Research Article

Fixed-Time Synchronization for Dynamical Complex Networks with Nonidentical Discontinuous Nodes

Xiaoliang Qian,¹ Qian Liu,¹ Qingbo Li,² Qi Yang,¹ Yuanyuan Wu ,³ and Wei Wang ¹

¹College of Electric and Information Engineering, Zhengzhou University of Light Industry, Zhengzhou 450002, China

²College of Mathematics and Information Science, Zhengzhou University of Light Industry, Zhengzhou 450002, China

³School of Information and Communication Engineering, Hainan University, Haikou 570228, China

Correspondence should be addressed to Yuanyuan Wu; wuyuanuan82@163.com and Wei Wang; wangwei-zzuli@zzuli.edu.cn

Received 18 December 2020; Revised 3 February 2021; Accepted 27 February 2021; Published 19 March 2021

Academic Editor: Shihong Ding

Copyright © 2021 Xiaoliang Qian et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This article investigates the fixed-time synchronization issue for linearly coupled complex networks with discontinuous non-identical nodes by employing state-feedback discontinuous controllers. Based on the fixed-time stability theorem and linear matrix inequality techniques, novel conditions are proposed for concerned complex networks, under which the fixed-time synchronization can be realized onto any target node by using a set of newly designed state-feedback discontinuous controllers. To some extent, this article extends and improves some existing results on the synchronization of complex networks. In the final numerical example section, the Chua circuit network is introduced to indicate the effectiveness of our method by showing its fixed-timely synchronization results with the proposed control scheme.

1. Introduction

As we know, in the last few decades, complex networks have been widely presented in our real world, for example, electrical power grids, metabolic pathways, neural networks, food webs, and World Wide Web [1–5]. Synchronization is a well-known crucial collective behavior for complex networks, so the synchronization of complex networks has received more attention due to many crucial applications [6–8] in information processing, secure communication, and biological systems [9–12]. Up to now, there are many research studies on complex network synchronization, most of which focus on asymptotic synchronization, mainly on asymptotic synchronization behavior [13–15] and exponential synchronization results [16], but the two kinds of synchronization belong to the infinite-time category [17–19].

Since it has been found that finite-time control ways will further enhance the rate of convergence greatly and synchronization will be performed in a settling time by designing appropriate finite-time synchronization controllers, the finite-time synchronization research [20–25] in complex

networks has been carried out one after another [26–30]. In [20], the issue of the finite-time synchronization is studied between complex networks with nondelay and delay coupling by using pulse control and periodic intermittent control. By use of aperiodically intermittent control, Liu et al. [21] considered the finite-time synchronization problem in dynamic networks with time delay. The global random finite-time synchronization issue is investigated in [22] for discontinuous semi-Markov switched neural networks with time delay and noise interference. The finite-time synchronization analysis of linear coupled complex networks is discussed [23] with discontinuous nonidentical nodes.

The convergence rate of classical finite-time synchronization is relatively fast in contrast to asymptotic synchronization and exponential synchronization. However, it has an obvious disadvantage that the synchronization convergence rate of complex networks depends on the initial states of all nodes. Unfortunately, it is very difficult or even impossible for some chaotic systems to know their state previously. In these results, the finite-time control methods may be ineffective. Taking advantage of the benefits of finite-

time control, a special finite-time synchronization is proposed in [20]. As for the novel fixed-time synchronization, the settling time has no relation with the initial conditions of the network system and only depends on the control parameters of the system controller, see [21, 22]. Thus, synchronization can be accomplished by using the fixed-time controller within a specified time. This remarkable characteristic makes fixed-time synchronization control more desirable than other synchronization controls and improves its practical application range. Therefore, fixed-time synchronization control of complex networks has received more attention [23–25, 31–34].

Moreover, if the dynamics of the nodes are different, then the synchronization issue will be more complex and challenging than the same node condition. By using the free matrix, the equilibrium solution synchronization is concerned on all alone nodes together with the average state trajectory synchronization of different nodes in [35]. The intermittent controller is employed to fix the complex network with different nodes in [36]. In [37], the cluster synchronization problem is investigated for complex dynamic networks with time-delay coupling and nonidentical nodes by the pinning control method. Furthermore, the finite-time synchronization issue is considered for coupled complex networks with discontinuous nonidentical nodes in [23].

Recently, the complex networks with perturbations have attracted more attention for their wide applications [38–43]. In [40], the global exponential synchronization issue is studied for linear coupled neural networks with impulsive disturbance and time-varying delay. The clustering synchronization scheme is deeply concerned with regard to uncertain delayed complex networks in [41]. The adaptive pinning control design is proposed in [42] for the clustering synchronization problem of coupled complex networks with uncertain disturbances.

Until now, there are several research results on the finite-time synchronization of complex networks with different nodes or uncertain disturbances, mostly about the asymptotic or exponential synchronization. However, it has not been fully investigated for the fixed-time synchronization analysis of heterogeneous networks with uncertain disturbances, and the relevant research results are rarely covered. In a word, it is indispensable and significant to consider the fixed-time synchronization problem of complex networks with different nodes and uncertain disturbances, which has profound theoretical and practical significance. From the above analysis, we face two difficulties: (i) what conditions are applicable and easy to verify for general complex networks with different nodes and uncertain disturbances? (ii) How to design the controller to overcome heterogeneity and uncertain disturbance of network nodes? This paper tries to conquer these two difficulties and realize the fixed-time synchronization of a certain kind of complex linear coupled networks with different nodes and uncertain disturbances, and then the theoretical results of network synchronization can be further enriched.

Applying the discontinuous control scheme, the fixed-time synchronization problem is analysed for complex

networks with uncertain disturbances and nonidentical nodes. Our main contributions here can be concluded as follows: (1) for a class of heterogeneous networks with uncertain disturbances, a novel state-feedback discontinuous controller is designed to get over the influence on the fixed-time synchronization from heterogeneous nodes and uncertain disturbances simultaneously; (2) several criteria are proposed to deduce the fixed-time synchronization for the considered networks. Unlike most existing results, the obtained fixed-time synchronization conditions are expressed by linear matrix inequality, which is easy to be verified; (3) as special cases, the fixed-time synchronization of complex networks without uncertain disturbances is also considered by employing some existing controllers, respectively, and the corresponding results are given in some corollaries.

The rest of the paper is arranged as follows. A network model is established with uncertain disturbances and nonidentical nodes, and then the problem of the fixed-time synchronization is described; meanwhile, some necessary definitions and assumptions are given in Section 2. The fixed-time synchronization conditions are achieved in Section 3. Several numerical examples are introduced in Section 4 to indicate the effectiveness of the proposed results. Section 5 summarizes the research conclusions of this paper and puts forward the future research directions.

2. Problem Formulation and Preliminaries

A kind of nonlinear system including N nonidentical nodes with diffusion linear coupling is considered, in which each node can be regarded as an n -dimensional dynamic system, as shown in the following:

$$\dot{x}_i(t) = A_i x_i(t) + f_i(t, x_i(t)) + h_i(t, x_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma x_j(t), \quad (1)$$

where $x_i(t) = [x_{i1}(t), \dots, x_{in}(t)]^T \in \mathbb{R}^n$ denotes the state vector of the i th dynamical node; the dynamics of the i th uncoupled node is $\dot{x}_i(t) = A_i x_i(t) + f_i(t, x_i(t)) + h_i(t, x_i(t))$ in which $A_i \in \mathbb{R}^{n \times n}$, and $f_i(t, x_i(t)) = [f_{i1}(t, x_i(t)), f_{i2}(t, x_i(t)), \dots, f_{in}(t, x_i(t))]^T: \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents a smooth nonlinear vector showing the node self-dynamics; moreover, $h_i(t, x_i(t)) = [h_{i1}(t, x_i(t)), h_{i2}(t, x_i(t)), \dots, h_{in}(t, x_i(t))]^T: \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is an uncertain vector and represents the disturbance. The constant $c > 0$ can be considered as the coupling strength of the concerned networks, and $\Gamma = (\gamma_{ij})_{ij} \in \mathbb{R}^{n \times n}$ is a matrix and denotes the inner coupling relation between the network nodes and indicates how the components of each pair of nodes are connected with each other, and $\gamma_{ij} \geq 0$; $G = (G_{ij})_{N \times N}$ is a coupling configuration constant matrix, which describes the topological structure and can be exhibited as the diffusion structure, i.e., $G_{ij} \geq 0$ and $G_{ii} = -\sum_{j=1, j \neq i}^N G_{ij}$. In this paper, the driven dynamical node of (1) satisfies

$$\dot{x}_0(t) = A_0 x_0(t) + f_0(t, x_0(t)) + h_0(t, x_0(t)), \quad (2)$$

where $A_0 \in \mathbb{R}^{n \times n}$, $f_0(t, x_0(t)) \in \mathbb{R}^n$, and $h_0(t, x_0(t)) \in \mathbb{R}^n$.

In fact, most of the well-known chaotic systems can be described by the above dynamical equation, such as Sprott circuit, Chua circuit, Rössler's systems, and Chen system [42].

Definition 1 (see [44]). Complex network (1) is said to be synchronized onto (2) in finite time if there exist a designed feedback controller to system (1) and a constant $t^* > 0$ such that

$$\lim_{t \rightarrow t^*} \|x_i(t) - x_0(t)\| = 0, \quad (3)$$

$$\|x_i(t) - x_0(t)\| \equiv 0, \quad i = 1, 2, \dots, N,$$

where $t^* > 0$ is called the settling time and often depends on the initial state vector value $X(0) = (x_1^T(0), \dots, x_N^T(0))^T$.

Definition 2 (see [44]). Complex network (1) is said to be synchronized onto (2) in fixed time if there exists a fixed settling time $T^* > 0$ such that

$$\lim_{t \rightarrow T^*} \|x_i(t) - x_0(t)\| = 0, \quad (4)$$

$$\|x_i(t) - x_0(t)\| \equiv 0, \quad i = 1, 2, \dots, N, t > T^*,$$

where $T^* > 0$ is called the settling time and is independent of the initial synchronization error $X(0) = (x_1^T(0), \dots, x_N^T(0))^T$.

In this paper, the goal is to fixed-timely synchronize the state of network (1) onto the driven one (2) by designing feedback controllers.

Obviously, controlled complex network (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + f_i(t, x_i(t)) + h_i(t, x_i(t)) \\ &+ c \sum_{j=1}^N G_{ij} \Gamma x_j(t) + u_i(t). \end{aligned} \quad (5)$$

Introduce the synchronization errors $e_i(t)$, which are defined as $e_i(t) = x_i(t) - x_0(t)$, $i = 1, 2, \dots, N$, $F_i(t) = f_i(t, x_i) - f_0(t, x_0) + (A_i - A_0)x_i(t)$, and $H_i(t, e_i(t)) = h_i(t, x_i(t)) - h_0(t, x_0(t))$. Subtracting (2) from (5), the error dynamical network model can be given by

$$\begin{aligned} \dot{e}_i(t) &= A_0 e_i(t) + F_i(t, e_i(t)) + H_i(t, e_i(t)) \\ &+ c \sum_{j=1}^N G_{ij} \Gamma e_j + u_i(t), \quad i = 1, \dots, N. \end{aligned} \quad (6)$$

In order to obtain our main results, some necessary assumptions are listed as follows.

Assumption 1. There exist constant $M_i > 0$ and uniformly symmetric positive definite matrix L_i , $i = 1, 2, \dots, n$, such that $f_i(t, x)$ satisfies

$$\begin{aligned} (y - x)^T (f_i(t, y) - f_i(t, x)) &\leq (y - x)^T L_i (y - x) \\ &+ M_i \sum_{j=1}^n |y_j - x_j|, \quad i = 1, \dots, n, \end{aligned} \quad (7)$$

for all $t \geq 0$, $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, and $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$.

Assumption 2. There exist constant $M_0 > 0$ and uniformly symmetric positive definite matrix L_0 such that $f_0(t, x)$ satisfies

$$\begin{aligned} (y - x)^T (f_0(t, y) - f_0(t, x)) &\leq (y - x)^T L_0 (y - x) \\ &+ M_0 \sum_{j=1}^n |y_j - x_j|, \end{aligned} \quad (8)$$

for all $t \geq 0$, $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, and $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$.

Assumption 3 (see [43]). There exists a time-varying function $\mu(t) \geq 0$ such that

$$|f_i(t, x) - f_0(t, x)| \leq \mu(t), \quad i, j = 1, \dots, N. \quad (9)$$

Assumption 4. For any i , $i = 1, 2, \dots, n$, the uncertain function vector $h_i(t, x_i(t))$ is assumed to be continuous at $t, x_i(t) \geq 0$ and bounded. Moreover, there is a known nonnegative number $h_{\max}$ such that

$$|h_i(t, x_i(t))| \leq h_{\max}, \quad i = 0, 1, \dots, N. \quad (10)$$

Remark 1. Assumptions 1 and 2 are general and satisfied with most of the well-known chaotic systems, for instance, Chua circuit [43], Rössler's systems, and discontinuous Chen system. In fact, the above systems meet the following conditions: there exist some positive constants $k_{ij} > 0, \beta_j$, $i, j = 1, 2, \dots, n$, satisfying

$$\begin{aligned} \|f_i(t, y) - f_i(t, x)\| &\leq \sum_{j=1}^n k_{ij} |y_j(t) - x_j(t)| + \beta_i, \\ i &= 1, 2, \dots, n, \end{aligned} \quad (11)$$

for any $t \geq 0$, $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, and $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$. Using condition (11), we have

$$\begin{aligned}
(y(t) - x(t))^T (f_i(t, y) - f_i(t, x)) &\leq \sum_{i=1}^n \sum_{j=1}^n k_{ij} |y_i(t) - x_i(t)| |y_j(t) - x_j(t)| + \sum_{j=1}^n \beta_i |y_j(t) - x_j(t)| \\
&= \sum_{i=1}^n \sum_{j=1}^n k_{ij}^{(1/2)} |y_i(t) - x_i(t)| k_{ij}^{(1/2)} |y_j(t) - x_j(t)| + \sum_{j=1}^n \beta_i |y_j(t) - x_j(t)| \\
&\leq \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2} (k_{ij} + k_{ji}) |y_i(t) - x_i(t)|^2 + \sum_{j=1}^n \beta_i |y_j(t) - x_j(t)| \\
&\leq (y(t) - x(t)) L_i (y(t) - x(t)) + M_i \sum_{j=1}^n |y_j(t) - x_j(t)|,
\end{aligned} \tag{12}$$

where $L_i = \text{diag}(l_{11}^i, l_{22}^i, \dots, l_{nn}^i)$, $l_{jj}^i = \sum_{j=1}^n (1/2)(k_{ij} + k_{ji})$, and $M_i = \beta_i$, $1 \leq i \leq n$. Clearly, if $\alpha = \max_{1 \leq i \leq n} \{l_{jj}^i, j = 1, 2, \dots, n\}$ and $M = \max_{1 \leq i \leq n} \{\beta_i\}$, then Assumptions 1 and 2 involve conditions (H2) and (H3) in [25]. Moreover, the continuous chaotic system is also a special case by setting $M_i = 0$ in Assumption 1 or $M_0 = 0$ in Assumption 2, for instance, the continuous Rössler system, Chua's circuit, Chen system, Lorenz system, and logistic differential system. Hence, Assumptions 1 and 2 are more general, and most popular chaotic systems are applicable. Assumptions 3 and 4 take advantage of conditions on the activation function, and it is seen that they are diffusely imposed in the literature [23, 35–37].

According to Definition 2, it is clear that the fixed-time synchronization of dynamical network (5) onto (2) can be degenerated into the fixed-time stabilization of error dynamic system (6).

3. Fixed-Time Synchronization Analysis

In this part, the controllers are designed for the fixed-time synchronization problem of complex network (1), and concerned complex network (5) can realize the fixed-timely synchronization under the appropriate designed controllers. Firstly, we give the synchronization controller design of complex network (1), and then fixed-time synchronization criteria can be obtained based on error system (6). Several corollaries are also obtained for (5) and (2) with identical nodes. For concerned complex network (1), the control input $u_i(t) \in \mathbb{R}^n$, $i = 1, \dots, N$, is designed as follows:

$$\begin{aligned}
u_i(t) &= (A_0 - A_i)x_0 - d_i e_i(t) - \eta_i(t) \text{sign}(e_i(t)) \\
&\quad - \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q),
\end{aligned} \tag{13}$$

where $d_1, \dots, d_N$ are positive constants, $\eta_i(t)$ is a function to be determined, a, b are positive constants, $\text{sign}(e_i(t)) = (\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{in}(t)))^T$, $|e_i(t)|^p = (|e_{i1}(t)|^p, \dots, |e_{in}(t)|^p)^T$, $|e_i(t)|^q = (|e_{i1}(t)|^q, \dots, |e_{in}(t)|^q)^T$, $\overline{\text{sign}}(e_i(t)) = \text{diag}(\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{in}(t)))$, and the real numbers p, q follow $0 < q < 1, p > 1$.

The following lemmas are necessary and given to derive the subsequent main results.

Lemma 1 (see [2, 45, 46]). Suppose that function $V(t): \mathbb{R}^n \rightarrow \mathbb{R}$ is C -regular and $x(t): [0, +\infty) \rightarrow \mathbb{R}^n$ is absolutely continuous on any compact interval $[0, +\infty)$. Denote $v(t) = V(x(t))$ if there exists a continuous function $\gamma: [0, +\infty) \rightarrow \mathbb{R}$ with $\gamma(\sigma) > 0$ for $\sigma \in (0, +\infty)$ such that

$$\dot{v}(t) \leq -\gamma(v), \tag{14}$$

for any $t > 0$ that $v(t) > 0$, and $v(t)$ is differentiable at t and satisfies

$$\int_0^{v(0)} \frac{1}{\gamma(v)} = t_1 < +\infty. \tag{15}$$

Then, we have $v(t) = 0$ for $t \geq t_1$. In particular, if $\gamma(v) = Qv^\mu$, where $\mu \in (0, 1)$ and $Q > 0$, then the setting time is estimated by

$$t_1 = \frac{v^{1-\mu}}{Q(1-\mu)}. \tag{16}$$

Lemma 2 (see [47]). For matrices A, B, C , and D with appropriate dimensions and a scalar α , the following assertions hold:

- (1) $(\alpha A) \otimes B = A \otimes (\alpha B)$,
- (2) $(A + B) \otimes C = A \otimes C + B \otimes C$,
- (3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$, and
- (4) $(A \otimes B)^T = A^T \otimes B^T$,

where $\otimes$ is the Kronecker product.

Lemma 3 (see [48]). Suppose there exists a continuous, positive-definite function $V(t)$ satisfying

$$\frac{dV(t)}{dt} \leq IV(t) - kV^\alpha(t), \quad \forall t \geq t_0, V^{1-\alpha}(t_0) \geq \frac{k}{I}, \tag{17}$$

where $k > 0$, $I > 0$, and $0 < \alpha < 1$ are three constants. Then, the following inequality is true:

$$V^{1-\alpha}(t) \geq V^{1-\alpha}(t_0) - k(1-\alpha)(t-t_0), \quad t_0 < t < t_1, \tag{18}$$

$V(t) \equiv 0$, $\forall t > t_1$, and the settling time t_1 is estimated by

$$t_1 = t_0 + \frac{\ln(1 - (I/k)V^{1-\alpha}(t_0))}{I(\alpha-1)}. \tag{19}$$

Lemma 4. Suppose there exists a continuous radially unbounded function $V(e(t)): \mathbb{R}^N \rightarrow [0, +\infty)$ satisfying the following two conditions:

- (i) If $e(t) \neq 0$, then $V(e(t)) > 0$ and $V(e(t)) = 0 \Leftrightarrow e(t) = 0$.
- (ii) Any solution $e(t)$ of system (6) satisfies

$$\frac{dV(e(t))}{dt} \leq -aV^p(e(t)) - bV^q(e(t)), \quad (20)$$

where $a > 0$, $b > 0$, $p > 1$, and $0 < q < 1$ are all constants. Then, $V(e(t))$ satisfies $V(e(t)) \equiv 0$, $\forall t > T^*$, and the fixed settling time is estimated by

$$T^* = \frac{1}{a(p-1)} + \frac{1}{b(1-q)}. \quad (21)$$

Lemma 5 (see [48]). Suppose that $a_i \geq 0$ ($i = 1, \dots, n$), $0 < p \leq 1$, and $0 < q < 2$; it follows that

$$\begin{aligned} \left(\sum_{i=1}^n a_i \right)^p &\leq \sum_{i=1}^n (a_i)^p, \\ \sum_{i=1}^n (a_i)^q &\geq \left(\sum_{i=1}^n a_i^2 \right)^{(q/2)}, \\ \left(\frac{1}{n} \sum_{i=1}^n (a_i)^q \right)^q &\geq \left(\frac{1}{n} \sum_{i=1}^n a_i^2 \right)^{(1/2)}. \end{aligned} \quad (22)$$

Theorem 1. For concerned complex network (1) with the control input, if Assumptions 1–4 hold and

$$\mu(t) + 2h_{\max} + M_i - \eta_i(t) < 0, \quad i = 1, 2, \dots, N, \quad (23)$$

$$\mathcal{L} + \mathcal{A} + cG^s \otimes \Gamma - D \otimes I_n < 0, \quad (24)$$

with $D = \text{diag}\{d_1, \dots, d_N\} > 0$, $G^s = ((G + G^T)/2)$, $\mathcal{L} = \text{diag}(L_1^s, L_2^s, \dots, L_N^s)$, and $\mathcal{A} = \text{diag}(A_1^s, A_2^s, \dots, A_N^s)$, then driven-response complex networks (1) and (2) can achieve fixed-time synchronization under controller (13), with the settling time

$$T^* = \frac{1}{2^{(1+p/2)} a (Nn)^{(1-p/2)} (p-1)} + \frac{1}{2^{(1+q/2)} b (1-q)}, \quad (25)$$

where $V(0) = (1/2) \sum_{i=1}^N e_i^T(0) e_i(0)$ and $e_i(0)$ is the initial value of $e_i(t) = x_i(t) - x_0(t)$ for $i = 1, \dots, N$. Proof.

For error dynamical system (6), a Lyapunov function is listed by

$$V(t) = \frac{1}{2} \sum_{i=1}^N e_i^T(t) e_i(t), \quad (26)$$

and the derivative of the above Lyapunov function along the trajectory of system (6) can be computed as

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T(t) \dot{e}_i(t) = \sum_{i=1}^N e_i^T(t) \left\{ A_0 e_i(t) + F_i(t, e_i(t)) + H_i(t, e_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma e_j(t) + u_i(t) \right\} \\ &= \sum_{i=1}^N e_i^T(t) \left\{ -d_i e_i(t) - \eta_i(t) \text{sign}(e_i(t)) - \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q) \right\} \\ &\quad + I_1(t) + I_2(t) + I_3(t), \end{aligned} \quad (27)$$

with $I_1(t) = \sum_{i=1}^N e_i^T(t) A_0 e_i(t) + c \sum_{i=1}^N e_i^T(t) \sum_{j=1}^N G_{ij} \Gamma e_j(t)$, $I_2(t) = \sum_{i=1}^N e_i^T(t) H_i(t, e_i(t))$, and $I_3(t) = \sum_{i=1}^N e_i^T(t) (F_i(t, e_i(t)) - (A_i - A_0)x_0(t))$.

According to Lemma 2, we know the following equation is true:

$$\begin{aligned} I_1(t) &= e^T(t) (I_N \otimes A_0) e(t) + c e^T(t) (G \otimes \Gamma) e(t) \\ &= e^T(t) (I_N \otimes A_0^s) e(t) + c e^T(t) \frac{G \otimes \Gamma + (G \otimes \Gamma)^T}{2} e(t) \\ &= e^T(t) (I_N \otimes A_0^s + c G^s \otimes \Gamma) e(t). \end{aligned} \quad (28)$$

For the term $I_2(t)$, using condition (10) in Assumption 4, we get the following inequality:

$$\begin{aligned} I_2(t) &= \sum_{i=1}^N e_i^T(t) (h_i(t, x_1(t)) - h_0(t, x_0(t))) \leq \sum_{i=1}^N |e_i(t)| |h_i(t, x_1(t)) - h_0(t, x_0(t))| \\ &\leq \sum_{i=1}^N |e_i(t)| (2h_{\max}) = \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| (2h_{\max}). \end{aligned} \quad (29)$$

Also, it can be seen that

$$\begin{aligned}
 I_3(t) &= \sum_{i=1}^N e_i^T(t) ((f_i(t, x_i(t)) - f_0(t, x_0(t))) + (A_i - A_0)x_i(t) - (A_i - A_0)x_0(t)) \\
 &= \sum_{i=1}^N e_i^T(t) ((f_i(t, x_i(t)) - f_i(t, x_0(t))) + (f_i(t, x_0(t)) - f_0(t, x_0(t))) + (A_i - A_0)e_i(t)) \\
 &= I_{31}(t) + I_{32}(t) + e(t)^T (\mathcal{A} - I_N \otimes A_0^s) e(t),
 \end{aligned} \tag{30}$$

with $I_{31}(t) = \sum_{i=1}^N e_i^T(t)(f_i(t, x_i(t)) - f_i(t, x_0(t)))$, $I_{32}(t) = \sum_{i=1}^N e_i^T(t)(f_i(t, x_0(t)) - f_0(t, x_0(t)))$, and $\mathcal{A} = \text{diag}(A_1^s, A_2^s, \dots, A_N^s)$.

From (7) in Assumption 1, it is known that

$$\begin{aligned}
 I_{31}(t) &\leq \sum_{i=1}^N e_i^T(t) L_i e_i(t) + \sum_{i=1}^N \left(M_i \sum_{j=1}^N |e_{ij}(t)| \right) \\
 &= e^T(t) \mathcal{L} e(t) + \sum_{i=1}^N \left(M_i \sum_{j=1}^N |e_{ij}(t)| \right),
 \end{aligned} \tag{31}$$

where $\mathcal{L} = \text{diag}(L_1^s, L_2^s, \dots, L_N^s)$.

By (9) in Assumption 3, one can get

$$\begin{aligned}
 I_{32}(t) &\leq \sum_{i=1}^N |e_i(t)| \|f_i(t, x_0(t)) - f_0(t, x_0(t))\| \\
 &\leq \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| \mu(t).
 \end{aligned} \tag{32}$$

Submitting (28)–(32) into (27) yields that

$$\begin{aligned}
 \dot{V}(t) &\leq e^T(t) \mathcal{L} e(t) + e^T(t) (I_N \otimes A_0^s + cG^s \otimes \Gamma) e(t) \\
 &\quad + \mu(t) \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| + \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| (2h_{\max}) + e(t)^T (\mathcal{A} - I_N \otimes (A_0)^s) e(t) + \sum_{i=1}^N \left(M_i \sum_{j=1}^N |e_{ij}(t)| \right) \\
 &\quad + \sum_{i=1}^N e_i^T(t) \{ -d_i e_i(t) - \eta_i(t) \text{sign}(e_i(t)) - \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q) \} \\
 &= W_1(t) + W_2(t) + W_3(t),
 \end{aligned} \tag{33}$$

where

$$\begin{aligned}
 W_1(t) &= e^T(t) \mathcal{L} e(t) + e^T(t) (I_N \otimes A_0^s + cG^s \otimes \Gamma) e(t) + e(t)^T (\mathcal{A} - I_N \otimes A_0^s) e(t) \\
 &\quad - \sum_{i=1}^N e_i^T(t) d_i e_i(t),
 \end{aligned} \tag{34}$$

$$\begin{aligned}
 W_2(t) &= \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| \mu(t) + \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| (2h_{\max}) + \sum_{i=1}^N \left(M_i \sum_{j=1}^N |e_{ij}(t)| \right) \\
 &\quad - \sum_{i=1}^N e_i^T(t) (\eta_i(t) \text{sign}(e_i(t))),
 \end{aligned}$$

$$W_3(t) = - \sum_{i=1}^N e_i^T(t) \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q). \tag{35}$$

By use of Lemma 2 and (24), it gives that

$$\begin{aligned}
W_1(t) &= e^T(t) (\mathcal{L} + I_N \otimes A_0^s + cG^s \otimes \Gamma + \mathcal{A}) e_i(t) \\
&\quad - e^T(t) (I_N \otimes (A_0^s)) e(t) - e^T(t) (D \otimes I_n) e_i(t) \\
&= e^T(t) (\mathcal{L} + \mathcal{A} + cG^s \otimes \Gamma - D \otimes I_n) e(t) \leq 0,
\end{aligned} \tag{36}$$

where $D = \text{diag}(d_1, d_2, \dots, d_N)$.

Obviously, $e_i^T(t) \text{sign}(e_i(t)) = (e_{i1}(t), \dots, e_{in}(t)) (\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{in}(t)))^T = \sum_{j=1}^n |e_{ij}(t)|$ and $\|e_i(t)\|_2 - e_i^T(t) \overline{\text{sign}}(e_i(t)) = (\sum_{j=1}^n e_{ij}^2(t))^{(1/2)} - \sum_{j=1}^n |e_{ij}(t)| \leq 0$, and applying Lemma 5, it gives that

$$\begin{aligned}
W_2(t) &= \sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)| \mu(t) + \sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)| (2h_{\max}) + \sum_{i=1}^N \left(M_i \sum_{j=1}^n |e_{ij}(t)| \right) \\
&\quad - \sum_{i=1}^N e_i^T(t) (\eta_i(t) \overline{\text{sign}}(e_i(t))) = \sum_{i=1}^N (\mu(t) + 2h_{\max} + M_i - \eta_i(t)) \sum_{j=1}^n |e_{ij}(t)| \leq 0.
\end{aligned} \tag{37}$$

Considering that $|e_i(t)| = (|e_{i1}(t)|, \dots, |e_{in}(t)|)^T$ and $|e_i(t)|^\beta = (|e_{i1}(t)|^\beta, \dots, |e_{in}(t)|^\beta)^T$, we have

$$\begin{aligned}
W_3(t) &= - \sum_{i=1}^N e_i^T(t) \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q) - \sum_{i=1}^N e_i^T(t) k \text{sign}(e_i(t)) |e_i(t)|^\beta \\
&= - \left(a \sum_{i=1}^N |e_i(t)|^T |e_i(t)|^p + b \sum_{i=1}^N |e_i(t)|^T |e_i(t)|^q \right) = - \left(a \sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^{1+p} + b \sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^{1+q} \right).
\end{aligned} \tag{38}$$

By Lemma 5, it can be obtained that

$$\left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^{1+q} \right)^{(1/(1+q))} \geq \left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^2 \right)^{(1/2)}, \tag{39}$$

and then

$$\left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^{1+p} \right)^{(1/(1+p))} \geq (Nn)^{(1/2)-(1/(1+p))} \left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^2 \right)^{(1/2)}. \tag{40}$$

Similarly, using Lemma 5, we have

Therefore,

$$a \sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^{1+p} \geq a (Nn)^{(1-p/2)} \left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^2 \right)^{(1+p/2)}, \tag{41}$$

which together with (38)–(42) implies that

$$\begin{aligned}
W_3(t) &\leq -a (Nn)^{(1-p/2)} \left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^2 \right)^{(1+p/2)} - b \left(\sum_{i=1}^N \sum_{j=1}^n |e_{ij}(t)|^2 \right)^{(1+p/2)} \\
&= -a (Nn)^{(1-p/2)} \left(\sum_{i=1}^N e_i(t)^T e_i(t) \right)^{(1+p/2)} - b \left(\sum_{i=1}^N e_i(t)^T e_i(t) \right)^{(1+p/2)} \\
&= -a (Nn)^{(1-p/2)} 2^{(1+p/2)} V(t)^{(1+p/2)} - b 2^{(1+p/2)} V(t)^{(1+p/2)}.
\end{aligned} \tag{42}$$

Submitting (36), (37), and (43) to (33), we can get

$$\dot{V}(t) \leq -2^{(1+p/2)}(Nn)^{(1-p/2)}aV(t)^{(1+p/2)} - 2^{(1+p/2)}bV(t)^{(1+p/2)}. \quad (44)$$

According to Lemma 4, $V(t)$ converges to zero within a settling time T^* , which is defined in Definition 2, and one can obtain that, by use of controller (13), the considered complex network (1) is fixed-timely synchronized onto driven node (2) within the fixed time T^* , which is given by

$$T^* = \frac{1}{2^{(1+p/2)}a(Nn)^{(1-p/2)}(p-1)} + \frac{1}{2^{(1+p/2)}b(1-q)}. \quad (45)$$

Therefore, it can be concluded that the error vector $e_i(t)$ converges to zero within T^* , and driven-response complex networks (1) and (2) are fixed-timely synchronized under controller (13) within the fixed time T^* . The proof is completed.

Remark 2. In recent years, a lot of extensive research has been conducted on the finite-time synchronization and fixed-time synchronization of complex networks, and many breakthroughs have been made. However, as far as we know, there are few published papers that deal with the fixed-time synchronization of heterogeneous complex networks. Theorem 1 suggests a way to choose the controller to realize the fixed-time synchronization for the heterogeneous complex network. The controller consists of three sections: the first two terms are used to overcome the influence from the linear condition of the nonlinear function, the second one $-\eta_i(t)\text{sign}(e_i(t))$ is introduced to compensate the influence of disturbance $h_i(t, x_i(t))$, and finally, the last section $\overline{\text{sign}}(e_i(t))(a|e_i(t)|^p + b|e_i(t)|^q)$ is employed to force the considered networks achieve the fixed-time synchronization.

Now, if $M = \max_{1 \leq i \leq n} M_i$ and $\eta(t) = \max_{1 \leq i \leq n} \eta_i(t)$, then the controllers $u_i(t) \in R^n$ can be designed as follows ($i = 1, \dots, N$):

$$u_i(t) = -(A_i - A_0)x_0 - d_i e_i(t) - \eta(t)\text{sign}(e_i(t)) - \overline{\text{sign}}(e_i(t))(a|e_i(t)|^p + b|e_i(t)|^q), \quad (46)$$

where the parameters d_i, a, b, p , and q are defined as the same as in (13).

Therefore, by using the same analysis method in Theorem 1, we can obtain Corollary 1 that is a similar conclusion with [25].

Corollary 1. For concerned complex networks (1) and (2) under controller (46), if Assumptions 1–4 hold and the control parameters $\eta_i(t)$ and d_i in (46) satisfy the following inequalities,

$$\mu(t) + 2h_{\max} + M - \eta(t) < 0, \quad (47)$$

$$\mathcal{L} + \mathcal{A} + cG^s \otimes \Gamma - D \otimes I_n < 0,$$

where $D = \text{diag}\{d_1, \dots, d_N\} > 0$, $G^s = ((G + G^T)/2)$, $\mathcal{L} = \text{diag}(L_1^s, L_2^s, \dots, L_N^s)$, and $\mathcal{A} = \text{diag}(A_1^s, A_2^s, \dots, A_N^s)$, then driven-response complex networks (1) and (2) can achieve fixed-time synchronization under controller (46), with the settling time

$$T^* = \frac{1}{2^{(1+p/2)}a(Nn)^{(1-p/2)}(p-1)} + \frac{1}{2^{(1+q/2)}b(1-q)}. \quad (48)$$

If the uncertain disturbance is not considered in the complex network model, i.e., $h_1 = h_2 = \dots = h_N = 0$, then the following network model is degenerated as

$$\dot{x}_i(t) = A_i x_i(t) + f_i(t, x_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma x_j(t), \quad (49)$$

$$i = 1, \dots, N.$$

Let $x_0|_{t=0} = x_0(0)$, and then the driven network node is governed by

$$\dot{x}_0(t) = A_0 x_0(t) + f_0(t, x_0(t)). \quad (50)$$

Then, the corresponding error dynamical system can be rewritten as follows:

$$\dot{e}_i(t) = F_i(t, e_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma e_j + u_i(t), \quad (51)$$

where $F_i(t) = f_i(t, x_i(t)) - f_0(t, x_0(t)) + (A_i - A_0)x_i(t)$.

The controllers are the same as before, and then a criterion can be obtained on the fixed-time synchronization of the concerned complex networks with nonidentical nodes. By taking $h_{\max} = 0$ in Theorem 1, one can easily get the following corollary, and its proof is omitted here.

Corollary 2. Consider complex network (49) with drive node (50) under the set of controllers (46). If Assumptions 1–3 hold and the controller parameters satisfy the following matrix inequalities,

$$\mu(t) + M - \eta(t) < 0, \quad (52)$$

$$\mathcal{L} + \mathcal{A} + cG^s \otimes \Gamma - D \otimes I_n < 0,$$

where $D = \text{diag}(d_1, \dots, d_N) > 0$, $G^s = ((G + G^T)/2)$, $\mathcal{L} = \text{diag}(L_1^s, L_2^s, \dots, L_N^s)$, and $\mathcal{A} = \text{diag}(A_1^s, A_2^s, \dots, A_N^s)$, then (49) can be synchronized to the state of drive node (50) within a fixed time T^* and the settling time

$$T^* = \frac{1}{2^{(1+p/2)}a(Nn)^{(1-p/2)}(p-1)} + \frac{1}{2^{(1+q/2)}b(1-q)}. \quad (53)$$

Furthermore, if $h_i = 0$, $A_i = 0$, and $f_i = f$ for $i = 0, 1, \dots, N$ in (49), then complex network (1) is further reduced to

$$\dot{x}_i(t) = f(t, x_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma x_j(t), \quad (54)$$

FIGURE 1: State responses without control. (a) State response x_{i1} . (b) State response x_{i2} . (c) State response x_{i3} .

and driven network node (2) is changed correspondingly into the following form:

$$\dot{x}_0(t) = f(t, x_0(t)). \quad (55)$$

In the issue, the proposed fixed-time synchronization scheme can be applied to the corresponding complex networks with identical nodes here, and the criteria are given in the following corollary.

Corollary 3. For concerned complex networks (54) and (55) under the controllers

$$u_i(t) = -d_i e_i(t) - \eta(t) \text{sign}(e_i(t)) - \overline{\text{sign}}(e_i(t)) (a|e_i(t)|^p + b|e_i(t)|^q), \quad i = 1, \dots, N, \quad (56)$$

where $d_i \geq 0, i = 1, 2, \dots, N$, $0 < q < 1, p > 1$, $|e_i(t)|^\beta = (|e_{i1}(t)|^\beta, \dots, |e_{in}(t)|^\beta)^T$, and $\text{sign}(e_i(t)) = \text{diag}(\text{sign}(e_{i1}(t)), \dots, \text{sign}(e_{in}(t)))$, if Assumptions 1 and 2 hold and the controller parameters satisfy the following matrix inequalities,

$$\begin{aligned} M - \eta(t) &< 0, \\ I_N \otimes L + c(G^s \otimes \Gamma) - D \otimes I_n &< 0, \end{aligned} \quad (57)$$

FIGURE 2: Synchronization error without control. (a) Synchronization error e_{i1} . (b) Synchronization error e_{i2} . (c) Synchronization error e_{i3} .

with $D = \text{diag}(d_1, d_2, \dots, d_N) > 0$ and $L_1 = L_2 = \dots = L_N = L$, then complex network (54) can be synchronized within a fixed time T^* :

$$T^* = \frac{1}{2^{(1+p/2)} a (Nn)^{(1-p/2)} (p-1)} + \frac{1}{2^{(1+q/2)} b (1-q)}. \quad (58)$$

4. Numerical Example

In this section, numerical simulation results are given to show that the proposed synchronization criterion is feasible. Consider the following discontinuous chaotic Chua circuit

with linear and diffusive coupling, where the dynamics of the i th node is described as follows:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + f_i(t, x_i) + h_i(t, x_i) + k \sum_{j=1}^5 G_{ij} \Gamma x_j(t), \\ i &= 1, \dots, 5, \end{aligned} \quad (59)$$

with $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$ and initial values $x_i(0)^T = (0, 2, 0)^T + (-1)^i ((i^2/5), |\sin(i)|, -(i^2/4))^T$. The inner coupling matrix Γ , activation matrix A_i , and Laplacian matrix G are defined as

FIGURE 3: State responses of fixed-time synchronization. (a) State response x_{i1} . (b) State response x_{i2} . (c) State response x_{i3} .

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_i = \begin{bmatrix} a_1 + \frac{i}{10} & b_1 - \frac{i}{10} & 0 \\ 1 & -1 & 1 \\ 0 & c_1 + \frac{i}{5} & -1 \end{bmatrix},$$

$$G = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & 0 & -3 \end{bmatrix}. \quad (60)$$

Its activation function $f_i(t, x_i(t)) = ((1 + (i/10))\theta \text{sign}(x_{i1}(t)), 0, 0)^T$, and its uncertain disturbance $h_i(t, x_i) = ((\sin(50t)/100)(\cos(50t)/100)(-\sin(50t)/100))^T$, where $a_1 = -2.75$, $b_1 = 9.0$, $c_1 = -17$, $\theta = 3.86$, and the coupling strength $k = 2.0$. The drive dynamical node is

FIGURE 4: Synchronization error of fixed-time synchronization. (a) Synchronization error e_{i1} . (b) Synchronization error e_{i2} . (c) Synchronization error e_{i3} .

$$\dot{x}_0(t) = A_0 x_0(t) + f_0(t, x_0) + h_0(t, x_0), \quad (61)$$

with initial values $x_0(0) = (0, 2, 0)^T$, where $f_0(t, x_0) = (\theta \text{sign}(x_{01}(t)), 0, 0)^T$ and $h_0(t, x_0) = (0, 0, 0)^T$.

By analyzing the state and error response trajectory of uncontrolled heterogeneous complex dynamic network (59) in Figures 1 and 2, we can easily draw the conclusion that the state of the nodes has not been synchronized, and the synchronization errors cannot tend to zero without the control input.

From the analysis of [13], systems (59) and (61) have chaotic behavior, and there exist positive constants $L_i = 0$ and $M_i = 2(1 + (i/10))\theta$ ($i = 0, 1, \dots, 5$) satisfying Assumptions 1 and 2.

Similarly, for $\mu = (2 + (5/10))\theta = 2.5\theta = 9.65$, it is easy to verify that systems (59) and (61) satisfy Assumption 3. Let $h_{\max} = (\sqrt[3]{3}/100)$; then, $\|h_i(t, x_i)\| \leq h_{\max} = (\sqrt[3]{3}/100)$ is right for $i = 1, 2, \dots, N$. So, systems (59) and (61) satisfy Assumption 4. With $D = \text{diag}(9, 6, 9, 3, 3)$ and $\eta_i(t) = 20 > \mu(t) + 2h_{\max} + \max\{M_i\} = 2.5\theta + 2(\sqrt[3]{3}/100) + 2(1 + (5/10))\theta = 5.5\theta + 2(\sqrt[3]{3}/100)$, $k = 1$, $\beta = 0.6$, it is known that $\lambda_{\max}(\mathcal{L} + \mathcal{A} + cG^s \otimes \Gamma - D \otimes I_n) = -3.6480$, which shows that condition (22) can be satisfied. Applying Theorem 1, network (59) under the set of controllers (13) with $a = 2$, $b = 3$, $p = 3$, $q = 0.5$, and $\eta_i(t) = 20$ can realize the synchronization within the fixed time $t^* = 1.3339$. As a matter of fact, the real time of the

synchronization is 0.0609 seconds in the numerical simulation, and the synchronization results of controlled networks are shown in Figures 3 and 4.

5. Conclusions

The fixed-time synchronization problem is studied for a type of dynamic complex networks with nonidentical nodes and uncertain disturbances. By employing the Lyapunov function theory, some novel sufficient conditions are provided and further applied to some special cases, such as the identical node issue. Future work may be centered on synchronous applications of complex networks with non-identical nodes and uncertain disturbances.

Data Availability

The data used to support the findings of this study are included within the article. No other data are used beyond this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Key Research and Development Plan of China (Grant no. 2018YFB1404400), National Science Foundation of China (Grant no. 62076223), Key Science and Technology Program of Henan Province (Grant nos. 202102210347 and 202102210143), Key Research Project of Henan Province University (Grant no. 19A413014), and Doctor Fund Project of Zhengzhou University of Light Industry (no. 2019BSJJ014).

References

- [1] W. Yu, G. Chen, and J. Lü, "On pinning synchronization of complex dynamical networks," *Automatica*, vol. 45, no. 2, pp. 429–435, 2009.
- [2] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," *Physics Reports*, vol. 469, no. 3, pp. 93–153, 2008.
- [3] X. Chen, T. Huang, J. Cao, J. H. Park, and J. Qiu, "Finite-time multi-switching sliding mode synchronisation for multiple uncertain complex chaotic systems with network transmission mode," *IET Control Theory & Applications*, vol. 13, no. 9, pp. 1246–1257, 2019.
- [4] Y. Chen, S. Fei, and Y. Li, "Robust stabilization for uncertain saturated time-delay systems: a distributed-delay-dependent polytopic approach," *IEEE Transactions on Automatic Control*, vol. 62, no. 7, pp. 3455–3460, 2017.
- [5] W. Sun, S.-F. Su, Y. Wu, J. Xia, and V.-T. Nguyen, "Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints," *IEEE Transactions on Cybernetics*, vol. 50, no. 8, pp. 3424–3432, 2020.
- [6] X. Song, J. Man, S. Song, Y. Zhang, and Z. Ning, "Finite/fixed-time synchronization for Markovian complex-valued memristive neural networks with reaction-diffusion terms and its application," *Neurocomputing*, vol. 414, pp. 131–142, 2020.
- [7] L. Liu, W. X. Zheng, and S. Ding, "An adaptive SOSM controller design by using a sliding-mode-based filter and its application to buck converter," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 67, no. 7, pp. 2409–2418, 2020.
- [8] X. Tang, Q. Zhang, X. Dai, and Y. Zou, "Neural membrane mutual coupling characterisation using entropy-based iterative learning identification," *IEEE Access*, vol. 8, pp. 205231–205243, 2020.
- [9] Y. Wu, J. Cao, A. Alofi, A. AL-Mazrooei, and A. Elaiw, "Finite-time boundedness and stabilization of uncertain switched neural networks with time-varying delay," *Neural Networks*, vol. 69, pp. 135–143, 2015.
- [10] Z. Ding and Z. Li, "Distributed adaptive consensus control of nonlinear output-feedback systems on directed graphs," *Automatica*, vol. 72, pp. 46–52, 2016.
- [11] Y. Wu, J. Cao, Q. Li, A. Alsaedi, and F. E. Alsaedi, "Finite-time synchronization of uncertain coupled switched neural networks under asynchronous switching," *Neural Networks*, vol. 85, pp. 128–139, 2017.
- [12] H.-L. Li, C. Hu, H. Jiang, Z. Teng, and Y.-L. Jiang, "Synchronization of fractional-order complex dynamical networks via periodically intermittent pinning control," *Chaos, Solitons & Fractals*, vol. 103, pp. 357–363, 2017.
- [13] X. Yang, J. Cao, and J. Lu, "Synchronization of delayed complex dynamical networks with impulsive and stochastic effects," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 4, pp. 2252–2266, 2011.
- [14] X. Gong, L. Gan, and Z. Wu, "Adaptive impulsive cluster synchronization in community network with nonidentical nodes," *International Journal of Modern Physics C*, vol. 27, no. 01, p. 1650010, 2016.
- [15] W. He, F. Qian, J. Lam, G. Chen, Q.-L. Han, and J. Kurths, "Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control: error estimation, optimization and design," *Automatica*, vol. 62, pp. 249–262, 2015.
- [16] X. Chen, J. H. Park, J. Cao, and J. Qiu, "Sliding mode synchronization of multiple chaotic systems with uncertainties and disturbances," *Applied Mathematics and Computation*, vol. 308, pp. 161–173, 2017.
- [17] J. Yuan, S. Ding, and K. Mei, "Fixed-time SOSM controller design with output constraint," *Nonlinear Dynamics*, vol. 102, no. 3, pp. 1567–1583, 2020.
- [18] X. Yin, Q. Zhang, H. Wang, and Z. Ding, "Rbfnn-based minimum entropy filtering for a class of stochastic nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 65, no. 1, pp. 376–381, 2020.
- [19] X. Chen, J. H. Park, J. Cao, and J. Qiu, "Adaptive synchronization of multiple uncertain coupled chaotic systems via sliding mode control," *Neurocomputing*, vol. 273, pp. 9–21, 2018.
- [20] J. Mei, M. Jiang, W. Xu, and B. Wang, "Finite-time synchronization control of complex dynamical networks with time delay," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 9, pp. 2462–2478, 2013.
- [21] M. Liu, H. J. Jiang, and C. Hu, "Finite-time synchronization of delayed dynamical networks via aperiodically intermittent control," *Journal of the Franklin Institute - Engineering and Applied Mathematics*, vol. 354, no. 13, pp. 5373–5397, 2017.
- [22] M. Liu and H. Wu, "Stochastic finite-time synchronization for discontinuous semi-Markovian switching neural networks with time delays and noise disturbance," *Neurocomputing*, vol. 310, pp. 246–264, 2018.

- [23] X. Yang, Z. Wu, and J. Cao, "Finite-time synchronization of complex networks with nonidentical discontinuous nodes," *Nonlinear Dynamics*, vol. 73, no. 4, pp. 2313–2327, 2013.
- [24] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2106–2110, 2012.
- [25] X. Yang, J. Lam, D. W. C. Ho, and Z. Feng, "Fixed-Time synchronization of complex networks with impulsive effects via nonchattering control," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5511–5521, 2017.
- [26] Q. Li, J. Guo, Y. Wu, and C.-Y. Sun, "Global asymptotical bounded synchronization for a class of coupled complex networks with nonidentical nodes," *Asian Journal of Control*, vol. 19, no. 4, pp. 1630–1640, 2017.
- [27] K. Q. Mei, L. Ma, R. X. He et al., "Finite-time controller design of multiple integrator nonlinear systems with input saturation," *Applied Mathematics and Computation*, vol. 372, 2020.
- [28] X. Tang, Q. Zhang, and L. Hu, "An EKF-based performance enhancement scheme for stochastic nonlinear systems by dynamic set-point Adjustment," *IEEE Access*, vol. 8, pp. 62261–62272, 2020.
- [29] M. Ren, Q. Zhang, and J. Zhang, "An introductory survey of probability density function control," *Systems Science & Control Engineering*, vol. 7, no. 1, pp. 158–170, 2019.
- [30] Q. Li, J. Guo, C. Sun, Y. Wu, and Z. Ding, "Finite-time synchronization for a class of dynamical complex networks with nonidentical nodes and uncertain disturbance," *Journal of Systems Science and Complexity*, vol. 32, no. 3, pp. 818–834, 2019.
- [31] W. Zhang, C. Li, T. Huang, and J. Huang, "Fixed-time synchronization of complex networks with nonidentical nodes and stochastic noise perturbations," *Physica A: Statistical Mechanics and Its Applications*, vol. 492, pp. 1531–1542, 2018.
- [32] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time stabilization: implicit Lyapunov function approach," *Automatica*, vol. 51, pp. 332–340, 2015.
- [33] X. Zhu, X. Yang, F. E. Alsaadi, and T. Hayat, "Fixed-time synchronization of coupled discontinuous neural networks with nonidentical perturbations," *Neural Processing Letters*, vol. 48, no. 2, pp. 1161–1174, 2018.
- [34] J. D. Cao and R. X. Li, "Fixed-time synchronization of delayed memristor-based recurrent neural networks," *Science China-Information Science*, vol. 60, no. 3, Article ID 032201, 2017.
- [35] J. Zhao, D. J. Hill, T. Liu et al., "Stability of dynamical networks with non-identical nodes: a multipleV-Lyapunov function method," *Automatica*, vol. 47, no. 12, pp. 2615–2625, 2011.
- [36] Q. Song, J. Cao, and F. Liu, "Synchronization of complex dynamical networks with nonidentical nodes," *Physics Letters A*, vol. 374, no. 4, pp. 544–551, 2010.
- [37] S. Wang, H. Yao, S. Zheng, and Y. Xie, "A novel criterion for cluster synchronization of complex dynamical networks with coupling time-varying delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 7, pp. 2997–3004, 2012.
- [38] X. Qian, J. Li, J. Cao, Y. Wu, and W. Wang, "Micro-cracks detection of solar cells surface via combining short-term and long-term deep features," *Neural Networks*, vol. 127, pp. 132–140, 2020.
- [39] C. Mu and H. He, "Dynamic behavior of terminal sliding mode control," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 4, pp. 3480–3490, 2018.
- [40] J. Lu, D. W. C. Ho, J. Cao, and J. Kurths, "Exponential synchronization of linearly coupled neural networks with impulsive disturbances," *IEEE Transactions on Neural Networks*, vol. 22, no. 2, pp. 329–336, 2011.
- [41] G. L. Cai, S. Q. Jiang, S. M. Cai et al., "Cluster synchronization of overlapping uncertain complex networks with time-varying impulse disturbances," *Nonlinear Dynamics*, vol. 80, no. 1–2, pp. 503–513, 2015.
- [42] S. Jiang, X. Lu, G. Cai, and S. Cai, "Adaptive fixed-time control for cluster synchronisation of coupled complex networks with uncertain disturbances," *International Journal of Systems Science*, vol. 48, no. 16, pp. 3382–3390, 2017.
- [43] C. Ma, T. Li, and J. Zhang, "Consensus control for leader-following multi-agent systems with measurement noises," *Journal of Systems Science and Complexity*, vol. 23, no. 1, pp. 35–49, 2010.
- [44] X. Yang and J. Cao, "Finite-time stochastic synchronization of complex networks," *Applied Mathematical Modelling*, vol. 34, no. 11, pp. 3631–3641, 2010.
- [45] G. Zong, H. Ren, and L. Hou, "Finite-time stability of interconnected impulsive switched systems," *IET Control Theory & Applications*, vol. 10, no. 6, pp. 648–654, 2016.
- [46] G. Zong, R. Wang, W. Zheng, and L. Hou, "Finite-time H_∞ control for discrete-time switched nonlinear systems with time delay," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 6, pp. 914–936, 2015.
- [47] A. N. Langville and W. J. Stewart, "The Kronecker product and stochastic automata networks," *Journal of Computational and Applied Mathematics*, vol. 167, no. 2, pp. 429–447, 2004.
- [48] Y. Wan, J. Cao, G. Wen, and W. Yu, "Robust fixed-time synchronization of delayed Cohen-Grossberg neural networks," *Neural Networks*, vol. 73, pp. 86–94, 2016.