Research Article

The Longitudinal Deformation Profile of a Rock Tunnel: An Elastic Analysis

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The longitudinal deformation profile (LDP) is the profile of wall displacement versus the distance from the tunnel face. To develop LDP equations, numerical methods and in situ experiments have been used to obtain the deformation of a tunnel in three-dimensional space. However, extant approaches are inadequate in terms of explaining the mechanical relation between the wall displacement and the conditions of a tunnel (e.g., properties of rock). In this paper, an analytical approach is proposed to develop a new LDP equation. First, on the basis of the axisymmetric elastic model of a tunnel, a closed-form solution of wall displacement is derived. Then, a new LDP equation is presented according to the solution developed above; the coefficient β , defined as the ratio of the effective range of the “face effect” to the radius of the tunnel, is proposed for the first time. Finally, a case study is proposed to validate the practicability of this equation.

1. Introduction

The distribution of wall displacement around a tunnel face is one of the most important topics in civil and mining engineering. According to the convergence-confinement method, the tunnel face itself carries a significant portion of the load of its surroundings, and this “face effect” decreases naturally along the tunnel axis [1]. Consequently, tunnel wall displacements increase gradually as the distance from the face increases. The profile of wall displacement versus the distance from the face is defined as the longitudinal deformation profile (LDP), as shown in Figure 1.

In practice, the wall displacements are usually normalized with respect to the maximum wall displacement at the far end of the tunnel in order to eliminate the effects of elastic properties (E and μ) and tunnel radius (r).

The mathematical difficulties associated with calculating the wall displacements in three-dimensional space have motivated researchers to the utilization of numerical

methods and in situ experiments. LEE [2] proposed a dimensionless equation for a circular opening excavated under hydrostatic stress conditions as follows:

$$u^* = 0.5 \cdot \left[1 - \tanh\left(\frac{1}{3} - \frac{x^*}{2}\right) \right], \quad (1)$$

where $u^* = u/u_{\max}$, in which u is the wall displacement at a specified x and $u_{\max}$ is the ultimate radial displacement far away from the tunnel face, and $x^* = x/r_0$, in which x is the distance from the face and r_0 is the radius of the tunnel.

Carranza-Torres and Fairhurst [1] proposed another empirical equation suggested by E. Hoek as follows:

$$u^* = \left[1 + \exp\left(\frac{-x^*}{1.10}\right) \right]^{-1.7}. \quad (2)$$

Based on linear elastic numerical models, Unlu and Gercek [3] developed equations considering Poisson's ratio as follows:

FIGURE 1: (a) A circular opening excavated in rock mass. (b) A form of the longitudinal deformation profile.

$$\left\{ \begin{array}{ll} u^* = u_0^* + A_a [1 - \exp(B_a x^*)] & (\text{ahead of the face, } x < 0), \\ u^* = u_0^* + A_b \left\{ 1 - \left[\frac{B_b}{(B_b + x^*)} \right]^2 \right\} & (\text{behind the face, } x > 0), \\ u_0^* = 0.22\mu + 0.19 & (\text{at the face, } x = 0), \\ A_a = -0.22\mu - 0.19, \\ B_a = 0.73\mu + 0.81, \\ A_b = -0.22\mu + 0.81, \\ B_b = 0.39\mu + 0.65, \end{array} \right. \quad (3)$$

where u_0^* is the accumulated displacement release rate at the face and μ is Poisson's ratio.

According to numerical models of elastic-perfect-plastic (EPP) rock masses, Vlachopoulos and Diederichs [4] proposed a series of exponential functions as follows:

$$\left\{ \begin{array}{ll} u^* = u_0^* \exp(x^*) & (\text{ahead of the face, } x < 0), \\ u^* = 1 - (1 - u_0^*) \exp\left(\frac{-1.5x^*}{R^*}\right) & (\text{behind the face, } x > 0), \\ u_0^* = \frac{1}{3} \exp(-0.15R^*) & (\text{at the face, } x = 0), \end{array} \right. \quad (4)$$

where $R^* = R_p/r_0$, in which R_p is the plastic radius of the tunnel and r_0 is the radius of the tunnel.

Based on numerical models of EPP rock masses, Basarir et al. [5] related to the radial displacement of a tunnel to the rock mass rating (RMR) was as follows:

$$\left\{ \begin{array}{ll} u = a \cdot r_0 \cdot \text{RMR}^b c^{(x/2r_0)} & (\text{ahead of the face, } x < 0), \\ u = a \cdot r_0 \cdot \text{RMR}^b \left(\frac{x}{2r_0}\right)^c & (\text{behind the face, } x > 0), \end{array} \right. \quad (5)$$

where a , b , and c are coefficients assessed by depth, RMR is the rock mass rating, and r_0 is the radius of the tunnel.

Shen et al. [6] performed an in situ test as part of the Dagangshan Hydropower Station project in China, and the authors suggested an equation as follows:

$$s = \frac{s_0}{2} \left[\tanh\left(\frac{ax}{2r_0} - c\right) + 1 \right], \quad (6)$$

where s_0 is the ultimate wall displacement observed at the far end of the tunnel, a and c are the best-fit coefficients given the measured data, and r_0 is the radius of the tunnel.

Alejano et al. [7] developed a simplified approximate equation of the plastic radius for a tunnel excavated in strain-softening (SS) rock masses as follows:

$$R_{ss}^* = R_{EPP}^* + \left[\frac{q_u/p_i^*}{(0.0244 \cdot GSI - 0.53) \cdot (1 - m/50)} \right]^{(-1/(0.0018 \cdot GSI + 0.0783) \cdot (1 + K_p))}, \quad (7)$$

where R_{ss}^* is the plastic radius for a tunnel excavated in SS material, R_{EPP}^* is the plastic radius for a tunnel excavated in EPP material, q_u is the unconfined compressive strength, K_p is the passive Earth pressure, p_i is the internal pressure, GSI is the geological strength index, and m is the Hoek–Brown parameter of intact rock.

Li et al. [8] performed another in situ test in the laboratory of Beishan Exploration Tunnel in China, and the authors recommended the following equation as the analytical equation of LDP on the basis of (2):

$$u^* = \left[1 + \exp\left(\frac{-x^*}{H_1}\right) \right]^{H_2}, \quad (8)$$

where H_1 and H_2 are the best-fit coefficients given the measured data.

The foregoing equations, which are acquired using numerical methods and in situ experiments, only reflect the quantitative relations between wall displacement and distance from the face. As a matter of fact, the wall displacement is independent from the distance from the face but strictly depends on the properties of rock and the radius of the tunnel [5, 7]. To reveal the core principle of wall displacement, it is essential to develop an analytical solution of wall displacement which considers those two aspects.

In this paper, a closed-form solution of wall displacement is developed based on an axisymmetric elastic model for the rock behind the face; then, an analytical equation of LDP is presented according to the solution developed above; finally, an in situ test is introduced to confirm the practicability of the proposed LDP equation.

2. Model Conditions

2.1. Geometry. In equations (3)–(5), the LDPs ahead of the face (i.e., $x < 0$) and the LDPs behind the face (i.e., $x > 0$) are expressed separately. In equations (1), (2), (6), and (8), only the LDPs behind the face are proposed. The LDP ahead of the face reflects the wall deformation that has already been released before any excavation.

It is not necessary to determine the exact LDP equation ahead of the face because only the LDP behind the face is considered in the calculation of actual wall displacement when the supports are installed [9, 10]. Therefore, this paper focuses on wall displacement behind the face, and the model for the rock behind the face is presented as follows.

For a circular opening excavated in rock, the rock behind the face is considered to be a hollow cylinder located in a cylindrical coordinate system with limited length and radius, as shown in Figure 2. The length of the model is L , the inner radius is r_1 , and the outer radius is r_2 . The rock is considered to be elastic homogeneous material, Poisson's ratio is μ , and elastic modulus is E .

2.2. Boundary Conditions. Carranza-Torres and Fairhurst [1] pointed out that the load carried by the face decreases as the tunnel advances, which also means that the load carried by the surrounding rock increases along the tunnel. It is assumed that the load carried by the surrounding rock is

FIGURE 2: Axisymmetric model for the rock behind the face.

expressed by a nonlinear monotonic increasing function, for instance, the sine function.

Figure 3 illustrates the hypothetical load carried by the surrounding rock. The load applied on the inner circular boundary (i.e., $r = r_1$) is expressed as $p_{in} = p_1 \cdot \sin(z \cdot \pi/2L)$, where z is the distance from the face, L is the length of the model (see Figure 2), and p_1 is the maximum pressure applied on the inner circular boundary at the end of the model (i.e., $z = L$). The load applied on the outer circular boundary (i.e., $r = r_2$) is expressed as $p_{out} = p_2 \cdot \sin(z \cdot \pi/2L)$, where p_2 is the maximum pressure applied on the outer circular boundary at the end of the model.

3. Solutions to the Proposed Model

3.1. The Solution Satisfies the Harmonic Equation. For the axisymmetric problem, a stress function satisfying the biharmonic equation is required, which is

$$\nabla^2 \nabla^2 \varphi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \varphi = 0. \quad (9)$$

At the beginning, it is more rational to find a solution that satisfies the harmonic equation:

$$\nabla^2 \varphi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \varphi = 0. \quad (10)$$

It is assumed that the function takes the separated form:

$$\varphi = g(r) \cos \lambda z, \quad (11)$$

where $\lambda = \pi/2L$. Substituting equation (11) into equation (10), and note that $g(r)$ depends only on r , then

$$\begin{aligned} \nabla^2 \varphi &= \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) [g(r) \cos \lambda z] \\ &= \left[\frac{\partial^2 g(r)}{\partial r^2} + \frac{1}{r} \frac{\partial g(r)}{\partial r} \right] \cdot \cos \lambda z - \lambda^2 \cdot g(r) \cdot \cos \lambda z \\ &= \left[\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 \right) g(r) \right] \cos \lambda z = 0. \end{aligned} \quad (12)$$

FIGURE 3: Hypothetical load carried by the surrounding rock.

Defining $f(\rho) = g(r)$, where $\rho = \lambda r, \lambda = \pi/2L$, the differential components of $g(r)$ can be written in terms of $f(\rho)$ as

$$\begin{aligned} \frac{dg(r)}{dr} &= \frac{df(\rho)}{d\rho} \cdot \frac{d\rho}{dr} = \frac{df(\rho)}{d\rho} \cdot \lambda, \\ \frac{d^2g(r)}{dr^2} &= \lambda \cdot \frac{d(df(\rho)/d\rho)}{d\rho} \cdot \frac{d\rho}{dr} = \lambda^2 \cdot \frac{d^2f(\rho)}{d\rho^2}. \end{aligned} \quad (13)$$

Thus, equation (12) can be expressed as

$$\begin{aligned} \nabla^2 \varphi &= \left[\lambda^2 \cdot \frac{d^2f(\rho)}{d\rho^2} + \lambda \cdot \frac{\lambda}{\rho} \cdot \frac{df(\rho)}{d\rho} - \lambda^2 f(\rho) \right] \cdot \cos \lambda z \\ &= \lambda^2 \cdot \left[\frac{d^2f(\rho)}{d\rho^2} + \frac{1}{\rho} \frac{df(\rho)}{d\rho} - f(\rho) \right] \cdot \cos \lambda z = 0. \end{aligned} \quad (14)$$

To find the solution to equation (14), a more general expression called Bessel equation with imaginary arguments is introduced as

$$\frac{d^2f(\rho)}{d\rho^2} + \frac{1}{\rho} \frac{df(\rho)}{d\rho} - \left(1 + \frac{m^2}{\rho^2}\right) f(\rho) = 0. \quad (15)$$

When the integral order m of equation (15) is 0, the correspondence answer is the solution to equation (10). The solution to equation (15) is known as

$$f(\rho) = c_1 I_m(\rho) + c_2 K_m(\rho),$$

$$\begin{cases} I_m(\rho) = \sum_{k=0}^{\infty} \frac{1}{k!(m+k)!} \left(\frac{\rho}{2}\right)^{m+2k}, \\ K_m(\rho) = \frac{\pi}{2} \cdot i \cdot e^{(\pi i/2)m} \cdot H_m^1(\rho), \end{cases} \quad (16)$$

where I_m functions are m -order Bessel functions with imaginary arguments and K_m functions are m -order Hankel functions with imaginary arguments.

Therefore, the solution to equation (10) can be written as

$$\varphi = [a_0 I_0(\lambda r) + b_0 K_0(\lambda r)] \cos \lambda z, \quad (17)$$

or

$$\varphi = [a_0 I_0(\rho) + b_0 K_0(\rho)] \cos \lambda z, \quad (18)$$

where a_0 and b_0 are coefficients determined by boundary conditions, $\rho = \lambda r$, $\lambda = \pi/2L$, and L is the length of the model.

3.2. Stress Function Satisfies the Biharmonic Equation.

The stress function satisfying the biharmonic equation is developed on the basis of equation (18). The stress function is further extended as

$$\varphi = [a_0 I_0(\rho) + b_0 K_0(\rho) + a_1 \rho I_1(\rho) + b_1 \rho K_1(\rho)] \cos \lambda z, \quad (19)$$

where $a_1 \rho I_1(\rho)$ is linearly independent from $a_0 I_0(\rho)$ and $b_1 \rho K_1(\rho)$ is linearly independent from $b_0 K_0(\rho)$, in which $\rho = \lambda r$, $\lambda = \pi/2L$, and L is the length of the model.

Substituting equation (19) into equation (9), the biharmonic equation is found to be

$$\begin{aligned} \nabla^2 \nabla^2 \varphi &= \nabla^2 \nabla^2 [a_0 I_0(\rho) + b_0 K_0(\rho)] \cos \lambda z \\ &\quad + \nabla^2 \nabla^2 [a_1 \rho I_1(\rho) + b_1 \rho K_1(\rho)] \cos \lambda z \\ &= \nabla^2 \nabla^2 [a_1 \rho I_1(\rho) + b_1 \rho K_1(\rho)] \cos \lambda z, \end{aligned} \quad (20)$$

while

$$\begin{aligned} \nabla^2 \nabla^2 [\rho I_1(\rho) \cos \lambda z] &= \nabla^2 \left[\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \rho I_1(\rho) \cos \lambda z \right] \\ &= \nabla^2 \left[\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 \right) [\rho I_1(\rho)] \cdot \cos \lambda z \right], \end{aligned} \quad (21)$$

$$\begin{aligned} \nabla^2 [\rho K_1(\rho) \cos \lambda z] &= \nabla^2 \left[\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \rho K_1(\rho) \cos \lambda z \right] \\ &= \nabla^2 \left[\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \lambda^2 \right) [\rho K_1(\rho)] \cdot \cos \lambda z \right]. \end{aligned} \quad (22)$$

For the I_η and K_η functions with general order η , the following differential cyclic relations are given:

$$\begin{cases} \frac{d}{d\rho} [\rho^\eta I_\eta(\rho)] = \rho^\eta \cdot I_{\eta-1}(\rho), \\ \frac{d}{d\rho} [\rho^{-\eta} I_\eta(\rho)] = \rho^{-\eta} \cdot I_{\eta+1}(\rho), \end{cases} \quad (23)$$

$$\begin{cases} \frac{d}{d\rho} [\rho^\eta K_\eta(\rho)] = -\rho^\eta \cdot K_{\eta-1}(\rho), \\ \frac{d}{d\rho} [\rho^{-\eta} K_\eta(\rho)] = -\rho^{-\eta} \cdot K_{\eta+1}(\rho). \end{cases} \quad (24)$$

According to equation (23), the differential relations between I_1 and I_0 are

$$\begin{cases} \frac{d}{d\rho} [\rho I_1(\rho)] = I_1(\rho) + \rho \cdot \frac{d}{d\rho} [I_1(\rho)] = \rho \cdot I_0(\rho), \\ \frac{d}{d\rho} [I_0(\rho)] = I_1(\rho). \end{cases} \quad (25)$$

Equation (25) can be written explicitly as

$$\begin{cases} \frac{d}{d\rho} I_1(\rho) = I_0(\rho) - \frac{I_1(\rho)}{\rho}, \\ \frac{d}{d\rho} I_0(\rho) = I_1(\rho). \end{cases} \quad (26)$$

Equation (26) is derived further by differential operator d/dr as

$$\begin{cases} \frac{d}{dr} I_1(\lambda r) = \lambda I_0(\lambda r) - \frac{I_1(\lambda r)}{r}, \\ \frac{d}{dr} I_0(\lambda r) = \lambda I_1(\lambda r). \end{cases} \quad (27)$$

The first-order and second-order differential equations of $\lambda r I_1(\lambda r)$ are expressed as

$$\begin{cases} \frac{d}{dr} \lambda r I_1(\lambda r) = \lambda I_1(\lambda r) + \lambda r \left[\lambda I_0(\lambda r) - \frac{I_1(\lambda r)}{r} \right] = \lambda^2 r I_0(\lambda r), \\ \frac{d^2}{dr^2} \lambda r I_1(\lambda r) = \frac{d}{dr} \lambda^2 r I_0(\lambda r) = \lambda^2 [I_0(\lambda r) + \lambda r I_1(\lambda r)]. \end{cases} \quad (28)$$

Substituting equation (28) into equation (21) gives

$$\begin{aligned} \nabla^2 \nabla^2 [\rho I_1(\rho) \cos \lambda z] &= \nabla^2 \left[\left(\frac{d^2 [\lambda r I_1(\lambda r)]}{dr^2} + \frac{1}{r} \frac{d [\lambda r I_1(\lambda r)]}{dr} - \lambda^3 r I_1(\lambda r) \right) \cdot \cos \lambda z \right] \\ &= \nabla^2 \{ [\lambda^2 I_0(\lambda r) + \lambda^3 r I_1(\lambda r) + \lambda^2 I_0(\lambda r) - \lambda^3 r I_1(\lambda r)] \cdot \cos \lambda z \} \\ &= \nabla^2 [2\lambda^2 I_0(\lambda r) \cos \lambda z]. \end{aligned} \quad (29)$$

Similarly, the first-order and second-order differential equations of $\lambda r K_1(\lambda r)$ can be found as

$$\begin{cases} \frac{d}{dr} \lambda r K_1(\lambda r) = \lambda K_1(\lambda r) + \lambda r \left[-\lambda K_0(\lambda r) - \frac{K_1(\lambda r)}{r} \right] = -\lambda^2 r K_0(\lambda r), \\ \frac{d^2}{dr^2} \lambda r K_1(\lambda r) = \frac{d}{dr} [-\lambda^2 r K_0(\lambda r)] = -\lambda^2 [K_0(\lambda r) - \lambda r K_1(\lambda r)]. \end{cases} \quad (30)$$

Substituting equation (30) into equation (22) gives

$$\begin{aligned} \nabla^2 [\rho K_1(\rho) \cos \lambda z] &= \nabla^2 \left[\left(\frac{d^2 [\lambda r K_1(\lambda r)]}{dr^2} + \frac{1}{r} \frac{d [\lambda r K_1(\lambda r)]}{dr} - \lambda^3 r K_1(\lambda r) \right) \cdot \cos \lambda z \right] \\ &= \nabla^2 \{ [-\lambda^2 K_0(\lambda r) + \lambda^3 r K_1(\lambda r) - \lambda^2 K_0(\lambda r) - \lambda^3 r K_1(\lambda r)] \cdot \cos \lambda z \} \\ &= \nabla^2 [-2\lambda^2 K_0(\lambda r) \cos \lambda z]. \end{aligned} \quad (31)$$

Finally, equation (20) can be simplified as

$$\begin{aligned} \nabla^2 \nabla^2 \varphi &= \nabla^2 \nabla^2 [a_1 \lambda r I_1(\lambda r) + b_1 \lambda r K_1(\lambda r)] \cos \lambda z \\ &= \nabla^2 [2\lambda^2 a_1 I_0(\lambda r) \cos \lambda z - 2\lambda^2 b_1 K_0(\lambda r) \cos \lambda z] \\ &= 2\lambda^2 \cdot \nabla^2 [a_1 I_0(\lambda r) \cos \lambda z - b_1 K_0(\lambda r) \cos \lambda z]. \end{aligned} \quad (32)$$

Recalling that the solution satisfies the harmonic equation, as suggested in equations (18), (32) equals zero.

Equation (19) is proved to be the stress function satisfying the biharmonic equation.

3.3. Radical Displacements. The radical displacement (behind the tunnel face) in terms of the stress function is solved by

$$u_r = -\frac{1+\mu}{E} \left[\frac{\partial^2}{\partial r \partial z} \right] \varphi. \quad (33)$$

Substituting equation (19) into equation (33), the radical displacement is expressed explicitly as

$$\begin{aligned}
 u_r &= -\frac{1+\mu}{E} \left[\frac{\partial^2}{\partial r \partial z} \right] [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho) + b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \\
 &= \frac{1+\mu}{E} \cdot \lambda \sin \lambda z \cdot \frac{d}{dr} [a_0 I_0(\lambda r) + a_1 \lambda r I_1(\lambda r) + b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \\
 &= \frac{1+\mu}{E} \sin \lambda z \cdot [a_0 \lambda^2 I_1(\lambda r) + a_1 \lambda^3 r I_0(\lambda r) - b_0 \lambda^2 K_1(\lambda r) - b_1 \lambda^3 r K_0(\lambda r)] \\
 &= \frac{1+\mu}{E} \sin \lambda z \cdot [a_0 A_0 + a_1 A_1 + b_0 B_0 + b_1 B_1],
 \end{aligned} \tag{34}$$

where $A_0 = \lambda^2 I_1$, $A_1 = \lambda^3 r I_0$, $B_0 = -\lambda^2 K_1$, and $B_1 = -\lambda^3 r K_0$; I_0 and I_1 are the Bessel functions with imaginary arguments; K_0 and K_1 are the Hankel functions with imaginary arguments; and a_0 , a_1 , b_0 , and b_1 are the coefficients determined by boundary conditions.

Identically, the stresses can also be solved by the stress function (Appendix A).

4. Case Study

4.1. An LDP Equation Based on the Proposed Displacement Solution. As a closed-form solution of displacement, the maximum displacement without considering the accumulated displacement ahead of the face can be solved from equation (34) as

$$u_{\max} = -\frac{1+\mu}{E} \cdot [a_0 A_0 + a_1 A_1 + b_0 B_0 + b_1 B_1]. \tag{35}$$

Without considering the accumulated displacement release rate ahead of the face, an LDP equation can be proposed as

$$u_{\text{behind}}^* = \frac{u}{u_{\max}} = \sin\left(\frac{\pi z}{2L}\right), \tag{36}$$

where z is the distance from the face and L is the effective range of the “face effect.”

At the face ($z=0$), the accumulated displacement release rate is estimated by equation (3) in the following form:

$$u_0^* = 0.22\mu + 0.19. \tag{37}$$

Considering the accumulated displacement release rate at the face, a total LDP equation can be proposed as

$$\begin{aligned}
 u^* &= u_0^* + (1 - u_0^*) \cdot u_{\text{behind}}^* \\
 &= u_0^* + (1 - u_0^*) \cdot \sin\left(\frac{\pi z}{2L}\right) \\
 &= u_0^* + (1 - u_0^*) \cdot \sin\left(\frac{\pi}{2\beta} \cdot \frac{z}{r_1}\right) \\
 &= u_0^* + (1 - u_0^*) \cdot \sin\left(\frac{\pi}{2\beta} \cdot z^*\right),
 \end{aligned} \tag{38}$$

where u_0^* is the displacement release rate at the face which can be estimated by equation (37), $z^* = z/r_1$, z is the distance from the face, r_1 is the radius of the tunnel, $\beta = L/r_1$, and L is the effective range of the “face effect.”

The coefficient β in equation (38) reflects the range of the “face effect”; this viewpoint has not hitherto been proposed in the literature. In future research, β could be related to the characteristics of rock mass (e.g., quality of rock mass and buried depth) as an empirical variant.

4.2. Practicability of the Proposed LDP Equation. The Beishan Exploration Tunnel is an underground laboratory in tonalite rock for the storage of radioactive waste in the Northwest of China. In this section, the in situ test performed by Li et al. [8] in the Beishan Exploration Tunnel is introduced to confirm the practicability of equation (38). The deformation of rock in this experiment was believed to be elastic.

As shown in Figure 4, a horseshoe-shaped tunnel (neighbor tunnel) had been excavated before the test. Three boreholes (B1, B2, and B3) were drilled from the neighbor tunnel to the sidewall of the test tunnel. In each borehole, three measuring points were installed to record rock deformation. The height and width of the test tunnel are 2.8 m and 3.0 m, respectively. According to Su et al. [11], the equivalent radius of the test tunnel is calculated by

$$r_{\text{eq}} = \frac{h + w}{4} = \frac{2.8 + 3.0}{4} = 1.45\text{m}. \tag{39}$$

The measuring points in borehole 1 and borehole 2 were installed immediately after the boreholes were created, but the measuring points in borehole 3 were installed until the advancing face of the test tunnel had reached borehole 3. Therefore, only the wall displacements recorded by the first measuring point in borehole 1 (B1-1) and the first measuring point in borehole 2 (B2-1) were supposed to be the complete deformation of rock.

Figure 5 shows the LDP data presented by Li et al. [8], and the effective range of the “face effect” is 10 times the equivalent radius. Poisson’s ratio of rock, which is not reported by Li et al. [8], is estimated as 0.30 (the typical Poisson’s ratio of rock). By substituting $\mu = 0.30$ and $\beta = 10.0$ into equation (38), the LDP equation is expressed as

FIGURE 4: The (a) plan view and (b) elevation view of the test performed by Li et al. [8].

FIGURE 5: Comparisons between the data reported by Li et al. [8] and the predicted LDPs.

$$u^* = 0.256 + 0.744 \cdot \sin\left(\frac{\pi}{20} \cdot x^*\right). \quad (40)$$

The LDPs predicted by equations (1)–(3) and (40) are also illustrated in Figure 5. Equations (1)–(3) clearly overestimate the displacement release rate (u^*) for a short distance from the face (e.g., $x^* < 3$), and equation (40) is reasonably consistent with the LDP data presented by Li et al. [8].

5. Conclusions

This paper proposes a closed-form solution of wall displacement on the basis of an axisymmetric elastic model in a cylindrical coordinate system. It was hypothesized that the load carried by the surrounding rock mass takes the form of a sine function.

Considering the stress function in the separated form written as equation (11), the closed-form solution of wall displacement to the proposed model is derived. According to this closed-form solution, an analytical LDP equation (equation (38)) is presented, where the coefficient β , defined as the ratio of the effective range of the “face effect” to the radius of the tunnel, is proposed for the first time.

Finally, the in situ test performed by Li et al. [8] is introduced; the LDP predicted by equation (38) is found to be consistent with the data presented in this test, and it is acceptable to estimate the LDP by the analytical equation proposed in this paper.

Appendix

A. Stress Equations Solved by the Stress Function

A.1. Radical Stress. The radical stress in terms of the stress function is solved by

$$\sigma_r = \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{\partial^2}{\partial r^2} \right] \varphi. \quad (A.1)$$

For expository purposes, the subentries of Bessel functions (I_η) and Hankel functions (K_η) are processed as the 1st and 2nd components, respectively. Substituting equation (19) into equation (A.1), the first component of radical stress is written as

$$\begin{aligned}
\sigma_{r,1st} &= \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{\partial^2}{\partial r^2} \right] [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z \\
&= \frac{\partial}{\partial z} \left[\mu \nabla^2 [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z - \frac{\partial^2}{\partial r^2} [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z \right] \\
&= \frac{\partial}{\partial z} \left[\mu \cdot \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right] \lambda r \cdot a_1 I_1(\lambda r) \cos \lambda z - a_0 \left[\lambda^2 I_0(\lambda r) - \frac{\lambda I_1(\lambda r)}{r} \right] \cos \lambda z - a_1 [\lambda^2 I_0(\lambda r) + \lambda^3 r I_1(\lambda r)] \cos \lambda z \right] \\
&= \frac{\partial}{\partial z} \left[\mu \cdot 2a_1 \lambda^2 I_0(\lambda r) \cos \lambda z - a_0 \left[\lambda^2 I_0(\lambda r) - \frac{\lambda I_1(\lambda r)}{r} \right] \cos \lambda z - a_1 [\lambda^2 I_0(\lambda r) + \lambda^3 r I_1(\lambda r)] \cos \lambda z \right] \\
&= \frac{\partial \cos \lambda z}{\partial z} \cdot \left[\mu \cdot 2a_1 \lambda^2 I_0(\lambda r) - a_0 \left[\lambda^2 I_0(\lambda r) - \frac{\lambda I_1(\lambda r)}{r} \right] - a_1 [\lambda^2 I_0(\lambda r) + \lambda^3 r I_1(\lambda r)] \right] \\
&= -\lambda \cdot \sin \lambda z \cdot \left[2\mu a_1 \lambda^2 I_0(\lambda r) - a_1 \lambda^2 I_0(\lambda r) - a_1 \lambda^3 r I_1(\lambda r) - a_0 \left[\lambda^2 I_0(\lambda r) - \frac{\lambda I_1(\lambda r)}{r} \right] \right] \\
&= -\sin \lambda z \cdot \left[a_1 \lambda^3 [(2\mu - 1)I_0(\lambda r) - \lambda r I_1(\lambda r)] - a_0 \left[\lambda^3 I_0(\lambda r) - \frac{\lambda^2 I_1(\lambda r)}{r} \right] \right] \\
&= -\sin \lambda z \cdot \left[a_1 \lambda^3 [(2\mu - 1)I_0(\lambda r) - \lambda r I_1(\lambda r)] - a_0 \lambda^3 \left(I_0(\lambda r) - \frac{I_1(\lambda r)}{\lambda r} \right) \right],
\end{aligned} \tag{A.2}$$

and the second component of radical stress is written as

$$\begin{aligned}
\sigma_{r,2nd} &= \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{\partial^2}{\partial r^2} \right] [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \\
&= \frac{\partial}{\partial z} \left[\mu \nabla^2 [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z - \frac{\partial^2}{\partial r^2} [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \right] \\
&= \frac{\partial}{\partial z} \left[\mu \cdot \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right] \lambda r \cdot b_1 K_1(\lambda r) \cos \lambda z - b_0 \left[\frac{\lambda K_1(\lambda r)}{r} + \lambda^2 K_0(\lambda r) \right] \cos \lambda z + b_1 \lambda^2 [K_0(\lambda r) - \lambda r K_1(\lambda r)] \cos \lambda z \right] \\
&= \frac{\partial}{\partial z} \left[-2\lambda^2 K_0(\lambda r) \cos \lambda z \cdot \mu b_1 - b_1 [-\lambda^2 K_0(\lambda r) + \lambda^3 r K_1(\lambda r)] \cos \lambda z - b_0 \left[\frac{\lambda K_1(\lambda r)}{r} + \lambda^2 K_0(\lambda r) \right] \cos \lambda z \right] \\
&= \frac{\partial \cos \lambda z}{\partial z} \cdot \left[-2\mu b_1 \lambda^2 K_0(\lambda r) - b_1 [-\lambda^2 K_0(\lambda r) + \lambda^3 r K_1(\lambda r)] - b_0 \left[\lambda^2 K_0(\lambda r) + \frac{\lambda K_1(\lambda r)}{r} \right] \right] \\
&= -\lambda \cdot \sin \lambda z \cdot \left[b_1 [-2\mu \lambda^2 K_0(\lambda r) + \lambda^2 K_0(\lambda r) - \lambda^3 r K_1(\lambda r)] - b_0 \left[\lambda^2 K_0(\lambda r) + \frac{\lambda K_1(\lambda r)}{r} \right] \right] \\
&= -\sin \lambda z \cdot \left[b_1 \lambda^3 [(1 - 2\mu)K_0(\lambda r) - \lambda r K_1(\lambda r)] - b_0 \lambda^3 \left(K_0(\lambda r) + \frac{K_1(\lambda r)}{\lambda r} \right) \right].
\end{aligned} \tag{A.3}$$

Combining equations (A.2) and (A.3), the radial stress is expressed ultimately as

$$\sigma_r = -\sin \lambda z \cdot [a_0 A_0 + a_1 A_1 + b_0 B_0 + b_1 B_1], \quad (\text{A.4})$$

where

$$\begin{cases} A_0 = -\lambda^3 \left(I_0 - \frac{I_1}{\lambda r} \right), \\ A_1 = \lambda^3 [(2\mu - 1)I_0 - \lambda r I_1], \\ B_0 = -\lambda^3 \left(K_0 + \frac{K_1}{\lambda r} \right), \\ B_1 = \lambda^3 [(1 - 2\mu)K_0 - \lambda r K_1]. \end{cases} \quad (\text{A.5})$$

A.2. Shear Stress. The shear stress in terms of the stress function is solved by

$$\tau_{rz} = \frac{\partial}{\partial r} \left[(1 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] \phi. \quad (\text{A.6})$$

Substituting equation (19) into equation (A.6), the first component of shear stress is written as

$$\begin{aligned} \tau_{rz,1st} &= \frac{\partial}{\partial r} \left[(1 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial r} \left[(1 - \mu) \cdot \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right] \lambda r a_1 I_1(\lambda r) \cos \lambda + \lambda^2 \cos \lambda z \cdot [a_0 I_0(\lambda r) + \lambda r a_1 I_1(\lambda r)] \right] \\ &= \frac{\partial}{\partial r} \left[(1 - \mu) a_1 \cos \lambda \cdot 2\lambda^2 I_0(\lambda r) + \lambda^2 \cos \lambda z \cdot [a_0 I_0(\lambda r) + \lambda r a_1 I_1(\lambda r)] \right] \\ &= \lambda^2 \cos \lambda z \cdot \frac{\partial}{\partial r} [2(1 - \mu) a_1 I_0(\lambda r) + a_0 I_0(\lambda r) + \lambda r a_1 I_1(\lambda r)] \\ &= \lambda^2 \cos \lambda z \cdot [2(1 - \mu) a_1 \lambda I_1(\lambda r) + a_0 \lambda I_1(\lambda r) + \lambda^2 r a_1 I_0(\lambda r)] \\ &= \lambda^2 \cos \lambda z \cdot [a_0 \lambda I_1(\lambda r) + a_1 [2(1 - \mu) \lambda I_1(\lambda r) + \lambda^2 r I_0(\lambda r)]] \\ &= \cos \lambda z \cdot [a_0 \lambda^3 I_1(\lambda r) + a_1 \lambda^3 [2(1 - \mu) I_1(\lambda r) + \lambda r I_0(\lambda r)]], \end{aligned} \quad (\text{A.7})$$

and the second component of shear stress is written as

$$\begin{aligned} \tau_{rz,2nd} &= \frac{\partial}{\partial r} \left[(1 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial r} \left[(1 - \mu) \cdot \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right] b_1 \lambda r K_1(\lambda r) \cos \lambda + \lambda^2 \cos \lambda z \cdot [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r)] \right] \\ &= \frac{\partial}{\partial r} \left[(1 - \mu) b_1 \cos \lambda \cdot [-2\lambda^2 K_0(\lambda r)] + \lambda^2 \cos \lambda z \cdot [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r)] \right] \\ &= \lambda^2 \cos \lambda z \cdot \frac{\partial}{\partial r} [b_0 K_0(\lambda r) + \lambda r b_1 K_1(\lambda r) - 2(1 - \mu) b_1 K_0(\lambda r)] \\ &= \lambda^2 \cos \lambda z \cdot [b_0 \cdot [-\lambda K_1(\lambda r)] + b_1 [-\lambda^2 r K_0(\lambda r)] + 2(1 - \mu) b_1 \lambda K_1(\lambda r)] \\ &= \lambda^2 \cos \lambda z \cdot [b_1 [2(1 - \mu) \lambda K_1(\lambda r) - \lambda^2 r K_0(\lambda r)] - b_0 \lambda K_1(\lambda r)] \\ &= \cos \lambda z \cdot [b_1 \lambda^3 [2(1 - \mu) K_1(\lambda r) - \lambda r K_0(\lambda r)] - b_0 \lambda^3 K_1(\lambda r)]. \end{aligned} \quad (\text{A.8})$$

Combining equations (A.7) and (A.8), the shear stress is expressed ultimately as

$$\tau_{rz} = \cos \lambda z \cdot [a_0 A_0 + a_1 A_1 + b_1 B_1 + b_0 B_0], \quad (\text{A.9})$$

where

$$\begin{cases} A_0 = \lambda^3 I_1, \\ A_1 = \lambda^3 [2(1 - \mu)I_1 + \lambda r I_0], \\ B_0 = -\lambda^3 K_1, \\ B_1 = \lambda^3 [2(1 - \mu)K_1 - \lambda r K_0]. \end{cases} \quad (\text{A.10})$$

A.3. Circumferential Stress. The circumferential stress in terms of the stress function is solved by

$$\sigma_\theta = \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{1}{r} \frac{\partial}{\partial r} \right] \varphi. \quad (\text{A.11})$$

Substituting equation (19) into equation (A.11), the first component of circumferential stress is written as

$$\begin{aligned} \sigma_{\theta,1st} &= \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{1}{r} \frac{\partial}{\partial r} \right] [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial z} \cdot \left[\mu a_1 \cos \lambda z \cdot \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right) \lambda r I_1(\lambda r) - \frac{1}{r} \frac{d}{dr} [a_0 I_0(\lambda r) + \lambda r a_1 I_1(\lambda r)] \cos \lambda z \right] \\ &= \frac{\partial}{\partial z} \cdot \left[\mu a_1 \cos \lambda z \cdot \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right) \lambda r I_1(\lambda r) - \frac{1}{r} [a_0 \lambda I_1(\lambda r) + \lambda^2 r a_1 I_0(\lambda r)] \cos \lambda z \right] \\ &= \frac{\partial \cos \lambda z}{\partial z} \cdot \left[\mu a_1 \cdot 2\lambda^2 I_0(\lambda r) - a_0 \lambda \frac{I_1(\lambda r)}{r} - a_1 \lambda^2 I_0(\lambda r) \right] \\ &= -\lambda \cdot \sin \lambda z \cdot \left[a_1 [2\mu \lambda^2 I_0(\lambda r) - \lambda^2 I_0(\lambda r)] - a_0 \lambda \frac{I_1(\lambda r)}{r} \right] \\ &= -\sin \lambda z \cdot \left[a_1 \lambda^3 [2\mu I_0(\lambda r) - I_0(\lambda r)] - a_0 \lambda^2 \frac{I_1(\lambda r)}{r} \right] \\ &= -\sin \lambda z \cdot \left[-a_1 \lambda^3 (1 - 2\mu) I_0(\lambda r) - a_0 \lambda^2 \frac{I_1(\lambda r)}{r} \right], \end{aligned} \quad (\text{A.12})$$

and the second component of circumferential stress is written as

$$\begin{aligned} \sigma_{\theta,2nd} &= \frac{\partial}{\partial z} \left[\mu \nabla^2 - \frac{1}{r} \frac{\partial}{\partial r} \right] [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial z} \cdot \left[\mu b_1 \cos \lambda z \cdot \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right) \lambda r K_1(\lambda r) - \frac{1}{r} \frac{d}{dr} [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r)] \cos \lambda z \right] \\ &= \frac{\partial}{\partial z} \cdot \left[\mu b_1 \cos \lambda z \cdot \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \lambda^2 \right) \lambda r K_1(\lambda r) - \frac{1}{r} [b_0 [-\lambda K_1(\lambda r)] + b_1 [-\lambda^2 r K_0(\lambda r)]] \cos \lambda z \right] \\ &= \frac{\partial \cos \lambda z}{\partial z} \cdot \left[\mu b_1 \cdot [-2\lambda^2 K_0(\lambda r)] + \frac{b_0 \lambda K_1(\lambda r)}{r} + b_1 \lambda^2 K_0(\lambda r) \right] \\ &= -\lambda \cdot \sin \lambda z \cdot \left[b_1 \lambda^2 K_0(\lambda r) \cdot (1 - 2\mu) + \frac{b_0 \lambda K_1(\lambda r)}{r} \right] \\ &= -\sin \lambda z \cdot \left[b_1 \lambda^3 (1 - 2\mu) K_0(\lambda r) + b_0 \lambda^2 \frac{K_1(\lambda r)}{r} \right]. \end{aligned} \quad (\text{A.13})$$

Combining equation (A.12) and equation (A.13), the circumferential stress is expressed ultimately as

$$\sigma_\theta = -\sin \lambda z \cdot [a_0 A_0 + a_1 A_1 + b_0 B_0 + b_1 B_1], \quad (\text{A.14})$$

where

$$\begin{cases} A_0 = -\lambda^2 \frac{I_1}{r}, \\ A_1 = -\lambda^3 (1 - 2\mu) I_0, \\ B_0 = \lambda^2 \frac{K_1}{r}, \\ B_1 = \lambda^3 (1 - 2\mu) K_0. \end{cases} \quad (\text{A.15})$$

A.4. Axial Stress. The axial stress in terms of the stress function is solved by

$$\sigma_z = \frac{\partial}{\partial z} \left[(2 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] \varphi. \quad (\text{A.16})$$

Substituting equation (19) into equation (A.16), the first component of axial stress is written as

$$\begin{aligned} \sigma_{z,1st} &= \frac{\partial}{\partial z} \left[(2 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] [a_0 I_0(\rho) + \rho \cdot a_1 I_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial z} [(2 - \mu) a_1 \cos \lambda z \cdot 2\lambda^2 I_0(\lambda r) + \lambda^2 \cos \lambda z [a_0 I_0(\lambda r) + a_1 \lambda r I_1(\lambda r)]] \\ &= \frac{\partial}{\partial z} [2(2 - \mu) a_1 I_0(\lambda r) \cdot \lambda^2 \cos \lambda z + \lambda^2 \cos \lambda z [a_0 I_0(\lambda r) + a_1 \lambda r I_1(\lambda r)]] \\ &= \frac{\partial}{\partial z} [\lambda^2 \cos \lambda z [2(2 - \mu) a_1 I_0(\lambda r) + a_0 I_0(\lambda r) + a_1 \lambda r I_1(\lambda r)]] \\ &= -\lambda^3 \sin \lambda z \cdot [a_1 [2(2 - \mu) I_0(\lambda r) + \lambda r I_1(\lambda r)] + a_0 I_0(\lambda r)], \end{aligned} \quad (\text{A.17})$$

and the second component of axial stress is written as

$$\begin{aligned} \sigma_{z,2nd} &= \frac{\partial}{\partial z} \left[(2 - \mu) \nabla^2 - \frac{\partial^2}{\partial z^2} \right] [b_0 K_0(\rho) + \rho \cdot b_1 K_1(\rho)] \cos \lambda z \\ &= \frac{\partial}{\partial z} [(2 - \mu) b_1 \cos \lambda z \cdot [-2\lambda^2 K_0(\lambda r)] + \lambda^2 \cos \lambda z [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r)]] \\ &= \frac{\partial}{\partial z} [-2(2 - \mu) b_1 K_0(\lambda r) \cdot \lambda^2 \cos \lambda z + \lambda^2 \cos \lambda z [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r)]] \\ &= \frac{\partial}{\partial z} [\lambda^2 \cos \lambda z [b_0 K_0(\lambda r) + b_1 \lambda r K_1(\lambda r) - 2(2 - \mu) b_1 K_0(\lambda r)]] \\ &= -\lambda^3 \sin \lambda z \cdot [b_0 K_0(\lambda r) + b_1 [\lambda r K_1(\lambda r) - 2(2 - \mu) K_0(\lambda r)]]. \end{aligned} \quad (\text{A.18})$$

Combining equations (A.17) and (A.18), the axial stress is expressed ultimately as

$$\sigma_z = -\sin \lambda z \cdot [a_0 A_0 + a_1 A_1 + b_0 B_0 + b_1 B_1], \quad (\text{A.19})$$

where

$$\begin{cases} A_0 = \lambda^3 I_0, \\ A_1 = \lambda^3 [2(2 - \mu) I_0 + \lambda r I_1], \\ B_0 = \lambda^3 K_0, \\ B_1 = \lambda^3 [\lambda r K_1 - 2(2 - \mu) K_0]. \end{cases} \quad (\text{A.20})$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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