Research Article

Comparison of the Fold and Cusp Catastrophe Models for Tensile Cracking and Sliding Rockburst

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Received 22 December 2020; Revised 17 March 2021; Accepted 19 March 2021; Published 28 March 2021

Academic Editor: Abdul Qadeer Khan

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The failure modes of rockburst in catastrophe theory play an essential role in both theoretical analysis and practical applications. The tensile cracking and sliding rockburst is studied by analyzing the stability of the simplified mechanical model based on the fold catastrophe model. Moreover, the theory of mechanical system stability, together with an engineering example, is introduced to verify the analysis accuracy. Additionally, the results of the fold catastrophe model are compared with that of the cusp catastrophe model, and the applicability of two catastrophe models is discussed. The results show that the analytical results of the fold catastrophe model are consistent with the solutions of the mechanical systems stability theory. Moreover, the critical loads calculated by two catastrophe models are both less than the sliding force, which conforms to the actual situations. Nevertheless, the critical loads calculated from the cusp catastrophe model are bigger than those obtained by the fold catastrophe model. In conclusion, a reasonable result of the critical load can be obtained by the fold catastrophe model rather than the cusp catastrophe model. Moreover, the fold catastrophe model has a much wider application. However, when the potential function of the system is a high-order function of the state variable, the fold catastrophe model can only be used to analyze local parts of the system, and using a more complex catastrophe model such as the cusp catastrophe model is recommended.

1. Introduction

Rockburst is a common geological disaster encountered during underground engineering excavations in high-stress areas. It may happen suddenly and is extremely destructive, seriously threatening the safety of on-site construction personnel and equipment.

In terms of the definition of the term rockburst, scholars hold different views on the following three points. First, whether rockburst only occurs in hard and brittle rock masses, second, whether static damages, such as spalling and slicing, belong to rockburst, and third, whether coal bump belongs to rockburst. Conclusions have been made according to increasing research and experience. First, rockburst not only occurs in hard and brittle rock masses but also in soft-rock formation [1–3]. Second, rockburst damage is categorized as a dynamic instability phenomenon, essentially different from static damage [4]. Thus, the static

damage does not belong to rockburst, but the rockburst's precursor phenomenon may manifest as static damage. Third, compared with rockburst in traffic and water conservancy projects, coal bump has the characteristics of high intensity, long lag time, and large impact range [1, 5, 6]. However, coal bump belongs to the broad sense of rockburst. In this paper, the definition of rockburst is clarified as an engineering geological hazard in which rock masses are ejected due to excavation disturbances in surrounding rocks that accumulate high elastic strain energy.

In recent years, various methods, such as theoretical analysis methods [6–8], artificial intelligence methods [9, 10], numerical simulation [11–13], laboratory tests [14, 15], and field monitoring methods [16, 17], are widely applied to study rockburst. Vardoulakis et al. treated rockburst as a surface instability phenomenon and studied rockburst through the “wedge-test” under laboratory conditions [6]. Based on a database consisting of 188 case

histories, Afraei et al. presented several statistical models for the preliminary prediction of rockburst occurrence and intensity potential and recognized the most significant predictor variables using statistical and data mining techniques [7]. Rudajev and Sileny held the opinion that the compressional component of the stress field around the loaded stress concentrator can participate in the rockburst event and then introduced and discussed the rockburst mechanism of an implosive character [8]. Driven by big data, artificial intelligence methods have become more and more popular and are applied in various areas, including rockburst. Zhou et al. employed support vector machines to determine the classification of long-term rockburst for underground openings [9]. Pu et al. surveyed the current applications of machine learning models in rockburst prediction and related mechanisms, technical details, and performance analysis [10]. Although numerical simulation for rockburst is challenging, the results can help researchers gain insights into the rockburst mechanism. Based on the nonlinear constitutive model, SATURN was utilized to conduct numerical simulation work on rockburst assessment [11]. Moreover, Gu used a numerical modeling program UDEC to identify failure stability in terms of manifestation of coal burst events in a stable and unstable manner. A laboratory-scale numerical model of a double shear test setup is developed and used to assess the ability of UDEC in detecting the failure stability (i.e., stable or unstable failure) of discontinuity shear failures [12]. Miao et al. improved the traditional overcoring technique and applied it to in situ stress measurement at deep positions in the Sanshandao Gold Mine. The potential location and intensity of rockburst during future mining activities are predicted with theoretical analysis and numerical simulation [13]. Except for theoretical analysis and numerical analysis, laboratory tests are also conducted by some scholars to study rockburst. By using the cyclic compression test in experiment analysis, Sirait et al. predict the potential hazard of rockburst [14]. He et al. used the strainburst testing machine and impact-induced rockburst testing machine to perform experimental investigation on rockbursts. Findings involving rockburst classification, rockburst failure criteria, and related control measures are presented [15]. Ma et al. employed the microseismic monitoring technology in the rockburst monitoring and early warning process, providing a new research idea in rockburst prediction and safe construction for tunnels [16]. Blake and Hedley introduced the rockburst phenomenon from different perspectives such as the definition, the classification and types, the causes, and so on. Then sixteen rockburst case histories from Canada, the USA, and several other countries were evaluated separately [17]. Zhou et al. briefly summarized rockburst classification and reviewed the research efforts about rockburst since 1965 comprehensively. Empirical, numerical, statistical, and intelligent classification methods were included [18]. Keneti and Sainsbury selected various published rockburst events and identified the in situ conditions that contribute to rockburst events. It has been found that the majority of rockburst cases occur by loading associated factors, such as overly stressed rock mass, and property associated factors,

such as sudden deformation in the forms leading to damage [19].

The research about rockburst can be categorized into three aspects: the damage mechanism of rockburst, the prediction of rockburst, and the prevention of rockburst. Among them, the damage mechanism is the most crucial problem. The prediction accuracy of rockburst can be promoted by understanding the damage mechanism. Moreover, more knowledge about the rockburst damage mechanism of rockburst can provide more proper rockburst prevention measures.

With regard to the damage mechanism of rockburst, two approaches can be utilized. First, based on theories such as the strength theory, rigidity theory, energy theory, the instability theory, the catastrophe theory, and some other theories, the source and damage mechanism of rockburst are studied [3, 18, 20]. Second, through detailed field investigation, the failure modes of rockburst were summarized, simplified mechanical models were established, and the critical conditions of stability were reached at last [21]. According to the damage mechanism, researchers have identified failure models for different types of rockburst. Ortlepp et al. proposed that rockburst can be classified into five types, including strain bursting, buckling, face crush, shear rupture, and fault-slip, according to source mechanisms [3, 20]. Based on that, Kaiser and Cai proposed that buckling rockburst can be considered as strainburst and shear rupture rockburst can be grouped into fault-slip rockburst, so three types of rockburst, i.e., strainburst, face crush/pillar burst, and fault-slip burst [22]. From a different perspective, Li et al. proposed six types of rockburst according to the geomechanical characteristics of rockburst and detailed field investigation of the Jinping II Hydropower Station. According to Li et al., rockburst is classified into tensile cracking and spalling, tensile cracking and toppling, tensile cracking and sliding, tensile shearing and bursting, buckling and breaking, and arc shearing and bursting [21].

The rockburst failure models listed above are the most commonly studied rockburst failure modes in the current research by other scholars. Among these failure modes, the tensile cracking and sliding rockburst proposed by Li et al. is chosen to be the research object in this paper.

The tensile cracking and sliding rockburst often occurs in brittle layered or massive rock bodies with microcracks, and the rockburst area is generally located at the tunnel sidewall. The procedure of the tensile cracking and sliding rockburst can be illustrated as follows: After the tunnel excavation, the stress relief of the surrounding rock proceeds along multiple sets of microscopic structural planes. Under the action of concentrated tangential stress, cracks parallel to the free surface are generated in the surrounding rock and connected with the existing discontinuous surface. Consequently, the discontinuous surfaces interconnect with each other, separating plate-like or massive rock bodies from the surrounding rock. When in a high-stress state, the internal and upper cracks of the cut rock bodies have been connected. Moreover, gravity and internal forces have a combined influence on the sliding mode of the rock mass; hence, the rock mass slides along the structural surface. When the

strain softening of the weak layer on the sliding surface occurs, the system composed of the rock mass and the weak layer is unstable, which leads to rockburst in the form of stratified peeling or wedge bursting. Also, the ladder-like and wedge-like fracture surfaces are formed after failure. The development of the tensile cracking and sliding rockburst is displayed in Figure 1.

Since the occurrence of rockburst is paroxysmal and influenced by many factors with complicated relations, it is difficult to use traditional theories and methods to analyze the rockburst mechanism. Rockburst is a catastrophe phenomenon of the system equilibrium state, and it is feasible to determine the critical load of failure by using the catastrophe theory. Catastrophe theory provides a mathematical method for analyzing phenomena of discontinuities and abrupt qualitative changes. The discontinuity process can be dealt with directly by catastrophe theory without taking into account the internal mechanism. Therefore, the catastrophe theory has been applied in engineering stability analysis [23, 24]. However, models need to be selected conditionally in the catastrophe theory, and there are relatively few studies on the applicability of catastrophe theory models.

Based on a brief overview of the known rockburst failure modes and the principle of catastrophe theory, this paper applied the catastrophe theory to calculate the critical loads of the tensile cracking and sliding rockburst. Seven elementary catastrophe models are summarized. Among the seven models, two catastrophe models, including the cusp catastrophe model and the fold catastrophe model, are utilized to calculate the critical loads, and the results are compared. According to the theory of mechanical system stability, the critical loads of the tensile cracking and sliding rockburst are calculated and compared with those calculated by the two catastrophe models to verify the application of these two models. Additionally, an engineering example is introduced to verify the correctness as well.

2. The basic Principle of Catastrophe Theory

Catastrophe theory is a branch of mathematics that studies the phenomena and laws of the transition of system states from one stable configuration to another. By using mathematical tools to describe the process of qualitative change caused by the sudden interruption of continuous action of the system, the parameter region within which the system is in a stable state is given.

The potential function of the system is Π . There are m_0 state variables $x_1, x_2, \dots, x_{m_0}$ and n_0 control parameters $c_1, c_2, \dots, c_{n_0}$. The system state changes with the control parameters, and when the values of the control parameters are some specific values, the state will change.

The $(m_0 + n_0)$ -dimensional space is formed by m_0 state variables and n_0 control parameters. Then, an $(m_0 + n_0 - 1)$ -dimensional curved surface in the state space is determined by the following equation:

$$\frac{\partial \Pi}{\partial x_i} = 0 \quad i = 1, 2, \dots, m_0. \quad (1)$$

FIGURE 1: Development of the tensile cracking and sliding rockburst [21].

This curved surface is called an equilibrium curved surface M .

By using equation (1) and the property that Hessian matrix determinant is zero at a critical point, namely,

$$\det \Pi_{ij} = 0, \quad (2)$$

the set S of singular points can be determined, where S is the subset of M , representing the set of coincident points on the equilibrium surface. Combining equations (1) with (2), one can eliminate the state variables and obtain equations with only control parameters. The points determined by these equations are called bifurcation set B_0 , which is the projection of singularities in the control parameter space.

When the dimension of the control parameters is less than four, there are seven kinds of elementary catastrophe models, namely, fold, cusp, swallowtail, butterfly, elliptic umbilicus, hyperbolic umbilicus, and parabolic umbilicus. The standard forms of the seven catastrophe models are shown in Table 1. Note that x is the state variable and a, b , and c are the control parameters.

In elementary catastrophe models, catastrophe theory uses the second derivative of the system potential function $\partial^2 \Pi / \partial x^2$ with respect to the system state variable x to determine the stability of the system equilibrium state: When the potential energy of the system takes the extreme value, if $\partial^2 \Pi / \partial x^2 > 0$, the system state is stable; if $\partial^2 \Pi / \partial x^2 < 0$, the system state is unstable; at $\partial^2 \Pi / \partial x^2 = 0$, i.e., when the control parameters satisfy the bifurcation set equation, the stability of the system equilibrium state will have a sudden change. Therefore the critical condition for the instability of the system equilibrium state can be determined. Note that there are two system equilibrium states in the catastrophe theory. They are the stable system state and unstable system state, respectively. When the system state is stable, the system is in an equilibrium state and is stable. When the system state is unstable, however, although the system is in an equilibrium state, some minor disturbance may lead to instability.

When using catastrophe models to explain the phenomenon of catastrophe, the selected model should

TABLE 1: Standard forms of the seven elementary catastrophe models.

Catastrophe model	Dimension of control parameters	Dimension of state variables	Expression of the potential function	Equilibrium surface M	Singular point set S
Fold	1	1	$(x^2/3) + ax$	$x^2 + a = 0$	$x = 0, a = 0$
Cusp	2	1	$(x^4/4) + (ax^2/2) + bx$	$x^3 + ax + b = 0$	$\begin{cases} x^3 + ax + b = 0 \\ 3x^2 + a = 0 \end{cases}$
Swallowtail	3	1	$(x^5/5) + (ax^3/3) + (bx^2/2) + cx$	$x^4 + ax^2 + bx + c = 0$	$\begin{cases} x^4 + ax^2 + bx + c = 0 \\ 4x^3 + 2ax + b = 0 \end{cases}$
Butterfly	4	1	$(x^6/6) + (ax^4/4) + (bx^3/3) + (cx^2/2) + dx$	$x^5 + ax^3 + bx^2 + cx + d = 0$	$\begin{cases} x^5 + ax^3 + bx^2 + cx + d = 0 \\ 5x^4 + 3ax^2 + 2bx + c = 0 \end{cases}$
Elliptic umbilicus	3	2	$x^3 - xy^2 + c(x^2 + y^2) + ax + bx$	$\begin{cases} 3x^2 - y^2 + 2cx + a = 0 \\ -2xy + 2cy + b = 0 \end{cases}$	$\begin{cases} 3x^2 - y^2 + 2cx + a = 0 \\ -2xy + 2cy + b = 0 \\ 2cx + c^2 - 3x^2 - y^2 = 0 \end{cases}$
Hyperbolic umbilicus	3	2	$x^3 + y^3 + cxy + ax^2 + bx$	$\begin{cases} 3x^2 + cy + a = 0 \\ 3y^2 + cx + b = 0 \end{cases}$	$\begin{cases} 3x^2 + cy + a = 0 \\ 3y^2 + cx + b = 0 \\ 36xy - c^2 = 0 \end{cases}$
Parabolic umbilicus	4	2	$x^4 - x^2y + cx^2 + dx^2 + ax + by$	$\begin{cases} 2xy + 2cx + a = 0 \\ 4y^3 + x^2 + 2dy + b = 0 \end{cases}$	$\begin{cases} 2xy + 2cx + a = 0 \\ 4y^3 + x^2 + 2dy + b = 0 \\ (y + c)(6y^2 + d) = 0 \end{cases}$

correspond to the main characteristics of the prototype of the research object. Among the seven elementary catastrophe models, the elliptic umbilicus model, hyperbolic umbilicus model, and parabolic umbilicus model are suitable for describing a system where the state variable is two dimensional. Among the models with one-dimensional state variables, the control parameter dimensions of the dovetail and butterfly catastrophe models are three dimensional and four dimensional, respectively, and there are multiple stable and unstable equilibrium positions in both of these two models. Hence, they are suitable for describing systems that have multiple independent control parameters. In contrast, the equilibrium of one branch of the fold catastrophe model is stable and the other branch is unstable. In terms of the cusp catastrophe model, its upper and lower leaf equilibrium is stable, and the middle leaf is unstable.

In the theoretical study of rockburst and other dynamic instability problems, the research object can usually be simplified as a plane problem, and the state variable dimension is mostly one dimensional [21]. Many observations and experimental studies have shown that the system has only two states before and after a sudden dynamic instability, namely, the unstable equilibrium before the instability and the new stable equilibrium after the instability [25]. Therefore, the fold and cusp catastrophe models are consistent with the failure mode of tensile cracking and sliding rockburst studied in this paper. Therefore, the choice of the fold and cusp catastrophe model is discussed to reasonably analyze the instability of the tensile cracking and sliding rockburst.

The expressions of the bifurcation set B_0 for the two models are

$$\begin{aligned} a &= 0, \\ 4a^3 + 27b^2 &= 0. \end{aligned} \quad (3)$$

The cusp catastrophe model has already been applied in the analysis of rockburst instability and failure. Zuo et al. built a cusp catastrophic model of plate beams based on the layer cracking and buckling model proposed by Feng and Pan [26] and analyzed the critical conditions for rockburst under static and dynamic disturbances [27]. For the failure mode of the tensile cracking and sliding rockburst, Li et al. [21] used the cusp catastrophe model to obtain the critical instability criterion and the quantitative equation of energy release under the condition that the weak layer medium is a single medium or double medium.

The fold catastrophe model is also used in the analysis of rockburst instability and failure. Pan and Wang employed the fold catastrophe model to analyze the instability conditions and quantitative equation of released energy of ore pillar rockburst and rockburst affected by faults [28]. Zhang et al. established the fold catastrophe model of dynamic instability of the two-body system and gave the general equation of dynamic instability of the rock mass [29]. However, the fold catastrophe model has not been used to analyze the instability of the tensile cracking and sliding rockburst.

Hence, the cusp catastrophe model and the fold catastrophe model are applied to calculate the critical loads for

instability failure of rockburst, thus analyzing the instability of rockburst. A comparison between these two models to analyze the instability of rockburst is conducted.

3. Instability Analysis of Tensile Cracking and Sliding Rockburst

3.1. Mechanical Model. According to the field investigation of rockburst in adits and auxiliary adits of the Jinping II Hydropower Station, the mechanism features of the tensile cracking and sliding rockburst were obtained. Thus, three stages of the tensile cracking and sliding rockburst, including initiation, propagation, and sliding, were summarized (see Figure 1). The mechanical model for the shearing slip is well defined by considering the fracture zone as a strain-softening medium. Li et al. (2017) have proposed a simplified mechanical model for the tensile cracking and sliding rockburst (see Figure 2) [21].

The rock separated from the surrounding rock slides along the sliding surface. The length of the weak layer on the sliding surface is B , the thickness is b , and b is much smaller than B . Thus, it can be simplified as a plane strain problem. The unit width of the sliding rock is taken into consideration, the length is B , and the thickness is h . The obliquity of the sliding surface is α . The mass of the block m_s equals $\rho B h g$, where ρ is the density of the rock mass. N is the pressure acting on the weak layer, and it is greater than or equal to $m_s g \cos \alpha$, where g is the acceleration of gravity. F is the downslide force, and it is greater than or equal to $m_s g \sin \alpha$.

For the weak layer on the sliding surface, the normal strain ε and shear strain γ are

$$\begin{aligned} \varepsilon &= \frac{v}{b}, \\ \gamma &= \frac{u}{b}, \end{aligned} \quad (4)$$

where v is the positive displacement and u is the tangential displacement.

The constitutive relationship of the weak layer can be expressed as [30]

$$\begin{aligned} \sigma &= E\varepsilon, \\ \tau &= G\gamma \exp\left[-\left(\frac{\gamma}{\gamma_0}\right)^m\right], \end{aligned} \quad (5)$$

where σ is the normal stress; E is the elastic modulus; τ is the shear stress; G is the shear modulus; γ_0 is the peak value of shear strain; and m is the constitutive parameter and is related to the brittleness degree of the rock mass. The larger m indicates the higher brittleness of the rock mass and the higher degree of postpeak softening.

By ignoring the small deformation of the rock block, the potential function of the mechanical system Π is expressed as

$$\begin{aligned} \Pi &= \int_V \left(\int_0^\varepsilon \sigma d\varepsilon + \int_0^\gamma \tau d\gamma \right) dV - Fu - Nv, \\ &= Bb \int_0^\gamma \tau d\gamma + \frac{BE\gamma^2}{2b} - Fu - Nv. \end{aligned} \quad (6)$$

FIGURE 2: Mechanical model of the tensile cracking and sliding rockburst [21].

The lateral displacement of the weak layer u is taken as the state variable. From $\partial\Pi/\partial u = 0$, the equation of the system equilibrium surface M can be obtained:

$$\frac{BG_u}{b} \exp\left[-\left(\frac{u}{u_0}\right)^m\right] - F = 0, \quad (7)$$

where $u_0 = b\gamma_0$.

If $f(u) = (BG_u/b)\exp[-(u/u_0)^m] - F$, $f'(u) = 0$ and $f''(u) = 0$ can be obtained. u_1 and u_2 can be given as

$$u_1 = u_0 \left(\frac{1}{m}\right)^{1/m}, \quad (8)$$

$$u_2 = u_0 \left(1 + \frac{1}{m}\right)^{1/m}.$$

Based on the mechanical model of the tensile cracking and sliding rockburst, the critical loads can be calculated using the cusp catastrophe model and the fold catastrophe model.

3.2. Calculation of Critical Loads for Instability Failure by Using the Cusp Catastrophe Model. When the weak layer medium is a single medium, we expand the equilibrium surface M equation (7) at $u = u_2$ by the Taylor series taken up to the cubic term:

$$\frac{BG}{b} \exp^{-(m+1/m)} \left[u_2 - m(u - u_2) + \frac{m(m+1)^2}{6u_2^2} (u - u_2)^3 \right] - F = 0. \quad (9)$$

If $x = u - u_2/u_2$, equation (9) can be transformed into the standard form of the cusp catastrophe model:

$$x^3 + px + q = 0, \quad (10)$$

where $p = -(6/(m+1)^2)$ and $q = (6/m(m+1)^2)(1 - (Fb/BGu_2)\exp^{m+1/m})$.

The analysis process of state mutation is not included here. As the control parameters satisfy the bifurcation set equation when the system catastrophe occurs, the critical load of instability and failure in the sliding process of the rock block is obtained:

$$F_1 = \frac{BG_u}{b} \exp^{-(m+1/m)} \left[1 + \frac{2\sqrt{2}m}{3(m+1)} \right]. \quad (11)$$

3.3. Calculation of Critical Loads for Instability Failure by Using the Fold Catastrophe Model. The equilibrium surface M equation (7) at $u = u_1$ is expanded by the Taylor series taken up to the quadratic term:

$$\left(\frac{BG_u}{b} \exp^{-1/m} - F \right) - \frac{BG \exp^{-1/m} m}{2bu_1} (u - u_1)^2 = 0. \quad (12)$$

If $x = (u - u_1)/u_1$, equation (12) can be transformed into a fold catastrophe model form (see Figure 3):

$$-x^2 + a = 0, \quad (13)$$

where $a = (2/m) - (2Fb/BG \exp^{-1/m} mu_1)$.

The effect of the F value on the stability of the equilibrium state is discussed. When the value of F is high enough and makes $a < 0$, the system cannot maintain balance; when the value of F is low enough and makes $a > 0$, the system has a maximum potential energy point and a minimum potential energy point according to equation (13). When the rock block just starts to slide, the state variable satisfies $x = (u - u_1)/u_1 < 0$ and $\partial^2 \Pi / \partial x^2 = -2x > 0$, the potential energy of the system is at the minimum value, and the equilibrium state is stable; when the value of F satisfies $a = 0$, the maximum value of the system potential energy coincides with the minimum value. In this case, $\partial^2 \Pi / \partial x^2 = 0$ and the system is in a critical state. Thus, the load at critical instability and destruction during the rock block sliding can be obtained:

$$F_2 = \frac{BG_u \exp^{-1/m}}{b}. \quad (14)$$

3.4. The Theory of Mechanical System Stability. In order to verify the analytical results of the fold catastrophe model, the theory of mechanical system stability is introduced [31]. According to this theory, if the potential energy of the system $\Pi(u)$ is smooth enough, the change of potential energy near the equilibrium state $\delta\Pi(u)$ can be expressed as follows:

FIGURE 3: The equilibrium surface, singular set, and bifurcation set of the fold catastrophe model.

$$\delta\Pi(u) = \Pi'(u) + \frac{1}{2!}\Pi''(u) + \dots + \frac{1}{m!}\Pi^{(m)}(u) + \dots \quad (15)$$

When the system is in equilibrium, the stability can be determined as follows: When k of the first nonzero term $\Pi^{(k)}(u)$ in equation (15) is odd, the equilibrium state of the system is unstable; when k is an even number, if $\Pi^{(k)}(u) > 0$, the equilibrium state is stable, and if $\Pi^{(k)}(u) < 0$, the equilibrium state is unstable.

For tensile cracking and sliding rockburst, the critical load can be determined by the abovementioned method. When the lateral displacement of the weak layer $u < u_1$, there are $\Pi'(u) = 0$ and $\Pi''(u) > 0$. Therefore, the equilibrium state of the system is stable. The displacement of the rock block on the weak layer is the quasistatic displacement. When the lateral displacement of the weak layer $u > u_1$, there are $\Pi'(u) = 0$ and $\Pi''(u) < 0$; hence, the equilibrium state of the system is unstable. At the extreme point $u = u_1$ of the stress-strain curve, it should satisfy equations $\Pi'(u) = 0$, $\Pi''(u) = 0$, and $\Pi'''(u) < 0$, that is, the system is in a critical equilibrium state. By substituting $u = u_1$ into the equilibrium surface equation, the critical load can be obtained according to the theory of mechanical system stability:

$$F_3 = \frac{BGu_1 \exp^{-1/m}}{b} \quad (16)$$

By comparing equations (14) and (16), it can be seen that the critical load obtained from the theory of mechanical system stability is consistent with that obtained from the fold catastrophe model. Therefore, the fold catastrophe model is concluded to be reliable from the theoretical perspective.

3.5. Engineering Example Verification. The Jinping II Hydropower Station is located in Liangshan, China. It is a

hydropower station with the highest water head and the largest installed capacity on the Yalong River. A total of seven parallel tunnels were built in the hydropower station, namely, 1–4 # diversion tunnels, construction drainage tunnel, and A, B auxiliary tunnels. The total length of the four diversion tunnels is about 17 km, and the excavation diameter is 12.40–13.00 m, with a generally buried depth of 1000–2000 m and the maximum buried depth of 2525 m. The maximum value of in situ stress measured is 46 MPa, and the maximum in situ stress by regression analysis can reach 72 MPa, which indicates this area belongs to the extremely high in-situ stress area [32]. Rockburst occurred many times during the excavation of the long exploratory tunnel in the survey phase. During the excavation of the two auxiliary tunnels, the frequency and intensity of rockburst occurred at a higher level. A rockburst occurred at stake number 3699.5 m of the PD1 long exploration tunnel of the Jinping II Hydropower Station. Based on the field investigation, the rockburst failure mode of this area can be regarded as tensile cracking and sliding rockburst. The field image of the rockburst is shown in Figure 4.

According to the laboratory test and field conditions, the geological condition parameters of the area can be determined [33]. The lithology is white marble, locally mixed with brecciated marble. It belongs to hard rock with a layered structure, and the surrounding rock grade is II. The properties of rock mass of the weak layer are as follows: elastic modulus (E) is 30 GPa, Poisson's ratio (ν) is 0.24, shear modulus (G) is 12.096 GPa and satisfying $G = E/2(1 + \nu)$, $\gamma_0 = 0.004$, and the rock bulk density is 26.8 kN/m³. Moreover, the obliquity of the structural plane (α) is 30°, the length of the sliding surface (B) is 1 m, the height of rock block (b) is 1 m, and the constitutive parameter (m) is 1.

By substituting the abovementioned parameters into equation (14), the critical load calculated by the fold catastrophe model can be obtained, i.e., $F_2 = 17.8$ kN/m. The downward sliding force of the rock block is 26.8 kN/m, which is higher than the critical load of rockburst. This phenomenon indicates that the tensile cracking and sliding rockburst may occur. Therefore, the theoretical analysis results are consistent with the actual situation in the field.

By substituting the abovementioned parameters into equation (11), the critical load calculated by the cusp catastrophe model can be obtained, i.e., $F_1 = 19.27$ kN/m. It is higher than the critical load of rockburst calculated by the fold catastrophe model but less than the downward sliding force. This phenomenon also indicates that the tensile cracking and sliding rockburst may occur there, and these theoretical analysis results are also consistent with the situation on site.

The analysis results show that both of the two models can predict the tensile cracking and sliding rockburst, but the critical loads determined by the two models are different.

4. Discussion on Comparison and Selection of Two Models

Compared with the critical load calculated by the fold catastrophe model, the critical load calculated by the cusp

FIGURE 4: Field image of the rockburst at PD1 3699.5 m [33].

catastrophe model is higher. The reasons for the differences between the results of these two models are discussed as follows.

According to the equation $\partial^3 \Pi / \partial u^3 = 0$, the equilibrium surface M , i.e., equation (7), can be expanded by utilizing the Taylor expansion when $u = u_2$. Then, the critical load can be obtained by applying the cusp catastrophe model. When $u < u_2$, $\partial^2 \Pi / \partial u^2$ decreases. When $u > u_2$, however, $\partial^2 \Pi / \partial u^2$ increases. In a sufficiently small neighborhood around $u = u_2$ that satisfies the Taylor expansion, the sign of $\partial^2 \Pi / \partial u^2$ does not change as suddenly. Actually, $\partial^2 \Pi / \partial u^2 < 0$ in this interval indicates that the expansion point is in an unstable state, showing that the equilibrium state of the system at $u = u_2$ on the stress decline curve is unstable, which is consistent with the results of the fold catastrophe model and the theory of mechanical system stability. When using the cusp catastrophe model to analyze the stability of the system state, the jump of state variables occurs at $x = [1 - \sqrt{2}/(m+1)]u_2$ and $x = [1 + \sqrt{2}/(m+1)]u_2$, which can be obtained from the singular point set equation of the cusp catastrophe model and equation (10). When studying the catastrophe process, the approximation interval of the Taylor expansion is extended from a sufficiently small neighborhood around $u = u_2$ to $[1 - \sqrt{2}/(m+1)]u_2, [1 + \sqrt{2}/(m+1)]u_2$. Within this interval, the consistency between equation (10) obtained by the Taylor expansion and the original equilibrium surface M equation (7) has declined; hence, the results are different from the situation on site. In the process of using the fold catastrophe model, the Taylor expansion points are the same as the state catastrophe points of the system, i.e., $u = u_1$; thus, the consistency between the expanded equation and the original equilibrium surface equation will not decrease.

The equilibrium surface equation of the simplified mechanical model system of the tensile cracking and sliding rockburst in this paper is an exponential function. When applying the catastrophe theory, the equilibrium surface equation needs to be expanded to meet the standard form of catastrophe models, and then, the critical condition of catastrophe can be reached according to the singular point set

FIGURE 5: Use of the fold catastrophe model to analyze problems applicable to the cusp catastrophe model.

equation. When the fold catastrophe model is used, the direct expansion at the catastrophe point can provide an accurate solution to the critical condition of catastrophe.

The fold catastrophe model is the simplest catastrophe model and is included in all the complex models. The fold catastrophe model can analyze all problems that can be solved by complex catastrophe models. Therefore, the cusp catastrophe model is used for analysis in the following part.

The expression of a system equilibrium surface is the standard form of the cusp catastrophe model, either before or after the Taylor expansion, namely,

$$f(x) = x^3 + ax + b = 0. \quad (17)$$

According to the catastrophe principle and the equation of a single point set, the stability of the system equilibrium state mutates at the state variable $x_{1,2} = \pm \sqrt{-a/3}$ ($a < 0$), and the control parameters satisfy

$$4a^3 + 27b^2 = 0. \quad (18)$$

For equation (17), by performing the Taylor expansion at $f'(x) = 0$ and intercepting it to the quadratic term, the expansion points are obtained at $x_{1,2} = \pm \sqrt{-a/3}$. At these points, the second derivative of potential energy is zero and corresponds to the catastrophe points. Taking $x_1 = \sqrt{-a/3}$ as an example, the expression of the equilibrium surface corresponding to it is

$$3x_1(x - x_1)^2 + (x_1^3 + ax_1 + b) = 0. \quad (19)$$

Equation (19) is simplified to the standard form of the fold catastrophe model, and the catastrophe conditions can be obtained directly utilizing the singular point set equation when

$$\begin{cases} x = x_1, \\ \frac{x_1^3 + ax_1 + b}{3x_1} = 0. \end{cases} \quad (20)$$

Since the expansion point $x = x_1$ is the state catastrophe point, the equilibrium surface equation (19) after the expansion is consistent with the original equation (17) in the neighborhood of $x = x_1$. By substituting the value $x_1 = \sqrt{-a/3}$ into equation (20), the same result as equation (18) calculated by the cusp catastrophe model can be obtained.

The abovementioned demonstration process can be illustrated in Figure 5.

5. Conclusions

In this paper, the critical loads and the instability of the tensile cracking and sliding rockburst have been investigated by applying two rockburst models based on the catastrophe theory. Moreover, the applicability of these two models, including the fold and the cusp catastrophe model, has been analyzed. The following conclusions can be obtained:

- (1) The critical load of instability has been obtained by applying the fold catastrophe model to analyze the simplified mechanical model of the tensile cracking and sliding rockburst. The results are consistent with the theoretical solutions from the theory of mechanical system stability, demonstrating that the proposed method is reliable.
- (2) The proposed method is verified by engineering application of the Jinping II Hydropower Station. It was found that the critical load is less than the sliding force of the rock block when the fold catastrophe model is used. This phenomenon illustrates that rockburst will occur in the field, which is consistent with the situation on site, validating the theoretical analysis's accuracy.
- (3) When utilizing the cusp catastrophe model, the critical load is relatively bigger than that obtained by using the fold catastrophe model. The approximation interval of the Taylor expansion of the equilibrium surface when analyzing system state catastrophe by using the cusp catastrophe model is bigger than that of the fold catastrophe model, which causes the difference between the critical loads calculated by these two models. It can be concluded that the fold catastrophe model is more suitable in analyzing the tensile cracking and sliding rockburst instability.

In conclusion, concerning the applicability of catastrophe models, the fold catastrophe model is the most basic. Problems that can be dealt with by the cusp catastrophe model can also be solved by the fold catastrophe model, and the same critical conditions can be given.

For problems where the form of the system equilibrium surface expression is not the standard form of catastrophe models, such as the tensile cracking and sliding rockburst in this paper, the inconsistency between the expansion point and the catastrophe point would cause a mismatch between the expanded equilibrium surface curve and the original surface curve. Applying the fold catastrophe model can

avoid this problem and can help obtain a more accurate solution. Therefore, the fold catastrophe model's application area is broader than that of the cusp catastrophe model. When the system's potential function is a quartic or higher-order function of the state variables, there will be many points where the second derivatives of the potential energy with respect to the state variable are zero. The fold catastrophe model can only be used to analyze the system locally due to its form restriction. Contrastingly, complex models such as the cusp catastrophe model can be applied to analyze systems with expressions of equilibrium surfaces quickly and obtain all the catastrophe points and critical conditions, which are more advantageous than the fold catastrophe model. In summary, when using the catastrophe theory to analyze problems, it is necessary to select the appropriate catastrophe model according to the actual situation. This paper's results may highlight further investigations on the applications of catastrophe models such as fold and cusp ones in tensile cracking and sliding rockburst.

Data Availability

All data generated or analyzed during this study are included in this article.

Conflicts of Interest

The authors declare no conflicts of interest.

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