Research Article

A Combined Convection Carreau–Yasuda Nanofluid Model over a Convective Heated Surface near a Stagnation Point: A Numerical Study

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The focus of this manuscript is on two-dimensional mixed convection non-Newtonian nanofluid flow near stagnation point over a stretched surface with convectively heated boundary conditions. The modeled equation representing nonlinear flow is transformed into a system of ordinary differential equations by implementing appropriate similarity transformations. The generated structure is numerically solved by applying the bvp4c method. Consequences of various involved parameters, e.g., stretching parameter, mixed convection parameter, thermophoresis parameter, Brownian movement parameter, Lewis number, Weissenberg number, Prandtl number, Biot number, buoyancy ratio parameter, mass and heat transport rates on temperature and velocity, the stretched surface, and nanoparticle concentration patterns are analyzed. Outcomes are shown graphically and displayed in tables. Velocity fluctuations are responded to by growing parameters of mixed convection and Weissenberg number. Concentration and thermal fields are also discovered for the Prandtl number. There are also flow line diagrams to analyze the behavior.

1. Introduction

Today, enhancement in the heat transfer mechanism using cooling fluids has become very significant in industrial applications such as machining and electronics, energy production industries, petrochemical, aeronautics, and transportation industries. The use of irreversible strength resources and the creation of environmental provisions have supported this improvement. The objective of manufacturing heat transfer devices requires reducing cost and increasing heat transfer in each surface area unit to achieve high efficiency. In recent years, technical advances have made improvements in the rheological

properties of cooling fluids, and the manufacture of a steady solid-liquid suspension called nanofluid has optimized the thermal performance of industrial instruments and heat exchangers [1]. Therefore, several researchers have reported enhancement in heat transfer through nanofluids [2–21].

The combination of forced and free convection is known as mixed convection. Extensive cases of mixed-convection are characterized as internal mass forces, where the flow is determined collectively by a few external forcing systems (i.e., external power supplied to the fluid-streamlined body system). This is a distribution of fluid in a gravity domain with differing densities. The acceleration of the Earth's temperature stratified

mass of air and water region is the realistic presentation of mixed convection that is conventionally studied in geophysics. However, in many engineering gadgets, mixed convection is found within the system of much smaller scales. This will be encapsulated on account of certain cases alluded to in channel flows. Mixed convection is often perceived in normal and traditional situations, i.e., heating or cooling of channel walls, limited velocities of a fluid flow that are characteristics of a laminar flow. Turbulent channel flows studies with considerable gravity area impacts have evolved since the 1960s after its essential use in industrial applications in nature, e.g., electronic appliances, cooling through electric fans, solar panels exposed to wind currents, atomic reactor cooled during emergency shutoff, flow in the ocean and the atmosphere, heat transmission placed in a low-velocity environment, and so on. Cesarano [22] discussed several fractional generalizations of different types, starting from the heat equation, and suggested a method useful for analytical/numerical solutions. Cesarano [23] introduced a nonconventional approach of multi-dimensional Chebyshev polynomials. Mixed convection flow was also reported by many researchers [24–30].

Commonly, fluids are categorized into two main classifications: non-Newtonian and Newtonian fluids; the main difference between Newtonian and non-Newtonian fluid is the connection between the deformation rate tensor and the extra stress tensor [31]. In a 3D free-form extrusion printing process, the rheological property plays an important role. But no rheological model was available in the open literature that could accurately consider the impacts of both the concentration of nano/microparticles and the non-Newtonian viscosity in the paste [32]. Zuo and Liu [32] introduced a fractal rheological model for 3D print pastes by applying a fractional derivative and it can be also used for other non-Newtonian fluids. The experimental findings are in strong agreement with the theoretical forecast, suggesting that the model is accurate and realistic. Defining the behaviors of melts and many polymer solutions, Carreau–Yasuda model was established successfully. It was accurately applicable to many experimental viscosity patterns with a wide range of shear rates [33]. Khan et al. [34] investigated the dissimilar solutions of MHD Carreau–Yasuda fluid with slip conditions flow past a rotating disk. Andrade et al. [35] presented the turbulent flow of the non-Newtonian Carreau–Yasuda fluid. Salahuddin et al. [36] examined the influence of transverse magnetic field on the squeezed two-dimensional flow of a free stream past a sensor surface of an electrically conducting Carreau–Yasuda fluid. Kumar et al. [37]

discussed pulsatile blood flow through the human carotid artery by applying computational fluid dynamics.

Inspired by the above research work, the ambition of the current investigation is to study the impact of mixed convection on Carreau–Yasuda stagnation point nanofluid flow over a vertical elastic surface by applying a mathematical nanofluid model introduced by Buongiorno [38]. In our point of view, the problem is new and original. The outcomes are discussed through various parameters such as the Brownian motion parameter, mixed convection parameter, thermophoresis parameter, buoyancy ratio parameter, stretching parameter, Lewis, Weissenberg, Prandtl, and Biot numbers on the mass and heat transfer.

2. Modeling

We inspect the incompressible, 2D, steady flow on a stagnation point attaining the stretched surface in a Carreau–Yasuda nanofluid as shown in Figure 1. Sheet is stretched with velocity $U_w(x) = ax$ and ambient fluid velocity is $U_E(x) = bx$ while the origin is fixed at S. It is assumed that the surface is warmed up by convection from a hot fluid at the temperature T_f and the sheet is warmed up by convection fluid at the temperature T_∞ which is by heat transport coefficient h_f . The surface motion will source the evolution of the boundary layer. Consider the Cartesian coordinates system; fluid flow velocity will change through x and y axis in a manner that the x -axis is taken vertically and the y -axis is taken horizontally.

The elemental mathematical form of Carreau–Yasuda fluid is [39]

$$\tau = \left[\mu_\infty + (\mu_0 - \mu_\infty) \left((1 + \Gamma \dot{\gamma})^d \right)^{(n-1)/d} \right] A_1, \quad (1)$$

where Γ and d are Carreau–Yasuda fluid parameters, A_1 is first Rivlin Ericksen tensor, μ_∞ is infinite shear rate viscosity, μ_0 is zero shear rate viscosity, τ is extra stress tensor, and $\dot{\gamma}$ is expressed as $\dot{\gamma} = \sqrt{\text{tr}(A_1^2)(1/2)}$, in which $A_1 = [(\text{grad}v)^t + \text{grad}v]$. Take into account that infinite shear rate viscosity $\mu_\infty = 0$ and then equation (1) reduces to the form

$$\tau = \left[\mu_0 \left((1 + \Gamma \dot{\gamma})^d \right)^{(n-1)/d} \right] A_1. \quad (2)$$

Under these considerations, governing equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3)$$

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & U_E \frac{dU_E}{dx} + \nu \frac{\partial^2 u}{\partial y^2} + \nu \left[\frac{(n-1)}{d} \cdot (d+1) \Gamma^d \left(\frac{\partial u}{\partial y} \right)^d \frac{\partial^2 u}{\partial y^2} \right] \\ & + g \left[\frac{(1-C)\beta \rho_{fm}}{\rho_f} (T - T_\infty) - \left(\frac{\rho_p - \rho_{fm}}{\rho_p} \right) (C - C_\infty) \right], \end{aligned} \quad (4)$$

FIGURE 1: Geometry of the physical problem.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \left(\frac{\rho c_p}{\rho c_p} \right)_p \left[D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \left(\frac{\partial T}{\partial y} \right)^2 \right], \quad (5)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2}, \quad (6)$$

Here, ν is the viscosity characteristics and u and v are the velocity components in the x and y directions, respectively, where $(\rho c_p)_p / (\rho c_p)_f$ is the proportion of nanoparticles capacity of heat to that of the base fluid capacity of heat, ρ_f is the density of the base fluid, $\alpha = K / (\rho c_p)_f$ is the thermal diffusibility of the base fluid, c_p is the constant pressure specific heat, D_B is the Brownian diffusion, D_T is the thermophoresis diffusion, and the subscripts np, ∞ , and f denote nanoparticles, a value very far away from the solid surface, and the base fluid, respectively.

The subjected boundary conditions are defined as

$$u = U_w(x) = ax,$$

$$v = 0,$$

$$-k \frac{\partial T}{\partial y} = h_f(T_f - T),$$

$$D_B \frac{\partial C}{\partial y} + \left(\frac{D_T}{T_\infty} \right) \frac{\partial T}{\partial y} = 0, \quad \text{at } y \rightarrow 0, \quad (7)$$

$$u = U_E(x) = bx,$$

$$T \rightarrow T_\infty,$$

$$C \rightarrow C_\infty, \quad \text{at } y \rightarrow \infty.$$

Appropriate similarity transformations are defined as

$$\psi = \sqrt{avx} f(\eta),$$

$$\eta = \sqrt{\frac{a}{\nu}} y,$$

$$u = \frac{\partial \psi}{\partial y} = ax f'(\eta), \quad (8)$$

$$v = -\frac{\partial \psi}{\partial x} = \sqrt{av} f(\eta),$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty},$$

$$\varphi(\eta) = \frac{C - C_\infty}{C_\infty}.$$

Applying the above relationship, the continuity equation (3) is satisfied. Equations (4)–(6) take the following form:

$$\left[1 + \frac{(n-1)}{d} (d+1) W_e^d (f'')^d \right] f'' + r^2 \quad (9)$$

$$+ f f'' - (f')^2 + \lambda \theta - N_r \varphi = 0,$$

$$\frac{1}{\text{Pr}} \theta'' + \theta' f + \text{Nb} \varphi' \theta' + \text{Nt} (\theta')^2 = 0, \quad (10)$$

$$\varphi'' + \text{LePr} f \varphi' + \frac{\text{Nt}}{\text{Nb}} \theta'' = 0,$$

$$(11)$$

with boundary conditions

$$\begin{aligned} f &= 0, \\ f' &= 1, \\ \theta' &= -\text{Bi}(1 - \theta), \\ \text{Nb}\varphi' + \text{Nt}\theta' &= 0, \quad \text{at } \eta \longrightarrow 0, \end{aligned} \quad (12)$$

$$\begin{aligned} f' &= r, \\ \theta &= 0, \\ \varphi &= 0, \quad \text{at } \eta \longrightarrow \infty. \end{aligned} \quad (13)$$

Here, prime denotes derivation for η , and further dimensionless parameters are defined as

$$\begin{aligned} \lambda &= \frac{(1-c)\beta\rho_{\text{fm}}(T_f - T_\infty)g}{a^2 x \rho_f}, \\ N_r &= \frac{(\rho_p - \rho_{\text{fm}})c_\infty g}{a^2 x \rho_f}, \\ r &= \frac{b}{a}, \\ W_e^d &= \left(\frac{a^{3/2} x \Gamma}{\sqrt{\gamma}} \right)^d, \\ \text{Nt} &= \frac{(\rho c_p)_p D_T (T_f - T_\infty)}{(\rho c_p)_f \gamma T_\infty}, \\ \text{Pr} &= \frac{\nu}{\alpha}, \\ \text{Nb} &= \frac{(\rho c_p)_p D_B C_\infty}{(\rho c_p)_f \gamma}, \\ \text{Le} &= \frac{\alpha}{D_B}, \\ \text{Bi} &= \frac{h_f}{k} \sqrt{\frac{\gamma}{a}}, \\ \alpha &= \frac{K}{(\rho c_p)_f}. \end{aligned} \quad (14)$$

Here, N_r , λ , Bi , W_e , and Le denote buoyancy ratio parameter, mixed convection parameter, Biot number, Lewis number, and Weissenberg number respectively. Pr represents the Prandtl number, r is the stretching characteristic, Nb is the Brownian motion characteristic, and Nt is the thermophoresis characteristic.

The skin friction coefficient C_f , local Nusselt number Nu_x , and local Sherwood number Sh_x are

$$\begin{aligned} C_f &= \frac{\tau_w}{\rho u_w^2}, \\ \text{Nu}_x &= \frac{x q_w}{k(T_f - T_\infty)}, \\ \text{Sh}_x &= \frac{x q_m}{D_B C_\infty}, \end{aligned} \quad (15)$$

where q_w presents surface heat flux, τ_w denotes surface shear stress, and q_m defines surface mass flux for Carreau–Yasuda fluid as follows:

$$\begin{aligned} \tau_w &= \left[\mu \left(1 + \left(\frac{n-1}{d} \right) \Gamma^d \left(\frac{\partial u}{\partial y} \right)^d \right) \frac{\partial u}{\partial y} \right]_{y=0}, \\ q_w &= -k \left[\frac{\partial T}{\partial y} \right]_{y=0}, \\ q_m &= -D_B \left[\frac{\partial C}{\partial y} \right]_{y=0}. \end{aligned} \quad (16)$$

By using appropriate similarity transformations (8), the expressions for dimensionless Nusselt number, skin friction, and the Sherwood number become

$$\begin{aligned} C_f \text{Re}_x^{1/2} &= \left[f''(0) + \left(\frac{n-1}{d} \right) W_e^d (f''(0))^{d+1} \right], \\ \text{Nu}_x \text{Re}_x^{-1/2} &= -\theta'(0), \\ \text{Sh}_x \text{Re}_x^{-1/2} &= -\varphi'(0). \end{aligned} \quad (17)$$

3. Results and Discussion

Nonlinear differential equations (9)–(11) with boundary conditions (12) and (13) are solved by using the MATLAB program `bvp4c` method. These ODEs (9)–(11) with boundary conditions (12) and (13) are also solvable by Homotopy perturbation and variational iteration method [40, 41]. For physical significance, the graphical and numerical outcomes of the solution will be taken into account. Figure 2 displays the effect of λ on the velocity patterns $f'(\eta)$ for distinct values of λ . It may be noticed that field of velocity increases when $(\lambda > 0)$ and decreases $(\lambda < 0)$. Figure 3 demonstrates the influence of Nt on $f'(\eta)$. It has been observed that the velocity curve increases while incrementing the values of Nt . The contrasting behaviour is shown for opposing flow in Figure 3. Figure 4 shows the effect of N_r on $f'(\eta)$, inclined value of N_r causing the boundary layer thickness upward and downward for reverse behavior. It is revealed that the value of n increases, and the flow of $f'(\eta)$ expands. The observation is the same for opposing flow in Figure 5. Figure 6 shows that the profile of velocity increases when the values of Biot number enhancing. In general,

FIGURE 2: Outcome of λ on $f'(\eta)$. FIGURE 3: Variation of Nt on $f'(\eta)$.

Prandtl number has an impact on $f'(\eta)$ reversed flow for the larger value of Pr in Figure 7. Figure 8 show that velocity distribution of the stretching parameter increases for expanding the value of r . It may be noted that W_e is enhanced, and the flow of velocity moves upward in Figure 9.

Deviation of temperature flow is discussed in Figures 10–14. Figure 10 represents the behavior of Biot number, the field of temperature inclined for a larger value of Bi . Figure 11 depicts that the larger value of the Prandtl number profile of temperature goes down rapidly. Figures 12 and 13 represent Le and Nb , deeper presentation shown for assisting flow and upper for opposing flow on the

temperature profile. Figure 14 shows the influence of the thermophoresis parameter on the thermal field; temperature flow moves downward for a larger value of Nt .

Nanoparticle concentration field dispute is shown in Figures 15–19. Figure 15 shows that boundary layer thickness declines by increasing the values of Le , and on the other hand, boundary layer thickness inclines for opposing flow. Figure 16 describes the impact of Biot number on the concentration field. Concentration field declines by increasing the values of Bi . Figure 17 demonstrates the impact of Pr on $\varphi(\eta)$ field; field rises with the increment of Pr . Figure 18 shows that concentration curve goes up for the

FIGURE 4: Impact of N_r on $f'(\eta)$. FIGURE 6: Result of Bi on $f'(\eta)$. FIGURE 5: Upshot of n on $f'(\eta)$. FIGURE 7: Effect of Pr on $f'(\eta)$.

inclined value of Nb and in the reverse flow curve reduces for enlarge value of Nb . It has been noted that in Figure 19 concentration distribution declines to expand the values of Nt . The opposite behavior is shown for opposing flow.

The results of the Nusselt number, skin friction, and Sherwood number are described numerically. Table 1 investigates the heat transfer parameter expanding if the values of $N_r, Nb, Le, Pr, \lambda, Bi, Nt$ inclined. Table 2 presents the impact of $N_r, Nb, Le, Pr, \lambda, Bi$, and Nt on the Sherwood number. The values of Nb increase, if the mass transfer number rapidly inclines. The numerical amount of the mass

transport coefficient diminishes if the values of these parameters $N_r, Nt, Le, Pr, \lambda, Bi$ rise. Table 3 represents the effect of a distinct amount of the stretching parameter and other parameters on the skin friction coefficient numerically. Skin friction enlarges when the values of N_r, W_e, Le , and λ increase and declines if the amount of Nb expands. In this paper, we cannot figure out the effect of the particle's size and distribution on the flow properties; the two-scale fractal calculus has to be adopted for this purpose [32, 42]. Furthermore, streamlines for distinct values of stretching parameter are shown in Figures 20–22, which show that fluid flows in the same direction.

FIGURE 8: Consequences of r on $f'(\eta)$. FIGURE 10: Upshot of Bi on $\theta(\eta)$. FIGURE 9: Influence of W_e on $f'(\eta)$. FIGURE 11: Result of Pr on $\theta(\eta)$.

FIGURE 12: Upshot of Le on $\theta(\eta)$. FIGURE 14: Deviation of Nt on $\theta(\eta)$. FIGURE 13: Influence of Nb on $\theta(\eta)$. FIGURE 15: Impact of Le on $\phi(\eta)$.

FIGURE 16: Effect of Bi on $\phi(\eta)$.

FIGURE 18: Consequence of Nb on $\phi(\eta)$.

FIGURE 17: Result of Pr on $\phi(\eta)$.

FIGURE 19: Influence of Nt on $\phi(\eta)$.

TABLE 1: Variation of $Nu_x(Re_x)^{-(1/2)}$ for distinct amounts of parameters.

N_r	Nb	Le	d	n	Pr	λ	W_e	Bi	Nt	$Nu_x(Re_x)^{-(1/2)}$					
										$r = 0.5$	$r = 1$	$r = 2$			
0.2	0.1	1	1	1	7	0	0.1	0.1	0.2	0.09529	0.09554	0.09600			
0.4										0.09536	0.09558	0.09603			
0.6										0.09541	0.09561	0.09604			
0.1	0.1	0.5	1	2	7	0.1	0.1	0.1	0.5	0.09522	0.09549	0.09596			
	0.6									0.09527	0.09554	0.09599			
	1									0.09531	0.09554	0.09602			
0.1	0.5	0.2	1	1	7	0.1	0.1	0.1	0.5	0.09522	0.09550	0.09599			
		0.4								0.09525	0.09552	0.09601			
		0.6								0.09528	0.09554	0.09602			
0.1	0.1	0.1	1	1	4.5	0.1	0.5	0.1	0.5	0.09401	0.09443	0.09511			
					6.5					0.09502	0.09532	0.09584			
					8.5					0.09565	0.09588	0.09631			
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.5	0.09520	0.09548	0.09598			
					1	1.5				0.09521	0.09548	0.09598			
										0.09521	0.09549	0.09599			
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.5	0.09520	0.09548	0.09598			
										0.2	0.18173	0.18273	0.18455		
										0.3	0.26075	0.26279	0.26655		
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.1	0.09520	0.09548	0.09598			
										0.4	0.09520	0.09548	0.09598		
										0.6	0.09521	0.09548	0.09598		

TABLE 2: Deviation of $Sh_x(Re_x)^{-(1/2)}$ for distinct values of parameters.

N_r	Nb	Le	d	n	Pr	λ	W_e	Bi	Nt	$Sh_x(Re_x)^{-(1/2)}$					
										$r = 0.5$	$r = 1$	$r = 2$			
0.2	0.1	1	1	1	7	0	0.1	0.1	0.2	-0.19059	-0.19108	-0.19202			
0.4										-0.19072	-0.19116	-0.19205			
0.6										-0.19083	-0.19123	-0.19209			
0.1	0.1	0.5	1	2	7	0.1	0.1	0.1	0.5	-0.47610	-0.47749	-0.47981			
	0.6									-0.07939	-0.07961	-0.07999			
	1									-0.04765	-0.04778	-0.04801			
0.1	0.5	0.2	1	1	7	0.1	0.1	0.1	0.5	-0.09522	-0.09550	-0.09599			
		0.4								-0.09525	-0.09552	-0.09601			
		0.6								-0.09528	-0.09554	-0.09602			
0.1	0.1	0.1	1	1	4.5	0.1	0.5	0.1	0.5	-0.47006	-0.47214	-0.47559			
					6.5					-0.47511	-0.47660	-0.47923			
					8.5					-0.47825	-0.47941	-0.48156			
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.5	-0.47602	0.47741	-0.47990			
					1	1.5				-0.47603	-0.47742	-0.47991			
										-0.47604	-0.47743	-0.47991			
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.5	-0.47602	-0.47741	-0.47990			
										0.2	-0.90864	-0.91367	-0.92278		
										0.3	-1.30372	-1.31398	-1.33279		
0.1	0.1	0.1	1	1	7	0.5	0.5	0.1	0.1	-0.09520	-0.09548	-0.09598			
										0.4	-0.38081	-0.38193	-0.38392		
										0.6	-0.57123	-0.57290	-0.57588		

TABLE 3: Deviation of $C_f(Re_x)^{1/2}$ for distinct values of parameters.

N_r	Nb	Le	d	n	Pr	λ	W_e	Bi	Nt	$C_f(Re_x)^{1/2}$		
										$r = 0.5$	$r = 1$	$r = 2$
0.2	0.1	1	1	1	7	0	0.1	0.1	0.2	-0.50648	0.13401	2.12364
0.4										-0.35370	0.26517	2.22901
0.6										-0.20696	0.39382	2.33364

TABLE 3: Continued.

N_r	Nb	Le	d	n	Pr	λ	W_e	Bi	Nt	$C_f(Re_x)^{1/2}$		
										$r = 0.5$	$r = 1$	$r = 2$
0.1	0.1	0.5	1	2	7	0.1	0.1	0.1	0.5	-0.71246	0.04259	2.02497
	0.6									-0.71949	0.03733	2.02458
	1									-0.72082	0.03637	2.02499
0.1	0.5	0.2	1	1	7	0.1	0.1	0.1	0.5	-0.64893	0.01511	2.02935
		0.4								-0.63330	0.02797	2.03953
		0.6								-0.61805	0.04060	2.04956
0	0.1	1	1	1	7	0.1	0.5	0.2	0.5	-0.47006	-0.47214	-0.47559
						0.5				-0.47511	-0.47660	-0.47923
						1				-0.47825	-0.47941	-0.48156
0.1	0.1	0.1	2	1.5	7	0.1	0.1	0.1	1	-0.80764	0.02186	2.02357
							0.7			-0.78202	-0.02187	2.02358
							1			-0.75996	-0.02187	2.02358

FIGURE 20: Streamlines for $r = 0.5$.

FIGURE 21: Streamlines for $r = 1$.

FIGURE 22: Streamlines for $r = 2$.

4. Concluding Remarks

In this manuscript, the problem of combined convection Carreau–Yasuda stagnation point nanofluid flow enclosed by a stretchable surface is discussed. The problem is solved numerically by applying the bvp4c technique through MATLAB. The main findings of the highlighted results are as follows:

- (i) Velocity profile accelerates by increasing the amount of λ , Nt , N_r , n , W_e , r , Bi and decelerates, when decreasing the amount of Prandtl number.
- (ii) It is observed that the concentration profile declined when Bi , Le , and Nt decreased. It is also revealed that, with the enlargement in Pr and Nb , the concentration pattern expands.
- (iii) The temperature profile increases with an increase in the value of Biot number (Bi) and falls when the values of Pr , Le , Nb , and Nt increase.
- (iv) Skin friction number increases when the values of N_r , W_e , Le , and λ increase and fall by increasing the values of Nb .
- (v) The local Nusselt number increases, on increasing the values of N_r , Nb , Le , Pr , λ , Bi , and Nt .
- (vi) Sherwood's number rapidly increases when the values of Nb shoot up. But increase in the values of N_r , Nt , Le , Pr , λ , and Bi then reduces the local Sherwood number.

Abbreviations

φ :	Dimensionless concentration function (–)
C_∞ :	Ambient fluid concentration (kg m^{-3})
b :	Body forces (N m^{-3})
C :	Nanoparticles concentration (kg m^{-3})
f :	Dimensionless velocity function (–)
h_f :	Convection coefficient (–)
Nb :	Brownian motion parameter (–)

T :	Temperature (–)
K :	Thermal conductivity (–)
Nu_x :	Local Nusselt number (–)
W_e :	Weissenberg number (–)
T_f :	Hot fluid temperature (K)
(u, v) :	Velocity components (m s^{-1})
d :	Fluid characteristic (–)
τ :	Extra stress tensor (–)
λ :	Mixed convection parameter (–)
Sh_x :	Local Sherwood number (–)
η :	Stretching parameter (–)
q_w :	Surface heat flux (–)
μ_∞ :	Infinite shear rate viscosity (–)
ρ_f :	The density of the base fluid (kg m^{-3})
A_1 :	Rivlin–Ericksen tensor (–)
np :	Nanoparticle (–)
Bi :	Biot number (–)
C_f :	Skin friction coefficient (–);
D_B :	Brownian diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
U_w :	Stretching sheet velocity (m s^{-1})
U_E :	External flow velocity (m s^{-1})
D_T :	Thermophoresis diffusion coefficient ($\text{m}^2 \text{s}^{-1}$)
Le :	Lewis number (–)
g :	Gravity acceleration (m s^{-2})
Nt :	Thermophoresis parameter (–)
Re_x :	Local Reynolds number (–)
Pr :	Prandtl number (–)
T_∞ :	Ambient temperature (–)
θ :	The dimensionless temperature function (–)
(x, y) :	Cartesian coordinate components (m)
τ_w :	Surface shear stress (–)
q_m :	Surface mass flux (–)
μ_0 :	Zero shear rate viscosity (–)
α :	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
ρ :	Density (kg m^{-3})
N_r :	Buoyancy ratio characteristics (–)
ρ_p :	The density of the nanoparticles (kg m^{-3})
r :	Stretching parameter (–).

Data Availability

All data are included within this manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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