

Research Article

Research on Array Structures of Acoustic Directional Transducer

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This paper focuses on the directivity design of array structures of acoustic directional transducers. Based on Huygens principles, the directivity formula of transducer arrays under random distribution in xyz space is derived when the circular piston transducers are used as the array element, which is used to analyze the directivity and acoustic pressure of conical transducer arrangements. In addition, a practical approach to analyze the directivity and acoustic pressure of transducer arrays under random arrangements is proposed. *Findings*. The conical transducer arrays show side lobes at higher frequency. Below the frequency of 2 kHz, array directivity shows rapid changes. Above the frequency of 2 kHz, array directivity varies slowly with frequency. Besides, the beam width is $\Theta_{-3\text{dB}} \leq 29.85^\circ$.

1. Theoretical Calculation of Transducer Array Directivity

For the single transducer, its directivity is decided by the ratio of sound wave length λ to size a . Take the circular piston source on the baffle as an example, as shown Figure 1. As for the single transducer, its directivity is expressed as the following formula [1–3]:

$$D(\theta) = \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| = \left| \left(\frac{2J_1(\pi d/\lambda) \sin \theta}{(\pi d/\lambda) \sin \theta} \right) \right|. \quad (1)$$

In this formula, J_1 is first-order Bessel function; wave number is $k = 2\pi/\lambda$; a represents sound source radius; and $d = 2a$ represents sound source diameter.

For linear arrays of point acoustic sources that consist of multiple transducers, the directivity is not decided by the ratio of sound wave length λ to transducer size a , but by array element arrangement. It is important to obtain the formula of directivity for transducer arrays in any random arrangements. The following shows the analysis of two relevant formulas [4–6].

2. Comparison of Directivity of Transducer Arrays and Random Array Configurations

According to Huygens principles, the linear array directivity function of n point acoustic sources is derived, as shown in Figure 1. It should be noted that at this point, each unit transducer is regarded as point with no radius, assuming $d = 2a$. It is possible to derive the directivity function of transducer arrays in N lines and M rows, as shown in Figure 2 [1]:

$$D(\alpha, \theta) = \frac{\sin((kMd_1/2)\cos \alpha \sin \theta)}{M \sin((kd_1/2)\cos \alpha \sin \theta)} \cdot \frac{\sin((kNd_2/2)\sin \alpha \sin \theta)}{N \sin((kd_2/2)\sin \alpha \sin \theta)}. \quad (2)$$

In this formula, d_1 and d_2 represent line space and space between columns; α is angle between the projection of vector \overline{OP} on XOY and positive x axis; and θ is the angle between vector \overline{OP} and z axis. The above formula obtains the following conclusion: for this function, it is necessary to demonstrate equidistant distribution in a certain direction (set the spacing distance in x axis or y axis); it needs to be a

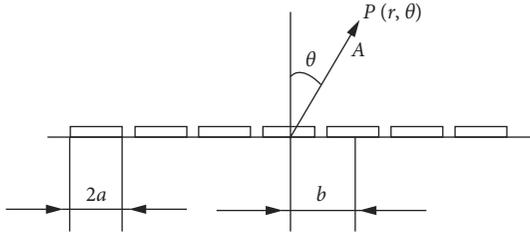


FIGURE 1: Transducer line arrays.

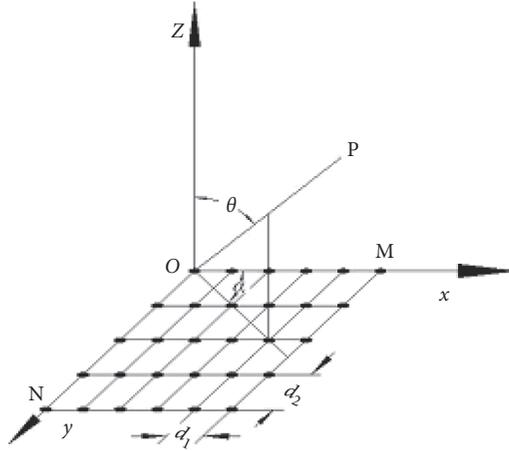


FIGURE 2: Transducer square arrays.

rectangular distribution instead of random arrays (such as polygon or circular array). When M or N is 1, this formula can calculate the array directivity. When $M=N=1$, the formula can calculate directivity of single transducer [7–9].

3. Theoretical Calculation of Random Transducer Array Directivity

Figure 3 shows the rectangular coordinate system of directional acoustic transducer arrays. The central point of transducer arrays O is the origin of coordinate. Assuming transducer array within the xy plane of the three-dimensional coordinate system, any single transducer is at the position $Q(x_0, y_0)$. In the sound field, the distance between any single observation point $P(x_0, y_0, z_0)$ and the origin of coordinate is r , the intersection angle between the point and z axis is θ , and rotation angle is φ . It is likely to obtain normalized directivity function of n circular piston transducer arrays with the radius a and random placements:

$$D(\theta, \varphi) = \frac{P_M(\theta, \varphi)}{P_M(\theta, \varphi)_{\theta=0}} = \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| \left| \sum_{i=1}^n e^{jk|x_i \sin \theta \sin \varphi + y_i \sin \theta \cos \varphi|} \right| \quad (3)$$

From formula (3), it can be concluded that, for this function, there is no need to set the distance between transducer array elements or rectangular arrangements to obtain the directivity of random array arrangements. It is only necessary to identify the coordinate of each array element. When $P(x_0, y_0)$ is determined, the directivity of random transducer plane layouts can be obtained. Besides,

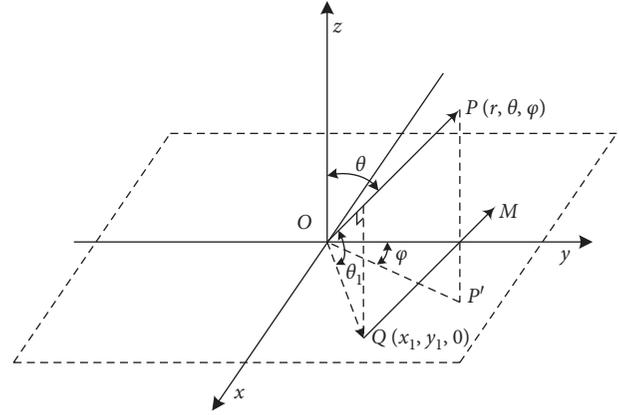


FIGURE 3: Coordinate system of transducer array.

this formula is relatively complex and difficult to obtain directivity patterns of arrays by conducting research on two-dimensional directivity. Hence, a research method of three-dimensional directivity is used to identify directivity [4–6].

4. Directivity Analysis of Three-Dimensional Transducer Arrangement

As shown in Figure 4, the three-dimensional space is established in the three-dimensional coordinate system with random transducers. O is the central point of the three-dimensional coordinate system. Assume transducer arrays are within the xyz plane of the three-dimensional coordinate system.

If the circular piston transducer with the radius a vibrates at the velocity, $u = u_0 e^{j\omega t}$, where u_0 is the velocity amplitude. Assume it is right at the origin of coordinate O . Then, its sound pressure is generated at observation point $P(x_0, y_0, z_0)$ (the distance from origin of coordinate is r . The intersection angle with z axis is θ . The rotation angle is the position of φ). The resulting sound pressure is

$$p = j\omega \frac{\rho_0 u_a a^2}{2r} \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| e^{j(\omega t - kr)}. \quad (4)$$

In this formula, ρ_0 is the density of medium. $k = 2\pi/\lambda$ is the wave number (λ is the sound wave length). Frequency $f = 10000$ Hz. Sound speed is $C_0 = 340$ m/s, $\lambda = C_0/f$.

When circular piston transducer is at any point of xyz space, set r_1 as the sonic path distance between Q and P . Likewise, for at any point of xyz space Q , the sound pressure generated by circular piston transducer with the radius a at point P is

$$p_1 = j\omega \frac{\rho_0 u_a a^2}{2r_1} \left| \frac{2J_1(ka \sin \theta_Q)}{ka \sin \theta_Q} \right| e^{j(\omega t - kr_1)}. \quad (5)$$

In this formula, θ_Q is the included angle between vector \overrightarrow{QP} and z . In the far field, it is approximately assumed as vector $\overrightarrow{OP}/\overrightarrow{QP}$. The connection between r_1 and r is shown in formula (4). $|\overrightarrow{OQ}|\cos \theta_1$ represents the projection of vector \overrightarrow{OQ} on vector \overrightarrow{OP} . From the projection relationship, it can be inferred

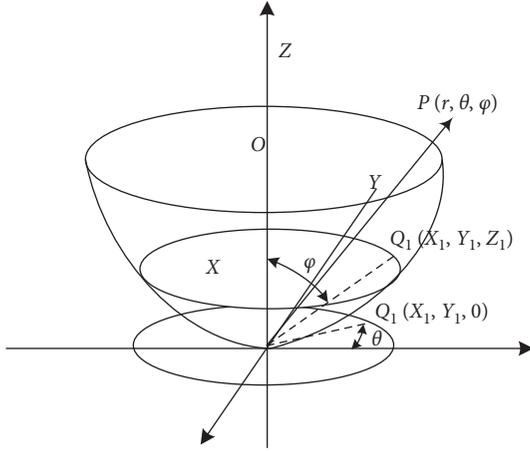


FIGURE 4: The three-dimensional coordinate system of transducer arrays.

$$r = r_1 + |\vec{OQ}| \cos \theta_1 = r_1 + r_Q \cos \theta_1. \quad (6)$$

In this formula, r_Q is the module of vector \vec{OQ} and θ_1 is the included angle between vectors \vec{OQ} and \vec{OP} . It should be noted that θ_1 may be an acute angle or an obtuse angle. The rectangular coordinate of point P is (x_0, y_0, z_0) . The rectangular coordinate of point Q is (x_1, y_1, z_1) . Vectors \vec{OP} and

\vec{OQ} are represented as $\{x_0, y_0, z_0\}$ and $\{x_1, y_1, z_1\}$. Then, included angle cosine in formula (4) is

$$\cos \theta_1 = \frac{x_0 x_1 + y_0 y_1 + z_0 z_1}{\sqrt{x_0^2 + y_0^2 + z_0^2} \sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{x_0 x_1 + y_0 y_1 + z_0 z_1}{r r_Q}. \quad (7)$$

Formulas (4) and (5) are combined and arranged to obtain

$$r_1 = r - \frac{x_0 x_1 + y_0 y_1 + z_0 z_1}{r}, \quad (8)$$

where, x_0, y_0 , and z_0 in formula (6) are converted to circular cylindrical coordinates as the following:

$$\begin{cases} x_0 = r \sin \theta \sin \phi, \\ y_0 = r \sin \theta \cos \phi, \\ z_0 = r \cos \theta. \end{cases} \quad (9)$$

Formula (7) is substituted to formula (6):

$$r_1 = r - (x_1 \sin \theta \sin \phi + y_1 \sin \theta \cos \phi + z_1 \cos \theta). \quad (10)$$

In the far field, the amplitude of formula (2) $r_1 \approx r$, $\theta_Q \approx \theta$. Formula (8) is substituted to formula (2) to obtain the following: the sound pressure generated by any point Q at point P in the xyz space is

$$p_1 = j\omega \frac{\rho_0 u_a a^2}{2r} \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| e^{j[\omega t - kr + k(x_1 \sin \theta \sin \phi + y_1 \sin \theta \cos \phi + z_1 \cos \theta)]}. \quad (11)$$

If planar transducer arrays consist of n circular piston transducers, the i transducer is in position (x_i, y_i, z_i) of the plane. Each transducer vibrates at the velocity $u = u_a e^{j\omega t}$.

The sound pressure generated by n circular piston transducers at point P in the xyz space is shown in the following:

$$p_M = j\omega \frac{\rho_0 u_a a^2}{2r} \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| e^{j(\omega t - kr)} \sum_{i=1}^n e^{jk(x_i \sin \theta \sin \phi + y_i \sin \theta \cos \phi + z_i \cos \theta)}. \quad (12)$$

According to Bessel function, when $x = 0$, $J_1(x)/x = (1/2)$. Based on formula (10), the normalized

directivity function of transducers in random space arrangement can be obtained:

$$D(\theta, \phi) = \left| \frac{p_M(\theta, \phi)_\theta}{p_M(\theta, \phi)_{\theta=0}} \right| = \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right| \left| \sum_{i=1}^M e^{jk(x_i \sin \theta \sin \phi + y_i \sin \theta \cos \phi + z_i \cos \theta)} \right|. \quad (13)$$

According to formula (11), it is feasible to calculate the feasibility of circular piston transducers with random arrangement in three-dimensional space. However, the above formula is a function concerned with variables θ and ϕ ,

which are hard to identify the directivity of the array. Hence, three-dimensional directivity research method is used in the process. To be more specific, conversion of coordinates is carried out in formula (11).

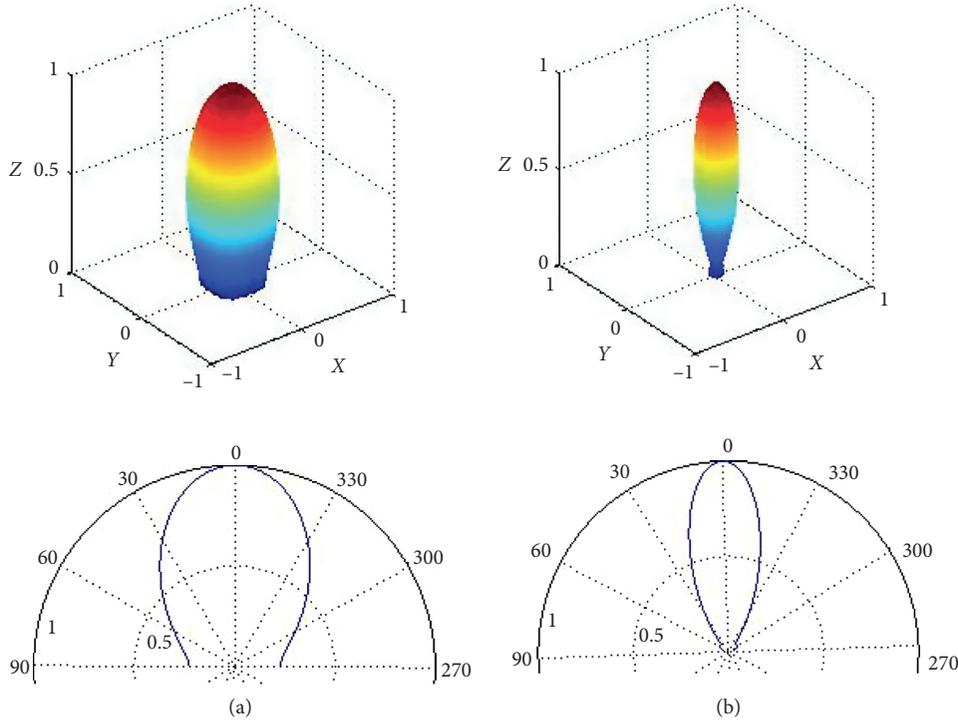


FIGURE 5: Directivity pattern of acoustic directional dispersion device. (a) 500 Hz. (b) 1 kHz.

$$\begin{cases} x_0 = \sin \theta \sin \phi \cdot D(\theta, \phi), \\ y_0 = \sin \theta \cos \phi \cdot D(\theta, \phi), \\ z_0 = \cos \theta \cdot D(\theta, \phi). \end{cases} \quad (14)$$

Assume the most central transducer is origin of coordinate $O(0, 0)$, then the position coordinate of acoustic directional dispersion system is shown in Figure 4. The rectangular coordinates of acoustic directional transducer array are substituted in formula (11) to calculate the directivity angle of transducer. By changing the frequency, it is feasible to get the result shown in Figure 3.

It is known from Figure 5 that acoustic directional transducer array shows weak directivity at the sound wave frequency level of 500 Hz. If the sound wave frequency level increases, the level of directivity also rises gradually. When the frequency level reaches 4 kHz, apparent sidelobe shows up. When the frequency level reaches 6 kHz, directivity becomes favorable, but sidelobe becomes more apparent as well.

The figure shows that beam width $\Theta_{-3\text{dB}}$ gradually narrows as frequency increases. Below the beam width of 2 kHz, frequency change is apparent. Above the beam width of 2 kHz, frequency change is slow. The frequency of dispersion sound wave ranges between 2.1 and 3.4 kHz. When it is above 2 kHz, the beam width of acoustic directional transducer is $\Theta_{-3\text{dB}} \leq 29.85^\circ$, directional acute angle $(\Theta_{-3\text{dB}}/2) \leq \pm 14.925^\circ$. Favorable directivity is shown.

5. Conclusion

Based on the Huygens principle of sound waves, this work has derived the formula for the directivity of transducer arrays in random arrangement when circular piston

transducers are used as array elements. Based on this formula, it studied the directivity and sound pressure of conical transducer array arrangements. The work provided a way to analyze directivity and sound pressure of transducer arrays in random arrangements for conical transducer and acoustic directional transducers. Findings: conical transducer arrays demonstrate sidelobe at high-frequency levels, but it can be overlooked compared to the main lobe. Below the frequency level of 2 kHz, array directivity changes rapidly. Above the frequency level of 2 kHz, array directivity changes more slowly, and the beam width is $\Theta_{-3\text{dB}} \leq 29.85^\circ$.

The work expands the formula of calculating the directivity of circular piston transducers with random array arrangements. Based on the digital simulation of computers, it resolves the difficult issues in the directivity design of three-dimensional arrays of acoustic directional transducers, providing positive significance for designing acoustic directional transducer arrays.

Data Availability

No data used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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