Research Article

A Novel Ship-Ship Distance Model in Restricted Channel via Gaussian-TRR Identification

Jie Zhang and Yingjun Zhang

Navigation College, Dalian Maritime University, Dalian 116026, China

Correspondence should be addressed to Jie Zhang; zhangjiedmu@163.com

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Very large ships are crucial cargo ships that are relatively difficult to manoeuvre, and ship-ship distance is a vital manoeuvring parameter in restricted channel. To ensure ship safety and improve scheduling efficiency, this study established a ship-ship distance model in restricted channel by ship manoeuvring motion simulation, collision detection, and identification modelling. Firstly, the ship manoeuvring model calculated the forces and moments of ship-ship interaction and ship-bank interaction. Then, the collision detection was applied to calculate the intersection area of ship collision. Secondly, the discrete numerical simulation approach was employed with varying speed and distance, and the intersection area was counted. Finally, the 3D Gaussian models of encountering and overtaking were identified by the trust-region-reflective (TRR) algorithm, and ship-ship distance and prohibited zone were proposed. The results show that the minimum ship-ship distance for encountering and overtaking is 1.50 and 2.4 ship beam, respectively, which is consistent with Japan’s standard. The numerical results revealed that the prohibited zone is an elliptical shape. The ship-ship distance and prohibited zone serve as ship safety domain for collision avoidance during harbor approaching.

1. Introduction

Approaching harbor is a difficult challenge for very large ships. Compared with open and deep seas, the water depth, traffic density, and offshore conditions of channels make it difficult to handling a ship. In maritime transportation, 400,000-ton oil tankers and 24,000 TEU container ships are undertaking crucial tasks [1]. These ships own obvious larger inertia, longer time delay, and higher nonlinearity. The emergence of very large ships has brought major safety challenges in maritime traffic. Due to close range encountering and overtaking, the ship-ship interaction and ship-bank may cause track offset and course changing. To ensure navigation safety and diminish ship-ship interaction, very large ships must maintain a proper distance. In particular, the ship-ship distance is also important to channel width design, the harbor approach navigation, and maritime supervision.

Firstly, due to the width of very large ships, the Maritime Safety Administration has implemented control measures, such as one-way channel and the prohibition of overtaking in traffic separation schemes [2]. Moreover, the Maritime Safety Administration has formulated a method for calculating the minimum ship-ship distance for very large ships, which aids the design of a one-way or two-way channel with a reasonable width. Secondly, the traffic flow is busy and the ship-ship distance is minor in the traffic separation scheme. Thus, the determination of minimum ship-ship distance can improve scheduling efficiency [3]. Thirdly, during navigation, keeping a reasonable distance between very large ships can reduce or avoid ship collisions to ensure the safety of maritime navigation.

In summary, determining the appropriate ship-ship distance of very large ships is a challenge and must be focused. Several studies have investigated the ship-ship distance. Studies on ship-ship distance of very large ships have been theoretically and practically valuable. However, the analytical ship-ship distance model has not been investigated. In order to ensure the ship safety in the channel encounter, the following issues should be solved:
(1) Is there a safety model for ship-ship distance in restricted channel?
(2) If yes, what factors impact the ship safety model and should be considered to establish the model? In other words, what is the structure and parameters of the model?
(3) How to identify the ship safety model? And how to apply the model for manoeuvring in channel while approaching harbor?

2. Literature Review

Methods for researching ship-ship interaction include model tests and numerical simulations. Taylor [4, 5] was the first to employ a ship model test for interaction experiments, using two ship models with the same displacement but slightly different ship types and no relative motion. Newton was the first to release data on tests of hydrodynamic interaction between two ships during overtaking in a deep-water situation [5]. Graff measured the hydrodynamic interaction experienced by a ship model moored on the tank bank when another ship passed by. Muller studied the encountering and overtaking of two ships in the restricted channel [6]. Oltmann studied the hydrodynamic force when an elliptical cylinder passes fixed in a two-dimensional flow. Remery compared the hydrodynamic interaction between a moored ship model and a passed one [7]. Dand investigated the hydrodynamic interaction of two ship models moved along parallel straight lines when encountering and overtaking [8]. Vantorre conducted a systematic study and comparison of the encountering and overtaking ship models with different ship types, water depths, and speeds [9, 10]. Kriebel focused on the effect of passing ships on the moored ships [11]. In a recent study, Mousaviraad examined the problem of ship disturbing force in calm water. However, that study only examined one loading condition [12]. Varyani conducted numerous theoretical and experimental studies [13–15] using captive models in a towing tank and achieved regression models. However, the complexity of numerical methods makes it not suitable for online calculation.

Furthermore, numerical simulations are a crucial means to explore the navigation safety of ships approaching a harbor. Sario used simulation methods to study the process of ships approaching the Istanbul waterways and analysed the relationship between ship size and navigation safety [16]. Lee formulated a manoeuvring motion simulation of ship overtaking in a restricted channel by using the course keeping control for ship-ship interaction [17]. Gucma used the model to simulate ship motion in a bend and performed various numerical simulations [18]. To explain the process of approaching a port, Quy established a general model to determine ship manoeuvrability. Research on ship-ship distance is relatively scarce. No research has investigated very large ships apart from 100,000-ton bulk carriers [19]. The World Association for Waterborne Transport Infrastructure (PIANC) gives the separation distance of two-way channels in their design guide and used the moulded beam as the design parameter [20]. Weng and Sun also used the ship beam as a research parameter to construct a Manoeuvring Modelling Group (MMG)

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important keystones. This study modelled the hydrodynamics of the ship hull, bank, shallow water, and ship-ship interaction.

3.1. Problem. This study determined the ship-ship distance between encountering ships in a two-way channel, judged the probability of very large ships colliding in the two-way channel, reducing waiting time of ships, and utilizing the channel resources. Generally, overtaking is not allowed in one-way and two-way channels, and encountering is not allowed in two-way channels. Because encountering has a shorter duration and smaller displacement due to ship-ship interaction, the hazard of encountering is smaller than that of overtaking in the two-way channel. Therefore, this study focused on the encountering and overtaking situation.

Information in nautical charts is highly accurate and essential for navigation [24]. As shown in Figure 1, the two-way channel is approximately 2 NM long, 0.38 NM wide, and 50 m deep. Very large ships are affected by ship-ship interaction, ship-bank interaction, and the shallow water effect when manoeuvring upon encountering.

3.2. Manoeuvring Motion Model. The motion of a manoeuvring ship can be mathematically modelled. The hydrodynamic forces experienced by the ship are related to the state of motion and the extent of control. This study ignored the ship oscillation and only considered the ship track and course deflection when the ship is manoeuvring in the channel. This study’s model had three degrees freedom, surge, sway, and yaw, which were enough to represent ship motion.

At present, classic simulation models, such as the Kijima model and whole model, are commonly used for ship manoeuvring [25, 26]. The whole model has been applied to Esso-type very large crude carriers (VLCCs) [26, 27]. The whole model Esso Osaka and Esso Bernicia described the deep-water and shallow water manoeuvring, respectively [27, 28]. References [27, 29] used optimization techniques to identify the better hydrodynamic coefficients of Esso Bernicia. The Esso Bernicia model is indicated in the following equation and used for simulation in the paper [27, 29, 30]:

\[\begin{align*}
\dot{u} - vr - x_G^r v^2 &= g X_h^r, \\
\dot{v} + ur + x_G^r v^2 &= g Y_h^r, \\
(L k_z^r)^2 \dot{r} + x_G^r (\dot{v} + ur) &= g L N_h^r,
\end{align*}\]  

where \(u, v\), and \(r\) are the surge velocity, sway velocity, and yaw velocity, O-XYZ is a cartesian coordinate system attached to ship to describe surge, sway, and yaw, respectively, \(L\) is the ship length between perpendiculars, \(x_G\) is the position of the gravity centre in the OX-direction, \(I_z\) is the inertial moment of a ship with respect to the OZ-axis, \(k_z^r = L^{-1} \sqrt{I_z/m}\) is the nondimensional radius of gyration, \(x_G^r = L^{-1} x_G\), and \(X_h^r, Y_h^r, N_h^r\) are nonlinear nondimensional functions. These functions consider ship propeller, rudder, and hull hydrodynamics. There are 34 hydrodynamic coefficients consist of these three functions [27]. The details of these functions and the reproduction of the whole model were recorded in Appendix A.

3.3. Ship-Ship Interaction Model. The ship-ship interaction force mainly comprises sway and yaw forces. Two previous studies [13, 14] studied the ship-ship interaction force in encountering and overtaking, respectively. Figure 2 illustrates ship-bank boundary and its hydrodynamic while encountering.

The magnitude of the ship-ship interaction force was related to the encountering situation, ship speed ratio, and ship-ship distance. The following equation details the ship’s interaction force and moment in encountering [10]:

\[
\begin{align*}
\frac{Y_{sh}}{0.5 p U_0 U_L L^2 d} &= CF = -0.47 \cos(0.86\pi \varepsilon) e^{-0.95\varepsilon} (1 + 0.18\varepsilon), \\
\frac{h/d}{1.5} &= 2.25 \left( \frac{2S_p}{L} \right)^{1.25} \left( \frac{I_1}{I_2} \right)^{2.25} 0.5 \left( 1 + \frac{U_1}{U_1} \right), \\
\frac{C_{Y_{sh}}}{0.5 p U_0 U_L L^2 d} &= CM = -0.47 \cos(0.86\pi \varepsilon) e^{-0.95\varepsilon} (1 + 0.18\varepsilon), \\
A(\varepsilon) \frac{h/d}{1.5} &= 2.25 \left( \frac{2S_p}{L} \right)^{1.25} \left( \frac{I_1}{I_2} \right)^{2.25} 0.5 \left( 1 + \frac{U_1}{U_1} \right),
\end{align*}
\]

(2)

where \(\varepsilon\) is the distance from the centre of the ship; \(S_p\) is the sway distance from the centre of the ship; \(U\) is ship speed; and \(h/d\) is the water depth to draft ratio. The formula accounts for the shallow water correction. Moreover, \(A(\varepsilon)\) represents the correction of moment disturbance when the two ships are abeam nearby. When \(\varepsilon\) is less than the ship length constant, the restricted channel causes the centre of rotation to be abnormal; this changes the moment, where this change must be corrected. The coefficients like 0.47 and 0.18 in equation (2) were identified by Varyani [10]. The reproduction of equation (2) was recorded in Appendix B.

3.4. Ship-Bank Interaction Model. Generally, when a ship navigates parallel to the bank along a channel, the ship-bank interaction causes ship drift and turning. The ship-bank interaction is not only related to the ship type, ship speed, and ship-bank distance but also to the bank slope. A steeper bank slope increases the distance between bank suction and bank cushion. This study used the results [31] to estimate the hydrodynamic force of the ship-bank interaction, as shown in Figure 3, where \(h\) is water depth; \(d\) is the ship draft; and \(y_{P1}, y_{S1}, y_P, y_S\) are the left and right widths of the bank.
Figure 1: The object extraction of Electronic Navigational Chart for harbor channel.

Figure 2: Ship-ship interaction force and coordinate system for encounter manoeuvre in a channel.

Figure 3: Ship-bank interaction coordinate system for a ship in channel.
The method for estimating the hydrodynamic force $Y_{bh}$ in ship-bank interaction is calculated by the following equation [10]:

$$
Y_{bh} = 0.001 \left( 0.5pU^2L^3 \right) \left( a_5y_{B3}^2 + a_6y_{B3}^2F_R + a_7y_{B3}^2F_T + a_8y_{B3}^2F_T + a_{13}y_{B3} + 0.011 \frac{y_B}{C_T} + 0.0006 \frac{y_B}{C_T^2} \right),
$$

(3)

The method for estimating the hydrodynamic moment $N_{bh}$ in ship-bank interaction is shown in the following equation [10]:

$$
N_{bh} = 0.001 \left( 0.5pU^2L^3 \right) \left( b_8y_{B3}^2 + b_{16}y_{B3}^2F_T^2 + b_{14}y_{B3}^2F_T^2 + b_16y_{B3}^2F_C^2 + 0.0009 \frac{y_B}{C_T^2} - 0.0044 \frac{y_B}{C_T^2} \right),
$$

(4)

where $U$ is ship speed; $L$ is ship length; $F_n$ is the Froude number on length (called Froude number in the following contexts), $F_n = U/V_B C_T$ is the propeller thrust coefficient; $y_B$ and $y_{B3}$ are obtained from $y_{p3}$, $y_{s3}$, $y_p$, and $y_s$; and the coefficients $a_5$, $a_6$, $a_7$, $a_{13}$, $b_8$, $b_{14}$, and $b_{16}$ are estimated using empirical formulas [32]. These coefficients like 0.001, 0.0006 and 0.0009 were obtained from [32].

4. Research Design

4.1. Technical Route. The research design of this study comprised experimental parameter setting, model simulation test, and evaluation of model performance. Because ship-ship interaction is related to the encountering situation, ship speed ratio, and maritime environment, the effect of these factors on ship-ship interaction must be studied. The minimum ship distance can be determined using a discrete numerical approach to simulate various manoeuvring scenarios. Detection can be used to determine whether a ship collision occurs in each scenario and to further propose the ship-ship distance of the very large ship.

The research process is shown in Figure 4. It is well known that ship collision avoidance is very important for ship navigation task [33]. In a certain sense, the calculation of ship-ship distance will be helpful for ship collision avoidance in restricted channel.

4.2. Test Condition Setting. According to the actual conditions and test requirements of the channel, the simulation conditions involved ship type, water depth, speed, ship control method, ship-ship distance, and ship-bank distance. The simulation scheme is illustrated in Figure 5. In Figure 5(a), the ship speed is set at full ahead, and the ship-ship distance varies from 0 ship beam to 2 ship beams. From Figure 5(a) to 5(b), the ship speed varies in full ahead, half ahead, slow ahead, slow ahead, and dead slow ahead.

The ship particulars are shown in Table 1. This ship displacement exceeds 200,000 tons. The ship length is large than 300 m. The ship speed is 16 knot or nearly 8 m/s. Then, in the test flow, the speed range was 1, 2, 3, . . . , 8 m/s. At the same time, the corresponding Froude numbers are 0.148, 0.130, 0.112, . . . , 0.020.

As the ship-ship distance and bank condition are significant factors for ship hydrodynamic, the settings were achieved from discreteness and Figure 1. In order to study the effect of ship-ship distance, different ship-ship distances were set. With the same conditions, the ship-ship distance was set to 0.1, 0.2, and up to 2 the ship beam. When the distance equals to $n$ ship beam, the ship moves along the boundary of channel. The bank condition is also an important setting for ship hydrodynamic condition. The parameters of bank conditions used in the simulation are shown in Figure 3, where $y_{p3} + y_{s3} = 400$ m and $y_p + y_{s} = 380$ m. $y_{p3}$, $y_{s3}$, $y_p$, and $y_s$ were cited in equations (3) and (4).

Regarding the ship path following method, it was used to control ship’s route. The ship motion control method included course control and path following control. Course control ensures that the ship travels along the course, avoids course deflection as much as possible, and prevents collision or grounding. The path following control maintains the course while avoiding yaw as much as possible to prevent collision or grounding. The path following ignored the human factors of seaman and pilot in the navigation tasks [34]. This study adopted a path following control method. The rudder angle was solved by the following equation:

$$
\delta = -0.0038 \Delta y - 0.9941 \Delta \psi - 1.9833 \sin \Delta \psi
- (65.6903 + 13.2221 \cos \Delta \psi)r + 62.7615r^3,
$$

(5)

where $\Delta y$ is the cross track error, $\Delta \psi$ is the course deviation, and $r$ is the yaw velocity.

5. Quantitative Results

5.1. Quantitative Example of Encounter Test. The test water depth was set to 26.7 m, ship speed to 7.377 m, propeller rotation to 80 RPM, ship-ship distance to 1 ship beam, and the initial course of Ships 1 and 2 to 0° and 180°, respectively, for ship manoeuvring motion simulation. Figure 6(a) shows the
Figure 4: The technology routing for ship-ship distance modelling based on collision detection and Gaussian identification.

Figure 5: The simulation process of discrete ship speed and discrete ship-ship distance: (a) ship encountering at full ahead; (b) ship overtaking at full ahead.

Table 1: Ship particulars.

<table>
<thead>
<tr>
<th>Ship name</th>
<th>Esso</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (Ton)</td>
<td>220,000</td>
</tr>
<tr>
<td>Ship type</td>
<td>Oil tanker</td>
</tr>
<tr>
<td>Length between perpendiculars (m)</td>
<td>304.8</td>
</tr>
<tr>
<td>Ship beam (m)</td>
<td>47.17</td>
</tr>
<tr>
<td>Ship draft (m)</td>
<td>18.46</td>
</tr>
<tr>
<td>Ship speed (knot)</td>
<td>16</td>
</tr>
<tr>
<td>Nominal propeller RPM</td>
<td>80</td>
</tr>
</tbody>
</table>
ship track, and the black part in the figure is the intersection area of ships. As indicated in the figure, Ship 1 and Ship 2 moved along a planned route, and the two ships approached each other due to the suction effect where a collision then occurred. In Figure 6(b), the ships encountering in restricted channel with ship-ship distance 1.50 ship beam was shown. There was no collision in the encountering, and the intersection area is 0 m². The collision area of the intersection area at each moment was calculated to obtain the time series of the intersection area. As illustrated in Figure 6(a), the intersection varied from small to large, with a maximum value of 1782 m². The intersection area was calculated by the algorithm from [35].

Figure 7 presents the dynamics simulation results, specifically the ship hull force $Y_h$ and moment $N_h$, ship-ship interaction $Y_{sh}$ and moment $N_{sh}$, and ship-bank force $Y_{bh}$ and moment $N_{bh}$. These forces and moments were all nondimensionalized. The nondimensionalized equation of the force is $F' = F (0.5 \rho U^2 L d)^{-1}$, and the nondimensionalized equation of the moment is $N' = N (0.5 \rho U^2 L^2 d)^{-1}$. According to equation (1), due to the path following control from autopilot, the manoeuvring caused the hull force $Y_h'$ and moment $N_h'$ to fluctuate to resist the ship-ship interaction and moment. The ship-ship interaction $Y_{sh}'$ was greater than the hull force $Y_h'$, and the moment of the ship-ship interaction $N_{sh}'$ was less than the hull moment. Moreover, the ship-bank force $Y_{bh}'$ and moment $N_{bh}'$ were the smallest.

Figure 8 presents the results of the kinematic simulation for speed, course, and position. As indicated in Figures 8(a) and 8(g), the ship speed decreased; as indicated in Figure 8(c), the sway velocity fluctuated; as indicated in Figure 8(b), the ship track remained largely unchanged; as indicated in Figure 8(d), the ship-ship distance was reduced by approximately 24 m; and as indicated in Figure 8(f), the ship turned 1.5° inward due to mutual attraction.

5.2. Analysis of Ship Intersection Area. This section analysed the results of encounter test continuously. In the encountering, the ship intersected each other. Figure 9 illustrates the time series of the intersection area corresponding to different ship-ship distances at different speeds. The Froude numbers for the different speeds were 0.020–0.148. As indicated in the figure, a lower Froude number $F_n$ has longer intersection time and larger intersection area. These phenomena have been caused by the low ship speed. In addition, the intersection area has a positive correlation with the initial ship-ship distance. If the initial ship-ship distance is larger, the intersection area is larger.

In Figure 9, when initial ship-ship distance is 0.70 ship beam and the Froude number $F_n$ is 0.020, ship 1 and ship 2 began to collide in 789 s and the intersection area was increasing to 11570 m² at 937 s. At last, the collision ended at 1067 s. The intersection area was treated as an important index of collision danger level. This intersection area will be used in the identification of the ship-ship distance model.

5.3. Model Identification of Ship-Ship Distance

5.3.1. Original Model. In order to describe the relationship between ship-ship distance $S_p$ and the intersection area $S_{total}$, the Gaussian function was proposed. The original function is written in equation (6), where parameter $A$ is the model shape amplitude; $(a, b)$ is the model shape centre; $\sigma_{sp}$
Figure 7: The force and moment of ship hull, ship-ship, and ship-bank for encounter manoeuvre with ship-ship distance 1.00 ship beam: (a) ship hull force; (b) ship hull moment; (c) ship-ship force; (d) ship-ship moment; (e) ship-bank force; (f) ship-bank moment.

Figure 8: The velocities, heading, and position for encounter manoeuvre with ship-ship distance 1.00 ship beam: (a) surge velocity; (b) horizontal value of ship track; (c) sway velocity; (d) vertical axis; (e) yaw velocity; (f) heading angle; (g) ship speed; (h) rudder angle.
and $\sigma_V$ are $Sp$ and $V$ spreads of the model shape; $\theta$ is a clockwise angle rotating the shape; and $c$ is the shift of the shape amplitude:

$$S_{\text{total}} = A \cdot \exp \left\{ \left[ \frac{(Sp - a) \cos \theta + (V - b) \sin \theta}{\sigma_{Sp}} \right]^2 - \left[ \frac{- (Sp - a) \cos \theta + (V - b) \sin \theta}{\sigma_{V}} \right]^2 \right\} + c. \quad (6)$$

The constant number $a$ in equation (6) is less than the max intersection area based on the characteristics of the normal distribution. Therefore, the max intersection area in Figure 9 is the upper bound. In the identification progress, the upper bound consists in the inequality constraint in nonlinear optimization. Since the Levenberg–Marquardt algorithm cannot handle bound constraints, the trust-region-reflective (TRR) algorithm was used for identification. $A, \theta, \sigma_{Sp}, \sigma_{V}, a, b,$ and $c$ are the unknown coefficients.

The relationships between ship-ship distance and intersection area at different speeds are displayed in Figure 9. A 3D ship-ship distance model in equation (6) will be established. The object function is present in equation (7). $(\bar{S}p_t, \bar{V}_q, \bar{h}S_{\text{total}})$ is a sample point in Figure 9. Set $x = (A, \theta, \sigma_{Sp}, \sigma_{V}, a, b, c), \quad l = (0, 0, 0, 0, \min(\bar{S}p), \min(\bar{V}), 0),$ and $u = (\infty, \pi, \infty, \infty, \max(\bar{S}p), \max(\bar{V}), \infty),$ then $l \leq x \leq u$ is the constraint.

$$\min_{x} (f(x)) = \min_{A, \theta, \sigma_{Sp}, \sigma_{V}, a, b, c} \left\{ A \cdot \exp \left\{ \left[ \frac{(\bar{S}p_t - na) \cos \theta + (V - b) \sin \theta}{\sigma_{Sp}} \right]^2 - \frac{- (\bar{S}p_t - na) \cos \theta + (V - b) \sin \theta}{\sigma_{V}} \right] \right\} + c - \bar{S}_{i,\text{total}} \right\}. \quad (7)$$

5.3.2. Identification Algorithm. The flow of the TRR algorithm is shown in Table 2. The TRR consist of the trust-region (TR) algorithm and interior reflective Newton (IRN) algorithm, proposed by Coleman & Li [36]. This TRR algorithm is powerful to address constraint bound nonlinear optimization problems. As shown in equation (7), $a$ is
bounded by maximum total intersection area \( \max(S_{\text{total}}) \) as upper bound and minimum total intersection area \( \min(S_{\text{total}}) \) as lower bound. Set \( x = (a, b, c, d) \), \( l = (\min(S_{\text{total}}), 0, 0, 0) \), and \( u = (\max(S_{\text{total}}), 0, 0, 0) \), then \( l \leq x \leq u \) is the constraint.

5.4. 3D Model of Encountering Distance. Based on the Gaussian-TRR algorithm, \( A, \theta, \sigma_{Sp}, \sigma_{V}, a, b, \) and \( c \) are identified. The training data can be found in Supplementary Material. The iteration progress is shown in Figure 10. It can be found that the iteration stopped at 62 and the objective function in equation (7) converged in Figure 10(h). Finally, \( A = 1.519 \times 10^7, \theta = 90.85^\circ, \sigma_{Sp} = 3.667, \sigma_{V} = 0.3255, a = 0.1786, b = -1.000, \) and \( c = 5.356 \times 10^5 \). According to the value of \( \theta \), the shape has been rotated by 90.85° clockwise.

\[
S_{\text{total}} = 1.519 \times 10^7 \cdot \exp \left\{ -\left( \frac{0.0021 \cdot Sp^2 + 9.5064 \cdot V^2 + 0.2808 \cdot Sp \cdot V}{+2.823 \cdot Sp + 1.881 \times 10^5 \cdot V + 9.385 \times 10^8} \right) \right\} + 5.356 \times 10^5. \tag{8}
\]

Based on the model in Figure 11(b), the ship safety domain for encountering in restricted channel is proposed in Figure 12. The limited ship-ship distance is 1.50 ship beam. In Japan’s standard, the ship-ship distance for encountering is 1.52 ship beam (0.76 × 2). The grid area is prohibited zone, while the blank area is approved area. The lowest speed is set as the minimum steering speed for rudder effect. Thus, the ship can use rudder to keep its course and the yaw changes with the rudder. The proposed ship safety domain acts as a key role in the ship collision avoidance [38]. However, the ship safety domain is not permitted to be invaded. Thus, it should be smaller than ship domain. In the ship collision avoidance field, an area called prohibited zone should be protected [33, 39]. The prohibited zone is set by manual. The novel determination method in this study solves the setting of the prohibited zone.

\[
S_{\text{total}} = 2.120 \times 10^7 \cdot \exp \left\{ -\left( \frac{0.0015 \cdot Sp^2 + 3.7257 \cdot V^2 - 0.1421 \cdot Sp \cdot V}{-0.1854 \cdot Sp + 9.7223 \cdot V + 6.3427} \right) \right\} + 3.037 \times 10^5. \tag{9}
\]

5.5. 3D Model of Overtaking Distance. The overtaking situation can be also taken into study on ship-ship distance. The bank, water depth, ship speed, and path following control can be referred to Figure 5 and Section 5.2. Based on the test result, the sample data were obtained. Then, the sample data were utilized for model identification, and the TRR algorithm was used for searching the parameters in equation (6). Finally, \( A = 2.120 \times 10^7, \theta = 88.90^\circ, \sigma_{Sp} = 3.270, \sigma_{V} = 0.5246, a = 0.2500, b = -1.300, \) and \( c = 3.037 \times 10^5 \). According to the value of \( \theta \), the shape has been rotated by 88.90° clockwise. Finally, the hot map and the safety domain are present in Figure 13. And, Japan’s standard [37] of overtaking was compared with the proposed value. In Japan’s standard, the ship-ship distance for overtaking is 2.60 ship beam (1.3 × 2). In this proposed study, the distance is 2.60 ship beam too.

Based on the proposed method, the other ships encountering distance has been solved. The ships contain general cargo ship, container ship, bulk ship, and LNG ship. The results can be found in Table 3. In the table, the proposed limited Sp distances have been compared with Japanese results in [37].

6. Application in Restricted Channel

In order to verify the limited distance \( Sp \), the ship arriving and departing operation have been carried out. The wind and current have been also taken into consideration. The situations included encountering and overtaking cases, respectively. The simulation data of these situations can be found in Supplementary Materials.

6.1. Encountering Case. In this subsection, 3 encountering cases were studied. The ship type, length, beam, and displacement are shown in Table 4. The ship length varied from 35,079 to 220,000. The limited \( Sp \) was calculated by threshold in Table 3.

Using the ship particulars in Table 4, the encountering cases were executed. The starting point was set randomly at the termination of the lane. The routes were along the
Gaussian-TRR algorithm: identification process of ship-ship distance

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Input:</strong> ( S_{p}, V, S_{\text{total}} ), maximum number of TRR iterations ( (K_{\text{max}}) ), minimum number of iteration accuracy ( (\varepsilon) )</td>
</tr>
<tr>
<td>2</td>
<td><strong>Output:</strong> ( a, b, c, d, )</td>
</tr>
<tr>
<td>3</td>
<td>( S_{\text{total}} = A \cdot \exp\left[-\left\lfloor (S_{p} - a) \cos \theta + (V - b) \sin \theta \sigma_{sp} \right\rfloor^2 - \left\lfloor -(S_{p} - a) \cos \theta + (V - b) \sin \theta \sigma_{V} \right\rfloor^2 + c )</td>
</tr>
<tr>
<td>4</td>
<td>Calculate function value ( f_{k} = f(x_{k}) ) at ( x_{k} ), and its gradient ( g_{k} );</td>
</tr>
<tr>
<td>5</td>
<td><strong>Initialize:</strong> ( \mu, \eta, \gamma_{1}, \gamma_{2}, \varepsilon, 0 &lt; \mu &lt; \eta &lt; 1, \mu = 0.2, \eta = 0.5, 0 &lt; \gamma_{1} &lt; \gamma_{2} &lt; 1, )</td>
</tr>
<tr>
<td>6</td>
<td>( x_{0} = (\max(S_{\text{total}}), 0, 0, 0) ), trust-region radius is ( \Delta_{0} )</td>
</tr>
<tr>
<td>7</td>
<td>Calculate function value ( f_{k} = f(x_{k}) ) at ( x_{k} ), and its gradient ( g_{k} );</td>
</tr>
<tr>
<td>8</td>
<td><strong>while</strong> ( k &lt; K_{\text{max}}, | g_{k} | \leq \varepsilon ) <strong>do</strong></td>
</tr>
<tr>
<td>9</td>
<td>calculate Hessian matrix ( H_{k} ), get the trust-region model (sub-question) as following:</td>
</tr>
<tr>
<td>10</td>
<td>( \min{\psi_{k}(s) = f_{k} + g_{k}^T \cdot s + 0.5 \cdot s^{T} \cdot H_{k} \cdot s} ), subject to ( | s | &lt; \Delta_{k} );</td>
</tr>
<tr>
<td>11</td>
<td>Solve the trust-region model, get the solution ( s_{k} );</td>
</tr>
<tr>
<td>12</td>
<td><strong>if</strong> ( s_{k} ) is within the boundary in TR; <strong>go to</strong> next step;</td>
</tr>
<tr>
<td>13</td>
<td><strong>else</strong> update ( s_{k} ) as following: ( s_{k} = \text{best} [\psi_{k}^{1}[s_{k}], \psi_{k}^{<em>}[\Delta k - \sigma_{V}], \psi_{k}^{</em>}[\sigma_{V}]] );</td>
</tr>
<tr>
<td>14</td>
<td><strong>D_k</strong> is a diagonal matrix of the vector about bounded ( x_{k} );</td>
</tr>
<tr>
<td>15</td>
<td>Calculate function value ( f(x_{k} + s_{k}) ) and ( \psi_{k}(s_{k}) ), calculate the following equation:</td>
</tr>
<tr>
<td>16</td>
<td>( \rho_{k} = [f(x_{k}) - f(x_{k} + s_{k})]/[f(x_{k}) - \psi_{k}]^{-1} );</td>
</tr>
<tr>
<td>17</td>
<td>Update next iteration point ( x_{k+1} );</td>
</tr>
<tr>
<td>18</td>
<td><strong>if</strong> ( \rho_{k} &lt; \mu ), then ( x_{k+1} = x_{k} + \Delta_{k} );</td>
</tr>
<tr>
<td>19</td>
<td><strong>else</strong> ( x_{k+1} = x_{k} );</td>
</tr>
<tr>
<td>20</td>
<td><strong>Update</strong> next iteration trust-region radius ( \Delta_{k+1} );</td>
</tr>
<tr>
<td>21</td>
<td><strong>if</strong> ( \rho_{k} &lt; \varepsilon ), then ( \Delta_{k+1} = 2\Delta_{k} );</td>
</tr>
<tr>
<td>22</td>
<td><strong>else</strong> if ( \mu &lt; \rho_{k} &lt; \varepsilon ), then ( \Delta_{k+1} = \Delta_{k} );</td>
</tr>
<tr>
<td>23</td>
<td><strong>else</strong> if ( \rho_{k} &lt; \mu ), then ( \Delta_{k+1} = 0.25 \cdot \Delta_{k} );</td>
</tr>
<tr>
<td>24</td>
<td><strong>Go to</strong> step 4 until ( | g_{k} | \leq \varepsilon ), and get the best ( s^{*} ).</td>
</tr>
</tbody>
</table>

**Figure 10:** The identification and iteration progress of 3D ship-ship distance model parameters of encountering manoeuvre for discrete ship speed and ship-ship distance: (a) parameter \( A \); (b) parameter \( \theta \); (c) parameter \( \sigma_{Sp} \); (d) parameter \( \sigma_{V} \); (e) parameter \( a \); (f) parameter \( b \); (g) parameter \( c \); (h) the norm of the residuals.
direction of the traffic lane. And, the sample data were collected and are shown in Figure 14. In the figure, the ships were labelled by sample time. Furthermore, the ship speed, course, and distance were also presented.

In Figure 14(b), the wind direction is 225°, wind speed is 5 m/s, current direction is 150°, and current speed is 1.5 m/s. As shown in the speed and course data, the rudder-plane controlled ship course and path and that made the ship speed varying. The distance between ship 1 and ship 2 was minimized as the ships being close to each other and reached the minimum value 200m at 1363s. These 3 cases in Figures 14(a)–14(c) showed the effectiveness of the limited distance in Table 3.

6.2. Overtaking Case. In this subsection, the limited Sp for overtaking cases in Table 5 was verified. The wind and current were also taken into consideration. The tanker ships and bulk ship have been used for validation. The limited Sp was calculated by threshold in Table 3. The 3 overtaking cases
and 2 kinds of VLCC have been used, and ship particulars and limited \( Sp \) were present in Table 5.

As shown in Figure 15, the tanker ships and bulk ship have been used for verification. The wind and current conditions were consistent with encountering cases. The starting points were labelled with yellow and set randomly at the termination of the traffic lane. The VLCC is taken as the main ship type in the overtaking case.

Note. the distance in Figure 15 is \( Sp \), while distance Figure 14 is ship point distance. So, the distance magnitude in Figure 14 is larger than that in Figure 15.

As shown in Figure 15(a), the distance began to increase at 900s and ship 1 moved ahead of ship 2, then the overtaking was completed. In Figure 15(b), the bulk ship and tanker ship moved against current and wind and the distance contained a varying value at 300m and reached minimum at 1800s. There is no collision occurred. In Figure 15(c), both tanker ships were used and made the situation much complicated than Figure 15(b). The current slowed down ship speed, and the larger size made ships much more difficult to control its path. Therefore the distance did not decrease before 1500s and reached minimum at 2700s with delay. These 3 cases certificated that limited \( Sp \) in Table 3 can be effective for approaching port setting.
Wind direction: 225º, speed: 5m/s.
Current: none.

10
Ship speed (s)
5
0 0 500 1000 1500
t (s)
2000 2500 3000

Course (º)
0 500 1000 1500
t (s)
2000 2500 3000

Distance (m)
×10^4
0 500 1000 1500
t (s)
2000 2500 3000

(a)

Star point
Ship 1
Ship 2

Sampling time (minute) 0, 1, 2...

(b)

(c)

Wind direction: 225º, speed: 5m/s.
Current: 150º, speed: 1.5m/s.

10
Ship speed (s)
5
0 0 500 1000 1500
t (s)
2000 2500 3000

Course (º)
0 500 1000 1500
t (s)
2000 2500 3000

Distance (m)
×10^4
0 500 1000 1500
t (s)
2000 2500 3000

(a)

Star point
Ship 1
Ship 2

Sampling time (minute) 0, 1, 2...

(b)

(c)

Figure 14: The limited distance verification case for encountering situations: (a) bulk ship and container ship in wind; (b) container ship and tanker ship in wind and current; (c) tanker ship and general cargo ship in wind and current.

Table 5: Ship particulars for overtaking case.

<table>
<thead>
<tr>
<th>Item</th>
<th>Name</th>
<th>Limited Sp (m)</th>
<th>Length (m)</th>
<th>Bream (m)</th>
<th>Displacement (Ton)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (a)</td>
<td>Ship 1</td>
<td>122</td>
<td>304.8</td>
<td>47.17</td>
<td>220,000</td>
<td>Tanker</td>
</tr>
<tr>
<td></td>
<td>Ship 2</td>
<td>122</td>
<td>304.8</td>
<td>47.17</td>
<td>220,000</td>
<td>Tanker</td>
</tr>
<tr>
<td>Case (b)</td>
<td>Ship 1</td>
<td>166</td>
<td>292</td>
<td>45</td>
<td>181,488</td>
<td>Bulk</td>
</tr>
<tr>
<td></td>
<td>Ship 2</td>
<td>122</td>
<td>304.8</td>
<td>47.17</td>
<td>220,000</td>
<td>Tanker</td>
</tr>
<tr>
<td>Case (c)</td>
<td>Ship 1</td>
<td>156</td>
<td>329.99</td>
<td>60</td>
<td>298,997</td>
<td>Tanker</td>
</tr>
<tr>
<td></td>
<td>Ship 2</td>
<td>122</td>
<td>304.8</td>
<td>47.17</td>
<td>220,000</td>
<td>Tanker</td>
</tr>
</tbody>
</table>
Figure 15: The limited distance verification case for overtaking situations: (a) both tanker ships in wind; (b) bulk ship and tanker ship in wind and current; (c) both tanker ships in wind and current.

Figure 16: Prediction of ship heading and comparison with [26] sea trials and for the Esso-type VLCC 20°/20° zigzag test.
7. Conclusion

This study investigated proper ship-ship distance in the restricted channel by using ship motion modelling, intersection area analysis, and Gaussian-TRR identification to solve the analytical expression of ship-ship distance. This study established 3D ship-ship distance models for encountering and overtaking, respectively. Finally, a ship safety domain for ships in restricted channel was proposed. The following conclusions are drawn:

1. A novel ship-ship distance model is present in this study. The proposed method solved two problems. The ship safety domain in restricted channel covers the shortage of ship domain proposed by [38]. The prohibited zone is a vital element for ship collision...
avoidance [33, 39]. And, the proposed method for the novel model can be used and expanded for any other ship-bottom and ship-bank distance, which avoids ship grounding.

(2) The ship-ship distance model can be expressed as 3D Gaussian formula. The formula is proposed as follows: 
\[ S_{\text{total}} = A \cdot \exp \left( -\frac{(S_p - a)\cos \theta + (V - b)}{\sin \theta/\sigma_a} \right)^2 - \left( (S_p - a)\cos \theta + (V - b)\sin \theta/\sigma_v \right)^2 + c. \] 
When the 3D model has been established, the 2 \cdot b is the limited value for overtaking and encounter and the elliptical section of the 3D model plays the role as prohibit zone in [33, 39].

(3) The minimum ship-ship distances of encountering and overtaking are 1.50 and 2.4 ship beam. Both distances are consistent with the ship interaction clearance standard “Technical Standards and Commentaries for Port and Harbour Facilities in Japan” [37].

(4) The TRR algorithm identifies the analytical expressions of ship-ship distance with constraint condition effectively. The analytical expressions revealed that the prohibited area is an elliptical shape. And as the ship speed and distance change, the intersection area value will be shown as the Gaussian distribution.

It should be noted that the minimum ship-ship distance was acquired with the path following control in this study. The control variables may affect the semimajor and semiminor axes of the ship-ship distance elliptical model. The proposed method can be used in other conditions. Subsequent studies can focus on the ship-ship and ship-bank distance during overtaking or investigate the other type ship’s distance’s problem. On the other hand, following the reference [27, 29] work, the next study can also dedicate to improve the ship-ship and ship-bank effect formulas in [10, 13, 14] by optimization technique.

**Nomenclature**

- \( u \): Ship surge velocity
- \( v \): Ship sway velocity
- \( r \): Ship yaw velocity
- \( U \): Ship speed
- \( \delta \): Rudder angle
- \( x \): Horizontal value of ship track
- \( y \): Vertical value of ship track
- \( \psi \): Heading angle ship track
- \( L \): Ship length
- \( B \): Ship bream
- \( d \): Ship draft
- \( h \): Water depth
- \( Y_h \): Ship sway force in open shallow water
- \( N_{h} \): Ship yaw moment in open shallow water
- \( Y_{Sh} \): Ship sway force of ship-ship effect
- \( N_{Sh} \): Ship yaw moment of ship-ship effect
- \( Y_{Bh} \): Ship sway force of ship-bank effect
- \( N_{Bh} \): Ship yaw moment of ship-bank effect
- \( S_p \): Ship-ship distance
- \( S \): Ship-ship intersection area
- \( F_n \): Froude number on length, \( F_n = v/\sqrt{gL} \)
- \( S_{\text{max}} \): Max ship-ship intersection area
- \( A \): Parameter of the model shape amplitude
- \( (a, b) \): The model shape centre
- \( \sigma_{Sp}, \sigma_{V} \): The spreads of the model shape
- \( \theta \): A clockwise angle rotating the shape
- \( c \): The shift of the shape amplitude.

**Appendix**

**A. Details of Ship Hull Hydrodynamic Functions and Its Verification**

The Esso Bernicia hydrodynamic functions consider shallow water effect and consist of 34 hydrodynamic coefficients. The structure of the functions is proposed in the following equation [27, 30]:

\[
\begin{align*}
  gX^i &= X^i_{ur} u + \frac{1}{L} X^i_{uy} |u| \frac{d}{dt} u + X^i_{uv} v r + \frac{1}{L} X^i_{vyr} |v| \frac{d}{dt} v + \frac{1}{L} X^i_{uc} c |c| \frac{d}{dt} c + \frac{1}{L} X^i_{uc} c |c| b \\
  &+ gT^i \left( 1 - t_d \right) + X^i_{ur} \frac{d}{dt} u \xi + \frac{1}{L} X^i_{uy} |u| \frac{d}{dt} \xi + X^i_{uv} v r \xi + \frac{1}{L} X^i_{vyr} v^2 \xi^2, \\
  gY^i &= Y^i_{ur} u v + \frac{1}{L} Y^i_{ur} |v| v r + \frac{1}{L} Y^i_{ur} |c| c \frac{d}{dt} c + Y^i_{ur} c \frac{d}{dt} c + \frac{1}{L} Y^i_{ur} |c| \frac{d}{dt} c + \frac{1}{L} Y^i_{ur} |c| \frac{d}{dt} c, \\
  &+ \frac{1}{L} Y^i_{ur} |c| \frac{d}{dt} c + \frac{1}{L} Y^i_{uv} u v \xi + \frac{1}{L} Y^i_{uv} v^2 \xi^2 + \frac{1}{L} Y^i_{uv} v^2 \xi^2 + \frac{1}{L} Y^i_{uv} v^2 \xi^2 \\
  \end{align*}
\]
(A.1)

\[
\begin{align*}
  gL^i &= L^2 N^i_{ur} r + N^i_{ur} u v + L N^i_{ur} |v| v r + N^i_{ur} |c| c \frac{d}{dt} c + L N^i_{ur} c \frac{d}{dt} c + \frac{1}{L} N^i_{ur} |c| \frac{d}{dt} c + \frac{1}{L} N^i_{ur} |c| \frac{d}{dt} c \\
  &+ \frac{1}{L} N^i_{ur} |c| \frac{d}{dt} c + \frac{1}{L} N^i_{uv} u v \xi + L^2 N^i_{ur} r \xi + N^i_{uv} u v \xi + L N^i_{uv} v^2 \xi^2 + \frac{1}{L} N^i_{uv} v^2 \xi^2, \\
\end{align*}
\]
where $\delta$ is the rudder angle, $n$ is the propeller rotation rate, $\beta = \arctan (-\nu/u)$ is the drift angle, $\xi = d/(h - d)$ is the depth factor, $h$ is water depth, $d$ is the ship draft, $X_{un}$, $Y_{un}$, ..., $N_{unun}$ are the nondimensional ship hydrodynamic, $T^d$ is the nondimensional propeller thrust, $c$ is the flow velocity at rudder, and $T^d$ and $c$ are given in the following equation:

$$
\begin{align*}
T^d &= L^{-1}T_{un}u^2 + T_{unu}un + LT_{unn}n^2, \\
c^2 &= c_{un}un + c_{un}n^2,
\end{align*}
$$

where $T_{un}^d$, $T_{unu}$, and $T_{unn}$ are the hydrodynamic coefficients of the propeller and $c_{un}$ and $c_{un}$ are the hydrodynamic coefficients of the rudder.

The hydrodynamic coefficients have been calculated from PMM tests at HyA. Results from these tests have been recalculated according to an essentially quadratic fit. The coefficients are given in the so-called “bis” system. This means that forces are nondimensionalized by dividing by the product $p_m g$ and moments by $p_m g L$. $\rho$ is the water density, $m$ is the ship mass, and $g$ is the gravitational acceleration [26].

Using the hydrodynamic coefficients in [30], the zigzag test and turning circle test are reproduced and compared with the sea trials and simulations from [26]. The results are shown Figures 16 and 17. Figure 16 is the 20°/20° zigzag test. Figure 17 is the 35° turn circle test. The comparisons show that the simulations fit the sea trials and reference [26] well to some a certain extent. The prediction errors of simulations are larger than reference [26] because the codes in [30] cannot reproduce the result of formula in [26] exactly. The errors can be diminished by the optimization method by [27, 29].

B. Details of Ship–Ship Effect and Its Verification

Figure 18(a) illustrates the verification results of the ship–ship interaction force in encountering. The water depth to draft ratio in the test varies from 1.2 to 2.0, ship–ship distance was 0.5 ship length, speed ratio was 1, and ship length ratio was 1. Figure 18(b) illustrates the verification results for ship–ship interaction moment in encountering. The moment described the yawing moment. The yawing moment made the ship turning in the encountering.

Data Availability

The Excel data in CSV used to support the findings of this study are included within the Supplemental Files.

Conflicts of Interest

The author(s) declare that they have no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

All authors contributed equally to this work.

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Supplementary Materials

Supplementary materials contain sample data for the training 3D model and simulation data for application in restricted channel. The sample data are called “3D model sample data for encountering.csv.txt” and can be used for the encountering model. The simulation data are called “Application data for overtaking and encountering cases—Encountering case (a).csv.txt,” “Application data for overtaking and encountering cases—Encountering case (b),csv.txt,” “Application data for overtaking and encountering cases—Encountering case (c).csv.txt,” “Application data for overtaking and encountering cases—Overtaking case (a).csv.txt,” “Application data for overtaking and encountering cases—Overtaking case (b),csv.txt,” and “Application data for overtaking and encountering cases—Overtaking case (c).csv.txt.” The simulation data are the record of applications for overtaking and encountering cases. (Supplementary Materials)

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