Research Article

Similarity Measures between Intuitionistic Fuzzy Credibility Sets and Their Multicriteria Decision-Making Method for the Performance Evaluation of Industrial Robots

Jun Ye ^{1,2}, Shigui Du,¹ and Rui Yong¹

¹School of Civil and Environmental Engineering, Ningbo University, Ningbo 315211, China

²Department of Electrical and Information Engineering, Shaoxing University, Shaoxing 312000, China

Correspondence should be addressed to Jun Ye; yehjun@aliyun.com

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To enhance the credibility level/measure of an intuitionistic fuzzy set (IFS), this study proposes the notion of an intuitionistic fuzzy credibility set (IFCS) to express the hybrid information of a pair of a membership degree and a credibility degree and a pair of a nonmembership degree and a credibility degree. Next, we propose generalized distance and similarity measures between IFCSs and then further generalize the weighted generalized distance measure of IFCSs to the trigonometric function-based similarity measures of IFCSs, including the cosine, sine, tangent, and cotangent similarity measures based on the weighted generalized distance measure of IFCSs. Then, a multicriteria decision making (MCDM) method using the proposed similarity measures is developed in the environment of IFCSs. An illustrative example about the performance evaluation of industrial robots and comparative analysis are presented to indicate the applicability and efficiency of the developed method in the setting of IFCSs.

1. Introduction

A similarity measure is a critical mathematical tool for judging the degree of similarity between objects and plays a key role in decision making, clustering analysis, pattern recognition, and medical diagnosis. Since a fuzzy set [1] is only depicted by a membership degree, Atanassov [2] extended the fuzzy set to an intuitionistic fuzzy set (IFS) by considering both a membership degree and a nonmembership degree, which is much more practical and flexible than the traditional fuzzy set in handling vagueness and uncertainty problems. Thus, various similarity measures of IFSs have been presented in the current literature and mainly applied in decision making, medical diagnosis, clustering analysis, and pattern recognition. For instance, the Hamming and Euclidean distances and similarity measures of IFSs [3–5] and similarity measures of IFSs based on cotangent function [6] were proposed and utilized in medical diagnosis problems. Then, some similarity measures of IFSs [7–10], similarity measures of IFSs based on the Hausdorff

distance [11], distance and similarity measures of IFSs [12], and cosine measures of IFSs in vector space [13] were introduced and applied in pattern recognition. Similarity measures of IFSs based on L_p metric [14], similarity measures of IFSs based on cotangent function and the Hausdorff distance [15, 16], the measure of inaccuracy of IFSs [17], similarity measures of IFSs based on cosine function [18], and the generalized Dice similarity measure of IFSs [19] were put forward and used in decision making problems. The similarity measures of IFSs and the Dice measure of IFSs also were utilized for clustering analysis [20–22]. As the extension of IFSs, the generalized Bonferroni mean operator of fuzzy number IFSs [23] and the generalized similarity measures and aggregation operators for Pythagorean fuzzy sets (PFSs) [24, 25] were used for decision making problems.

Due to the uncertain knowledge and judgments of human cognitions in complicated decision making problems, the intuitionistic fuzzy evaluation value should be closed related to its measure/degree of credibility in the vagueness and uncertain environment to enhance the

credibility level/degree of the intuitionistic fuzzy value. Clearly, IFSS and PFSs lack the credibility level/measure, which implies their insufficiency in the uncertain information measure and expression. To overcome their insufficiency and enhance their credibility level, the intuitionistic fuzzy concept should be combined with the degrees/levels of credibility to effectively express the hybrid information of a pair of a membership degree and a credibility degree and a pair of a nonmembership degree and a credibility degree. Hence, we need to propose a new intuitionistic fuzzy framework combining the credibility degrees of the membership and the nonmembership so as to enhance the credibility level of IFSS. Motivated by this new intuitionistic fuzzy framework with the credibility level, this study firstly proposes an intuitionistic fuzzy credibility set (IFCS) notion to make IFS more credible and generalizes distance and similarity measures between IFCSs and then further generalizes the weighted generalized distance measure of IFCSs to the trigonometric function-based similarity measures of IFCSs, including the cosine, sine, tangent, and cotangent similarity measures based on the weighted generalized distance measure of IFCSs. Next, a multicriteria decision making (MCDM) method using the proposed similarity measures is developed in the environment of IFCSs. Finally, an illustrative example about the performance evaluation of industrial robots and comparative analysis are presented to indicate the applicability and efficiency of the developed method.

In this original study, the novelty and advantages are summarized as follows:

- The proposed IFCS enhances the credibility level of the intuitionistic fuzzy information with the help of the credibility degrees of a membership and a nonmembership.
- The proposed various similarity measures of IFCSs provide effective measure tools for MCDM problems with IFCS information.
- As the generalization/improvement of existing intuitionistic fuzzy MCDM methods, the developed MCDM method based on the proposed similarity measures not only strengthens the credibility level/degree of existing intuitionistic fuzzy MCDM methods but also overcomes the insufficiency of existing intuitionistic fuzzy MCDM methods.

The framework of this study is arranged as follows. Section 2 proposes a new IFCS notion and operational relations of intuitionistic fuzzy credibility numbers (IFCNs). Next, generalized distance and similarity measures between IFCSs are presented, and then the weighted generalized distance measure of IFCSs is further generalized to the trigonometric function-based similarity measures of IFCSs, including the cosine, sine, tangent, and cotangent similarity measures based on the weighted generalized distance measure of IFCSs in Section 3. In Section 4, an MCDM method using the proposed similarity measures is developed in the environment of IFCSs. In Section 5, the developed MCDM method is applied to an illustrative example about

the performance evaluation of industrial robots to show its applicability and efficiency in the environment of IFCSs, and then the comparative analysis is given to demonstrate the main highlights of working in the environment of IFCSs. Conclusions and future work are included in Section 6.

2. Intuitionistic Fuzzy Credibility Sets (IFCSs)

To depict the hybrid information of a pair of a membership degree and a credibility degree and a pair of a nonmembership degree and a credibility degree, this section proposes the definition of an IFCS as the generalization of the IFS notion.

An IFS [2] in a universe set X is denoted by $B = \{ \langle x, t_B(x), f_B(x) \rangle | x \in X \}$, where $t_B(x): X \rightarrow [0, 1]$ and $f_B(x): X \rightarrow [0, 1]$ are the membership degree and nonmembership degree of the element x to B with $0 \leq t_B(x) + f_B(x) \leq 1$ for $x \in X$.

When we consider the credibility levels/measures of the membership degree and the nonmembership degree in IFS, we can give the definition of an IFCS below.

Definition 1. Let X be a universe set. Then, an IFCS in X is defined as

$$R = \{ \langle x, (t_R(x), c_t(x)), (f_R(x), c_f(x)) \rangle | x \in X \}, \quad (1)$$

where $(t_R(x), c_t(x)): X \rightarrow [0, 1]^2$ is a pair of the membership degree $t_R(x)$ of the element x to R and the credibility degree $c_t(x)$ to $t_R(x)$ and $(f_R(x), c_f(x)): X \rightarrow [0, 1]^2$ is a pair of the nonmembership degree $f_R(x)$ of the element x to R and the credibility degree $c_f(x)$ to $f_R(x)$ for $x \in X$, along with the condition $t_R(x), c_t(x), f_R(x), c_f(x) \in [0, 1]$ and $0 \leq t_R(x) + f_R(x) \leq 1$.

For the convenient representation, the component $\langle x, (t_R(x), c_t(x)), (f_R(x), c_f(x)) \rangle$ in R is simply denoted as $r = \langle (t, c_t), (f, c_f) \rangle$, which is named IFCN.

Definition 2. Set $r_1 = \langle (t_1, c_{t1}), (f_1, c_{f1}) \rangle$ and $r_2 = \langle (t_2, c_{t2}), (f_2, c_{f2}) \rangle$ as two IFCNs and $\eta > 0$. Then, their operational relations are given as follows:

- $r_1 \supseteq r_2 \Leftrightarrow t_1 \geq t_2, c_{t1} \geq c_{t2}, f_1 \leq f_2, \text{ and } c_{f1} \leq c_{f2}$
- $r_1 = r_2 \Leftrightarrow r_1 \supseteq r_2 \text{ and } r_2 \supseteq r_1$
- $r_1 \cup r_2 = \langle (t_1 \vee t_2, c_{t1} \vee c_{t2}), (f_1 \wedge f_2, c_{f1} \wedge c_{f2}) \rangle$
- $r_1 \cap r_2 = \langle (t_1 \wedge t_2, c_{t1} \wedge c_{t2}), (f_1 \vee f_2, c_{f1} \vee c_{f2}) \rangle$
- $(r_1)^c = \langle (f_1, c_{f1}), (t_1, c_{t1}) \rangle$ (the complement of r_1)
- $r_1 \oplus r_2 = \langle (t_1 + t_2 - t_1 t_2, c_{t1} + c_{t2} - c_{t1} c_{t2}), (f_1 f_2, c_{f1} c_{f2}) \rangle$
- $r_1 \otimes r_2 = \langle (t_1 t_2, c_{t1} c_{t2}), (f_1 + f_2 - f_1 f_2, c_{f1} + c_{f2} - c_{f1} c_{f2}) \rangle$
- $\eta r_1 = \langle (1 - (1 - t_1)^\eta, 1 - (1 - c_{t1})^\eta), ((f_1)^\eta, (c_{f1})^\eta) \rangle$
- $(r_1)^\eta = \langle ((t_1)^\eta, (c_{t1})^\eta), (1 - (1 - f_1)^\eta, 1 - (1 - c_{f1})^\eta) \rangle$

3. Similarity Measures Based on the Generalized Distance of IFCSs

3.1. Generalized Distance Measure of IFCSs. For two IFCSs, we give the definition of the generalized distance measure of IFCSs below.

Definition 3. Set $R_1 = \{r_{11}, r_{12}, \dots, r_{1n}\}$ and $R_2 = \{r_{21}, r_{22}, \dots, r_{2n}\}$ as two IFCSSs, where $r_{1k} = \langle (t_{1k}, c_{t1k}), (f_{1k}, c_{f1k}) \rangle$ and $r_{2k} = \langle (t_{2k}, c_{t2k}), (f_{2k}, c_{f2k}) \rangle$ are two IFCNs, and let $\lambda \geq 1$ be

any integer value. Then, the generalized distance measure between R_1 and R_2 is defined as follows:

$$d_\lambda(R_1, R_2) = \frac{1}{2} \left\{ \left[\frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} + \left[\frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t2k}|^\lambda + |c_{f1k} - c_{f2k}|^\lambda \right) \right]^{1/\lambda} \right\}. \quad (2)$$

Especially when $\lambda = 1, 2$, equation (2) reduces to the Hamming and Euclidean distances:

$$d_1(R_1, R_2) = \frac{1}{4n} \sum_{k=1}^n (|t_{1k} - t_{2k}| + |f_{1k} - f_{2k}|) + \frac{1}{4n} \sum_{k=1}^n (|c_{t1k} - c_{t2k}| + |c_{f1k} - c_{f2k}|),$$

$$d_2(R_1, R_2) = \frac{1}{2} \left\{ \left[\frac{1}{2n} \sum_{k=1}^n (|t_{1k} - t_{2k}|^2 + |f_{1k} - f_{2k}|^2) \right]^{1/2} + \left[\frac{1}{2n} \sum_{k=1}^n (|c_{t1k} - c_{t2k}|^2 + |c_{f1k} - c_{f2k}|^2) \right]^{1/2} \right\}. \quad (3)$$

Based on the properties of distance measures [12], the generalized distance measure of IFCSSs implies the following proposition.

Proposition 1. The generalized distance measure $d_\lambda(R_1, R_2)$ for any integer value of $\lambda \geq 1$ contains the following properties:

- (d1) $d_\lambda(R_1, R_2) \in [0, 1]$
- (d2) $d_\lambda(R_1, R_2) = 0$ iff $R_1 = R_2$
- (d3) $d_\lambda(R_1, R_2) = d_\lambda(R_2, R_1)$
- (d4) If $R_1 \subseteq R_2 \subseteq R_3$ for an IFCSS R_3 , then $d_\lambda(R_1, R_3) \geq d_\lambda(R_1, R_2)$ and $d_\lambda(R_1, R_3) \geq d_\lambda(R_2, R_3)$

Proof. It is obvious that $d_\lambda(R_1, R_2)$ can satisfy the properties (d1) – (d3). Thus, we only verify the property (d4).

If $R_1 \subseteq R_2 \subseteq R_3$, then $t_3 \geq t_2 \geq t_1$, $c_{t3} \geq c_{t2} \geq c_{t1}$, $f_3 \leq f_2 \leq f_1$, and $c_{f3} \leq c_{f2} \leq c_{f1}$. Hence, there are the following inequalities:

$$\begin{aligned} |t_{1k} - t_{2k}|^\lambda &\leq |t_{1k} - t_{3k}|^\lambda, \\ |c_{t1k} - c_{t2k}|^\lambda &\leq |c_{t1k} - c_{t3k}|^\lambda, \\ |f_{1k} - f_{2k}|^\lambda &\leq |f_{1k} - f_{3k}|^\lambda, \\ |c_{f1k} - c_{f2k}|^\lambda &\leq |c_{f1k} - c_{f3k}|^\lambda, \\ |t_{2k} - t_{3k}|^\lambda &\leq |t_{1k} - t_{3k}|^\lambda, \\ |c_{t2k} - c_{t3k}|^\lambda &\leq |c_{t1k} - c_{t3k}|^\lambda, \\ |f_{2k} - f_{3k}|^\lambda &\leq |f_{1k} - f_{3k}|^\lambda, \\ |c_{f2k} - c_{f3k}|^\lambda &\leq |c_{f1k} - c_{f3k}|^\lambda. \end{aligned} \quad (4)$$

Then, we give the following results:

$$\begin{aligned}
 \frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) &\leq \frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{3k}|^\lambda + |f_{1k} - f_{3k}|^\lambda \right), \\
 \frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t2k}|^\lambda + |c_{f1k} - c_{f2k}|^\lambda \right) &\leq \frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t3k}|^\lambda + |c_{f1k} - c_{f3k}|^\lambda \right), \\
 \frac{1}{2n} \sum_{k=1}^n \left(|t_{2k} - t_{3k}|^\lambda + |f_{2k} - f_{3k}|^\lambda \right) &\leq \frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{3k}|^\lambda + |f_{1k} - f_{3k}|^\lambda \right), \\
 \frac{1}{2n} \sum_{k=1}^n \left(|c_{t2k} - c_{t3k}|^\lambda + |c_{f2k} - c_{f3k}|^\lambda \right) &\leq \frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t3k}|^\lambda + |c_{f1k} - c_{f3k}|^\lambda \right).
 \end{aligned} \tag{5}$$

Hence, there also exist the following results:

$$\begin{aligned}
 \left[\frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} &\leq \left[\frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{3k}|^\lambda + |f_{1k} - f_{3k}|^\lambda \right) \right]^{1/\lambda}, \\
 \left[\frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t2k}|^\lambda + |c_{f1k} - c_{f2k}|^\lambda \right) \right]^{1/\lambda} &\leq \left[\frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t3k}|^\lambda + |c_{f1k} - c_{f3k}|^\lambda \right) \right]^{1/\lambda}, \\
 \left[\frac{1}{2n} \sum_{k=1}^n \left(|t_{2k} - t_{3k}|^\lambda + |f_{2k} - f_{3k}|^\lambda \right) \right]^{1/\lambda} &\leq \left[\frac{1}{2n} \sum_{k=1}^n \left(|t_{1k} - t_{3k}|^\lambda + |f_{1k} - f_{3k}|^\lambda \right) \right]^{1/\lambda}, \\
 \left[\frac{1}{2n} \sum_{k=1}^n \left(|c_{t2k} - c_{t3k}|^\lambda + |c_{f2k} - c_{f3k}|^\lambda \right) \right]^{1/\lambda} &\leq \left[\frac{1}{2n} \sum_{k=1}^n \left(|c_{t1k} - c_{t3k}|^\lambda + |c_{f1k} - c_{f3k}|^\lambda \right) \right]^{1/\lambda}.
 \end{aligned} \tag{6}$$

Consequently, there are $d_\lambda(R_1, R_3) \geq d_\lambda(R_1, R_2)$ and $d_\lambda(R_1, R_3) \geq d_\lambda(R_2, R_3)$.

The verification is completed.

When one considers the importance of r_{1k} and r_{2k} ($k = 1, 2, \dots, n$), the weight of r_{1k} and r_{2k} is specified as $\eta_k \in [0, 1]$

with $\sum_{k=1}^n \eta_k = 1$. Thus, the weighted generalized distance measure between R_1 and R_2 for any integer value of $\lambda \geq 1$ is presented as follows:

$$d_{w\lambda}(R_1, R_2) = \frac{1}{2} \left\{ \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} + \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|c_{t1k} - c_{t2k}|^\lambda + |c_{f1k} - c_{f2k}|^\lambda \right) \right]^{1/\lambda} \right\}. \tag{7}$$

Clearly, equation (7) also implies the following proposition.

Proposition 2. The weighted generalized distance measure $d_{w\lambda}(R_1, R_2)$ for any integer value of $\lambda \geq 1$ contains the following properties:

- (d1) $d_{w\lambda}(R_1, R_2) \in [0, 1]$
- (d2) $d_{w\lambda}(R_1, R_2) = 0$ iff $R_1 = R_2$
- (d3) $d_{w\lambda}(R_1, R_2) = d_{w\lambda}(R_2, R_1)$
- (d4) If $R_1 \subseteq R_2 \subseteq R_3$ for an IFCS R_3 , then $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_1, R_2)$ and $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_2, R_3)$

Similar to the proof of Proposition 1, we can easily verify the properties (d1)–(d4) in Proposition 2, which are not repeated.

3.2. Similarity Measures Based on the Generalized Distance of IFCSs. Since there is the complementary relationship

$$M_{\lambda}(R_1, R_2) = 1 - d_{\lambda}(R_1, R_2) = 1 - \frac{1}{2} \left\{ \left[\frac{1}{2n} \sum_{k=1}^n (|t_{1k} - t_{2k}|^{\lambda} + |f_{1k} - f_{2k}|^{\lambda}) \right]^{1/\lambda} + \left[\frac{1}{2n} \sum_{k=1}^n (|c_{t1k} - c_{t2k}|^{\lambda} + |c_{f1k} - c_{f2k}|^{\lambda}) \right]^{1/\lambda} \right\}. \quad (8)$$

Corresponding to the complementary relationship between the distance measure and the similarity measure, the generalized distance-based similarity measure $M_{\lambda}(R_1, R_2)$ implies the following proposition.

Proposition 3. *The generalized distance-based similarity measure $M_{\lambda}(R_1, R_2)$ for any integer value of $\lambda \geq 1$ contains the following properties:*

- (M1) $M_{\lambda}(R_1, R_2) \in [0, 1]$
- (M2) $M_{\lambda}(R_1, R_2) = 1$ iff $R_1 = R_2$
- (M3) $M_{\lambda}(R_1, R_2) = M_{\lambda}(R_2, R_1)$

between the distance measure and the similarity measure [12], we can introduce the similarity measures of IFCSs.

Set $R_1 = \{r_{11}, r_{12}, \dots, r_{1n}\}$ and $R_2 = \{r_{21}, r_{22}, \dots, r_{2n}\}$ as two IFCSs, where $r_{1k} = \langle (t_{1k}, c_{t1k}), (f_{1k}, c_{f1k}) \rangle$ and $r_{2k} = \langle (t_{2k}, c_{t2k}), (f_{2k}, c_{f2k}) \rangle$ are two IFCSNs, and let $\lambda \geq 1$ be any integer value. Then, we propose the similarity measure based on the generalized distance measure between R_1 and R_2 :

(M4) If $R_1 \subseteq R_2 \subseteq R_3$ for an IFCS R_3 , then $M_{\lambda}(R_1, R_3) \leq M_{\lambda}(R_1, R_2)$ and $M_{\lambda}(R_1, R_3) \leq M_{\lambda}(R_2, R_3)$

Obviously, by equation (8) we can easily verify the above properties (M1)–(M4) regarding the complementary relationship between the similarity measure and the generalized distance measure. Hence, the proof is straightforward.

When one considers the importance of r_{1k} and r_{2k} ($k = 1, 2, \dots, n$), the weight of r_{1k} and r_{2k} is specified as $\eta_k \in [0, 1]$ with $\sum_{k=1}^n \eta_k = 1$. Thus, the similarity measure based on the weighted generalized distance with any integer value of $\lambda \geq 1$ is presented as the following formula:

$$M_{w\lambda}(R_1, R_2) = 1 - d_{w\lambda}(R_1, R_2) = 1 - \frac{1}{2} \left\{ \left[\frac{1}{2} \sum_{k=1}^n \eta_k (|t_{1k} - t_{2k}|^{\lambda} + |f_{1k} - f_{2k}|^{\lambda}) \right]^{1/\lambda} + \left[\frac{1}{2} \sum_{k=1}^n \eta_k (|c_{t1k} - c_{t2k}|^{\lambda} + |c_{f1k} - c_{f2k}|^{\lambda}) \right]^{1/\lambda} \right\}. \quad (9)$$

Obviously, the similarity measure $M_{w\lambda}(R_1, R_2)$ also implies the following proposition.

Proposition 4. *The similarity measure $M_{w\lambda}(R_1, R_2)$ for any integer value of $\lambda \geq 1$ contains the following properties:*

- (M1) $M_{w\lambda}(R_1, R_2) \in [0, 1]$
- (M2) $M_{w\lambda}(R_1, R_2) = 1$ iff $R_1 = R_2$
- (M3) $M_{w\lambda}(R_1, R_2) = M_{w\lambda}(R_2, R_1)$

(M4) If $R_1 \subseteq R_2 \subseteq R_3$ for an IFCS R_3 , then $M_{w\lambda}(R_1, R_3) \leq M_{w\lambda}(R_1, R_2)$ and $M_{w\lambda}(R_1, R_3) \leq M_{w\lambda}(R_2, R_3)$

Obviously, by equation (9), we can easily verify the properties (M1)–(M4) in Proposition 4 regarding the complementary relationship between the similarity measure and the distance measure. Thus, the proof is straightforward.

When we further generalize the weighted generalized distance measure of IFCSs to the trigonometric function-based similarity measures, we can introduce the cosine, sine,

tangent, and cotangent similarity measures based on the weighted generalized distance measure for any integer value of $\lambda \geq 1$, respectively, as follows:

$$C_{w\lambda}(R_1, R_2) = \cos\left\{\frac{\pi}{2}d_{w\lambda}(R_1, R_2)\right\} = \cos\left\{\frac{\pi}{4}\left\{\left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|t_{1k}-t_{2k}|^\lambda + |f_{1k}-f_{2k}|^\lambda\right)\right]^{1/\lambda} + \left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|c_{t1k}-c_{t2k}|^\lambda + |c_{f1k}-c_{f2k}|^\lambda\right)\right]^{1/\lambda}\right\}\right\}, \quad (10)$$

$$S_{w\lambda}(R_1, R_2) = 1 - \sin\left\{\frac{\pi}{2}d_{w\lambda}(R_1, R_2)\right\} = 1 - \sin\left\{\frac{\pi}{4}\left\{\left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|t_{1k}-t_{2k}|^\lambda + |f_{1k}-f_{2k}|^\lambda\right)\right]^{1/\lambda} + \left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|c_{t1k}-c_{t2k}|^\lambda + |c_{f1k}-c_{f2k}|^\lambda\right)\right]^{1/\lambda}\right\}\right\}, \quad (11)$$

$$T_{w\lambda}(R_1, R_2) = 1 - \tan\left\{\frac{\pi}{4}d_{w\lambda}(R_1, R_2)\right\} = 1 - \tan\left\{\frac{\pi}{8}\left\{\left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|t_{1k}-t_{2k}|^\lambda + |f_{1k}-f_{2k}|^\lambda\right)\right]^{1/\lambda} + \left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|c_{t1k}-c_{t2k}|^\lambda + |c_{f1k}-c_{f2k}|^\lambda\right)\right]^{1/\lambda}\right\}\right\}, \quad (12)$$

$$CT_{w\lambda}(R_1, R_2) = \cot\left\{\frac{\pi}{4} + \frac{\pi}{4}d_{w\lambda}(R_1, R_2)\right\} = \cot\left\{\frac{\pi}{4} + \frac{\pi}{8}\left\{\left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|t_{1k}-t_{2k}|^\lambda + |f_{1k}-f_{2k}|^\lambda\right)\right]^{1/\lambda} + \left[\frac{1}{2}\sum_{k=1}^n \eta_k\left(|c_{t1k}-c_{t2k}|^\lambda + |c_{f1k}-c_{f2k}|^\lambda\right)\right]^{1/\lambda}\right\}\right\}. \quad (13)$$

Obviously, the trigonometric function-based similarity measures also indicate the following proposition.

$T_{w\lambda}(R_1, R_2)$ and $T_{w\lambda}(R_1, R_3) \leq T_{w\lambda}(R_2, R_3)$; and $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_1, R_2)$ and $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_2, R_3)$

Proposition 5. The trigonometric function-based similarity measures $C_{w\lambda}(R_1, R_2)$, $S_{w\lambda}(R_1, R_2)$, $T_{w\lambda}(R_1, R_2)$, and $CT_{w\lambda}(R_1, R_2)$ for any integer value of $\lambda \geq 1$ contain the following properties:

(M1) $C_{w\lambda}(R_1, R_2) \in [0, 1]$, $S_{w\lambda}(R_1, R_2) \in [0, 1]$, $T_{w\lambda}(R_1, R_2) \in [0, 1]$, and $CT_{w\lambda}(R_1, R_2) \in [0, 1]$

(M2) $C_{w\lambda}(R_1, R_2) = S_{w\lambda}(R_1, R_2) = T_{w\lambda}(R_1, R_2) = CT_{w\lambda}(R_1, R_2) = 1$ iff $R_1 = R_2$

(M3) $C_{w\lambda}(R_1, R_2) = C_{w\lambda}(R_2, R_1)$, $S_{w\lambda}(R_1, R_2) = S_{w\lambda}(R_2, R_1)$, $T_{w\lambda}(R_1, R_2) = T_{w\lambda}(R_2, R_1)$, and $CT_{w\lambda}(R_1, R_2) = CT_{w\lambda}(R_2, R_1)$

(M4) If $R_1 \subseteq R_2 \subseteq R_3$ for an IFCS R_3 , then $C_{w\lambda}(R_1, R_3) \leq C_{w\lambda}(R_1, R_2)$ and $C_{w\lambda}(R_1, R_3) \leq C_{w\lambda}(R_2, R_3)$; $S_{w\lambda}(R_1, R_3) \leq S_{w\lambda}(R_1, R_2)$ and $S_{w\lambda}(R_1, R_3) \leq S_{w\lambda}(R_2, R_3)$; $T_{w\lambda}(R_1, R_3) \leq T_{w\lambda}(R_1, R_2)$ and $T_{w\lambda}(R_1, R_3) \leq T_{w\lambda}(R_2, R_3)$; $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_1, R_2)$ and $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_2, R_3)$

Proof. It is obvious that $C_{w\lambda}(R_1, R_2)$, $S_{w\lambda}(R_1, R_2)$, $T_{w\lambda}(R_1, R_2)$, and $CT_{w\lambda}(R_1, R_2)$ can satisfy the properties (M1)–(M3). Thus, we only verify the property (M4).

Since $\cos(x)$ for $x \in [0, \pi/2]$ is a decreasing function with $1 \geq \cos(x) \geq 0$, there are $C_{w\lambda}(R_1, R_3) \leq C_{w\lambda}(R_1, R_2)$ and $C_{w\lambda}(R_1, R_3) \leq C_{w\lambda}(R_2, R_3)$ based on $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_1, R_2)$, $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_2, R_3)$, and equation (10).

Since $\sin(x)$ for $x \in [0, \pi/2]$ is an increasing function with $0 \leq \sin(x) \leq 1$, there are $S_{w\lambda}(R_1, R_3) \leq S_{w\lambda}(R_1, R_2)$ and $S_{w\lambda}(R_1, R_3) \leq S_{w\lambda}(R_2, R_3)$ based on $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_1, R_2)$, $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_2, R_3)$, and equation (11).

Since $\tan(x)$ for $x \in [0, \pi/4]$ is an increasing function with $0 \leq \tan(x) \leq 1$, there are $T_{w\lambda}(R_1, R_3) \leq T_{w\lambda}(R_1, R_2)$ and $T_{w\lambda}(R_1, R_3) \leq T_{w\lambda}(R_2, R_3)$ based on $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_1, R_2)$, $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_2, R_3)$, and equation (12).

Since $\cot(x)$ for $x \in [\pi/4, \pi/2]$ is a decreasing function with $1 \geq \cot(x) \geq 0$, there are $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_1, R_2)$ and $CT_{w\lambda}(R_1, R_3) \leq CT_{w\lambda}(R_2, R_3)$ based on $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_1, R_2)$, $d_{w\lambda}(R_1, R_3) \geq d_{w\lambda}(R_2, R_3)$, and equation (13).

Thus, we complete the proof. $\square$

4. MCDM Method Based on the Proposed Similarity Measures of IFCSs

In this section, an MCDM method is developed by using the proposed similarity measures of IFCSs to choose the best alternative in a set of finite alternatives.

Assume the collection of m alternatives be denoted by $P = \{P_1, P_2, \dots, P_m\}$ in an MCDM problem, which is assessed by the set of n criteria $X = \{x_1, x_2, \dots, x_n\}$, along with the

specific weight vector $\eta = (\eta_1, \eta_2, \dots, \eta_n)$. Then, the decision makers give the evaluation value of each alternative over the criteria by the IFCN form that is composed of a pair of a membership degree and a credibility degree and a pair of a nonmembership degree and a credibility degree. Then, all IFCNs can be constructed as their decision matrix $A = (r_{jk})_{m \times n}$, where $r_{jk} = \langle (t_{jk}, c_{tjk}), (f_{jk}, c_{fjk}) \rangle$ ($j = 1, 2, \dots, m$; $k = 1, 2, \dots, n$) are IFCNs and forms the IFCS $R_j = \{r_{j1}, r_{j2}, \dots, r_{jn}\}$ for P_j .

Thus, the MCDM algorithm using one of the proposed similarity measures is demonstrated as follows:

Step 1. From the decision matrix $A = (r_{jk})_{m \times n}$, we obtain the ideal solution (the ideal IFCS) by the following form:

$$R^* = \left\{ \langle (t_1^*, c_{t1}^*), (f_1^*, c_{f1}^*) \rangle, \langle (t_2^*, c_{t2}^*), (f_2^*, c_{f2}^*) \rangle, \dots, \langle (t_n^*, c_{tn}^*), (f_n^*, c_{fn}^*) \rangle \right\} \\ \left\{ \left\langle \max_j(t_{j1}), \max_j(c_{tj1}) \right\rangle, \left\langle \min_j(f_{j1}), \min_j(c_{fj1}) \right\rangle \right\}, \\ \left\{ \left\langle \max_j(t_{j2}), \max_j(c_{tj2}) \right\rangle, \left\langle \min_j(f_{j2}), \min_j(c_{fj2}) \right\rangle \right\}, \\ \dots, \left\{ \left\langle \max_j(t_{jn}), \max_j(c_{tjn}) \right\rangle, \left\langle \min_j(f_{jn}), \min_j(c_{fjn}) \right\rangle \right\} \quad (14)$$

Step 2. Based on one of equations (9)–(13), we obtain the values of similarity measures between R_j and

Step 3. The alternatives are ranked, and the best one is chosen

Step 4. End

5. Illustrative Example and Comparative Analysis

5.1. Illustrative Example about the Performance Evaluation of Industrial Robots. In this section, the developed MCDM method is applied to the performance evaluation of industrial robots to show its applicability in the environment of IFCSs.

Suppose that there are four kinds of industrial robots (four alternatives), which are denoted by their set $P = \{P_1, P_2, P_3, P_4\}$. In the evaluation of the industrial robots, the four criteria that contain the principal performances include x_1 (dynamic accuracy), x_2 (dexterity), x_3 (working ability), and x_4 (communication and control capability). Then, the weight vector for the four criteria is specified as $\eta = (0.3, 0.25, 0.2, 0.25)$ by experts. The evaluation values of four kinds of industrial robots over the four criteria are provided by the IFCN form that is composed of a pair of a membership degree and a credibility degree and a pair of a nonmembership degree and a credibility degree, and then all IFCNs can be constructed as the following decision matrix $A = (r_{jk})_{4 \times 4}$:

$$A = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} \langle (0.65, 0.85), (0.3, 0.7) \rangle & \langle (0.72, 0.8), (0.25, 0.75) \rangle & \langle (0.6, 0.78), (0.35, 0.7) \rangle & \langle (0.6, 0.88), (0.3, 0.76) \rangle \\ \langle (0.7, 0.7), (0.2, 0.8) \rangle & \langle (0.65, 0.7), (0.3, 0.85) \rangle & \langle (0.86, 0.8), (0.1, 0.8) \rangle & \langle (0.8, 0.8), (0.1, 0.8) \rangle \\ \langle (0.6, 0.8), (0.3, 0.85) \rangle & \langle (0.75, 0.8), (0.2, 0.7) \rangle & \langle (0.85, 0.7), (0.12, 0.75) \rangle & \langle (0.7, 0.8), (0.2, 0.86) \rangle \\ \langle (0.8, 0.8), (0.1, 0.8) \rangle & \langle (0.9, 0.8), (0.1, 0.8) \rangle & \langle (0.7, 0.8), (0.25, 0.8) \rangle & \langle (0.7, 0.75), (0.2, 0.8) \rangle \end{bmatrix}. \quad (15)$$

From the decision matrix A , we first give the ideal solution (the ideal IFCs):

$$R^* = \{ \langle (0.8, 0.85), (0.1, 0.7) \rangle, \langle (0.9, 0.8), (0.1, 0.7) \rangle, \langle (0.86, 0.8), (0.1, 0.7) \rangle, \langle (0.8, 0.88), (0.1, 0.76) \rangle \}. \quad (16)$$

Then, based on equations (9)–(13) for $\lambda = 1, 2$, we obtain the values of the proposed similarity measures and decision results, which are shown in Tables 1 and 2 for the convenient comparison.

When $\lambda = 1$, all the ranking results in Table 1 are identical and the best one is P_4 , which show the effectiveness of the developed MCDM method based on the proposed similarity measures. When $\lambda = 2$, all the ranking orders in Table 2 are identical and the best one also is also P_4 , which also show the effectiveness of the developed MCDM method based on the proposed similarity measures. Then, when $\lambda = 1, 2$, the ranking results between the Hamming distance-based similarity measures and the Euclidean distance-based similarity measures indicate some difference, but the best one is

till P_4 . It is obvious that different values of λ may affect the ranking results, which are dependent on the preference value of λ chosen by experts/decision makers in actual applications.

5.2. Comparative Analysis. To conveniently compare the proposed MCDM method with the intuitionistic fuzzy MCDM method for demonstrating the importance of the credibility measure in the decision making application, we ignore all measure values of the credibility and only keep intuitionistic fuzzy values in IFCNs. In this case, the decision matrix of IFCs in the above example is reduced to the decision matrix of IFSs:

$$A' = \begin{bmatrix} R'_1 \\ R'_2 \\ R'_3 \\ R'_4 \end{bmatrix} = \begin{bmatrix} \langle (0.65, 0.3) & \langle 0.72, 0.25 \rangle & \langle 0.6, 0.35 \rangle & \langle 0.6, 0.3 \rangle \\ \langle (0.7, 0.2) & \langle 0.65, 0.3 \rangle & \langle 0.86, 0.1 \rangle & \langle 0.8, 0.1 \rangle \\ \langle 0.6, 0.3 \rangle & \langle 0.75, 0.2 \rangle & \langle 0.85, 0.12 \rangle & \langle 0.7, 0.2 \rangle \\ \langle 0.8, 0.1 \rangle & \langle 0.9, 0.1 \rangle & \langle 0.7, 0.25 \rangle & \langle 0.7, 0.2 \rangle \end{bmatrix}. \quad (17)$$

Consequently, the ideal solution (the ideal IFS) is $R'^* = \{ \langle 0.8, 0.1 \rangle, \langle 0.9, 0.1 \rangle, \langle 0.86, 0.1 \rangle, \langle 0.8, 0.1 \rangle \}$.

Thus, equations (9)–(13) in the setting of IFSs are reduced to the following similarity measures between R'_j ($j = 1, 2, 3, 4$) and R'^* :

$$M'_{w\lambda}(R'_j, R'^*) = 1 - \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{jk} - t_k^*|^\lambda + |f_{jk} - f_k^*|^\lambda \right) \right]^{1/\lambda}, \quad (18)$$

$$C'_{w\lambda}(R'_j, R'^*) = \cos \left\{ \frac{\pi}{2} \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{1k} - t_k^*|^\lambda + |f_{1k} - f_k^*|^\lambda \right) \right]^{1/\lambda} \right\}, \quad (19)$$

$$S'_{w\lambda}(R'_j, R'^*) = 1 - \sin \left\{ \frac{\pi}{2} \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} \right\}, \quad (20)$$

$$T'_{w\lambda}(R'_j, R'^*) = 1 - \tan \left\{ \frac{\pi}{4} \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} \right\}, \quad (21)$$

$$CT'_{w\lambda}(R'_j, R'^*) = \cot \left\{ \frac{\pi}{4} + \frac{\pi}{4} \left[\frac{1}{2} \sum_{k=1}^n \eta_k \left(|t_{1k} - t_{2k}|^\lambda + |f_{1k} - f_{2k}|^\lambda \right) \right]^{1/\lambda} \right\}. \quad (22)$$

Based on equations (18)–(22), for $\lambda = 1, 2$, we obtain the values of the similarity measures between R'_j ($j = 1, 2, 3, 4$) and R'^* and decision results, which are shown in Table 3.

In Table 3, all ranking orders and the best one regarding the special case of the illustrative example in the setting of IFSs are identical. Then, the ranking orders based on the

TABLE 1: The values of the proposed similarity measures and decision results for $\lambda = 1$.

Similarity measure	Measure value	Ranking	The best one
$M_{w1}(R_j, R^*)$	0.8985, 0.9100, 0.9066, 0.9389	$P_4 > P_2 > P_3 > P_1$	P_4
$C_{w1}(R_j, R^*)$	0.9873, 0.9900, 0.9893, 0.9954	$P_4 > P_2 > P_3 > P_1$	P_4
$S_{w1}(R_j, R^*)$	0.8412, 0.8591, 0.8539, 0.9041	$P_4 > P_2 > P_3 > P_1$	P_4
$T_{w1}(R_j, R^*)$	0.9201, 0.9292, 0.9265, 0.9520	$P_4 > P_2 > P_3 > P_1$	P_4
$CT_{w1}(R_j, R^*)$	0.8520, 0.8678, 0.8631, 0.9083	$P_4 > P_2 > P_3 > P_1$	P_4

TABLE 2: The values of the proposed similarity measures and decision results for $\lambda = 2$.

Similarity measure	Measure value	Ranking	The best one
$M_{w2}(R_j, R^*)$	0.8916, 0.8848, 0.8898, 0.9171	$P_4 > P_1 > P_3 > P_2$	P_4
$C_{w2}(R_j, R^*)$	0.9855, 0.9837, 0.9851, 0.9915	$P_4 > P_1 > P_3 > P_2$	P_4
$S_{w2}(R_j, R^*)$	0.8305, 0.8201, 0.8278, 0.8702	$P_4 > P_1 > P_3 > P_2$	P_4
$T_{w2}(R_j, R^*)$	0.9146, 0.9093, 0.9132, 0.9348	$P_4 > P_1 > P_3 > P_2$	P_4
$CT_{w2}(R_j, R^*)$	0.8427, 0.8337, 0.8403, 0.8776	$P_4 > P_1 > P_3 > P_2$	P_4

TABLE 3: The values of the similarity measures between R'_j and R'^* and decision results.

Similarity measure	Measure value for $\lambda = 1$	Measure value for $\lambda = 2$	Ranking	The best one
$M_{w\lambda}'(R'_j, R'^*)$	0.8053, 0.9137, 0.8808, 0.9440	0.8019, 0.8743, 0.8636, 0.9145	$P_4 > P_2 > P_3 > P_1$	P_4
$C_{w\lambda}'(R'_j, R'^*)$	0.9536, 0.9908, 0.9825, 0.9961	0.9520, 0.9806, 0.9771, 0.9910	$P_4 > P_2 > P_3 > P_1$	P_4
$S_{w\lambda}'(R'_j, R'^*)$	0.6988, 0.8649, 0.8138, 0.9121	0.6938, 0.8038, 0.7873, 0.8661	$P_4 > P_2 > P_3 > P_1$	P_4
$T_{w\lambda}'(R'_j, R'^*)$	0.8458, 0.9322, 0.9061, 0.9560	0.8431, 0.9009, 0.8924, 0.9327	$P_4 > P_2 > P_3 > P_1$	P_4
$CT_{w\lambda}'(R'_j, R'^*)$	0.7329, 0.8729, 0.8283, 0.9157	0.7288, 0.8197, 0.8058, 0.8740	$P_4 > P_2 > P_3 > P_1$	P_4

proposed MCDM method using the similarity measures of IFCSs in Table 2 are different from ones based on the MCDM method using the weighted similarity measures of IFCSs in Table 3 when $\lambda = 2$. Hence, the credibility level/measure can impact on the ranking of alternatives and indicate its importance in MCDM applications.

On the other hand, various decision making methods in the environments of IFs [3–5, 11, 12, 14], fuzzy number IFs [23], and PFSs [24, 25] cannot handle the MCDM problems with IFCS information because IFs, number IFs, and PFSs lack their credibility measures in general. Then, the proposed MCDM method based on the similarity measures of IFCSs can be used for the MCDM problems with IFCSs and make the decision results more credible and more reasonable in decision making applications.

The main advantages of working in the environment of IFCSs are reflected as follows:

- (1) IFCS enhances the credibility measures of the membership degree and the nonmembership degree to make IFs more credible. This means that the proposed IFCS contains much more useful information than IFs.
- (2) The MCDM method using the proposed similarity measures of IFCSs in this study strengthens the credibility and rationality of decision making results and demonstrates its superiority over existing intuitionistic fuzzy MDM methods without

considering their credibility measures since the credibility degrees can affect ranking results of alternatives.

6. Conclusion

To enhance the credibility level/measure of both a membership degree and a nonmembership degree, this paper proposed a new IFCS notion, the generalized distance of IFCSs, and the generalized distance-based similarity measures and then further proposed the cosine, sine, tangent, and cotangent similarity measures based on the weighted generalized distance measure of IFCSs. Next, an MCDM method using the proposed similarity measures is developed in the environment of IFCSs. Finally, the developed MCDM method is applied to an illustrative example about the performance evaluation of industrial robots to show its applicability and efficiency in the environment of IFCSs. By comparative analysis, the main highlights of working in the environment of IFCSs were indicated as follows:

- (a) In decision making process, the information expression of IFCS can enhance the credibility level of the intuitionistic fuzzy evaluation information with the help of the credibility measure
- (b) The proposed similarity measures of IFCSs can provide new mathematical tools for the effective

modeling of MCDM problems with IFCS information

- (c) The developed MCDM method based on the proposed similarity measures can strengthen the credibility of decision results and extend existing intuitionistic fuzzy MCDM methods
- (d) In decision making process, the credibility degree can affect the ranking of alternatives, which reflects the importance

In the future work, we should further extend new similarity measures between IFCSs and utilize them in pattern recognition, image processing, clustering analysis, medical diagnosis, and so on in the environment of IFCSs.

Data Availability

There is no data used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References

- [1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
- [2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, pp. 87–96, 1986.
- [3] E. Szmidt and J. Kacprzyk, "On measuring distances between intuitionistic fuzzy sets," *Notes on Intuitionistic Fuzzy Sets*, vol. 3, no. 4, pp. 1–13, 1997.
- [4] E. Szmidt and J. Kacprzyk, "Distances between intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 114, no. 3, pp. 505–518, 2000.
- [5] E. Szmidt and J. Kacprzyk, "A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning," *Lecture Notes in Computer Science*, vol. 3070, pp. 388–393, 2004.
- [6] M. Y. Tian, "A new fuzzy similarity based on cotangent function for medical diagnosis," *Advanced Modeling and Optimization*, vol. 15, no. 2, pp. 151–156, 2013.
- [7] D. Li and C. Cheng, "New similarity measures of intuitionistic fuzzy sets and application to pattern recognition," *Pattern Recognition Letters*, vol. 23, pp. 221–225, 2002.
- [8] Z. Liang and P. Shi, "Similarity measures on intuitionistic fuzzy sets," *Pattern Recognition Letters*, vol. 24, no. 15, pp. 2687–2693, 2003.
- [9] H. B. Mitchell, "On the Dengfeng-Chuntian similarity measure and its application to pattern recognition," *Pattern Recognition Letters*, vol. 24, no. 16, pp. 3101–3104, 2003.
- [10] H.-W. Liu, "New similarity measures between intuitionistic fuzzy sets and between elements," *Mathematical and Computer Modelling*, vol. 42, no. 1–2, pp. 61–70, 2005.
- [11] W.-L. Hung and M.-S. Yang, "Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance," *Pattern Recognition Letters*, vol. 25, no. 14, pp. 1603–1611, 2004.
- [12] Z. S. Xu and J. Chen, "An overview of distance and similarity measures of intuitionistic fuzzy sets," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 16, no. 04, pp. 529–555, 2008.
- [13] J. Ye, "Cosine similarity measures for intuitionistic fuzzy sets and their applications," *Mathematical and Computer Modelling*, vol. 53, no. 1–2, pp. 91–97, 2011.
- [14] W.-L. Hung and M.-S. Yang, "Similarity measures of intuitionistic fuzzy sets based on L_p metric," *International Journal of Approximate Reasoning*, vol. 46, no. 1, pp. 120–136, 2007.
- [15] M. Xia and Z. Xu, "Some new similarity measures for intuitionistic fuzzy values and their application in group decision making," *Journal of Systems Science and Systems Engineering*, vol. 19, no. 4, pp. 430–452, 2010.
- [16] P. Rajarajeswari and N. Uma, "Intuitionistic fuzzy multi similarity measure based on cotangent function," *International Journal of Engineering Research & Technology*, vol. 2, no. 11, pp. 1323–1329, 2013.
- [17] R. Verma and B. D. Sharma, "A new measure of inaccuracy with its application to multi-criteria decision making under intuitionistic fuzzy environment," *Journal of Intelligent & Fuzzy Systems*, vol. 27, no. 4, pp. 1811–1824, 2014.
- [18] J. Ye, "Similarity measures of intuitionistic fuzzy sets based on cosine function for the decision making of mechanical design schemes," *Journal of Intelligent & Fuzzy Systems*, vol. 30, pp. 151–158, 2016.
- [19] J. Ye, "Generalized Dice measures for multiple attribute decision making under intuitionistic and interval-valued intuitionistic fuzzy environments," *Neural Computing and Applications*, vol. 30, no. 12, pp. 3623–3632, 2018.
- [20] Z. S. Xu, J. Chen, and J. Wu, "Clustering algorithm for intuitionistic fuzzy sets," *Information Sciences*, vol. 19, no. 178, pp. 3775–3790, 2008.
- [21] Z. Wang, Z. Xu, S. Liu, and J. Tang, "A netting clustering analysis method under intuitionistic fuzzy environment," *Applied Soft Computing*, vol. 11, no. 8, pp. 5558–5564, 2011.
- [22] R. Yong, J. Ye, S.-G. Du, H. Zhang, L. Gu, and H. Lin, "A Dice similarity measure for TBM penetrability classification in hard rock condition with the intuitionistic fuzzy information of rock mass properties," *European Journal of Environmental and Civil Engineering*, 2019.
- [23] R. Verma, "Generalized Bonferroni mean operator for fuzzy number intuitionistic fuzzy sets and its application to multiattribute decision making," *International Journal of Intelligent Systems*, vol. 30, no. 5, pp. 499–519, 2015.
- [24] R. Verma and J. M. Merigó, "On generalized similarity measures for Pythagorean fuzzy sets and their applications to multiple attribute decision-making," *International Journal of Intelligent Systems*, vol. 34, no. 10, pp. 2556–2583, 2019.
- [25] L. Wang and N. Li, "Pythagorean fuzzy interaction power Bonferroni mean aggregation operators in multiple attribute decision making," *International Journal of Intelligent Systems*, vol. 35, no. 1, pp. 150–183, 2020.