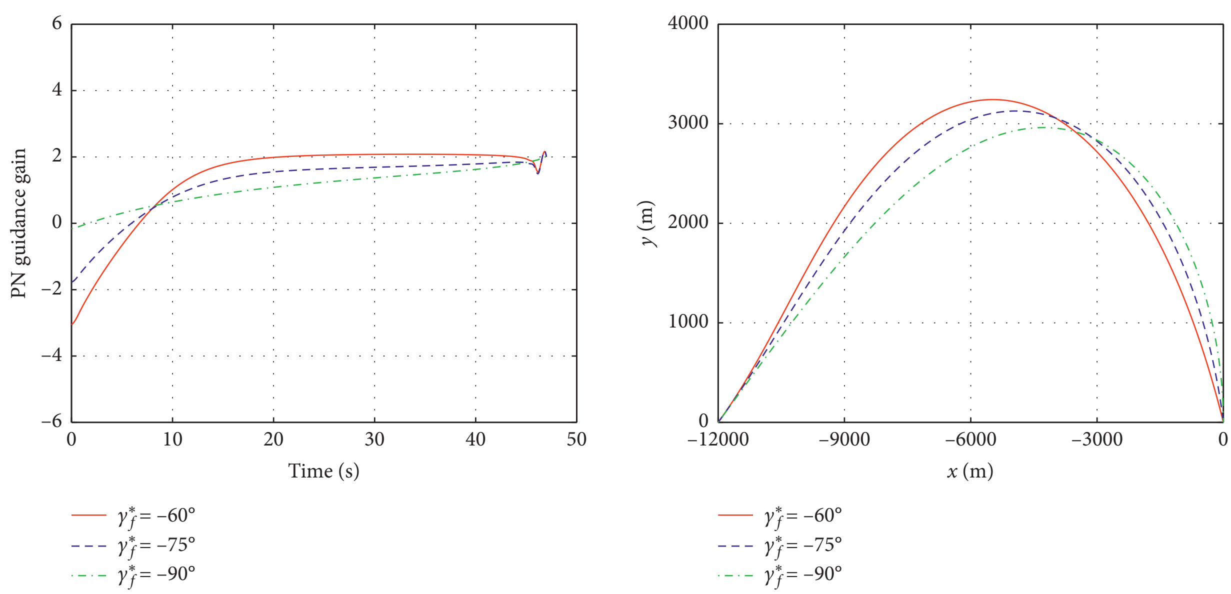
(d)

Figure 7: Continued.



(e) (f)

Figure 7: (a) PN guidance gains, (b) trajectory, (c) lateral acceleration, (d) LOS angle, (e) look angle, and (f) performance index values of different impact times.



(a) (b)

Figure 8: Continued.

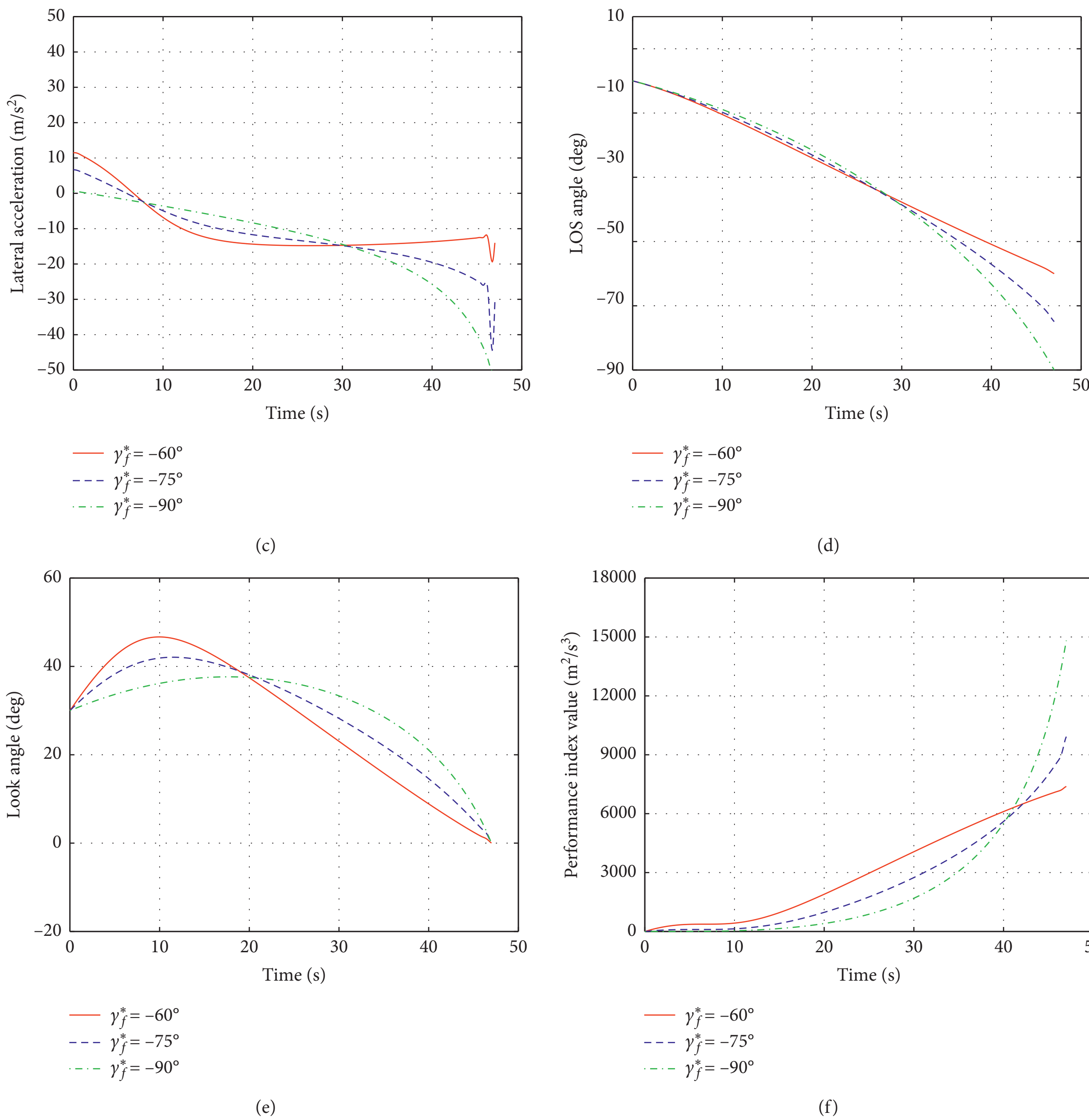


FIGURE 8: (a) PN guidance gains, (b) trajectory, (c) lateral acceleration, (d) LOS angle, (e) look angle, and (f) performance index values of different impact angles.

5. Conclusion

In this paper, a sequential convex programming algorithm is developed for time-varying PN guidance law design under impact time and impact angle constraints, as well as field-of-view and acceleration limits. This guidance problem is formulated as a nonlinear optimization problem and then as linearized and discretized to be solved by sequential convex programming. Numerical simulations are compared with GPOPS and GOED guidance law to validate the convergence and optimality of the proposed algorithm. Furthermore, simulation results prove that the proposed algorithm can generate optimal time-varying guidance law to achieve different impact times and impact angles.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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