Purpose. The main purpose of this study is to investigate the unsteady flow behavior of second-grade inviscid fluid between parallel plates. The effects on the flow are explored through modeling of continuity, momentum, and energy equations. Graphical and tabular exploration has been made to analyze the impact of several influential variables on the dimensionless temperature and velocity profiles. Three-dimensional graphs and streamlines are also mentioned.

Findings. The graphs for the squeezing number, Prandtl number, and Eckert number are decreasing by increasing the values of these parameters. The graphs of skin friction coefficient and Nusselt number are increasing by changing the values of both parameters. Originality/Value. The significances of an unsteady squeezed flow of a nonviscous second-grade fluid between parallel plates by using boundary layer phenomenon are discussed.

1. Introduction

Some important applications of non-Newtonian fluids are introduced to enhance the research interest in food preservation, polymeric substitutions, nuclear fuels, liquid metals, paints, and blood flow. In non-Newtonian fluids, mixed convection phenomenon has enchanted scientific experts because of its momentousness real-world applications, for example, solar energy, electronic appliances cooled with fans, and cooling of nuclear reactors. Complicated comparison of viscous fluids is formed due to strange nature between shear stress and strain rate in such fluids. The criterion of viscoelasticity contributed further complexities in the governing equations at the time of comparison with Navier–Stokes equations. Most of the investigations [1–25], worked on such problems by assuming different types of flows and effects on various fluids. The flow squeezed between parallel walls happens in many biological and industrial systems. The nonsteady viscous flow fluid squeezed between parallel plates is a great subject of interest in hydrodynamic machines due to their motion normal to their own surfaces. The initiate work and the fundamental formulation of under lubrication squeezing flows were assumed by Stefan [14]. In previous literature, over few decades, the flow squeezed by elliptic plates was discussed by Reynolds [15] while Archibald [16] suggested the same inquisition for rectangular plates.

The evaluation of boundary layer squeezed flow is an interesting research matter due to its wide range of applications in industry and engineering. The most common scientific and engineering applications are in the drawing of plastic wires and films, extrusion of a polymer in a melting-spinning process, manufacturing of foods, crystal growing, liquid film in condensation process, electrochemical process, paper and glass fiber production, thermal energy storage, electronic chips, flow through filtering devices, food processing, cooling towers, marine engineering, hydro towers, distillation columns, and so on. The viscosity and thermic conductivity are presumed as a function of temperature. Unsteadiness is the loss of equilibrium with environment, usually with an affection of almost falling, or the consequences of bumping into objects. There are numerous reasons for unsteadiness, together with the problems in the cerebral or cerebellar sections of the spinal cord, brain, inner
ear, or vestibular system. Unsteady flow of the fluid is the one where properties of the fluid vary with time. It is worthless to say that any beginning procedure is unsteady.

Now, the development in industries has motivated the researchers to discover non-Newtonian fluids properties in a more organized way. In nature, a number of fluids show non-Newtonian behavior, i.e., slurries, honey, glue, gels, toothpaste, ketchup, etc. Various paradigms of unsteadiness can be found from our daily life like the flow of water out from a tap which has been just opened. This is unsteady flow in the start, but it becomes steady with time. In the current problem, the boundary layer approximation is utilized to construct an unsteady second-grade fluid flow model. The obtained coupled partial differential equations are simplified by using suitable mathematical techniques. The dimensionless equations are being solved by using numerical techniques, i.e., shooting technique. A comprehensive graphical and tabular study is constructed to check the convergence of the obtained results.

2. Mathematical Description of the Flow Phenomenon

The stress tensor [14] for the current problem is given by

\[ T = -P I + \mu A_1 + \alpha_1 A_2 + \alpha_2 A^2_1. \]  

We have restriction as

\[ \mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0, \quad \alpha_1 = -\alpha_2. \]  

So, equation (1) reduces to

\[ T = -P I + \mu A_1 + \alpha_1 \left( A_2 - A^2_1 \right). \]  

\[ A_1 \] is Rivlin–Ericksen material expansion of the strain rate tensor as the derivative rotates and translates with flow. The non viscous squeezed flow of an unsteady second grade fluid between parallel plates segregated by a distance \( z = \pm l (1 - \alpha t)^{1/2} \), where characteristic parameter is \( \alpha \) and length \( l \) at \( t = 0 \) is considered. Furthermore, \( \alpha > 0 \) is relative to the motion of both squeezed plates till they connect each other at \( t = 1/\alpha \), for when \( \alpha < 0 \), the plates are separated.

The fields of the flows corresponding to Cartesian coordinates [16] are

\[ V = u(x, y, t) + v(x, y, t), \]  

\[ \theta = \theta(x, y, t). \]  

The momentum equations which govern for the problem become
where specific heat is \( C_P \).

By using boundary layer flow equations, (3)–(9) become

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial^2 u}{\partial x^2} + \frac{\alpha_1}{\rho} \frac{\partial^3 u}{\partial t \partial y^2} - \frac{\alpha_1}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\alpha_1}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{k}{\rho C_P} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \frac{\partial^2 u}{\partial x \partial y} \right)^2 - \frac{\alpha_1}{\rho} \frac{\partial u}{\partial x} \left( \frac{\partial^2 u}{\partial y} \right)^2.
\]

(9)

Conforming to layout of the inquisition in Figure 1, the boundary conditions’ suits for the flow are

\[
u = 0, \quad \nu = \frac{dh}{dt}, \quad T = T_H \text{ at } y = h(t),
\]

\[
\frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y}, \quad \nu = 0 \text{ at } y = 0.
\]

(10)

### 2.1. Nondimensional Equations.

Using transformation [11],

\[
u = \frac{ax}{2(1 - at)} f'(\eta), \quad \nu = \frac{-al}{2(1 - at)^{1/2}} F(\eta),
\]

where

\[
\eta = \frac{y}{l(1 - at)^{1/2}},
\]

\[
\theta(\eta) = \frac{T}{T_H}.
\]

(13)

(14)

(15)

Applying equations (15)–(17) in equations (10)–(12), we obtain

\[
2\lambda \delta^4 - 3 \delta^2 F'' + 4 \lambda^2 F'' - SBF''
\]

\[
- \delta \delta^4 \left( \eta F'' - 2F'' + FF'' - 2F'' \right) = 0,
\]

\[
\delta^3 \lambda^2 \theta'' - 2\eta \lambda S^2 \Pr \delta \theta' + \delta Ec Pr \left( \lambda^5 F''^2 - 2 \delta S^2 F'' F'' \right) = 0,
\]

(16)

(17)

where

\[
\delta = \frac{l(1 - at)}{\alpha}, \quad S = \frac{al^2}{2mu} Pr = \frac{C_P k}{E c} = \frac{1}{C_P T_H} \left( \frac{ax}{2(1 - at)} \right).
\]

(18)

Here, nondimensional length is \( \delta \), Ec is Eckert number, \( S \) is squeezing number, and \( Pr \) is Prandtl number is dimensionless.

The nondimensional boundary conditions are

\[
F(0) = 0, \quad F''(0) = 0,
\]

\[
F(1) = 1, \quad F'(1) = 0,
\]

\[
\theta'(0) = 0,
\]

\[
\theta(1) = 1.
\]

(19)

(20)

(21)

(22)

### 3. Numerical Solution

The highly nonlinear partial differential equations are changed to ordinary differential equations by using transformations by shooting technique along with Runge–Kutta scheme are numerically solved with the aid of Maple software equations (34)–(45).

New variables are used to lessen the higher order ODEs into 1st order equations, i.e.,

\[
F = w_1, \quad F' = w_2, \quad F'' = w_3, \quad F''' = w_4, \quad F'''' = w_5,
\]

\[
\theta = w_6, \quad \theta' = w_7, \quad \theta'' = w_8.
\]

(23)

New system of ODEs by using equation (23) is formed, i.e.,

\[
w_1' = w_2, \quad w_2' = w_3, \quad w_3' = w_4, \quad w_4' = w_5,
\]

\[
w_5' = \frac{1}{2\lambda S} \left[ -\delta^2 F'' - 4 \lambda^2 F'' + SBF''
\]

\[
+ \delta \delta^4 \left( \eta F'' - 2F'' + FF'' - 2F'' \right) \right],
\]

\[
w_1 = \frac{1}{\delta^3 \lambda^2} \left[ 2\lambda S^2 \Pr \delta \theta' - \delta Ec Pr \left( \lambda^5 F''^2 - 2 \delta S^2 F'' F'' \right) \right],
\]

(24)

(25)

(26)

along with boundary conditions

\[
a_1(0) = 0, a_1(\infty) = 1, a_2(0) = 1, a_2(\infty) = 0.
\]

(27)
4. Graphical Discussion

The graphical discussion of the problem described is as given below. Figure 2 shows the results of $\delta$ for velocity by assuming the values of $\delta$ as $\delta = 0.1, 1.5, 1.8, 2.0$ which give a decline in the graph. Figures 3–5 give the temperature profile for the Eckert number, Prandtl number, and squeezing number by using different values of all numbers as $Ec = 11.5, 11.8, 12.1, 12.4$ and $Pr = 7.3, 7.5, 7.7, 7.9$, and we have used such values of the Prandtl number due to non-Newtonian fluid flow and similarly for $S = 0.1, 0.11, 0.12, 0.13$. The graph decreases for $Ec$ and $Pr$ while increases for the squeezing number. Figure 6 shows skin friction graph for $\lambda$ and $\delta$ for the values $\lambda = 0.1, 0.3, 0.5$ with positively

**Figure 2:** Impact of decreasing effects of $\delta$ for velocity portray.

**Figure 3:** Illustration of decreasing effect of $Ec$ on the temperature distribution.

**Figure 4:** Decreasing effect of $Pr$ on temperature portray.

**Figure 5:** Influence of increasing effects of the squeezing number on temperature.
Figure 6: Increasing effects of \( \lambda \) on the skin friction factor.

Figure 7: Impact of increasing effects of \( \lambda \) for the Nusselt number.

Figure 8: Stream lines for \( \delta = 0.1 \).
increasing values. Figure 7 explicates the graphical representations of the Nusselt number for $\lambda$ and $\delta$ with $\lambda = 1.1, 2.1, 3.1$ with increment in the graph, as the Nusselt number is the ratio of convective to conductive heat transfer at the boundary in a fluid. Figures 8–10 illustrate the stream lines for the values of $\delta = 0.1, 0.3, 0.5$. Figures 11–13 demonstrate the three-dimensional graph for $\delta$. Table 1 gives different values for skin friction for $\delta, \lambda, S$, and $B$. 
Figure 11: 3D graph for $\delta = 0.1$.

Figure 12: 3D graph for $\delta = 0.3$.

Figure 13: 3D graph for $\delta = 0.5$. 
Table 1: Values of $\delta, \lambda, S,$ and $B$ for the skin friction coefficient.

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<th>$\lambda$</th>
<th>$B$</th>
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5. Concluding Remarks

Non-Newtonian fluid flow is very persuasive topic since many years because it has an extensive use in many applications such as mining industry, chemical engineering, petroleum engineering, and plastic processing industry. Now, the development in industries has motivated the researchers to discover non-Newtonian fluids’ properties in a more organized way. We have discussed the unsteady boundary layer flow of a second-grade fluid. The discussion is significantly influenced by the fluid which is constructed to check the obtained results which are given below:

1. The velocity with respect to delta and temperature for Ec and Pr are decreasing
2. We have increasing graphs of the skin friction, Nusselt number, and squeezing number of the fluid by increasing the values of their parameters

Abbreviations

- $C_p$: Specific heat at constant pressure $JK^{-1}kg^{-1}$
- $t$: Time $[T]$
- $\alpha$: Squeezed strength $[l]$  
- $f$: Body force $[N/m^3]$
- $P$: Pressure field $[ML/T^2]$
- $\lambda$: Viscosity ratio
- $T$: Temperature $[K]$
- $\rho$: Density $[kg/m^3]$
- $\theta$: Temperature $[K]$
- $Pr$: Prandtl number $Ns/m^2$
- $\tau$: Stress tensor $N/m^2$
- $V$: Velocity of the fluid $[m^3/s]$
- $\mu_0$: Viscosity of the fluid $N/m^2s^{-1}$
- $y$: Dimensionless number
- $N(\tau)$: Distribution function
- $A_i$: Rivlin–Ericksen tensor
- $Ec$: Eckert number $m^2/s$
- $u, v$: Velocity components along $x, y$ direction $[L/T]$
- $I$: Identity tensor
- $x, y, z$: Spatial coordinates $[L]$

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


