Research Article

A Hybrid Whale Optimization with Seagull Algorithm for Global Optimization Problems

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Seagull optimization algorithm (SOA) inspired by the migration and attack behavior of seagulls in nature is used to solve the global optimization problem. However, like other well-known metaheuristic algorithms, SOA has low computational accuracy and premature convergence. Therefore, in the current work, these problems are solved by proposing the modified version of SOA. This paper proposes a novel hybrid algorithm, called whale optimization with seagull algorithm (WSOA), for solving global optimization problems. The main reason is that the spiral attack prey of seagulls is very similar to the predation behavior of whale bubble net, and the WOA has strong global search ability. Therefore, firstly, this paper combines WOA's contraction surrounding mechanism with SOA's spiral attack behavior to improve the calculation accuracy of SOA. Secondly, the levy flight strategy is introduced into the search formula of SOA, which can effectively avoid premature convergence of algorithms and balance exploration and exploitation among algorithms more effectively. In order to evaluate the effectiveness of solving global optimization problems, 25 benchmark test functions are tested, and WSOA is compared with seven famous metaheuristic algorithms. Statistical analysis and results comparison show that WSOA has obvious advantages compared with other algorithms. Finally, four engineering examples are tested with the proposed algorithm, and the effectiveness and feasibility of WSOA are verified.

1. Introduction

Over the past ten years, the metaheuristic algorithm has become very popular. For example, it has been widely used to find solutions to many complex problems in engineering and computer science. The main reasons for its popularity are flexibility, gradient-free mechanism, and avoiding falling into local optimization. In other words, metaheuristic algorithms only need to look at the input and output to consider optimization problems and do not need to calculate the derivative of search space, which makes them more flexible in solving various optimization problems.

Compared with traditional optimization algorithms, the metaheuristic algorithm can solve various optimization problems more effectively. Inspired by the intelligent behaviors and evolution laws of natural swarm, this kind of algorithm has attracted the attention of scholars, so many well-known swarm intelligence algorithms have been

proposed. For example, Genetic Algorithm (GA) [1] is a random search algorithm, which mainly imitates the natural selection in the biological world. The main reason for the success of GA is that the behaviors of selection, replication, and mutation are random, which can help the algorithm avoid falling into local optimum. Particle Swarm Optimization (PSO) algorithm [2] simulates the foraging behaviors of birds or fish in nature. Ant Colony Optimization (ACO) algorithm [3] is inspired by the behaviors of ants to find the shortest path to food. Grey Wolf Optimization (GWO) algorithm [4] simulates the hierarchical mechanism and predation behaviors of grey wolf population in nature. Moth-Flame Optimization (MFO) algorithm [5] is mainly derived from the simulation of the lateral positioning flight mode of moths. Whale Optimization Algorithm (WOA) [6] simulates the predatory behaviors of humpback whale populations in nature. Squirrel Search Algorithm (SqSA) [7] is mainly inspired by squirrel's foraging behavior.

The inspiration of Sea Lion Optimization (SLnO) algorithm [8] mainly comes from the hunting behavior of sea lions. Bald Eagle Search (BES) [9] is mainly inspired by vulture hunting strategy. Chimp optimization algorithm (COA) [10] is inspired by the behavior of chimpanzee's individual intelligence and sexual motivation in hunting. Artificial Electric Field Algorithm (AEFA) [11] is mainly inspired by Coulomb's electrostatic force law and Newton's law of motion. Invasive Weed Optimization (IWO) [12] algorithm is mainly inspired by the process of weed invasion in nature. Artificial Bee Colony (ABC) [13] simulates the hunting honey behaviors of bees in nature.

Although each metaheuristic algorithm has its own unique advantages, according to the theorem of no free lunch [14], no algorithm can solve all optimization problems [15]. The performance of the metaheuristic algorithm mainly depends on its exploration and exploitation ability. Therefore, many researchers constantly propose new algorithms and improve the original algorithm. For example, in order to improve the searching ability of grey wolves, Gupta and Deep proposed a novel random walk grey wolf optimizer in 2018 [16]. In 2019, the improved SSA tried to balance the exploration and exploitation of the algorithm to avoid the disadvantages of suboptimal solution and low accuracy [17]. In 2020, a new grey wolf optimization algorithm based on memory was proposed [18]. In 2020, a memory-guided sine and cosine algorithm was improved [19]. In 2020, an Archimedes Optimization Algorithm (AOA) was proposed to solve the optimization problem [20]. Hussien et al. proposed a new binary whale optimization algorithm for discrete optimization problems in 2020 [21]. Among them, combining two or more metaheuristic algorithms into a hybrid algorithm has attracted more and more attention in the field of optimization algorithms. For example, Gupta and Deep proposed a hybrid adaptive sine and cosine algorithm based on reverse learning in 2019 to solve the defects of the original algorithm such as low diversity and local optimal stagnation [22]. In 2019, a hybrid grey wolf optimizer with mutation operator was proposed to avoid the algorithm falling into local optimal solution [23]. Compared with the single algorithm, the hybrid algorithm can find the global optimal solution more effectively. In this paper, the contraction surrounding mechanism of WOA is combined with the spiral attack behaviors of SOA in order to obtain better performance.

Inspired by seagull migration and attack behaviors in nature, Indian scholar Gaurav Dhiman proposed a novel bionic algorithm called seagull optimization algorithm (SOA) in 2019 [24]. The most important things about seagulls are their migration and attack behaviors. Migration behavior is that seagulls migrate from one place to another full of food. Attack behavior is that seagulls often attack other birds at sea during migration, and they will attack their prey in a spiral natural shape. Since the migration behaviors of seagulls are a large-scale flight, they are regarded as global search. In contrast, the attack behaviors of seagulls are considered as local search. Therefore, SOA based on simulating seagull migration and attack behaviors is also regarded as global search and local searching process.

In recent years, scholars have conducted more extensive research on SOA. Jiang et al. studied a hybrid classification method based on the oppositional seagull optimization algorithm [25]. Jia et al. proposed a new hybrid seagull optimization algorithm for feature selection [26]. Dhiman et al. studied Emo SOA: a new evolutionary multitarget gull global optimization algorithm [27]. Panagant et al. studied the seagull optimization algorithm to solve the design optimization problem in the real world [28].

Based on the characteristics of SOA and WOA, a hybrid whale optimization with seagull algorithm (WSOA) is proposed to improve the potential of SOA. In view of the shortcomings of SOA, such as premature convergence and low computational accuracy, this paper combines the contraction surrounding mechanism of whale optimization algorithm with the spiral attack behavior of seagull optimization algorithm to improve the computational accuracy of SOA. The core idea of this hybrid technology is to make use of the strong global search ability of WOA and the fast convergence ability of SOA and effectively accelerate the search ability of WSOA. In addition, the levy flight strategy is introduced into the search formula of SOA, which can effectively avoid premature convergence of algorithms and balance exploration and exploitation among algorithms more effectively.

The main contributions of this paper can be summarized as follows: Firstly, a new hybrid whale optimization with seagull algorithm (WSOA) for optimization problems is proposed. WSOA takes advantage of the strong global search ability of WOA and the fast convergence ability of SOA. In addition, the levy flight strategy is introduced into the search formula of SOA, which can effectively avoid premature convergence of algorithms and balance exploration and exploitation among algorithms more effectively. Secondly, in order to verify the performance of WSOA, this paper verifies 25 benchmark test functions. Finally, the proposed WSOA is used to solve four constrained engineering optimization problems, which show that the proposed algorithm has strong performance.

The rest of this paper is structured as follows: Section 2 briefly introduces the original seagull optimization algorithm; Section 3 briefly introduces the original whale optimization algorithm; Section 4 introduces the hybrid whale optimization with seagull algorithm (WSOA); Section 5 is the experimental results and analysis; Section 6 introduces the limitations of WSOA; Section 7 is the conclusion of this paper and future work.

2. Seagull Optimization Algorithm (SOA)

Seagull optimization algorithm mainly simulates the migration and attack behaviors of seagulls in nature. Seagulls are a kind of seabird all over the world, with various species, which mainly feed on insects, fish, reptiles, amphibians, and earthworms. Seagulls are very clever birds that use their wisdom to find food and attack prey. For example, they use breadcrumbs to attract fish and their feet to imitate the sound of rain to lure earthworms hidden underground. Migration and attack are the most important behaviors of

seagulls. Migration behaviors are defined as the source of food for seagulls. Attack behaviors are defined as the attack behaviors from the seagulls against the migrating birds at sea.

During the migration process, the algorithm simulates how seagulls move from one place to another. At this stage, seagulls must meet the following conditions.

In order to avoid collisions between seagulls and adjacent seagulls, variable A is added to calculate the new search agent position.

$$\vec{C}_s = A \times \vec{P}_s(x), \quad (1)$$

where $\vec{C}_s$ represents the location of the search agent that does not collide with other search agents, $\vec{P}_s$ represents the current location of the search agent, x represents the current iteration times, and A represents the mobile behavior of the search agent in a given search space.

$$A = f_c - \left(x \times \left(\frac{f_c}{\text{Max}_{\text{iteration}}} \right) \right) \quad (2)$$

where: $x = 0, 1, 2, \dots, \text{Max}_{\text{iteration}}$,

where f_c is introduced to control the use frequency of variable A , which is linearly reduced from f_c to 0, and in this paper f_c is set to 2.

After avoiding collisions between seagulls, the search agent moves in the direction of the best neighbor.

$$\vec{M}_s = B \times (\vec{P}_{bs}(x) - \vec{P}_s(x)), \quad (3)$$

where $\vec{M}_s$ represents the position of search agent $\vec{P}_s$ towards the best-fit search agent $\vec{P}_{bs}$ (i.e., the best seagull with a small fitness value). The behavior of B is randomized, which is responsible for proper balancing between exploration and exploitation. B is calculated as

$$B = 2 \times A^2 \times \text{rd}, \quad (4)$$

where rd is a random number that lies in the range of $[0, 1]$.

Finally, the search agent can update its position relative to the best search agent.

$$\vec{D}_s = |\vec{C}_s + \vec{M}_s|, \quad (5)$$

where $\vec{D}_s$ represents the distance between the search agent and best-fit search agent (i.e., best seagull whose fitness value is less).

In the process of attacking prey, seagulls will spiral motion behavior in the air. This behavior in x , y , and z planes is described as follows:

$$x' = r \times \cos(k), \quad (6)$$

$$y' = r \times \sin(k), \quad (7)$$

$$z' = r \times k, \quad (8)$$

$$r = u \times e^{kv}, \quad (9)$$

where r is the radius of each turn of the spiral and k is a random number in the range $[0, 2\pi]$. u and v are constants to define the spiral shape, and e is the base of the natural logarithm. The updated position of search agent is calculated using equations (5)–(9).

$$\vec{P}_s(x) = (\vec{D}_s \times x' \times y' \times z') + \vec{P}_{bs}(x), \quad (10)$$

where $\vec{P}_s(x)$ saves the best solution and updates the position of other search agents. The pseudocode for the SOA is provided in Algorithm 1.

3. Whale Optimization Algorithm (WOA)

Whale optimization algorithm (WOA) is a new meta-heuristic algorithm that was proposed by Australian scholar Mirjalili and others in 2016. The main inspiration of the algorithm is to simulate the predation behaviors of hump-back whale population and update the position of candidate solution through the process of whale population encircling prey, spiral updating position, and finding prey. The first stage of the algorithm is to surround prey and spiral bubble net attack (exploitation phase); the second stage is whales randomly looking for food (exploration phase). The following describes the detailed steps of the WOA.

3.1. Encircling Prey: If $p < 0.5$ and $|A| < 1$. Whales can identify the location of prey and surround them. Assuming that the optimal position in the current population is prey, other whale individuals surround the optimal individuals. Use formula (10) to update the position:

$$\vec{D} = |\vec{C} \cdot \vec{X}^*(t) - \vec{X}(t)|, \quad (11)$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D}, \quad (12)$$

where t is the current iteration number; $\vec{X}^*$ is the prey position; and $\vec{A}$ and $\vec{C}$ are coefficient vectors, and $\vec{A}$ and $\vec{C}$ can be expressed as

$$\begin{aligned} \vec{A} &= 2\vec{a} \cdot \vec{r}_1 - \vec{a}, \\ \vec{C} &= 2 \cdot \vec{r}_2, \end{aligned} \quad (13)$$

where $\vec{r}_1$ and $\vec{r}_2$ are the random vectors between $[0, 1]$; is the convergence factor, which linearly decreases from 2 to 0 as the number of iterations increases; the formula is defined as

Input: Seagull population $\vec{P}_s$

Output: Optimal search agent $\vec{P}_{bs}$

```

(1) Procedure SOA
(2) Initialize the parameters  $A$ ,  $B$  and  $\text{Max}_{\text{iteration}}$ 
(3) Set  $f_c \leftarrow 2$ 
(4) Set  $u \leftarrow 1$ 
(5) Set  $v \leftarrow 1$ 
(6) While ( $x < \text{Max}_{\text{iteration}}$ ) do
(7)  $\vec{P}_{bs} \leftarrow \text{ComputeFitness}(\vec{P}_s)$  / * Calculate fitness values of search agent using ComputeFitness * // * Migration behavior * /
(8)  $\text{rd} \leftarrow \text{Rand}(0, 1)$  / * To generate the random number in range  $[0, 1]$  * /
(9)  $k \leftarrow \text{Rand}(0, 2\pi)$  / * To generate the random number in range  $[0, 2\pi]$  * // * Attacking behavior * /
(10)  $r \leftarrow u \times e^{kv}$  / * To generate the spiral behavior during migration * /
(11) Calculate the distance  $\vec{D}_s$  using equation (5)
(12)  $P \leftarrow x' \times y' \times z'$  / * Compute  $x$ ,  $y$ ,  $z$  planes using equations (6)–(9) * /
(13)  $\vec{P}_s(x) \leftarrow (\vec{D}_s \times P) + \vec{P}_{bs}$ 
(14)  $x \leftarrow x + 1$ 
(15) end while
(16) return  $\vec{P}_{bs}$ 
(17) end procedure

(1) Procedure  $\text{ComputeFitness}(\vec{P}_s)$ 
(2) for  $i \leftarrow 1$  to  $n$  do / * Here,  $n$  represents the dimension of a given problem * /
(3)  $\text{FIT}[i] \leftarrow \text{FitnessFunction}(\vec{P}_s(i, :))$  / * Calculate the best fitness of each individual * /
(4) end for
(5)  $\text{FIT}_{s_{\text{best}}} \leftarrow \text{BEST}(\text{FIT}_s)$  / * Calculate the best fitness value using BEST function * /
(6) return  $\text{FIT}_{s_{\text{best}}}$ 
(7) end procedure

(1) Procedure  $\text{BEST}(\text{FIT}_s)$ 
(2)  $\text{BEST} \leftarrow \text{FIT}_s[0]$ 
(3) for  $i \leftarrow 1$  to  $n$  do
(4) if ( $\text{FIT}_s[i] < \text{Best}$ ) then
(5)  $\text{Best} \leftarrow \text{FIT}_s[i]$ 
(6) end if
(7) end for
(8) return  $\text{Best}$  / * Return the best fitness value * /
(9) end procedure

```

ALGORITHM 1: Pseudocode of the SOA.

```

(1) Generate the initial population  $X_i (i = 1, 2, \dots, NP)$ 
(2) Evaluate the fitness for each candidate solutions in  $X_i$ 
(3)  $X^*$  = the best candidate solutions
(4) while the halting criterion is not satisfied do
(5) for  $i = 1$  to  $NP$  do
(6) Update  $a$ ,  $A$ ,  $C$ ,  $l$  and  $P$ ;
(7) for  $1$  to  $n$  do
(8) if  $P < 0.5$  then
(9) if ( $|A| < 1$ )
(10)  $D = |C \cdot X^* - X_t|$ 
(11)  $X_i(j) = |X^*(j) - A \cdot D|$ 
(12) else if ( $|A| \geq 1$ )
(13) select a random individual  $X_{\text{rand}}$ 
(14)  $D = |C \cdot X_{\text{rand}} - X_t|$ 
(15)  $X_i(j) = X_{\text{rand}}(j) - A \cdot D$ 
(16) end if
(17) else if  $P \geq 0.5$  then
(18)  $D_l = |X^* - X_t|$ 
(19)  $X_i(j) = D_l \cdot e^{bl} \cdot \cos(2\pi l) + X^*$ 
(20) end if
(21) end for
(22) Evaluate the fitness for  $X_i$ 
(23) end for
(24) end while

```

ALGORITHM 2: Pseudocode of the WOA.

FIGURE 1: Flow chart of the WSOA.

$$\vec{a} = 2 - \frac{2t}{t_{\max}}, \quad (14)$$

where $t_{\max}$ is the maximum number of iterations.

3.2. Search for Prey: If $p < 0.5$ and $|A| \geq 1$. In addition to bubble net predation behavior, whales can also randomly search for food, and the process of searching for food is the exploration stage of the algorithm. That is to say, whale individuals randomly search according to each other's position, and the mathematical model can be expressed as follows:

$$\vec{D} = |\vec{C} \cdot \vec{X}_{\text{rand}} - \vec{X}|, \quad (15)$$

$$\vec{X}(t+1) = \vec{X}_{\text{rand}} - \vec{A} \cdot \vec{D}, \quad (16)$$

where $\vec{X}_{\text{rand}}$ is the position vector of individual whales randomly selected from the current population.

3.3. Spiral Updating Position: If $p \geq 0.5$. Encircling prey and spiral update position methods are utilized during the exploitation phase of the WOA. The spiral update position formula is as follows:

Input: Seagull population $\vec{P}_s$
Output: Optimal search agent $\vec{P}_{bs}$

```

(1) Procedure WSOA
(2) Initialize the SOA parameters and WOA parameters
(3)   Set  $f_c \leftarrow 2$ 
(4)   Set  $u \leftarrow 1$ 
(5)   Set  $v \leftarrow 1$ 
(6)   while ( $x < \text{Max}_{\text{iteration}}$ ) do
(7)      $\vec{P}_{bs} \leftarrow \text{ComputeFitness}(\vec{P}_s)$  / * Migration behavior * /
(8)      $\text{rd} \leftarrow \text{Rand}(0, 1)$  / * To generate the random number in range [0, 1] * /
(9)      $k \leftarrow \text{Rand}(0, 2\pi)$  / * To generate the random number in range [0, 2π] * // * Attacking behavior * /
(10)     $r \leftarrow u \times e^{kv}$  / * To generate the spiral behavior during migration * /
(11)    if 1  $p < 0.5$ 
(12)      if 2  $|A| < 1$ 
(13)        update the position of the current search agent by the equation (18)
(14)      else if 2  $|A| \geq 1$ 
(15)        update the position of the current search agent by the equation (15)
(16)      end if 2
(17)    else if 1  $p \geq 0.5$ 
(18)      Calculate the distance  $\vec{D}_s$  using equation (15)
(19)       $\vec{p} \leftarrow x' \times y' \times z'$  / * Compute  $x, y, z$  planes using equations (6)–(9) * /
(20)       $\vec{P}_s(x) \leftarrow (\vec{D}_s \times \vec{p}) + \vec{P}_{bs}$ 
(21)       $x \leftarrow x + 1$ 
(22)    end if 1
(23)  end while
(24) return  $\vec{P}_{bs}$ 
(25) end procedure

(1) Procedure ComputeFitness ( $\vec{P}_s$ )
(2)   for  $i \leftarrow 1$  to  $n$  do / * Here,  $n$  represents the dimension of a given problem * /
(3)      $\text{FIT}[i] \leftarrow \text{Fitness Function}(\vec{P}_s(i, :))$  / * Calculate the best fitness of each individual * /
(4)   end for
(5)    $\text{FIT}_{s_{\text{best}}} \leftarrow \text{BEST}(\text{FIT}_s)$  / * Calculate the best fitness value using BEST function * /
(6) return  $\text{FIT}_{s_{\text{best}}}$ 
(7) end procedure

(1) Procedure BEST ( $\text{FIT}_{s_{\text{best}}}$ )
(2)    $\text{Best} \leftarrow \text{FIT}_s[0]$ 
(3)   for  $i \leftarrow 1$  to  $n$  do
(4)     if ( $\text{FIT}_s[i] < \text{Best}$ ) then
(5)        $\text{Best} \leftarrow \text{FIT}_s[i]$ 
(6)     end if
(7)   end for
(8) return Best / * Return the best fitness value * /
(9) end procedure

```

ALGORITHM 3: Pseudocode of the WSOA.

$$\vec{X}(t+1) = \vec{D}' \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t), \quad (17)$$

where $\vec{D}' = |\vec{X}_p(t) - \vec{X}(t)|$, which is the distance between the i th candidate solution and the best solution in the current generation. b is a constant and $l \in [-1, 1]$. The specific process of the WOA is reflected in the pseudocode of Algorithm 2.

4. Hybrid Whale Optimization With Seagull Algorithm (WSOA)

Many creatures in nature have similarities in the process of foraging. For example, whales have a unique predation behavior in foraging, that is, bubble net foraging behavior.

The WOA designs shrinking encircling mechanism and upward spiral way to attack prey by simulating predation behavior of whales. Seagulls migrate from one place to another to find abundant food when they are looking for food. Once rich prey is found, they will attack prey in a spiral shape in the air. This paper proposes a hybrid whale optimization with seagull algorithm (WSOA) for global optimization problems, which mainly combines the shrinking encircling mechanism of the WOA with the spiral attack behaviors of the SOA, thus greatly improving the local searching and global searching ability of the algorithm. Levy flight is a mechanism of random walking behaviors, which can properly control local search. However, seagull optimization algorithm will converge prematurely. Therefore, this paper also considers the levy flight strategy to be

TABLE 1: Parameter setting value of algorithm.

Algorithm	Parameter	Value
WSOA	Control parameter (A)	[2, 0]
	f_c	2
SOA	Control parameter (A)	[2, 0]
	f_c	2
WOA	A	Decreased from 2 to 0
PSO	Inertia coefficient	0.75
	Cognitive and social coeff.	1.8 and 2
GWO	Control parameter ($\vec{a}$)	[2, 0]
	Number of initial populations (N_0)	5
IWO	Maximum number of plant populations	10
	Maximum number of seeds	2
	Minimum number of seeds	0
MFO	Convergence constant	[-1, -2]
	Logarithmic spiral	0.75
	Number of onlookers	50% of the swarm
ABC	Number of employed bees	50% of the swarm
	Swarm size	20
	Number of scouts	1

TABLE 2: Unimodal function.

Benchmark test functions	Dim	Range	$f_{\min}$
$f_{01} = \sum_{i=1}^n x_i^2$	30	[-100, 100]	0
$f_{02}(x) = \sum_{i=1}^n x_i + \prod_{i=1}^n x_i $	30	[-10, 10]	0
$f_{03}(x) = \sum_{i=1}^n (\sum_{j=1}^i x_j)^2$	30	[-100, 100]	0
$f_{04}(x) = \max_i \{ x_i , 1 \leq i \leq D\}$	30	[-100, 100]	0
$f_{05}(x) = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30	[-30, 30]	0
$f_{06}(x) = \sum_{i=1}^n (x_i + 0.5)^2$	30	[-100, 100]	0
$f_{07}(x) = \sum_{i=1}^n x_i^4 + \text{random}(0, 1)$	30	[-1.28, 1.28]	0

introduced into the contraction surrounding mechanism of the WOA and the local searching stage of the SOA, which improves the exploitation ability and avoids the premature convergence of the algorithm. The update formula of contraction surrounding mechanism of the WOA with levy flight strategy is as follows:

$$\begin{aligned} \vec{D}_s &= \text{levy}(d) \times (\text{levy}(d) \times \vec{P}_{bs}(x) - \vec{P}_s(x)), \\ \vec{P}_s(x+1) &= \vec{P}_{bs}(x) - \vec{A} \times \vec{D}_s, \end{aligned} \quad (18)$$

where d is the dimension of the current position vector, x is the current number of iterations, and levy flight [29] can be expressed as

$$\text{levy}(x) = 0.01 \times \frac{r_1 \times \sigma}{|r_2|^{(1/\beta)}}, \quad (19)$$

where r_1 and r_2 are random numbers in [0, 1], β is the constant, and σ can be expressed as

$$\sigma = \left(\frac{\Gamma(1+\beta) \times \sin(\pi\beta/2)}{\Gamma(1+\beta/2) \times \beta \times 2^{(\beta-1/2)}} \right)^{(1/\beta)}, \quad (20)$$

where $\Gamma(x) = (x-1)!$.

The improvement of spiral attack method of the SOA is as follows:

$$\begin{aligned} \vec{D}_s &= \vec{C}_s + B \times \text{levy}(d) \times (\vec{P}_{bs}(x) - \vec{P}_s(x)), \\ \vec{P}_s(x) &= (\vec{D}_s \times x' \times y' \times z') + \vec{P}_{bs}(x), \end{aligned} \quad (21)$$

where $\vec{C}_s$ represents the location of the search agent that does not collide with other search agents, $\vec{P}_s$ represents the current location of the search agent, represents the distance between the search agent and best-fit search agent, and represents the best-fit search agent. Figure 1 describes the flow chart of WSOA. The pseudocode of the WSOA is shown in Algorithm 3.

5. Experimental Results and Analysis

In this section, 25 benchmark functions are used to test the performance of the proposed WSOA. This section is divided into five parts. In Section 5.1, the benchmark function is introduced in detail. Performance analysis of the proposed WSOA and comparison algorithm is shown in Section 5.2. In Section 5.3, statistical test of the algorithm is provided. Section 5.4 analyzes the results. Finally, the proposed WSOA is applied to solve engineering optimization problems in Section 5.5.

TABLE 3: Multimodal function.

Benchmark test functions	Dim	Range	$f_{\min}$
$f_{08}(x) = -\sum_{i=1}^n x_i \sin(\sqrt{ x_i })$	30	$[-500, 500]$	$-418.9829 * \text{Dim}$
$f_{09}(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]$	0
$f_{10}(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}) - \exp(\frac{1}{n} \sum_{i=1}^n \cos 2\pi x_i) + 20 + e$	30	$[-32, 32]$	0
$f_{11}(x) = (1/4000) \sum_{i=1}^n (x_i^2) - \prod_{i=1}^n \cos(x_i/\sqrt{i}) + 1$	30	$[-600, 600]$	0
$f_{12}(x) = (\pi/D) \left\{ 10 \sin^2(\pi y_1) + \sum_{i=1}^{D-1} (y-1)^2 [1 + 10 \sin^2(\pi y_i)] \right\} \\ + \sum_{i=1}^D u(x_i, 10, 100, 4) \\ y_i = 1 + (x_i + 1/4) \\ u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x^i > a \\ 0, & -a \leq x_i \leq a \\ k(-x_i - z)^m, & x_i < a \end{cases}$	30	$[-50, 50]$	0
$f_{13}(x) = 0.1 \left\{ \sin^2 \left(3\pi x_1 + \sum_{i=1}^n (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] \right) \right\} \\ + \sum_{i=1}^n u(x_i, 5, 100, 4)$	30	$[-50, 50]$	0

TABLE 4: Fixed-dimensional multimodal function.

Benchmark test functions	Dim	Range	$f_{\min}$
$f_{14}(x) = (1/500 + \sum_{j=1}^{25} 1/j + \sum_{i=1}^2 (x_i - a_{ij})^6)^{-1}$	2	$[-65, 65]$	1
$f_{15}(x) = \sum_{i=1}^{11} [a_i - (x_1(b_i^2 + b_i x_2)/b_i^2 + b_i x_3 + x_4)]^2$	4	$[-5, 5]$	0.0003075
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + (1/3)x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]$	-1.0316285
$f_{17}(x) = -(1 + \cos(12\sqrt{x_1^2 + x_2^2})/0.5(x_1^2 + x_2^2) + 2)$	2	$[-5.12, 5.12]$	-1
$f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	2	$[-5, 5]$	3
$f_{19}(x) = -\sum_{i=1}^4 c_i \exp(-\sum_{j=1}^6 a_{ij}(x_j - p_{ij})^2)$	6	$[0, 1]$	-3.32
$f_{20}(x) = -\sum_{i=1}^5 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.1532
$f_{21}(x) = -\sum_{i=1}^7 [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.4029
$f_{22}(x) = -\sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	$[0, 10]$	-10.5364
$f_{23}(x) = -\cos(x_1)\cos(x_2)\exp(-(x_1 - \pi)^2 - (x_2 - \pi)^2)$	2	$[-100, 100]$	-1
$f_{24}(x) = 0.5 + (\sin^2(\sqrt{x_1^2 + x_2^2}) - 0.5/(1 + 0.001(x_1^2 + x_2^2)))^2$	2	$[-100, 100]$	-1
$f_{25}(x) = \sum_{i=1}^n x_i \sin(x_i) + 0.1x_i$	10	$[-10, 10]$	0

TABLE 5: Test results of unimodal test function.

Functions	Algorithms	Mean	Best	Worst	Std.
f_1	WSOA	8.47E-219	6.06E-241	2.40E-217	0
	SOA	3.82E-28	2.90E-31	3.66E-27	7.17E-28
	WOA	2.71E-147	5.60E-173	7.03E-146	1.29E-146
	PSO	7.37E-14	1.20E-16	1.09E-12	2.07E-13
	GWO	9.65E-59	3.72E-61	1.01E-57	2.06E-58
	IWO	1259.1673	14.367	4578.0546	974.9639
	MFO	1666.6669	1.06E-05	10000.0001	3790.4901
	ABC	6.9723	0.0075248	60.4577	15.8377
f_2	WSOA	2.36E-112	1.68E-129	4.52E-111	9.30E-112
	SOA	5.31E-18	3.89E-19	3.05E-17	6.93E-18
	WOA	6.02E-101	5.56E-116	1.81E-99	3.30E-100
	PSO	0.0063353	6.60E-08	0.18941	0.034578
	GWO	6.95E-35	8.01E-36	2.59E-34	6.01E-35
	IWO	0.032918	0.026614	0.043399	0.0038925
	MFO	34.6673	0.00053366	100	20.4653
	ABC	0.49868	0.029053	1.8489	0.45169
f_3	WSOA	2.46E-153	2.54E-203	7.37E-152	1.34E-152
	SOA	3.27E-13	5.75E-18	6.58E-12	1.24E-12
	WOA	18928.8069	3503.1755	38099.6547	9210.6917
	PSO	110.9042	8.0146	427.2507	108.2086
	GWO	1.60E-15	4.72E-20	3.17E-14	5.84E-15
	IWO	8484.0385	4740.4266	13760.1827	2323.4503
	MFO	19544.2017	465.4813	55008.1336	15635.4144
	ABC	43922.9306	30253.8188	65581.9318	8548.9204
f_4	WSOA	7.93E-98	1.37E-119	2.20E-96	4.01E-97
	SOA	6.71E-08	8.42E-11	1.24E-06	2.27E-07
	WOA	22.1714	0.048485	76.5243	20.0257
	PSO	3.6386	1.0381	8.1432	1.7835
	GWO	1.44E-14	1.18E-15	1.14E-13	2.30E-14
	IWO	38.0993	28.1075	49.4369	5.754
	MFO	69.0251	52.7901	85.3707	8.6404
	ABC	85.9882	79.2378	92.8272	3.3293
f_5	WSOA	4.7201	0.0069459	27.7934	10.3352
	SOA	28.1858	27.1821	28.8496	0.52153
	WOA	27.2119	26.3476	28.7456	0.63476
	PSO	136.1758	7.1025	2495.505	450.714
	GWO	26.8744	25.9231	28.54	0.64642
	IWO	118.1505	22.5334	1030.8263	233.6927
	MFO	18555.9331	6.81	90085.3516	36367.0326
	ABC	3413.6593	512.2443	18158.4492	3832.0029

TABLE 5: Continued.

Functions	Algorithms	Mean	Best	Worst	Std.
f_6	WSOA	0.002881	0.00070095	0.0070674	0.001618
	SOA	3.1986	2.6531	3.9467	0.36716
	WOA	0.10824	0.008272	0.3296	0.1094
	PSO	9.21E-14	3.62E-16	7.66E-13	1.73E-13
	GWO	0.6821	1.01E-05	1.5062	0.37642
	IWO	1148.6679	4.65E-05	3399.7094	988.8429
	MFO	2336.7252	2.08E-05	10100.2501	4308.358
	ABC	6.7192	0.0052413	97.6377	18.5232
f_7	WSOA	0.00016715	3.22E-06	0.0011171	0.00020972
	SOA	0.0010557	2.15E-05	0.0039959	0.00088306
	WOA	0.0021791	1.31E-05	0.0092708	0.0023497
	PSO	0.58978	0.081789	1.1513	0.29928
	GWO	0.00074907	0.00013394	0.0019163	0.0072772
	IWO	0.021927	0.010026	0.036071	0.0072772
	MFO	4.1674	0.056477	24.2912	6.6287
	ABC	0.38307	0.23624	0.59655	0.095506

TABLE 6: Test results of multimodal test function.

Functions	Algorithms	Mean	Best	Worst	Std.
f_8	WSOA	-12273.3151	-12569.3181	-10662.1204	457.2487
	SOA	-5532.3042	-7878.2295	-4484.3954	868.0981
	WOA	-10934.7761	-12569.4762	-8367.04	1620.5229
	PSO	-5.95E+228	-1.69E+230	-3.74E+195	Inf
	GWO	-6139.4458	-7537.5931	-3849.2598	801.0418
	IWO	-6762.544	-7868.1968	-6036.2644	525.7308
	MFO	-8563.3627	-10255.3797	-7319.1662	750.0895
	ABC	-10807.4564	-11773.7621	-9961.6702	537.5941
f_9	WSOA	0	0	0	0
	SOA	1.7804	0	24.7269	6.1853
	WOA	0	0	0	0
	PSO	72.4328	38.8034	122.3795	20.99
	GWO	8.0806	0	221.0579	40.2696
	IWO	74.6642	38.8146	137.3125	19.5274
	MFO	146.8836	68.652	248.809	37.6504
	ABC	76.4702	43.3295	117.2736	15.6098
f_{10}	WSOA	8.88E-16	8.88E-16	8.88E-16	0
	SOA	19.9593	19.9555	19.9614	0.0014469
	WOA	4.09E-15	8.88E-16	7.99E-15	2.35E-15
	PSO	1.5522	8.19E-09	4.5048	1.1664
	GWO	1.69E-14	1.15E-14	2.93E-14	4.25E-15
	IWO	19.0729	17.905	19.5174	0.37948
	MFO	16.2462	0.0083099	19.963	6.3179
	ABC	14.2383	10.3727	17.8378	2.128
f_{11}	WSOA	0	0	0	0
	SOA	0.0046933	0	0.061194	0.013801
	WOA	0.0020213	0	0.06064	0.011071
	PSO	0.041701	1.33E-15	0.20776	0.045345
	GWO	0.0046648	0	0.02939	0.0086959
	IWO	482.8997	356.8605	635.4635	67.1807
	MFO	12.0909	2.57E-05	180.9617	39.301
	ABC	0.76722	0.038514	1.5517	0.42278

TABLE 6: Continued.

Functions	Algorithms	Mean	Best	Worst	Std.
f_{12}	WSOA	0.00032222	$3.56E-05$	0.00098051	0.00023705
	SOA	0.30919	0.13453	0.82893	0.12059
	WOA	0.0066089	0.0006208	0.02561	0.0069502
	PSO	0.34246	2.16E-13	1.5421	0.44148
	GWO	0.036269	0.01308	0.098627	0.019866
	IWO	20.185	10.9406	31.9883	5.3077
	MFO	8533335.445	$6.93E-06$	256000015.7	46738994.14
	ABC	0.46742	0.020219	1.1086	0.33933
f_{13}	WSOA	0.0093616	0.0010055	0.051019	0.01256
	SOA	1.9594	1.4542	2.4358	0.23239
	WOA	0.24625	0.026198	0.5863	0.15422
	PSO	0.38429	1.15E-16	3.5975	1.0906
	GWO	0.48834	$7.24E-05$	1.1269	0.22795
	IWO	0.00037624	$6.61E-06$	0.011	0.0020065
	MFO	13668758.95	0.00012905	410062760.4	74866874.57
	ABC	1.6428	0.16393	5.2668	1.2614

TABLE 7: Test results of fixed-dimensional multimodal test function.

Functions	Algorithms	Mean	Best	Worst	Std.
f_{14}	WSOA	0.998	0.998	0.998	$1.85E-13$
	SOA	2.3207	0.998	2.9821	1.1455
	WOA	2.3207	0.998	2.9821	1.1455
	PSO	0.998	0.998	0.9980	0
	GWO	0.998	0.998	0.998	$1.16E-10$
	IWO	11.6069	5.9288	21.9884	9.0039
	MFO	1.3293	0.998	1.992	0.5739
	ABC	0.998	0.998	0.998	$1.17E-13$
f_{15}	WSOA	0.00056838	0.00030764	0.0015736	0.00036169
	SOA	0.0012309	0.00066377	0.0013874	0.0001195
	WOA	0.0010588	0.00030758	0.013197	0.0023246
	PSO	0.00070654	0.00030749	0.0010683	0.00029363
	GWO	0.0024196	0.00030749	0.020363	0.0060904
	IWO	0.0013891	0.0003075	0.020363	0.0036025
	MFO	0.0022639	0.0003704	0.020363	0.0049334
	ABC	0.0012599	0.00062549	0.0019125	0.00031881
f_{16}	WSOA	-1.0316	-1.0316	-1.0316	$1.07E-08$
	SOA	-1.0316	-1.0316	-1.0316	$4.41E-07$
	WOA	-1.0316	-1.0316	-1.0316	$2.29E-10$
	PSO	-1.0316	-1.0316	-1.0316	6.71E-16
	GWO	-1.0316	-1.0316	-1.0316	$4.23E-09$
	IWO	-1.0316	-1.0316	-1.0316	$4.51E-09$
	MFO	-1.0316	-1.0316	-1.0316	$6.78E-16$
	ABC	-1.0316	-1.0316	-1.0316	$1.16E-15$
f_{17}	WSOA	-1	-1	-1	0
	SOA	-1	-1	-1	0
	WOA	-0.97237	-1	-0.93625	0.032133
	PSO	-0.99787	-1	-0.93625	0.01164
	GWO	-1	-1	-1	0
	IWO	-1	-1	-1	$3.60E-08$
	MFO	-0.9745	-1	-0.93625	0.031767
	ABC	-0.99878	-1	-0.99583	0.0011721

TABLE 7: Continued.

Functions	Algorithms	Mean	Best	Worst	Std.
f_{18}	WSOA	3	3	3	$7.28E-05$
	SOA	3	3	3.0001	$2.19E-05$
	WOA	3	3	3.0002	$3.51E-05$
	PSO	3	3	3	$1.37E-15$
	GWO	5.7	3	84	14.7885
	IWO	3	3	3	$3.46E-07$
	MFO	3	3	3	$1.98E-15$
	ABC	3.0016	3	3.0227	0.0043786
f_{19}	WSOA	-3.322	-3.322	-3.322	$3.31E-05$
	SOA	-3.0507	-3.121	-3.0156	0.060884
	WOA	-3.3218	-3.3219	-3.3216	0.00015049
	PSO	-3.2824	-3.322	-3.2031	0.068643
	GWO	-3.2409	-3.322	-3.1978	0.070281
	IWO	-3.2031	-3.2031	-3.2031	$2.42E-06$
	MFO	-3.1785	-3.2031	-3.1327	0.039669
	ABC	-3.322	-3.322	-3.322	$1.48E-06$
f_{20}	WSOA	-10.1532	-10.1532	-10.1531	$3.45E-05$
	SOA	-6.8513	-10.138	-0.35136	5.6292
	WOA	-8.453	-10.152	-5.0551	2.9427
	PSO	-5.0856	-5.1008	-5.0552	0.026312
	GWO	-10.1527	-10.1528	-10.1524	0.0002039
	IWO	-5.138	-10.1532	-2.6305	4.3432
	MFO	-5.9615	-10.1532	-2.6305	3.8345
	ABC	-10.1166	-10.1489	-10.0844	0.032264
f_{21}	WSOA	-10.4029	-10.4029	-10.4027	$5.51E-05$
	SOA	-5.8125	-10.3992	-0.37244	4.3645
	WOA	-8.7562	-10.4029	-3.7242	2.5549
	PSO	-5.9382	-10.4029	-2.7519	3.3146
	GWO	-10.4025	-10.4029	-10.4015	0.00032137
	IWO	-6.4519	-10.4029	-2.7519	3.594
	MFO	-7.7868	-10.4029	-2.7519	3.3175
	ABC	-10.3656	-10.4029	-10.1998	0.058166
f_{22}	WSOA	-10.5364	-10.5364	-10.5364	$1.19E-05$
	SOA	-7.3236	-10.5326	-0.94707	5.5222
	WOA	-6.4993	-10.5342	-3.8352	3.5537
	PSO	-10.5364	-10.5364	-10.5364	0
	GWO	-10.536	-10.5364	-10.5357	0.00034804
	IWO	-2.6783	-2.8066	-2.4217	0.22222
	MFO	-6.0691	-10.5364	-3.8354	3.8688
	ABC	-10.5247	-10.5364	-10.5183	0.010085
f_{23}	WSOA	-1	-1	-1	$8.23E-09$
	SOA	-0.99997	-1	-0.99993	$1.92E-05$
	WOA	-1	-1	-1	$8.15E-08$
	PSO	-1	-1	-1	0
	GWO	-1	-1	-1	$2.81E-07$
	IWO	-1	-1	-1	$1.29E-09$
	MFO	-1	-1	-1	0
	ABC	-0.99999	-1	-0.99984	$2.90E-05$
f_{24}	WSOA	-1	-1	-1	0
	SOA	-0.99806	-1	-0.99028	0.0039528
	WOA	-0.99514	-1	-0.99028	0.004941
	PSO	-0.99437	-1	-0.99028	0.004798
	GWO	-0.99644	-1	-0.99028	0.0047621
	IWO	-0.61609	-0.99028	-0.50405	0.13302
	MFO	-0.9919	-1	-0.99028	0.0036828
	ABC	-0.99159	-0.99998	-0.99024	0.0027293

TABLE 7: Continued.

Functions	Algorithms	Mean	Best	Worst	Std.
f_{25}	WSOA	$2.86E-98$	$3.27E-133$	$8.57E-97$	$1.56E-97$
	SOA	$1.21E-24$	$1.43E-35$	$3.64E-23$	$6.65E-24$
	WOA	0.42502	$5.15E-120$	3.6239	0.98594
	PSO	$2.98E-15$	$2.29E-28$	$1.85E-14$	$3.89E-15$
	GWO	0.00010204	$8.13E-68$	0.00096615	0.00021053
	IWO	0.026443	0.0090162	0.066102	0.011482
	MFO	0.14801	$2.88E-22$	4.4402	0.81067
	ABC	0.048445	0.0042035	0.17205	0.046518

FIGURE 2: Convergence curve of f_1 . FIGURE 4: Convergence curve of f_2 . FIGURE 3: Variance analysis diagram of f_1 . FIGURE 5: Variance analysis diagram of f_2 .

The experiment is described in detail below. This article selects the seven following algorithms (SOA, WOA, PSO, GWO, IWO, MFO, and ABC) for comparative research. During the experiment, the population size N was set to be

30, the maximum number of iterations was 1000, and each benchmark function was run independently for 30 times. All experiments are implemented in MATLAB R2018b and are carried out on a computer with Intel (R) Core (TM) i5-

FIGURE 6: Convergence curve of f_3 . FIGURE 8: Convergence curve of f_4 . FIGURE 7: Variance analysis diagram of f_3 . FIGURE 9: Variance analysis diagram of f_4 .

10210U CPU @ 2.11 GHz system type as 64-bit Windows 10 operating system. In addition, in order to make the comparison more convenient, the specific parameters of these seven algorithms can be obtained according to the original paper, as shown in Table 1.

5.1. Benchmark Function. According to the references, we extracted 25 benchmark functions to verify the algorithm [30–32]. These benchmark functions are mainly divided into three categories: unimodal function (Table 2), multimodal function (Table 3), and fixed-dimensional multimodal function (Table 4). Unimodal function has only one optimal solution in a given interval, and these functions can test the exploitation ability of the algorithm. Multimodal function

has multiple optimal solutions, among which there is only one global optimal solution, and these functions can test the exploration ability of the algorithm. The detailed information of these 25 benchmark functions is shown in Tables 2–4.

5.2. Performance Comparison and Analysis with Other Algorithms. In this experiment, mean, best, worst, and standard deviation (Std) are used as evaluation indexes. The experimental results of unimodal function are shown in Table 5, those of multimodal function in Table 6, and those of fixed multimodal function in Table 7; bold numbers indicate the optimal solution of comparison algorithm.

FIGURE 10: Convergence curve of f_5 . FIGURE 12: Convergence curve of f_6 . FIGURE 11: Variance analysis diagram of f_5 . FIGURE 13: Variance analysis diagram of f_6 .

5.2.1. Analysis of Unimodal Function Test Results.

Table 5 shows the test results of 8 metaheuristic algorithms in testing unimodal functions $f_1 - f_7$. It can be seen from the table that the WSOA has better performance than other algorithms in most of the tested unimodal functions. Except for function f_6 , which ranks second after PSO, the WSOA ranks first among the other six unimodal test functions $f_1 - f_5$ and f_7 . The variances of functions $f_1 - f_4$ and f_7 are all in the first place, indicating that the WSOA has better stability. Generally speaking, the WSOA can find the optimal solution faster than other algorithms, which shows that the WSOA has better convergence speed.

Figures 2–15 are the convergence curves and variance analysis diagram of unimodal test functions $f_1 - f_7$. The convergence curve here is drawn according to the average

value of these eight metaheuristic algorithms and is the result of each algorithm running independently for 30 times. It can be seen from the figure that the WSOA converges faster than the other seven algorithms and can find the near-optimal solution faster. It can be seen from Figure 12 that the convergence speed of the WSOA in the early stage is faster than that of the other seven algorithms, and the convergence speed in the later stage is slightly inferior to that of PSO algorithm. The ANOVA chart can directly see the stability of the unimodal function tested by the algorithm; that is to say, the lower the box chart is, the more stable it is. It can be seen from the figure that the variance analysis chart except f_5 is worse than SOA, WOA, and GWO, ranking fourth. In the ANOVA diagram of test functions $f_1 - f_4$, f_6 , and f_7 , the variance value of the WSOA is the smallest. Generally speaking, the WSOA has good stability in testing unimodal functions. Through the

FIGURE 14: Convergence curve of f_7 . FIGURE 16: Convergence curve of f_8 . FIGURE 15: Variance analysis diagram of f_7 . FIGURE 17: Variance analysis diagram of f_8 .

above experiments, it can be proved that the WSOA has strong convergence speed in solving unimodal test functions, which shows that the algorithm has strong global search ability.

5.2.2. Analysis of Multimodal Function Test Results. Table 6 shows the test results of eight metaheuristic algorithms for multimodal test functions $f_8 - f_{13}$. It can be seen from the table that the WSOA's test results for other multimodal test functions $f_8 - f_{12}$ are in the first place except that the test results of function f_{13} are slightly inferior to IWO and rank second. Furthermore, the WSOA can accurately find the corresponding theoretical optimal solution in the multimodal test functions f_9 and f_{11} , which fully proves that the WSOA has strong optimization ability. Although the best values of multimodal test functions f_8 and f_{12} rank

second, on the whole, the WSOA is superior to the other seven metaheuristic algorithms, and the proposed algorithm has better performance.

Figures 16–27 are the convergence curves and variance analysis graph of the multimodal test functions $f_8 - f_{13}$. From the convergence graph, it can be seen that the convergence speed and convergence accuracy of the WSOA in multimodal test function are superior to seven other metaheuristic algorithms. In the multimodal test function f_{13} (Figure 26), although the IWO algorithm gets smaller value than the WSOA in the later stage, the convergence speed of the WSOA is obviously better than that of the IWO algorithm in the early stage. On the whole, the WSOA has stronger convergence speed in testing multimodal functions. It can be seen from the analysis of variance that the WSOA can almost get the minimum value in the

FIGURE 18: Convergence curve of f_9 . FIGURE 20: Convergence curve of f_{10} . FIGURE 19: Variance analysis diagram of f_9 . FIGURE 21: Variance analysis diagram of f_{10} .

multimodal test function, which shows that it has good stability in the test of multimodal test function.

5.2.3. Analysis of Test Results of Fixed-Dimensional Multimodal Function. The fixed-dimensional multimodal test function is the same as the image of multimodal test function, but their dimensions are different, and the former's dimension is a fixed value. Table 7 shows the test results of eight metaheuristic algorithms on the fixed-dimensional multimodal test functions. It can be seen that the WSOA shows that its average value ranks first in the test of the fixed-dimensional multimodal test functions $f_{14} - f_{25}$ and the best values of the other fixed-dimensional multimodal test functions f_{14} and $f_{16} - f_{25}$ rank first except the best value of function f_{15} . Among

them, the corresponding theoretical optimum values can be found for functions f_{16} , f_{17} , f_{18} , $f_{20} - f_{24}$, and so forth.

Figures 28–51 are the convergence curves and variance analysis graph of the test results of optimizing the fixed-dimensional multimodal function with 8 metaheuristic algorithms. It can be seen from the convergence graph that the convergence speed of the WSOA algorithm is better than that of seven other algorithms, and the convergence accuracy is also very good, which shows the effectiveness of this algorithm. It can be seen from the analysis of variance that most of the variance values in the fixed-dimensional multimodal functions $f_{14} - f_{25}$ are the smallest, which shows that the algorithm is stable. Generally speaking, the WSOA has better convergence speed than seven other metaheuristic algorithms.

FIGURE 22: Convergence curve of f_{11} . FIGURE 24: Convergence curve of f_{12} . FIGURE 23: Variance analysis diagram of f_{11} . FIGURE 25: Variance analysis diagram of f_{12} .

5.3. Statistical Tests. In order to verify the noncontingency of the results, Wilcoxon signed-rank test [33] is carried out in this paper. Wilcoxon signed-rank test compares the performance of the proposed SOA with those of other algorithms, and it is widely used to compare the performances of two optimization algorithms [34–36]. In this experiment, the mean value obtained by running the WSOA algorithm independently for 30 times is compared with the mean values of seven other algorithms, and the Wilcoxon signed-rank test result with significance level of $\alpha = 0.05$ is obtained.

Table 8 shows the Wilcoxon signed-rank test results between the WSOA and seven comparison algorithms. In the table, “H” is marked with “1,” which indicates that there is a significant difference between the WSOA and the comparative algorithm, and it has statistical advantage. “H” marking “0” indicates that there is no obvious difference

between the WSOA and the comparison algorithm. “S” marking “+” indicates that the performance of the WSOA is better than that of the comparison algorithm. “S” marking “=” indicates that the performance of the WSOA is the same as that of the comparison algorithm. In addition, the last column of each table under the heading w/t/l represents the win, flat, and lose counts of the WSOA relative to the comparison algorithm. It can be seen from the table that the ABC algorithm is worse than the WSOA in other test functions except that the test function f_{23} is equal to the WSOA. The WSOA is superior to or equal to SOA, WOA, PSO, GWO, IWO, and MFO in all test functions. More specifically, in the unimodal test functions f_1 – f_7 , the WSOA is superior to the comparison algorithm, which shows that the performance of the WSOA is superior to the

FIGURE 26: Convergence curve of f_{13} . FIGURE 28: Convergence curve of f_{14} . FIGURE 27: Variance analysis diagram of f_{13} . FIGURE 29: Variance analysis diagram of f_{14} .

comparison algorithm, and the exploitation ability of the WSOA is superior to the comparison algorithm. Among the multimodal test functions $f_8 - f_{13}$, the WSOA is superior to GWO, IWO, MFO, and ABC. Except that f_9 and f_{11} are equal to the WSOA, SOA and WOA are worse than the WSOA in other test functions. In the fixed multippeak test functions $f_{14} - f_{25}$, the WSOA is superior to or equal to the comparison algorithm. To sum up, the proposed SOA is superior to the comparative algorithm in the stability of global search capability.

In order to further verify the reliability of the algorithm, the Friedman rank test is carried out on the experimental results [37]. Table 9 shows the results of the Friedman rank test. It can be seen from the table that the WSOA ranks first, which shows that the performance of this algorithm is better than that of the comparison algorithm.

5.4. Result Analysis. Through the above analysis, compared with other comparison algorithms, the proposed WSOA shows superior performance, which shows that the combination of the SOA and the WOA is very successful in solving optimization problems. The reasons for the superior performance can be summarized as follows: Firstly, the local search ability of the WSOA is effectively enhanced by using the fast search ability of contraction and encirclement mechanism of the WOA and spiral attack behavior of the SOA. Secondly, levy flight strategy is introduced, which makes use of the random flight mode characteristics of levy's long-term small-scale search and occasionally large-scale exploration, which is beneficial to the exploration and development of balanced algorithm and avoids premature convergence. Finally, the proposed SOA retains the original migration behavior, which is helpful to improve the search

FIGURE 30: Convergence curve of f_{15} . FIGURE 32: Convergence curve of f_{16} . FIGURE 31: Variance analysis diagram of f_{15} . FIGURE 33: Variance analysis diagram of f_{16} .

ability of the algorithm. Generally speaking, the performance of the WSOA is far superior to the original SOA in testing 25 benchmark functions, which shows that the proposed algorithm has strong competitiveness.

5.5. Solving Engineering Optimization Problems. Engineering optimization problems are common in people's lives, but many optimization problems have complex constraints, especially the optimization of engineering structure design. In order to verify the performance of the WSOA in

solving the optimization problem of engineering structure design, the design problems such as tension/compression spring, welded beam, multiplate clutch separator, and pressure vessel are verified in this experiment.

5.5.1. Tension/Compression Spring Design Problem. In real life, the optimization goal of many problems is to minimize the weight of the problem. For example, the design problem of tension/compression spring is a common example. The variables of this problem include coil diameter (x_1), spring

FIGURE 34: Convergence curve of f_{17} . FIGURE 35: Variance analysis diagram of f_{17} .

coil diameter (x_2), and the number of coils (x_3). The constraint conditions are minimum deflection ($g_1(X)$), shear stress ($g_2(X)$), impact frequency ($g_3(X)$), and outer diameter limit ($g_4(X)$). The spring design pattern is shown in Figure 52, and the mathematical formula can be expressed as follows:

consider: $\vec{x} = [x_1 x_2 x_3] = [dDN]$,

minimize: $f(X) = (x_3 + 2)x_2x_1^2$,

$$g_2(X) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5180x_1^2} - 1 \leq 0,$$

$$g_3(X) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0,$$

$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \leq 0,$$

$$0.05 \leq x_1 \leq 2,$$

$$0.25 \leq x_2 \leq 1.3,$$

$$2 \leq x_3 \leq 15.$$

(22)

Table 10 shows the comparison results of tension/compression spring design problems. It can be seen from the table that the WSOA proposed in this paper has better optimization performance, and the optimal function value obtained in solving spring design problems is lower than

FIGURE 36: Convergence curve of f_{18} . FIGURE 38: Convergence curve of f_{19} . FIGURE 37: Variance analysis diagram of f_{18} .

those of previous studies. The algorithm of this experiment is the result of running independently for 30 times. The optimal fitness value of the WSOA is $f(x) = 0.01267$, and the corresponding optimal solution is $\vec{x} = [0.0512, 0.3441, 12.0663]$. It can be seen that the WSOA has better optimization accuracy in solving the design problem of tension/compression spring.

5.5.2. Welded Beam Design Problem. The goal of welding beam design problem is to minimize the manufacturing cost, which must satisfy the related constraints such as shear stress (τ), beam bending stress (θ), bar buckling load (P_c), beam end deflection (δ), normal stress (σ), and boundary. The set variables include welded beam thickness (x_1), welded joint length (x_2), welded beam width (x_3), and beam thickness

(x_4). See Figure 53 for the design pattern of welded beam and the mathematical formula can be expressed as follows:

consider: $\vec{x} = [x_1 x_2 x_3 x_4] = [hltb]$,

minimize: $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2)$,

s.t. $g_1(X) = \tau(X) - \tau_{\max}$,

$g_2(X) = \sigma(X) - \sigma_{\max}$,

$g_4(X) = 0.125 - x_1 \leq 0$,

$g_5(X) = \delta(X) - 0.25 \leq 0$,

$g_6(X) = P - P_c(X) \leq 0$,

$g_7(X) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0$,

$0.1 \leq x_1 \leq 2$,

$0.1 \leq x_2 \leq 10$,

$0.1 \leq x_3 \leq 10$,

$0.1 \leq x_4 \leq 2$,

(23)

where $\tau_{\max}$ is the maximum acceptable shear stress, $\sigma_{\max}$ is the maximum acceptable normal stress, and P is the load. We have the following:

$$\tau(X) = \sqrt{\tau_1^2 + 2\tau_1\tau_2\left(\frac{x_2}{2R}\right) + \tau_2^2},$$

$$\tau_1 = \frac{P}{\sqrt{2}x_1x_2}, \quad (24)$$

$$\tau_2 = \frac{MR}{J},$$

$$M = P\left(L + \frac{x_2}{2}\right), \quad (25)$$

FIGURE 39: Variance analysis diagram of f_{19} . FIGURE 40: Convergence curve of f_{20} . FIGURE 41: Variance analysis diagram of f_{20} .

— WSOA - - - GWO
 - - - SOA - - - IWO
 - - - WOA - - - MFO
 - - - PSO - - - ABC

FIGURE 42: Convergence curve of f_{21} . FIGURE 43: Variance analysis diagram of f_{21} .

$$J(X) = 2 \left\{ \sqrt{2} x_1 x_2 \left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2} \right)^2 \right] \right\}. \quad (26)$$

M in equation (25) and $J(X)$ in equation (26) represent moment of inertia and polarity, respectively. The remaining parameters are shown in equation (27) to equation (31).

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2} \right)^2}. \quad (27)$$

$$\sigma(X) = \frac{6PL}{x_4 x_3^2}, \quad (28)$$

FIGURE 44: Convergence curve of f_{22} . FIGURE 46: Convergence curve of f_{23} . FIGURE 45: Variance analysis diagram of f_{22} . FIGURE 47: Variance analysis diagram of f_{23} .

$$\delta(X) = \frac{6PL^3}{Ex_3^3x_4}, \quad (29)$$

$$P_c(X) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2} \left(1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}} \right), \quad (30)$$

$$\begin{aligned} G &= 12 \times 10^6 \text{ psi}, \\ E &= 30 \times 10^6 \text{ psi}, \\ P &= 6000 \text{ lb}, \\ L &= 14 \text{ in}. \end{aligned} \quad (31)$$

Table 11 shows the research results of welding beam design. It can be seen from the table that the function value of the WSOA in solving the design problem of welded beam

is lower than that of previous research. The algorithm of this experiment is the result of running independently for 30 times. The optimal fitness value of the WSOA is $f(x) = 1.7519$, and the corresponding optimal solution is $\vec{x} = [0.1919, 3.7633, 9.109, 0.2054]$. Therefore, it can be shown that the WSOA has better optimization accuracy in solving the design problem of welded beams.

5.5.3. Multiplate Disc Clutch Brake Design Problem. The purpose of multiplate disc clutch brake is to optimize the total weight of multiplate disc clutch brake, in which the variables are driving force (F), inner and outer radius (r_i and r_o), friction surface number (Z), and disc thickness (t). Because the problem contains eight different constraints, the feasible region in the solution space only accounts for 70%, which makes it more difficult to solve the problem. See Figure 54 for the design pattern of multiplate disc clutch

FIGURE 48: Convergence curve of f_{24} . FIGURE 50: Convergence curve of f_{25} . FIGURE 49: Variance analysis diagram of f_{24} . FIGURE 51: Variance analysis diagram of f_{25} .

brake, and the mathematical formula can be expressed as follows:

$$\begin{aligned}
 &\text{minimize: } f(x) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho, \\
 &\text{s.t. } g_1(x) = x_2 - x_1 - \Delta R \geq 0, \\
 &\quad g_2(x) = L_{\max} - (x_5 + 1)(x_3 + \delta) \geq 0, \\
 &\quad g_3(x) = P_{\max} - P_{rz} \geq 0, \\
 &\quad g_4(x) = P_{\max}v_{sr\max} - P_{rz}v_{sr} \geq 0, \\
 &\quad g_5(x) = v_{sr\max} - v_{sr} \geq 0, \\
 &\quad g_6(x) = T_{\max} - T \geq 0, \\
 &\quad g_7(x) = M_h - sM_s \geq 0, \\
 &\quad g_8(x) = T \geq 0,
 \end{aligned} \tag{32}$$

where $M_h = (2/3)\mu x_4 x_5 (x_2^3 - x_1^3/x_2^2 - x_1^2)$ and $w = (\pi n/30)\text{rad/s}$.

$$A = \pi(x_2^2 - x_1^2)\text{mm}^2,$$

$$P_{rz} = \frac{x_4}{A}N/\text{mm}^2,$$

$$V_{sr} = \frac{\pi R_{sr} n}{30} \text{mm/s},$$

$$R_{sr} = \frac{2}{3} \frac{x_2^2 - x_1^2}{x_2^2 x_1^2} \text{mm},$$

$$T = \frac{I_z \pi n}{30(M_h + M_f)} \text{mm},$$

TABLE 8: Wilcoxon signed-rank test results between the WSOA and seven compared algorithms.

WSOA versus																					
No.	SOA			WOA			PSO			GWO			IWO			MFO			ABC		
	P value	H	S	P value	H	S	P value	H	S	P value	H	S	P value	H	S	Pvalue	H	S	P value	H	S
f_1	1.73E-06	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
f_2	1.73E-06	1	+	3.47E-10	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
f_3	1.73E-06	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
f_4	1.73E-06	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
f_5	2.60E-06	1	+	6.05E-07	1	+	4.69E-08	1	+	1.03E-06	1	+	2.02E-08	1	+	8.15E-11	1	+	3.02E-11	1	+
f_6	1.73E-06	1	+	3.02E-11	1	+	3.02E-11	1	+	5.57E-10	1	+	5.57E-10	1	+	3.99E-04	1	+	3.69E-11	1	+
f_7	1.36E-05	1	+	1.73E-07	1	+	3.02E-11	1	+	5.97E-09	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
f_8	1.73E-06	1	+	3.37E-04	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	2.61E-10	1	+
f_9	1.25E-01	0	=	1.00E+00	0	=	1.21E-12	1	+	1.36E-03	1	+	1.21E-12	1	+	1.21E-12	1	+	1.21E-12	1	+
f_{10}	1.73E-06	1	+	1.01E-08	1	+	1.21E-12	1	+	2.05E-13	1	+	1.21E-12	1	+	1.21E-12	1	+	1.21E-12	1	+
f_{11}	1.25E-01	0	=	3.34E-01	0	=	1.21E-12	1	+	2.79E-03	1	+	1.21E-12	1	+	1.21E-12	1	+	1.21E-12	1	+
f_{12}	1.73E-06	1	+	5.49E-11	1	+	1.86E-01	0	=	3.02E-11	1	+	3.02E-11	1	+	5.09E-08	1	+	3.02E-11	1	+
f_{13}	1.73E-06	1	+	4.08E-11	1	+	9.59E-01	0	=	5.57E-10	1	+	2.87E-10	1	+	1.00E-03	1	+	3.02E-11	1	+
f_{14}	8.13E-01	0	=	1.70E-02	1	+	1.21E-12	1	+	1.44E-03	1	+	6.69E-11	1	+	3.08E-01	0	=	3.34E-03	1	+
f_{15}	3.52E-06	1	+	2.52E-01	0	=	2.34E-01	0	=	3.56E-04	1	+	1.63E-02	1	+	4.12E-06	1	+	1.47E-07	1	+
f_{16}	1.92E-06	1	+	1.86E-09	1	+	1.72E-12	1	+	1.44E-03	1	+	4.86E-03	1	+	1.21E-12	1	+	1.75E-11	1	+
f_{17}	1.00E+00	0	=	6.42E-05	1	+	3.34E-01	0	=	1.00E+00	0	=	1.21E-12	1	+	1.40E-04	1	+	1.21E-12	1	+
f_{18}	3.49E-01	0	=	1.58E-01	0	=	3.16E-12	1	+	2.17E-01	0	=	3.20E-09	1	+	2.33E-11	1	+	1.78E-04	1	+
f_{19}	4.29E-06	1	+	1.76E-01	0	=	1.07E-04	1	+	2.32E-02	1	+	3.79E-01	0	=	1.54E-01	0	=	1.61E-10	1	+
f_{20}	8.47E-06	1	+	3.50E-09	1	+	5.51E-02	0	=	1.70E-08	1	+	1.60E-03	1	+	5.27E-01	0	=	2.62E-03	1	+
f_{21}	1.73E-06	1	+	7.39E-11	1	+	7.86E-03	1	+	2.67E-09	1	+	2.51E-02	1	+	1.83E-01	0	=	1.03E-06	1	+
f_{22}	1.36E-05	1	+	8.10E-10	1	+	2.22E-02	1	+	1.60E-07	1	+	1.54E-01	0	=	6.16E-03	1	+	2.15E-06	1	+
f_{23}	1.73E-06	1	+	9.21E-05	1	+	1.21E-12	1	+	3.34E-11	1	+	9.71E-01	0	=	1.21E-12	1	+	2.12E-01	0	=
f_{24}	3.13E-02	1	+	1.23E-05	1	+	5.66E-07	1	+	3.13E-04	1	+	1.21E-12	1	+	8.99E-11	1	+	1.21E-12	1	+
f_{25}	1.73E-06	1	+	3.16E-10	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+	3.02E-11	1	+
w/t/l	20/5/0			20/5/0			20/5/0			23/2/0			22/3/0			21/4/0			24/1/0		

TABLE 9: Friedman rank test results of the algorithm.

Algorithms	Mean rank	Rank
WSOA	2.51	1
SOA	5.21	6
WOA	3.96	4
PSO	3.84	3
GWO	3.82	2
IWO	5.81	7
MFO	4.98	5
ABC	5.87	8

$$\Delta r = 20\text{mm},$$

$$L_{\max} = 30\text{mm},$$

$$\mu = 0.6,$$

$$T_{\max} = 15\text{s},$$

$$\mu = 0.5,$$

$$s = 1.5,$$

$$M_s = 40\text{Nm},$$

$$p_{\max} = 1\text{Mp}_a,$$

$$\rho = 0.0000078\text{kg/mm}^3,$$

$$v_{sr\max} = 10\text{m/s},$$

$$\delta = 0.5\text{mm},$$

$$s = 1.5,$$

$$T_{\max} = 15\text{s},$$

$$n = 250\text{rpm},$$

$$I_z = 55\text{kg/m}^2,$$

$$M_s = 40\text{Nm},$$

$$M_f = 3\text{Nm},$$

$$60 \leq x_1 \leq 80,$$

$$90 \leq x_2 \leq 110,$$

$$1 \leq x_3 \leq 3,$$

$$60 \leq x_4 \leq 1000,$$

$$2 \leq x_5 \leq 9,$$

$$i = 1, 2, 3, 4, 5.$$

(33)

FIGURE 52: Structure of the TCS design.

TABLE 10: Comparative results for the tension/compression spring problem.

Algorithms	Optimum variables			Optimum weight
	D	D	N	
WSOA	0.0512	0.3441	12.0663	0.01267
GDA [38]	0.0516925	0.3568108	11.2835059	0.012875
GA [1]	0.05148	0.351661	11.632201	0.0127048
GPSO [39]	0.0517	0.3573	11.254	0.0127
SES [40]	N/A	N/A	N/A	0.012732
SCPSO [41]	0.051688	0.356705	11.289687	0.0127576
AATM [42]	0.051813	0.35969	11.119253	0.0127081
UABC [43]	0.051691	0.356769	11.285988	0.012683
MVO [44]	0.05	0.315956	14.22623	0.0144644
TEO [45]	0.051775	0.358792	11.16839	0.012685
MGWO [46]	0.051334	0.348197	11.80809	0.0126799
EEGWO [47]	0.051673	0.35634	11.3113	0.012685

FIGURE 53: The structure of welded beam design.

The comparison results of design problems of multiplate disc clutch brake are shown in Table 12. It can be seen from the table that the WSOA shows superiority in solving this problem, and the obtained function value is lower than those of previous studies. The experiment was run independently for 30 times. The optimal fitness value of the algorithm is

TABLE 11: Comparative results for the welded beam design problem.

Algorithms	Optimum variables				Optimum cost
	h	L	T	b	
WSOA	0.1919	3.7633	9.109	0.2054	1.7519
SBM [48]	0.2407	6.485	8.2399	0.2497	2.442
SA [49]	0.247	6.145	8.272	0.249	2.4148
SCA [50]	0.244	6.238	8.2886	0.2446	2.3854
BFOA [51]	0.205	3.4711	9.0367	0.2057	2.3868
EA [52]	0.2443	6.2201	8.294	0.2444	2.3816
T-cell [53]	0.2444	6.1286	8.2915	0.2444	2.3811
FSA [54]	0.2444	6.1258	8.2939	0.2444	2.3811
GSA [55]	0.182129	3.856979	10	0.202376	1.879952
HS [56]	0.2442	6.2231	8.2915	0.2443	2.3807
SBO [57]	0.21421	3.4924	8.55771	0.22945	1.8496

FIGURE 54: Multiplate disc clutch brake problem.

$f(x) = 0.23525$, and the optimal solution is $\vec{x} = [69.9996, 90, 1, 600, 2]$. It can be shown that the WSOA has good optimization accuracy in solving the design problem of multiplate disc clutch brake.

5.5.4. Pressure Vessel Design Problem. The optimization purpose of pressure vessel problem is to minimize the total cost. Constraints include material cost, forming cost, and welding cost. The variables of this problem are shell thickness (T_s), head thickness (T_h), inner diameter (R), and cylindrical section length (L) [61]. See Figure 55 for the

TABLE 12: Comparative results for multiplate disc clutch brake problem.

Algorithms	$ri(x_1)$	$r0(x_2)$	$t(x_3)$	$F(x_4)$	$Z(x_5)$	Optimal cost
WSOA	69.9996	90	1	600	2	0.23525
TLBO [58]	70	90	1	810	3	0.313656
WCA [59]	70	90	1	910	3	0.313656
PVS [60]	70	90	1	980	3	0.31366
HHO [58]	69.999999992493	90	1	1000	3	0.259768993

FIGURE 55: Pressure vessel design and its features.

TABLE 13: Comparative results for the pressure vessel design problem.

Algorithms	Optimum variables				Optimum cost
	T_s	T_h	R	L	
WSOA	0.8056	0.4081	41.7401	181.1285	5964.6114
GWO [4]	0.8125	0.4345	42.089181	176.758731	6051.5639
MFO [5]	0.8125	0.4375	42.098445	176.636596	6059.7143
CPSO [62]	0.8125	0.4375	42.091266	176.7465	6061.0777
HAIS-GA [63]	0.8125	0.4375	42.0931	176.7031	6060.367
ES [64]	0.8125	0.4375	42.098087	176.640518	6059.745
CDE [65]	0.8125	0.4375	42.098411	176.63769	6059.734
BA [66]	0.8125	0.4375	42.098445	176.636595	6059.7143
WOA [6]	0.8125	0.4375	42.0982699	176.638998	6059.741
DTS-GA [67]	0.8125	0.4375	42.097398	176.654047	6059.9463
GeneAS [68]	0.9375	0.5	48.329	112.679	6410.3811
WEO [69]	0.8125	0.4375	42.098444	176.636622	6059.71
HHO [70]	0.81758383	0.4072927	42.09174576	176.7196352	6000.46259

design of pressure vessel, and the mathematical formula can be expressed as follows:

consider: $Z = [z_1, z_2, z_3, z_4] = [h, l, t, b]$,

minimize: $f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^3$
 $+ 3.1661x_1^2x_4 + 19.84x_1^2x_3$,

subject to

$$g_1(x) = -x_1 + 0.0193x_3 \leq 0, \quad (34)$$

$$g_2(x) = -x_3 + 0.00954z_3 \leq 0,$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1,296,00 \leq 0,$$

$$g_4(x) = x_4 - 240 \leq 0,$$

where $0 \leq x_1, x_2 \leq 99, 0 \leq x_3$, and $x_4 \leq 200$.

Table 13 shows the research results of the WSOA and other algorithms on pressure vessel design. It can be seen from the table that the results obtained by the WSOA algorithm are better than those obtained by other algorithms, in which the optimal fitness value is $f(x) = 5964.6114$ and the optimal solution is $\vec{x} = [0.8056, 0.4081, 41.7401, 181.1285]$. It is shown that the algorithm has better optimization performance and accuracy in solving the design problem of pressure vessels.

6. Research Limitations

Although the above experimental results show that the WSOA has better performance than most comparative algorithms, it also has some shortcomings. There is a room for improvement in the performance of the WSOA. For example, in the unimodal test function, it is found that the number of theoretical optimum values obtained by the WSOA is small, which shows that the exploitation ability of the WSOA can be further improved in the test unimodal function. After testing the multimodal test function, it is

found that the number of theoretical optimal values obtained is small, which indicates that the exploration and exploitation ability of the WSOA under multiple optimal solutions needs to be further improved. Moreover, the WSOA has not been further verified in a higher dimension. Finally, because the WSOA combines the WOA with the SOA, the structure of this algorithm is more complex than the original algorithm, so the flexibility of the WSOA is worse than that of the original algorithm.

7. Conclusion and Future Work

In this paper, an improved seagull optimization algorithm (WSOA) is proposed to solve the global optimization problem. The combination of contraction and encirclement mechanism of the WOA and spiral attack behavior of the SOA can improve the calculation accuracy of the SOA and balance the exploitation and exploration capabilities more effectively. In addition, introducing levy flight strategy into the search formula can avoid premature convergence of the SOA. In the aspect of performance evaluation, 25 benchmark test functions are used for verification. Comparing the performance of the WSOA with the performances of seven famous metaheuristic algorithms, the results show that the algorithm has strong competitiveness. Statistical test analysis and results comparison show that the improved SOA has better performance than the SOA. In this paper, four engineering examples are also used to test the performance of the proposed algorithm, which proves that the WSOA has better performance.

For the WSOA, further research is needed in future work. First of all, the WSOA can be applied to target allocation problem and more constrained engineering examples. Second, the WSOA is applied to solve real-life problems, such as medical data.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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