Research Article

A Random Access Protocol Approach to Stochastic Jumping Systems with Singular Perturbations

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This paper focuses on the stochastic jumping systems with singular perturbation subject to a random access protocol. The key challenge with controller design issue of stochastic jumping systems is how to assess the coordination of communication orders. In this study, a joint Markov process is established, and a novel control law is proposed. In contrast with the existing methods, the developed controller is more general. Finally, a practical example is exhibited to show the effectiveness of the achieved theories.

1. Introduction

In practical, random faults, random communication failures, and so on are commonly encountered. In these systems, the effect of unpredictable random abrupt events can be modeled as a Markov chain model. Markov jumping systems (MJSs) consist of a series of submodels, in which the switching is determined by the Markov process. In recent years, MJSs have been successfully applied in various areas such as biological systems, networked control systems, hybrid systems, and economic systems [1–3]. Due to the great advantage of MJSs, up to now, many valuable results on the issues of MJSs have been published [4–8], including state estimation [5] and robust control [7, 8].

On the contrary, there exists a set of practical systems, whose dynamic behavior is characterized by the multiple time scales (MTSs) property. Especially, in the MTS-based systems, the small parameters lead to difficulty in system performance analysis. Therefore, it is unsolvable by means of the conventional single time-scale technique. It is well known that MTS-based systems can be described by singularly perturbed systems (SPSs). In light of its unique characteristic, a singularly perturbed parameter (SPP) is employed, where the MTS-based systems can be divided into fast and slow dynamic states [9–11]. Recently, quantities of excellent results on SPSs have been exhibited [12]. However,

when it comes to Markov jumping SPSs (MJSPSs), the investigation of relevant problems is far away from maturity. How to tackle the sophisticated systems with both SPP and transition probabilities partly is the motive of this study.

Note that the information exchange in the networked control systems may result in the heavy communication burden. These limited bandwidths may result in network-induced phenomena, such as data collisions, packet losses, and network-induced delay [13]. Recently, the protocol-based networked systems have attracted increasing interest, for instance, try-once-discard protocol [14] and round-robin protocol (RRP) [15]. Note that the RRP is a static transmission mechanism, which limits the practical application. Compared with the RRP, the random access protocol (RAP) is a dynamic case, in which the sensor node will get access to transmit data in a randomly way [16, 17]. However, to the best of our knowledge, the RAP has not been applied in MJSPSs yet, which partly motives us to fill this gap.

Inspired by the above discussion, it is imperative to study the problem of MJSPSs with RAP. To avoid data collisions, the RAP is introduced to regulate the data transmission order. With respect to the dynamic behavior of the MJSPS and the RAP, a joint Markov process is proposed. Therefore, a novel control design technique is presented, which covers the conventional case. Finally, a practical simulation is given to show the effectiveness of the

obtained theories. *Notations.* The notations are fairly standard. $\mathbb{R}^m$ refers to the m -dimensional Euclidean space. $\lambda_{\min}(L)$ denotes the minimum eigenvalue of L . $\text{diag}\{\dots\}$ represents a block diagonal matrix. $\text{mod}(y_1, y_2)$ stands for the nonnegative remainder on division of y_1 by y_2 .

2. Problem Formulations

Consider the following MSSPS described as follows:

$$\begin{cases} \alpha_1(k+1) = A_{\mu(k)}^{11}\alpha_1(k) + \varepsilon A_{\mu(k)}^{12}\alpha_2(k) + B_{\mu(k)}^1 u(k) + C_{\mu(k)}^1 v(k), \\ \alpha_2(k+1) = A_{\mu(k)}^{21}\alpha_1(k) + \varepsilon A_{\mu(k)}^{22}\alpha_2(k) + B_{\mu(k)}^2 u(k) + C_{\mu(k)}^2 v(k), \\ z(k) = D_{\mu(k)}^1 \alpha_1(k) + \varepsilon D_{\mu(k)}^2 \alpha_2(k) + F_{\mu(k)} u(k) + H_{\mu(k)} v(k), \end{cases} \quad (1)$$

where $\alpha_1(k) \in \mathbb{R}^{n_s}$ means the fast vector and $\alpha_2(k) \in \mathbb{R}^{n_f}$ stands for the slow vector, respectively. $u(k) \in \mathbb{R}^{n_u}$ symbolizes the control input, $z(k) \in \mathbb{R}^{n_z}$ signifies the output state, $\varepsilon > 0$ symbolizes a singular perturbation parameter, and $v(k) \in l_2[0, \infty)$ renders the disturbance signal. The stochastic variable $\{\mu(k), k \geq 0\}$ indicates a discrete-time Markov chain (DTMC) obeying a finite set $\mathcal{N} = \{1, 2, \dots, N\}$. Clearly, the transition probability matrix (TPM) of $\mu(k)$ is defined by $\Pi = [\varphi_{pq}]_{\mathcal{N} \times \mathcal{N}}$, where

$$\varphi_{pq} = \Pr\{\mu(k+1) = q | \mu(k) = p\}, \quad (2)$$

where $\varphi_{pq} \geq 0$ and $\sum_{q \in \mathcal{N}} \varphi_{pq} = 1, \forall p, q \in \mathcal{N}$. Therefore, it is easy to derive that $\forall p \in \mathcal{N}$, and $A_{\mu(k)}^{11}, A_{\mu(k)}^{12}, A_{\mu(k)}^{21}, A_{\mu(k)}^{22}, B_{\mu(k)}^1, B_{\mu(k)}^2, C_{\mu(k)}^1, C_{\mu(k)}^2, D_{\mu(k)}^1, D_{\mu(k)}^2, F_{\mu(k)}$, and $H_{\mu(k)}$ can be represented by $A_p^{11}, A_p^{12}, A_p^{21}, A_p^{22}, B_p^1, B_p^2, C_p^1, C_p^2, D_p^1, D_p^2, M_p$, and G_p , respectively.

Let $\alpha(k) = [\alpha_1^\top(k) \alpha_2^\top(k)]^\top$, and the MSSPS (1) can be rewritten as

$$\begin{cases} \alpha(k+1) = A_p E_\varepsilon \alpha(k) + B_p u(k) + C_p v(k), \\ z(k) = D_p E_\varepsilon \alpha(k) + M_p u(k) + G_p v(k), \end{cases} \quad (3)$$

where

$$\begin{aligned} E_\varepsilon &= \text{diag}\{I_{n_s}, \varepsilon I_{n_f}\}, \\ A_p &= \begin{bmatrix} A_p^{11} & A_p^{12} \\ A_p^{21} & A_p^{22} \end{bmatrix}, \\ B_p &= \begin{bmatrix} B_p^1 \\ B_p^2 \end{bmatrix}, \\ C_p &= \begin{bmatrix} C_p^1 \\ C_p^2 \end{bmatrix}, \\ D_p &= \begin{bmatrix} D_p^1 & D_p^2 \end{bmatrix}. \end{aligned} \quad (4)$$

The signal transmission between the plant and the controller via a shared communication networks, which may result in many network-induced phenomena, for example, data collisions. To regulate the data transmission, the RAP is applied between the sensors and the actuators. When the

RAP is triggered, only one node can access network at each time instant. Note that the RAP is a dynamic scheduling agreement. Employ a stochastic variable $v(k) \in \{1, 2, \dots, M\}$ to schedule network resources. Here, $v(k)$ is known as a homogeneous DTMC obeying a set $\mathcal{N}_c = \{1, 2, \dots, M\}$, and the TPM $\Phi = [\psi_{fg}]_{M \times M}$ is inferred as

$$\psi_{fg} = \Pr\{v(k+1) = g | v(k) = f\}, \quad (5)$$

where $\forall f, g \in \mathcal{M}$, $\psi_{fg} \in [0, 1]$, and $\sum_{g \in \mathcal{M}} \psi_{fg} = 1$.

Denoting $v(k) = [v_1^\top(k) \ v_2^\top(k) \ \dots \ v_{n_u}^\top(k)]$ and $u(k) = [u_1^\top(k) \ u_2^\top(k) \ \dots \ u_{n_u}^\top(k)]$, where $v_m(k)$ signifies the f^{th} control input state and u_n indicates the n^{th} actuator. By considering the zero-order hold technique, the f^{th} actuator $u_f(k)$ can be updated as

$$u_f(k) = \begin{cases} v_f(k), & \text{if } v(k) = f, \\ u_f(k-1), & \text{otherwise.} \end{cases} \quad (6)$$

In what follows, a Kronecker sign function is given by

$$\delta(F - \sqsupset) = \begin{cases} 1, & \text{if } F = \sqsupset, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Therefore, by the updated rule (6), for $\forall f \in \mathcal{M}$, the actuator $u(k)$ is formulated as

$$u(k) = \Phi_{v(k)} v(k) + (I - \Phi_{v(k)}) u(k-1), \quad (8)$$

where $\Phi_f \triangleq \text{diag}\{\delta_f^1, \delta_f^2, \dots, \delta_f^{n_u}\}$ ($f = 1, 2, \dots, n_u$) and $\delta_f^\sqsupset = \delta(F - \sqsupset)$. Clearly, it can be devised from (8) that $u(k) = [u_1(k-1) \ u_2(k-1) \ \dots \ u_{f-1}(k) \ v_f(k) \ u_{f+1}(k) \ \dots \ u_{n_u}(k)]^\top$.

It can be observed from (3) and (8) that two Markov processes $\mu(k)$ and $v(k)$ are involved in this paper. To reveal the relationship between the Markov processes (MPs) $\mu(k)$ and $v(k)$, the merging strategy is absorbed in Lemma 1.

Lemma 1 (see [4]). *The MPs $\mu(k)$ and $v(k)$ can be mapped to a novel MP $r(k) \in \{1, 2, \dots, S\}$, which can be described as follows:*

$$r(k) = R(\mu(k), v(k)) = \mu(k) + (v(k) - 1)N, \quad (9)$$

where $S = NM$.

In virtue of $\xi(k)$, the values of $\mu(k)$ and $v(k)$ could be acquired by $\zeta_1(r(k))$ and $\zeta_2(r(k))$:

$$\begin{aligned}\mu(k) &= \zeta_1(r(k)) = \text{mod}(r(k) - 1, N) + 1, \\ \nu(k) &= \zeta_2(r(k)) = \lfloor \frac{r(k) - 1}{N} \rfloor + 1.\end{aligned}\quad (10)$$

Obviously, the values of $r(k)$ can be determined by the pair $(\mu(k), \nu(k))$. The joint TPM of the MP $r(k)$ is expressed as

$$\begin{aligned}\theta_{hl} &= \Pr\{r(k+1) = l | r(k) = h\} \\ &= \Pr\{\varphi(k+1) = \zeta_1(l) | \varphi(k) = \zeta_1(h)\} \\ &\quad \times \Pr\{\psi(k+1) = \zeta_2(l) | \psi(k) = \zeta_2(h)\} \\ &= \pi_{\zeta_1(h)\zeta_1(l)} \tau_{\zeta_2(h)\zeta_2(l)},\end{aligned}\quad (11)$$

where $\pi_{\zeta_1(h)\zeta_1(l)}$ and $\tau_{\zeta_2(h)\zeta_2(l)}$ are in (2) and (4).

By resorting to the joint MP, the control law $v(k)$ is designed as

$$v(k) = K_{r(k)} E_\epsilon x(k), \quad (12)$$

where $K_{r(k)}$ are matrices being solved.

Remark 1. The matrix $K_{r(k)}$ in (12) is assumed to be dependent on the joint Markov process $r(k)$, which implies that $K_{r(k)}$ is characterized by both the corresponding mode $\mu(k)$ and the RAP case $\nu(k)$. Otherwise, the controller gain is reduced to $K_{\mu(k), \nu(k)}$, which will rise the difficulty in the MSSPS analysis.

According to (3), (8), and (12), the augmented MSSPS (13) is established:

$$\begin{cases} x(k+1) = \mathcal{A}_h E_\epsilon x(k) + B_i (I - \Psi_m) u(k-1) + C_i v(k), \\ z(k) = \mathcal{D}_h E_\epsilon x(k) + F_i (I - \Psi_m) u(k-1) + H_i v(k), \end{cases} \quad (13)$$

where

$$\mathcal{A}_h = \mathcal{A}_{im} = A_i + B_i \Psi_m K_h, \mathcal{D}_h = \mathcal{D}_{im} = D_i + F_i \Psi_m K_h. \quad (14)$$

Definition 1 (see [18]). The MSSPS (13) with $v(k) = 0$ is said to be stochastic stable (SS), if for any (δ_0, ϑ_0) , such that

$$\mathcal{E} \left\{ \sum_{k=0}^{\infty} \|\delta(k)\|^2 | \delta_0, \vartheta_0 \right\} < \infty. \quad (15)$$

Definition 2 (see [18]). The MSSPS (13) with is called SS with a $\mathcal{H}_\infty$ performance level γ , if the MSSPS (11) is SS, and under zero initial condition, one gets

$$\sum_{k=0}^{\infty} \mathcal{E} \{ \|\delta(k)\|^2 \} < \gamma^2 \sum_{k=0}^{\infty} \mathcal{E} \{ \|q(k)\|^2 \}. \quad (16)$$

Lemma 2 (see [11]). For symmetric matrices $\mathcal{X}_t (t = 1, 2, 3, 4)$ and matrix $\mathcal{X}_5$ satisfies

$$\begin{aligned}\mathcal{X}_1 &> 0, \\ \begin{bmatrix} \mathcal{X}_1 + \bar{\epsilon} \mathcal{X}_3 & \bar{\epsilon} \mathcal{X}_5^\top \\ * & \bar{\epsilon} \mathcal{X}_2 \end{bmatrix} &> 0, \\ \begin{bmatrix} \mathcal{X}_1 + \bar{\epsilon} \mathcal{X}_3 & \bar{\epsilon} \mathcal{X}_5^\top \\ * & \bar{\epsilon} \mathcal{X}_2 + \bar{\epsilon}^2 \mathcal{X}_4 \end{bmatrix} &> 0,\end{aligned}\quad (17)$$

such that $E_\epsilon \mathcal{X}_\epsilon = \mathcal{X}_\epsilon^\top E_\epsilon > 0$ for all $\epsilon \in (0, \bar{\epsilon}]$, where

$$\mathcal{X}_\epsilon = \begin{bmatrix} \mathcal{X}_1 + \epsilon \mathcal{X}_3 & \epsilon \mathcal{X}_5^\top \\ * & \epsilon \mathcal{X}_2 + \epsilon^2 \mathcal{X}_4 \end{bmatrix}. \quad (18)$$

Lemma 3 (see [11]). For given a scalar $\bar{\epsilon} > 0$, $\mathcal{W}_1$, $\mathcal{W}_2$, and $\mathcal{W}_3$ are with suitable dimensions. For all $\epsilon \in (0, \bar{\epsilon}]$, if $\mathcal{W}_1 + \epsilon \mathcal{W}_1 + \epsilon^2 \mathcal{W}_1 > 0$, one gets

$$\begin{aligned}\mathcal{W}_1 &> 0, \\ \mathcal{W}_1 + \bar{\epsilon} \mathcal{W}_1 &> 0, \\ \mathcal{W}_1 + \bar{\epsilon} \mathcal{W}_2 + \bar{\epsilon}^2 \mathcal{W}_3 &> 0.\end{aligned}\quad (19)$$

3. Main Results

Theorem 1. The augmented MSSPS (13) is said to be SS with a given $\mathcal{H}_\infty$ performance, if there exists symmetric matrices $\bar{P}_h > 0$, $\bar{Q}_h > 0$, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$, $\mathcal{X}_4$, and $\mathcal{X}_5$, $\bar{K}_{\epsilon f}$, one has

$$\mathcal{X}_1 > 0, \quad (20)$$

$$\begin{bmatrix} \mathcal{X}_1 + \bar{\epsilon} \mathcal{X}_3 & \bar{\epsilon} \mathcal{X}_5^\top \\ \bar{\epsilon} \mathcal{X}_5 & \bar{\epsilon} \mathcal{X}_2 \end{bmatrix} > 0, \quad (21)$$

$$\begin{bmatrix} \mathcal{X}_1 + \bar{\epsilon} \mathcal{X}_3 & \bar{\epsilon} \mathcal{X}_5^\top \\ \bar{\epsilon} \mathcal{X}_5 & \bar{\epsilon} \mathcal{X}_2 + \bar{\epsilon}^2 \mathcal{X}_4 \end{bmatrix} > 0, \quad (22)$$

$$\begin{bmatrix} \Theta_h^{1(t)} & \Theta_h^{2(t)\top} \mathcal{T}_h & \Theta_h^{3(t)\top} \mathcal{T}_h & \Theta_h^{4(t)\top} \\ * & \Theta_h^5 & 0 & 0 \\ * & * & \Theta_h^6 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (t = \ell, 1, j). \quad (23)$$

Meanwhile, the controller gains can be achieved:

$$K_h = \bar{K}_{\epsilon h} (\mathcal{Y}_1^\top + \epsilon \mathcal{Y}_1^\top)^{-1}, \quad (h \in \mathcal{S}), \quad (24)$$

where

$$\begin{aligned}
\Theta_h^{1(\ell)} &= \text{diag}\{\bar{P}_h - (\mathcal{V}_1 + \mathcal{V}_1^\top), \bar{Q}_h - (Y_h + Y_h^\top), -\gamma^2 I\}, \\
\Theta_h^{1(i)} &= \text{diag}\{\bar{P}_h - (\mathcal{X}_\varepsilon + \mathcal{X}_\varepsilon^\top), \bar{Q}_h - (Y_h + Y_h^\top), -\gamma^2 I\}, \\
\Theta_h^{1(j)} &= \text{diag}\{\bar{P}_h - (\mathcal{X}_\varepsilon + \mathcal{X}_\varepsilon^\top), \bar{Q}_h - (Y_h + Y_h^\top), -\gamma^2 I\}, \\
\Theta_h^{2(\ell)} &= [A_i \mathcal{Y}_1 + B_i \Psi_h \bar{K}_{\varepsilon f} E_0 \quad B_i(I - \Psi_h)Y_h \quad C_i], \\
\Theta_h^{2(i)} &= [A_i \mathcal{Y}_2 + B_i \Psi_h \bar{K}_{\varepsilon f} E_\varepsilon \quad B_i(I - \Psi_h)Y_h \quad C_i], \\
\Theta_h^{2(j)} &= [A_i \mathcal{Y}_3 + B_i \Psi_h \bar{K}_{\varepsilon f} E_\varepsilon \quad B_i(I - \Psi_h)Y_h \quad C_i], \\
\Theta_h^{3(\ell)} &= [\Psi_h \bar{K}_{\varepsilon f} E_0 \quad (I - \Psi_h)Y_h \quad 0], \\
\Theta_h^{3(i)} &= \Theta_h^{3(j)} \\
\Theta_h^{4(\ell)} &= [D_i \mathcal{Y}_1 + F_i \Psi_h \bar{K}_{\varepsilon f} E_0 \quad F_i(I - \Psi_h)Y_h \quad H_i], \\
\Theta_h^{4(i)} &= [D_i \mathcal{Y}_2 + F_i \Psi_h \bar{K}_{\varepsilon f} E_\varepsilon \quad F_i(I - \Psi_h)Y_h \quad H_i], \\
\Theta_h^{4(j)} &= [D_i \mathcal{Y}_3 + F_i \Psi_h \bar{K}_{\varepsilon f} E_\varepsilon \quad F_i(I - \Psi_h)Y_h \quad H_i], \\
\Theta_h^5 &= \text{diag}\{-\bar{P}_1, -\bar{P}_2, \dots, -\bar{P}_S\}, \\
\Theta_h^6 &= \text{diag}\{-\bar{Q}_1, -\bar{Q}_2, \dots, -\bar{Q}_S\}, \\
\mathcal{T}_h &= [\sqrt{\theta_{f1}} I \quad \sqrt{\theta_{f2}} I \quad \dots \quad \sqrt{\theta_{fN_r}} I], \\
\mathcal{X}_\varepsilon &= \mathcal{V}_1 + \varepsilon \mathcal{V}_2, \\
\mathcal{Y}_1 &= \mathcal{W}_1, \\
\mathcal{Y}_2 &= \mathcal{W}_1 + \varepsilon \mathcal{W}_2, \\
\mathcal{Y}_3 &= \mathcal{W}_1 + \varepsilon \mathcal{W}_2 + \varepsilon^2 \mathcal{W}_3, \\
\mathcal{V}_1 &= \begin{bmatrix} \mathcal{X}_1 & 0 \\ \mathcal{X}_5 & \mathcal{X}_2 \end{bmatrix}, \\
\mathcal{V}_2 &= \begin{bmatrix} \mathcal{X}_3 & \mathcal{X}_5^\top \\ 0 & \mathcal{X}_4 \end{bmatrix}, \\
\mathcal{W}_1 &= \begin{bmatrix} \mathcal{X}_1 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathcal{W}_2 &= \begin{bmatrix} \mathcal{X}_3 & \mathcal{X}_5^\top \\ \mathcal{X}_5 & \mathcal{X}_2 \end{bmatrix}, \\
\mathcal{W}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{X}_4 \end{bmatrix}, \\
E_0 &= \begin{bmatrix} I_{n_s} & 0 \\ 0 & 0 \end{bmatrix}, \\
E_\varepsilon &= \begin{bmatrix} I_{n_s} & 0 \\ 0 & \varepsilon I_{n_h} \end{bmatrix}.
\end{aligned} \tag{25}$$

Proof. Recalling the LMIs (20)–(22) and by means of Lemma 2, for all $\varepsilon \in (0, \bar{\varepsilon}]$, we have

$$\begin{bmatrix} \tilde{\Theta}_h^1 & \tilde{\Theta}_h^{2\top} \mathcal{T}_h & \tilde{\Theta}_h^{3\top} \mathcal{T}_h & \tilde{\Theta}_h^{4\top} \\ * & \Theta_h^5 & 0 & 0 \\ * & * & \Theta_h^6 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}
\tilde{\Theta}_h^1 &= \text{diag}\{\bar{P}_h - (\mathcal{X}_\varepsilon + \mathcal{X}_\varepsilon^\top), \bar{Q}_h - (\mathcal{X}_\varepsilon + \mathcal{X}_\varepsilon^\top), -\gamma^2 I\}, \\
\tilde{\Theta}_h^2 &= [A_i E_\varepsilon \mathcal{X}_\varepsilon + B_i \Psi_h K_h \mathcal{X}_\varepsilon^\top E_\varepsilon \quad B_i(I - \Psi_h)\mathcal{X}_\varepsilon \quad C_i], \\
\tilde{\Theta}_h^3 &= [\Psi_h K_h \mathcal{X}_\varepsilon^\top E_\varepsilon \quad (I - \Psi_h)\mathcal{X}_\varepsilon \quad 0], \\
\tilde{\Theta}_h^4 &= [D_i E_\varepsilon \mathcal{X}_\varepsilon + F_i \Psi_h K_h \mathcal{X}_\varepsilon^\top E_\varepsilon \quad F_i(I - \Psi_h)\mathcal{X}_\varepsilon \quad H_i].
\end{aligned} \tag{27}$$

By virtue of Lemma 3 and the LMIs (13)–(21), for all $\varepsilon \in (0, \bar{\varepsilon}]$, one derives the fact that $E_\varepsilon \mathcal{X}_\varepsilon = \mathcal{X}_\varepsilon^\top E_\varepsilon > 0$. Furthermore, it is well recognized that the following inequalities $(\mathcal{X}_\varepsilon^\top - \bar{P}_h)P_h(\mathcal{X}_\varepsilon - \bar{P}_h) \geq 0$ and $(\mathcal{X}_\varepsilon^\top - \bar{Q}_h)Q_h(\mathcal{X}_\varepsilon - \bar{Q}_h) \geq 0$ hold. Denote $\bar{P}_h = P_h^{-1}$ and $\bar{Q}_h = Q_h^{-1}$, and we obtain

$$\begin{bmatrix} \hat{\Theta}_h^1 & \hat{\Theta}_h^{2\top} \mathcal{T}_h & \hat{\Theta}_h^{3\top} \mathcal{T}_h & \hat{\Theta}_h^{4\top} \\ * & \Theta_h^5 & 0 & 0 \\ * & * & \Theta_h^6 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{28}$$

where

$$\begin{aligned}
\hat{\Theta}_h^1 &= \text{diag}\{-\mathcal{X}_\varepsilon^\top P_h \mathcal{X}_\varepsilon, -\mathcal{X}_\varepsilon^\top Q_h \mathcal{X}_\varepsilon, -\gamma^2 I\}, \\
\hat{\Theta}_h^2 &= [\mathcal{A}_h E_\varepsilon \mathcal{X}_\varepsilon \quad B_i(I - \Psi_h)\mathcal{X}_\varepsilon \quad C_i], \\
\hat{\Theta}_h^3 &= [\Psi_h K_h E_\varepsilon \mathcal{X}_\varepsilon \quad (I - \Psi_h)\mathcal{X}_\varepsilon \quad 0], \\
\hat{\Theta}_h^4 &= [\mathcal{D}_h E_\varepsilon \mathcal{X}_\varepsilon \quad F_i(I - \Psi_h)\mathcal{X}_\varepsilon \quad H_i], \\
\hat{\Theta}_h^5 &= \text{diag}\{-P_1^{-1}, -P_2^{-1}, \dots, -P_S^{-1}\}, \\
\hat{\Theta}_h^6 &= \text{diag}\{-Q_1^{-1}, -Q_2^{-1}, \dots, -Q_S^{-1}\}.
\end{aligned} \tag{29}$$

Recalling (28), with the help of term, $\text{diag}\{\mathcal{X}_\varepsilon^{-\top}, \mathcal{X}_\varepsilon^{-\top}, I, \dots, I\}$ and its transpose, for any $\mathcal{X}_\varepsilon = \mathcal{Y}_1 + \varepsilon \mathcal{Y}_2$, the following inequality holds:

$$\begin{bmatrix} \bar{\Theta}_h^1 & \bar{\Theta}_h^{2\top} \mathcal{T}_h & \bar{\Theta}_h^{3\top} \mathcal{T}_h & \bar{\Theta}_h^{4\top} \\ * & \Theta_h^5 & 0 & 0 \\ * & * & \Theta_h^6 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{30}$$

where

$$\begin{aligned}
\bar{\Theta}_h^1 &= \text{diag}\{-P_h, -Q_h, -\gamma^2 I\}, \\
\bar{\Theta}_h^2 &= [\mathcal{A}_h E_\epsilon \quad B_i(I - \Psi_h) \quad C_i], \\
\bar{\Theta}_h^3 &= [\Psi_h K_h E_\epsilon \quad (I - \Psi_h) \quad 0], \\
\bar{\Theta}_h^4 &= [\mathcal{D}_h E_\epsilon \quad F_i(I - \Psi_h) \quad H_i].
\end{aligned} \tag{31}$$

Next, choose a Lyapunov functional candidate as

$$V(k, x(k), u(k), r(k)) = x^\top(k)P(r(k))x(k) + u^\top(k-1)Q(r(k))u(k-1). \tag{32}$$

Calculating the time derivative of (32), one gets $V(k, x(k), u(k), r(k))$, and one has

$$\begin{aligned}
&\mathcal{E}\{V(k+1, x(k+1), u(k+1), r(k+1) = g|k, x(k), u(k), f)\}, \\
&\quad - V(k, x(k), u(k), r(k)), \\
&x^\top(k+1)\mathcal{P}_h x(k+1) - x^\top(k)P_h x(k) + u^\top(k)\mathcal{Q}_h u(k), \\
&\quad - u^\top(k-1)Q_h u(k-1), \\
&x^\top(k)(P_h + E_\epsilon \mathcal{A}_h^\top \mathcal{P}_h \mathcal{A}_h E_\epsilon)x(k) + x^\top(k)E_\epsilon \mathcal{A}_h^\top \mathcal{P}_h B_i(I - \Psi_h)u(k-1), \\
&\quad + u^\top(k-1)(I - \Psi_h)B_i^\top \mathcal{P}_h \mathcal{A}_h E_\epsilon x(k), \\
&\quad + x^\top(k)E_\epsilon \mathcal{A}_h^\top \mathcal{P}_h C_i v(k) + v^\top(k)C_i^\top \mathcal{P}_h \mathcal{A}_h E_\epsilon x(k), \\
&\quad + u^\top(k-1)(I - \Psi_h)^\top B_i^\top \mathcal{P}_h B_i(I - \Psi_h)u(k-1), \\
&\quad + u^\top(k-1)(I - \Psi_h)B_i^\top \mathcal{P}_h C_i v(k), \\
&\quad + v^\top(k)C_i^\top \mathcal{P}_h B_i(I - \Psi_h)u(k-1) + v^\top(k)C_i^\top \mathcal{P}_h C_i v(k), \\
&\quad + [\Psi_h K_h E_\epsilon x(k) + (I - \Psi_h)u(k-1)]^\top \mathcal{Q}_h, \\
&\quad \times [\Psi_h K_h E_\epsilon x(k) + (I - \Psi_h)u(k-1)], \\
&\quad + x^\top(k)E_\epsilon K_h^\top \Psi_h^\top \mathcal{Q}_h \Psi_h K_h E_\epsilon x(k), \\
&\quad + x^\top(k)E_\epsilon K_h^\top \Psi_h^\top \mathcal{Q}_h (I - \Psi_h)u(k-1), \\
&\quad + u^\top(k-1)(I - \Psi_h)\mathcal{Q}_h \Psi_h K_h E_\epsilon x(k), \\
&\quad + u^\top(k-1)(I - \Psi_h)\mathcal{Q}_h (I - \Psi_h)u(k-1).
\end{aligned} \tag{33}$$

where $\mathcal{P}_h = \sum_{g \in \mathcal{N}_r} \theta_{hl} P_g$, $\mathcal{Q}_h = \sum_{g \in \mathcal{N}_r} \theta_{hl} Q_g$.

Note that inequality (33) can be rewritten as

$$\mathcal{E}\{\Delta V(k)\} = \vartheta^\top(k) \bar{\Theta}_h \vartheta(k), \tag{34}$$

where

$$\begin{aligned}
\vartheta(k) &= [x^\top(k) \quad u^\top(k-1) \quad v^\top(k)], \\
\bar{\Theta}_h^1 &= \text{diag}\{-P_h, -Q_h, 0\}, \\
\bar{\Theta}_h &= \bar{\Theta}_h^1 + \bar{\Theta}_h^{2\top} \mathcal{P}_h \bar{\Theta}_h^2 + \bar{\Theta}_h^{3\top} \mathcal{Q}_h \bar{\Theta}_h^3.
\end{aligned} \tag{35}$$

In case of $v(k) = 0$, by resorting to (34), it is clear that

$$\mathcal{E}\{\Delta V(k)\} \leq \underline{\vartheta}^\top(k) \underline{\Theta}_h \underline{\vartheta}(k) \leq -\chi \mathcal{E}\{\|x(k)\|^2\}, \tag{36}$$

where $\underline{\vartheta}(k) = [x^\top(k) \quad u^\top(k-1)]$, $\underline{\Theta}_h = \underline{\Theta}_h^1 + \underline{\Theta}_h^{2\top} \mathcal{P}_h \underline{\Theta}_h^2 + \underline{\Theta}_h^{3\top} \mathcal{Q}_h \underline{\Theta}_h^3$, $\underline{\Theta}_h^1 = \text{diag}\{-P_h, -Q_h\}$, $\underline{\Theta}_h^2 = [\mathcal{A}_h E_\epsilon \quad B_i(I - \Psi_h)]$, $\underline{\Theta}_h^3 = [\mathcal{D}_h E_\epsilon \quad F_i(I - \Psi_h)]$, and $\omega = \min_{f \in \mathcal{S}, p, q \in \{1, 2, \dots, r\}} \{\lambda_{\min}(\underline{\Theta}_h)\}$. Apparently, condition (30) implies $\omega > 0$. Therefore, we have

$$\begin{aligned}
&\mathcal{E}\left\{\sum_{k=0}^{\infty} \|x(k)\|^2\right\} < -\frac{1}{\omega} \mathcal{E}\left\{\sum_{k=0}^{\infty} \Delta V(k)\right\}, \\
&\leq \frac{1}{\omega} \mathcal{E}\{V(0, x(0), u(0), r(0))\} < \infty.
\end{aligned} \tag{37}$$

By Definition 1, in case of $v(k) = 0$, one concludes that the MSSPS (13) is SS.

In the following, when $v(k) \neq 0$, we first denoting the H_∞ performance as

$$\mathcal{J}(T) = \mathcal{E} \left\{ \sum_{k=0}^T z^\top(k) z(k) - \gamma^2 v^\top(k) v(k) \right\}. \quad (38)$$

Taking (34) and (38) into consideration, $J(T)$ is shown as follows:

$$\mathcal{J}(T) \leq \mathcal{E} \left\{ \sum_{k=0}^T [z^\top(k) z(k) - \gamma^2 v^\top(k) v(k) + \Delta V(k)] \right\} \leq \vartheta^\top(k) \Theta_h' \vartheta(k), \quad (39)$$

where

$$\begin{aligned} \Theta_h' &= \vec{\Theta}_h + \Theta_h^{4\top} \Theta_h^4, \\ \Theta_h^4 &= [\mathcal{D}_h E_\epsilon \quad F_i(I - \Psi_h) \quad H_i]. \end{aligned} \quad (40)$$

Clearly, it is easy to derive the following condition from (30) and (39) that

$$\mathcal{J}(T) < 0. \quad (41)$$

Letting $T \rightarrow \infty$, it can be derived from (41) that

$$\sum_{k=0}^{\infty} \mathcal{E} \{ \|z(k)\|^2 \} \leq \gamma^2 \sum_{k=0}^{\infty} \mathcal{E} \{ \|v(k)\|^2 \}. \quad (42)$$

Therefore, recalling the Definition 2, we can conclude that the MSSPS (13) is SS with $\mathcal{H}_\infty$ performance. This ends our proof. $\square$

Remark 2. Remarkably, in updated actuator $u(k)$, the compensation strategy is applied to schedule the information exchange. By omitting the compensation scheme, $\forall m = 1, 2, \dots, n_u$, the received measurement signal (RMS) can be debased into

$$u_f(k) = \begin{cases} v_f(k), & \text{if } v(k) = m, \\ 0, & \text{otherwise.} \end{cases} \quad (43)$$

Consequently, the RMS is degraded as

$$u(k) = \Psi_{v(k)} v(k). \quad (44)$$

Accordingly, the closed-loop FMSSPS (13) will be reformulated as

$$\begin{cases} x(k+1) = A_i E_\epsilon x(k) + B_i \Psi_h K_h E_\epsilon x(k) + C_i v(k), \\ z(k) = D_i E_\epsilon x(k) + F_i \Psi_h K_h E_\epsilon x(k) + H_i v(k). \end{cases} \quad (45)$$

To exploit the dynamic behavior of the augmented MSSPS (45), the sufficient condition is forwarded in Corollary 1.

Corollary 1. *The augmented MSSPS (45) is called SS with a prescribed $\mathcal{H}_\infty$ performance level γ , if there exists symmetric matrices $\bar{P}_h > 0$, $\bar{Q}_h > 0$, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$, $\mathcal{X}_4$, and matrices $\mathcal{X}_5$, $\bar{K}_{\epsilon f}$, such that (20)–(22) hold and*

$$\begin{bmatrix} \Theta_h^{1(t)} & \Theta_h^{2(t)\top} \mathcal{T}_h & \Theta_h^{4(t)\top} \\ * & \Theta_h^5 & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (t = \ell, 1, j). \quad (46)$$

Meanwhile, the ϵ -dependent controller gains are achieved as

$$K_h = \bar{K}_{\epsilon f} (\mathcal{Y}_1^\top + \epsilon \mathcal{Y}_1^\top)^{-1}, \quad (47)$$

where

$$\begin{aligned} \Theta_h^{2(\ell)} &= [A_i \mathcal{Y}_1 + B_i \Psi_h \bar{K}_{\epsilon f} E_0 \quad C_i], \\ \Theta_h^{2(i)} &= [A_i \mathcal{Y}_2 + B_i \Psi_h \bar{K}_{\epsilon f} E_\epsilon \quad C_i], \\ \Theta_h^{2(j)} &= [A_i \mathcal{Y}_3 + B_i \Psi_h \bar{K}_{\epsilon f} E_\epsilon \quad C_i], \\ \Theta_h^{4(\ell)} &= [D_i \mathcal{Y}_1 + F_i \Psi_h \bar{K}_{\epsilon f} \mathcal{X}_\epsilon^\top E_0 \quad H_i], \\ \Theta_h^{4(i)} &= [D_i \mathcal{Y}_2 + F_i \Psi_h \bar{K}_{\epsilon f} \mathcal{X}_\epsilon^\top E_\epsilon \quad H_i], \\ \Theta_h^{4(j)} &= [D_i \mathcal{Y}_3 + F_i \Psi_h \bar{K}_{\epsilon f} \mathcal{X}_\epsilon^\top E_\epsilon \quad H_i]. \end{aligned} \quad (48)$$

4. Numerical Examples

Example 1. (a dc motor via gear train model (DMGTM)).

To explain the validity of the attained scheme, a practical DMGTM is borrowed from [19]. The dynamic equation is governed by

$$\begin{cases} \dot{\theta}_p(t) = x_2(t), \\ \ddot{\theta}_p(t) = \frac{g}{l} \sin(\theta_p(t)) + \frac{NK_h}{ml^2} I_a(t), \\ L_a \dot{I}_a(t) = -K_b N \dot{\theta} - R_i I_a(t) + u_1(t) + u_2(t) + \omega(t), \end{cases} \quad (49)$$

where the physical meaning of (49) is expressed in Table 1. The parameters are elicited as $g = 9.8 \text{ h/s}^2$, $K_h = 0.1 \text{ Nm/s}$, $K_b = 0.1 \text{ Vs/rad}$, $N_r = 10$, $l = 1 \text{ h}$, and $L_a = 50 \text{ mH}$. For any $i = 1, 2$, letting $x_1(t) = \theta_p(t)$, $x_2(t) = \dot{\theta}_p(t)$, and $x_3(t) = I_a(t)$, $x(t) = [x_1^\top(t) \quad x_2^\top(t) \quad x_3^\top(t)]^\top$ and $\epsilon = L_a$, and one gets

$$E_e \dot{x}(t) = A_i x(t) + B_i u(t) + C_i \omega(t), \quad (50)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{l} & 0 & \frac{NK_h}{ml^2} \\ 0 & -K_b N_r & -R_i \end{bmatrix}, \quad (51)$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix},$$

$$C_i = [0 \ 0 \ 1]^\top, \quad (i = 1, 2),$$

$$E_e = \text{diag}\{1, 1, \epsilon\}.$$

By means of a zero-order and sampling period $T = 0.05\text{s}$, (49) can be discretized into the following model:

$$x(k+1) = A_i E_e x(k) + B_i u(k) + C_i v(k), \quad (52)$$

where

$$A_1 = \begin{bmatrix} 1.0122 & 0.0499 & 0.0184 \\ 0.4888 & 0.9939 & 0.6302 \\ -0.1800 & -0.6302 & 7.0938 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.0122 & 0.0499 & 0.0142 \\ 0.4894 & 0.9981 & 0.4316 \\ -0.1389 & -0.4316 & 2.5584 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.0003 & 0.0003 \\ 0.0184 & 0.0184 \\ 0.6269 & 0.6269 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.0003 & 0.0003 \\ 0.0142 & 0.0142 \\ 0.4290 & 0.4290 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.0003 \\ 0.0184 \\ 0.6269 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.0003 \\ 0.0184 \\ 0.6269 \end{bmatrix},$$

$$D_1 = [1 \ 1 \ 1],$$

$$D_2 = [1 \ 1 \ 1],$$

$$F_1 = [0.2 \ 0.3],$$

$$F_2 = [0.3 \ 0.7],$$

$$H_1 = 0.5,$$

$$H_2 = 0.4.$$

TABLE 1: The meaning of parameters.

$\theta_p(t)$	Angle of the pendulum
$I_a(t)$	Current of the motor
K_m	Motor torque constant
l	Length of shaft
$R_i (i = 1, 2)$	Resistance
$\dot{\theta}_p(t)$	Angular velocity
g	Gravitational constant
K_b	Back emf constant
N_r	Gear ratio
L_a	Inductance

FIGURE 1: The possible mode switching of MP $r(k)$.

FIGURE 2: The evolution of states $x_1(k)$.

The TPM of the corresponding FMSSPS (1) is selected as $\Pi = \begin{bmatrix} 0.25 & 0.75 \\ 0.65 & 0.35 \end{bmatrix}$. For another MP Φ in the RAP (6), the

FIGURE 3: The evolution of states $x_2(k)$ over 50 realizations. FIGURE 4: The evolution of states $x_3(k)$ over 50 realizations.

TPM is chosen as $\Phi = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$. Consequently, by virtue of Lemma 1, the joint TPM can be inferred as

$$\Theta = \begin{bmatrix} 0.1 & 0.3 & 0.15 & 0.45 \\ 0.26 & 0.14 & 0.39 & 0.21 \\ 0.125 & 0.375 & 0.125 & 0.375 \\ 0.325 & 0.175 & 0.325 & 0.175 \end{bmatrix}. \quad (54)$$

Letting $\epsilon = 0.05$ and $\gamma = 5$. In view of LMIs of Theorem 1, the controller gains K_h can be listed as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} -143.2309 & 30.8397 & 9.2890 \\ 0 & 0 & 0 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -149.0814 & 27.7277 & 7.4494 \\ 0 & 0 & 0 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} 0 & 0 & 0 \\ -151.4691 & 35.2964 & 9.9320 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} 0 & 0 & 0 \\ -154.4086 & 28.5113 & 7.4207 \end{bmatrix}. \end{aligned} \quad (55)$$

In this section, according to the TPMs of corresponding plant and RAP, Figure 1 depicts the possible mode switching of Markov process $r(k)$. The evolution of states $x_1(k)$, $x_2(k)$, and $x_3(k)$ over 50 realizations is depicted in Figures 2–4,

FIGURE 5: The control input $u(k)$ over 50 realizations.

FIGURE 6: The output $z(k)$ over 50 realizations.

respectively. With the designed control law, the control input over 50 realizations is shown in Figure 5. Meanwhile, the evolution of the output $z(k)$ over 50 realizations is shown in Figure 6. From the above results, it is obvious that the RAP is effective in regulating the data transmission.

5. Conclusions

This paper has studied a type of MJSPS with RAP. A dynamic RAP was adopted to regulate the data transmission, in which only one sensor node has gained the access to transmit data each time. Furthermore, a joint Markov process is established, and sufficient conditions are achieved. Finally, a practical example has been exhibited to show the effectiveness of the achieved theories. Besides, to extend the

achieved results to sliding mode-based filter/controller is an issue [20, 21], and we will devote ourselves to tackling in near future.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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