Research Article

Fuzzy-Model-Based Control for Markov Switching Singularly Perturbed Systems with the Stochastic Communication Protocol

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Received 16 December 2020; Revised 21 January 2021; Accepted 1 February 2021; Published 18 February 2021

Academic Editor: Shihong Ding

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This work is concerned with the $H_\infty$ control for Markov switching singularly perturbed systems with the stochastic communication protocol. To coordinate the data transmission and save the bandwidth usage, the stochastic communication protocol with a compensator is applied to schedule the information exchange. The goal of this work is to design a joint-Markov-process-based controller such that the resulting system is stochastically stable with prescribed performance. Based on the Lyapunov functional technique, a sufficient condition is derived to ensure the existence of the achieved controller. Finally, the effectiveness and correctness of the developed results are verified by the simulation example.

1. Introduction

As a significant component of hybrid systems, Markov switching systems (MSSs) have gained extensive interest due to their capability in modeling subsystems [1–4]. Note that MSSs consist of a finite number of subsystems, and some abrupt variations can be depicted by a Markov process, which is recognized as a key feature of MSSs. Nowadays, owing to their potential practical application, much effort has been devoted, and wonderful fruitful achievements have been gained for both continuous-time MSSs and discrete-time cases [5–8]. Nevertheless, as pointed out in [5, 6], the most existing results are concerned with Markov switching linear systems. Due to the widespread of nonlinear characteristics, it is natural to investigate the Markov switching nonlinear systems. Compared with the standard MSSs, the Markov switching nonlinear systems are more general as they contain high nonlinearity. Lately, the T-S fuzzy model has been tendered to deal with the system’s nonlinearities [9, 10]. Benefit from the T-S fuzzy model, many Markov switching nonlinear systems can be approximated as T-S fuzzy MSSs (FMSSs). Following this excellent result, quantities of valuable results have been forwarded on T-S FMSSs [11–13]. For instance, in [11], a dropout compensation approach has been studied for T-S FMSSs. With respect to the network-induced phenomena, the cyber attack has been considered in FMSSs [13].

In many dynamic systems, the system behaviors are involved in multiple-time-scale property. The parasitic parameters, for instance, small-time constants and inducances, may result in the numerical ill-conditioned issues of physical systems. In this regard, the singular perturbation strategy has been employed to tackle the above obstacles. Thanks to singularly perturbed systems (SPSs), the multiple-time-scale-based systems can be transformed into a framework model. Note that the examples of SPSs can be widely found in power systems, airplane systems, etc. Recently, many scholars have drawn their attention to both continuous-time SPSs and discrete-time cases [14–16]. When investigating the SPSs, an extra phenomenon can be encountered, for example, the sudden changes of parameters. To tackle this occurrence, Markov switching SPSs have been studied in [17, 18]. However, the aforementioned results are concerned with linear systems, little attention has been devoted to T-S fuzzy Markov switching SPSs (FMSSPS) except for [19, 20], and this issue remains open and a challenge, which deserves further research.
In the networked control systems (NCSs), massive signals are communicated through a shared wireless network. As an unavoidable phenomenon, the NCSs always experience data collisions, fading channels, and input saturation [21]. To prevent the above shortage and mitigate the side effects, many communication protocols have been addressed to govern which sensors can obtain access to send signals such as the popular communication schedule called round-robin protocol [22, 23], try-once-discard protocol [24], and stochastic communication protocol (SCP) [25, 26]. Among them, the SCP is known as an effective method to schedule the signal exchange via a shared channel, in which only one sensor is activated to transmit data. Nevertheless, to our knowledge, no one carries out the exploration of FMSSPSs with the SCP mechanism, which motivates us to this work.

Inspired by the aforementioned discussions, our attention focuses on the control issue for FMSSPSs with the communication protocol. The main contributions are outlined as follows: in light of discrete-time FMSSPSs, to coordinate the data transmission and save the bandwidth usage, the SCP is applied to schedule the information exchange. Benefiting from the novel Markov process, a mode-dependent Lyapunov functional is formulated such that the resulting system is stochastically stable, and the controller is designed.

## 2. Problem Formulations

Consider the $i$th discrete-time Markov switching system modeled by the T-S fuzzy model.

Plant Rule $p$: IF $\xi_w(k) = M_{p1}$, and $\xi_z(k) = M_{p2}$, and, $\xi_a(k) = M_{pp}$, THEN

$$
\begin{align*}
\dot{x}(t+1) & = A_{p\phi}(t)x(t) + \varepsilon A_{p\phi}(t)x_2(t) + B_{p\phi}(t)u(t) + C_{p\phi}(t)\omega(t), \\
\dot{x}_2(t+1) & = A_{p\phi}(t)x_1(t) + \varepsilon A_{p\phi}(t)x_2(t) + B_{p\phi}(t)u(t) + C_{p\phi}(t)\omega(t), \\
z(t) & = D_{p\phi}(t)x_1(t) + \varepsilon D_{p\phi}(t)x_2(t) + M_{p\phi}(t)u(t) + G_{p\phi}(t)\omega(t),
\end{align*}
$$

where $x_1(t) \in \mathbb{R}^n$ and $x_2(t) \in \mathbb{R}^n$ are the fast state and the slow state, respectively. $z(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^n$ are the output vector and control input, respectively. $\omega(t) \in \mathcal{L}_2[0, \infty)$ means the disturbance signal. The sequence $\{\varphi(t), \tau \geq 0\}$ renders a discrete-time Markov chain (DTMC) subject to a finite set $\mathcal{M}_{\omega} = \{1, 2, \ldots, N_{\omega}\}$. Here, $\varphi(t)$ describes a homogenous DTMC with the transition probability matrix of FMSSPS (1) inferred as

$$
\pi_{ij} = \Pr(\varphi(t+1) = j|\varphi(t) = i),
$$

where $\pi_{ij} \geq 0$, $\sum_{j \in \mathcal{M}_{\omega}} \pi_{ij} = 1$, $\forall i \in \mathcal{M}_{\omega}$, and TPM $\Pi = [\pi_{ij}]_{\mathcal{M}_{\omega} \times \mathcal{M}_{\omega}}$.

For technique analysis, $\forall i \in \mathcal{M}_{\omega}$, $A_{pi\phi}$, $A_{p2\phi}$, $A_{p3\phi}$, $B_{p\phi}$, $C_{p\phi}$, $D_{p\phi}$, $M_{p\phi}$, and $G_{p\phi}$ are denoted by $A_{pi1}$, $A_{pi2}$, $A_{pi3}$, $B_{pi}$, $C_{pi}$, $D_{pi}$, $M_{pi}$, and $G_{pi}$, respectively.

Recall the fuzzy weighting function $h_p(\xi(t)) = (\sum_{i=1}^{N_{\omega}} M_{pi}(\xi_i(t))) / (\sum_{i=1}^{N_{\omega}} M_{pi}(\xi_0(t)))$, where $M_{pi}(\xi_0(t))$ refers to the grade of the membership degree of $\xi_0(t)$ in $M_{pi}$. In general, assume $\sum_{i=1}^{N_{\omega}} h_p(\xi_0(t)) = 1$ and $h_p(\xi_0(t)) \geq 0$.

Let $x(i) = [x_2(i) \ x_2(i)]^T$; by virtue of T-S fuzzy technique, FMSSPS (1) is derived as

$$
\begin{align*}
x(t+1) & = \sum_{p=1}^{r} h_p(\xi(t))[A_{pi}E_{\varepsilon}x(t) + B_{pi}u(t) + C_{pi}\omega(t)], \\
z(t) & = \sum_{p=1}^{r} h_p(\xi(t))[D_{pi}E_{\varepsilon}x(t) + M_{pi}u(t) + G_{pi}\omega(t)],
\end{align*}
$$

where $E_{\varepsilon} = \text{diag}\{I_{n1}, \varepsilon I_{n1}\}$, $A_{pi} = \begin{bmatrix} A_{pi1} & A_{pi2} \\ A_{pi1} & A_{pi2} \end{bmatrix}$, $B_{pi} = \begin{bmatrix} B_{pi1} \\ B_{pi1} \end{bmatrix}$, $C_{pi} = \begin{bmatrix} C_{pi1} \\ C_{pi2} \end{bmatrix}$, and $D_{pi} = \begin{bmatrix} D_{pi1} \\ D_{pi1} \end{bmatrix}$.

In the NCSs, some redundant signals are communicated in the conventional data transmission manner, which may result in unfavorable phenomena, for instance, data collisions. The control signal $v(k)$ and the actuators $u(k)$ share the same communication network (CN). To prevent such unfavorable factors, the SCP scheduling is used to regulate the node order in transmitting data. Note that only one sensor is borrowed to release the signal each time, and the sensors are chosen in a stochastic way. In general, letting $\psi(t) \in \{1, 2, \ldots, N_{\psi}\}$ signifies the chosen actuator which gains the permission to access the CN at the time interval $t$. Notably, $\psi(t)$ can be recognized as a stochastic process regulated by another DTMC obeying a set $\mathcal{M}_{\psi} = \{1, 2, \ldots, N_{\psi}\}$, and TPM $\Psi = [\tau_{mn}]_{N_{\psi} \times N_{\psi}}$, is determined by

$$
\tau_{mn} = \Pr(\psi(t+1) = n|\psi(t) = m),
$$

where $\forall m, n \in \mathcal{M}_{\psi}$, $\tau_{mn} \in [0, 1]$, and $\sum_{m \in \mathcal{M}_{\psi}} \tau_{mn} = 1$. 
Let \( v(i) = \begin{bmatrix} v_1(i) \ v_2(i) \ \cdots \ v_n(i) \end{bmatrix} \) and \( u(i) = \begin{bmatrix} u_1(i) \ u_2(i) \ \cdots \ u_n(i) \end{bmatrix} \), where \( v_n(i) \) denotes the \( n \)th control input vector and \( u_n \) signifies the \( n \)th actuator. Firstly, assume that a set of zero-order hold is employed in the signal transmission. Accordingly, the \( m \)th actuator \( u_m(i) \) is updated by the following principle:

\[
u_m(i) = \begin{cases} v_m(i), & \text{if } \psi(i) = m, \\ u_m(i - 1), & \text{otherwise.} \end{cases}
\]

Aiming at describing the data transmission strategy of actuators mathematically, a Kronecker sign function is inferred as

\[
\delta(x - y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{otherwise.} \end{cases}
\]

Before proceeding further, some lemmas and definitions are provided.

**Definition 1** (see [27]). The FMSPPS (9) with \( \omega(k) = 0 \) is named stochastic stable (SS) if for any \((\delta_0, \theta_0)\), one has

\[
E\left\{ \sum_{k=0}^{\infty} \| \delta(k) \|^2 | \delta_0, \theta_0 \right\} < \infty.
\]

**Definition 2** (see [27]). The FMSPPS (9) is named SS with a prescribed \( \mathcal{H}_\infty \) performance level \( \gamma \) if the FMSPPS (18) is SS and under zero initial condition such that

\[
\sum_{k=0}^{\infty} E\left\{ \| \delta(k) \|^2 \right\} < \gamma^2 \sum_{k=0}^{\infty} E\left\{ \| \theta(k) \|^2 \right\}.
\]

**Lemma 1** (see [18]). For given a scalar \( \tau > 0 \), \( \mathcal{W}_1 \), \( \mathcal{W}_2 \), and \( \mathcal{W}_3 \) are matrices with suitable dimensions. For any \( \epsilon \in (0, \tau] \), \( \mathcal{W}_1 + \epsilon \mathcal{W}_1 + \epsilon^2 \mathcal{W}_1 > 0 \) such that

\[
\mathcal{R}_1 > 0, \\
\mathcal{W}_1 + \epsilon \mathcal{W}_1 > 0, \\
\mathcal{W}_1 + \epsilon \mathcal{W}_2 + \epsilon^2 \mathcal{W}_3 > 0.
\]

As indicated from the updating principle (5), the \( m \)th actuator \( u_m(i) \) is updated when \( \psi(i) = m \). Consequently, for \( \forall i \), the updated actuator \( u(i) \) can be devised as

\[
u(i) = \psi(i)v(i) + (1 - \psi(i))u(i - 1),
\]

where \( \psi_m \) \( m = 1, 2, \ldots, n_u \) and \( \delta^T_x = (x - y) \).

The control law \( v(k) \) in this work is constructed as follows:

\[
v(i) = \sum_{q=1}^{r} h_q(\xi(i))K_{\delta \phi(i)}E_{c}x(i),
\]

where \( K_{\delta \phi(i)} \) are matrices to be designed.

Substituting (8) into (3), the closed-loop FMSPPS (9) is formulated as

\[
\mathcal{L}_2 \text{ (see [18]). For any symmetric matrices } \mathcal{R}_1 (t = 1, 2, 3, 4) \text{ and matrix } \mathcal{R}_5 \text{ which meets}
\]

\[
\mathcal{R}_1 > 0,
\]

\[
\begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 \end{bmatrix} > 0,
\]

\[
\begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 + \epsilon^2 \mathcal{R}_4 \end{bmatrix} > 0,
\]

one has \( E_{c}\mathcal{R}_c = E_{c}\mathcal{R}_c > 0 \) for any \( \epsilon \in (0, \tau] \), where

\[
\mathcal{R}_c = \begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 + \epsilon^2 \mathcal{R}_4 \end{bmatrix}.
\]

3. Main Results

In this section, sufficient conditions are elicited to ensure the SS and a prescribed \( \mathcal{H}_\infty \) performance level of the FMSPPS (9).

**Theorem 1.** The closed FMSPPS (9) is called SS with a prescribed \( \mathcal{H}_\infty \) performance level \( \gamma \) if there exist symmetric matrices \( \mathcal{R}_1 > 0, \mathcal{Q}_1 > 0, \mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3, \) and \( \mathcal{R}_4 \) and matrices \( \mathcal{R}_5, \mathcal{K}_{\delta \phi(i)}, \mathcal{U}_1, \) and \( \mathcal{U}_2 \) such that

\[
\mathcal{R}_1 > 0,
\]

\[
\begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 \end{bmatrix} > 0,
\]

\[
\begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 + \epsilon^2 \mathcal{R}_4 \end{bmatrix} > 0,
\]

\[
\begin{bmatrix} \mathcal{R}_1 + \epsilon \mathcal{R}_3 & \epsilon \mathcal{R}_5 \\ * & \epsilon \mathcal{R}_2 + \epsilon^2 \mathcal{R}_4 \end{bmatrix} > 0.
\]
Meanwhile, the $\epsilon$-dependent controller gains are achieved as

$$K_{qi} = \mathcal{K}_{eqi} (U_1 + \epsilon \mathcal{U}_2)^{-1}, \quad (q = 1, 2, \ldots, r, i \in \mathcal{N}_r),$$

(21)

where
In the following, a Lyapunov functional for FMSSPS (9) is established:

\[ V(i, x(i), u(i), \phi(i)) = x^T(i)P(\phi(i))x(i) + u^T(k-1)Q(\phi(i))u(k-1). \]  

(29)

By calculating the difference of \( V(i, x(i), u(i), \phi(i)) \), one has

\[ \mathcal{E}[\Delta V(i)] = \mathcal{E}[V(i+1, x(i+1), u(i+1)), \phi(i)] - V(i, x(i), u(i), \phi(i)). \]  

(30)

Recalling FMSSPS (9), \( \mathcal{E}[\Delta V(i)] \) can be derived as

\[ \mathcal{E}[\Delta V(i)] = x^T(i+1)\mathcal{P}_i x(i+1) - x^T(i)P_i x(i) + u^T(i)\mathcal{Q}_i u(i) - u^T(k-1)Q_i u(k-1), \]  

(31)

where

\[ \mathcal{P}_i = \sum_{j \in \mathcal{S}_i} \pi_{ij}P_j, \]

\[ \mathcal{Q}_i = \sum_{j \in \mathcal{S}_i} \theta_{ij}Q_j. \]

(32)

In (31), the first term can be further devised as

\[ \begin{bmatrix} \Gamma_{pp}^1 \Gamma_{pqim}^{2T} \Gamma_{pqim}^{3T} \Gamma_{pqim}^{4T} \end{bmatrix} \begin{bmatrix} \Gamma_{pp}^5 & 0 & 0 & 0 \\ \Gamma_{pp}^6 & 0 & 0 & -I \end{bmatrix} < 0, \]

(25)

where

\[ \Gamma_{pp}^1 = \text{diag}\{-P, -Q, -\gamma^2I\}, \]

\[ \Gamma_{pqim}^2 = [\mathcal{A}_{pqim} \mathcal{E}_m \mathcal{B}_{pi}(I - \Psi_m) \mathcal{C}_{pi}], \]

\[ \Gamma_{pqim}^3 = [\Psi_m \mathcal{K}_q \mathcal{E}_m (I - \Psi_m) 0], \]

\[ \Gamma_{pqim}^4 = [\mathcal{D}_{pqim} \mathcal{E}_m \mathcal{M}_{pi}(I - \Psi_m) \mathcal{G}_{pi}], \]

\[ \Gamma_{pqim}^5 = [\Psi_m \mathcal{K}_q \mathcal{E}_m (I - \Psi_m) 0], \]

\[ \Gamma_{pqim}^6 = [\mathcal{D}_{pqim} \mathcal{E}_m \mathcal{M}_{pi}(I - \Psi_m) \mathcal{G}_{pi}], \]

(26)

Premultiplying and postmultiplying (25) with \( \text{diag}\{\mathcal{R}_1^{-1}, \mathcal{R}_2^{-1}, \ldots, 1\} \) and its transpose, where \( \mathcal{R}_e = \mathcal{U}_1 + e\mathcal{U}_2 \), yield

\[ \begin{bmatrix} \Gamma_{pp}^1 \Gamma_{pqim}^{2T} \Gamma_{pqim}^{3T} \Gamma_{pqim}^{4T} \end{bmatrix} \begin{bmatrix} \Gamma_{pp}^5 & 0 & 0 & 0 \\ \Gamma_{pp}^6 & 0 & 0 & -I \end{bmatrix} < 0, \]

(27)

where

\[ \Gamma_{pp}^1 = \text{diag}\{-P, -Q, -\gamma^2I\}, \]

\[ \Gamma_{pqim}^2 = [\mathcal{A}_{pqim} \mathcal{E}_m \mathcal{B}_{pi}(I - \Psi_m) \mathcal{C}_{pi}], \]

\[ \Gamma_{pqim}^3 = [\Psi_m \mathcal{K}_q \mathcal{E}_m (I - \Psi_m) 0], \]

\[ \Gamma_{pqim}^4 = [\mathcal{D}_{pqim} \mathcal{E}_m \mathcal{M}_{pi}(I - \Psi_m) \mathcal{G}_{pi}], \]

\[ \Gamma_{pqim}^5 = [\Psi_m \mathcal{K}_q \mathcal{E}_m (I - \Psi_m) 0], \]

\[ \Gamma_{pqim}^6 = [\mathcal{D}_{pqim} \mathcal{E}_m \mathcal{M}_{pi}(I - \Psi_m) \mathcal{G}_{pi}], \]

(28)

\[ x^T(i + 1)\mathcal{P}_i x(i + 1) = \sum_{p=1}^{r} h_p(x(i)) \sum_{q=1}^{r} h_q(x(i)) \]

\[ x^T(i)(P_i + E_{pqim}^T \mathcal{P}_i \mathcal{A}_{pqim} \mathcal{E}_m) x(i) + x^T(i)E_{pqim}^T \mathcal{P}_i \mathcal{B}_{pi}(I - \Psi_m) u(i - 1) + u^T(i - 1)(I - \Psi_m) \mathcal{B}_{pi}^T \mathcal{P}_i \mathcal{A}_{pqim} \mathcal{E}_m x(i) + x^T(i)E_{pqim}^T \mathcal{P}_i \mathcal{C}_{pi} \omega(k) + \omega^T(k) \mathcal{C}_{pi}^T \mathcal{P}_i \mathcal{A}_{pqim} \mathcal{E}_m x(i) + u^T(i - 1)(I - \Psi_m) \mathcal{B}_{pi}^T \mathcal{P}_i \mathcal{B}_{pi}(I - \Psi_m) u(i - 1) + u^T(i - 1)(I - \Psi_m) \mathcal{B}_{pi}^T \mathcal{P}_i \mathcal{C}_{pi} \omega(k) + \omega^T(k) \mathcal{C}_{pi}^T \mathcal{P}_i \mathcal{B}_{pi}(I - \Psi_m) u(i - 1) + \omega^T(k) \mathcal{C}_{pi}^T \mathcal{P}_i \mathcal{C}_{pi} \omega(k) \]

(33)
Besides, the third term in (31) can be rewritten as

\[ u^\top(i) \xi_i u(i) = \sum_{q=1}^{r} h_q(\xi(i)) \left[ \Psi_m K_q E_c x(i) + (I - \Psi_m) u(t-1) \right]^\top \xi_i \]

\[ \times \left[ \Psi_m K_q E_c x(i) + (I - \Psi_m) u(t-1) \right] \]

When \( \omega(k) \neq 0 \), it follows from inequality (35) that

\[ \mathcal{E}[\Delta V(i)] \leq \sum_{q=1}^{r} \frac{\mathcal{E}}{\vartheta_q(\xi(i))} \sum_{q=1}^{r} h_q(\xi(i)) \Gamma_{pqim} \Theta(k) \]

\[ \leq - \chi \mathcal{E} \left\{ \|x(i)\|^2 \right\}, \]

where

\[ \vartheta(k) = \left[ x^\top(i) u^\top(i-1) \omega^\top(k) \right], \]

\[ \Gamma_{pqim} = \text{diag}[-P_i, -Q_i, 0], \]

\[ \Gamma_{pqim} = \Gamma_{pqim}^{1} + \Gamma_{pqim}^{2} \Gamma_{pqim}^{3} + \Gamma_{pqim}^{4} \Gamma_{pqim}^{5} \].

When \( \omega(k) = 0 \), Combining (29)–(34) yields

\[ \mathcal{E}[\Delta V(i)] = \mathcal{E} \left\{ \sum_{i=0}^{\infty} \|x(i)\|^2 \right\} \leq - \frac{1}{\chi} \mathcal{E} \left\{ \sum_{i=0}^{\infty} \Delta V(i) \right\} < \infty. \]
Additionally, by applying the Schur complement to (27) and (40), one gets
\[ \mathcal{F}(T) < 0. \]
4. Simulation Examples

Example 1. Consider FMSSPS (16) with the following parameters:

\[
A_{11} = \begin{bmatrix} 0.3 & 1.7 & 0.3 \\ 0.8 & 0.1 & 0.3 \\ 1.4 & 0.2 & 0.9 \end{bmatrix},
B_{11} = \begin{bmatrix} 0.5 & 0.4 \\ 1.2 & 0.2 \\ 0.2 & 0.3 \end{bmatrix},
C_{11} = \begin{bmatrix} 0.6 & 0.2 & 0.7 \end{bmatrix}^T,
D_{11} = \begin{bmatrix} 0.3 & 0.4 & 0.9 \end{bmatrix},
M_{11} = \begin{bmatrix} 0.5 & 0.8 \end{bmatrix},
G_{11} = 0.7,
A_{12} = \begin{bmatrix} 1.5 & 0.4 & 1.1 \\ 0.8 & 1.1 & 0.6 \end{bmatrix},
B_{12} = \begin{bmatrix} 1.4 & 0.6 \\ 0.7 & 0.1 \\ 0.5 & 0.8 \end{bmatrix},
C_{12} = \begin{bmatrix} 0.3 & 0.5 & 1 \end{bmatrix}^T,
D_{12} = \begin{bmatrix} 0.6 & 0.2 & 0.3 \end{bmatrix},
M_{12} = \begin{bmatrix} 1.5 & 0.5 \end{bmatrix},
G_{12} = 0.4,
A_{21} = \begin{bmatrix} 0.2 & 1.1 & 0.1 \\ 0.2 & 1.1 & 1.2 \\ 0.6 & 0.9 & 1.1 \end{bmatrix},
B_{21} = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.5 \\ 0.2 & 0.3 \end{bmatrix},
C_{21} = \begin{bmatrix} 1.8 & 0.7 & 0.2 \end{bmatrix}^T,
D_{21} = \begin{bmatrix} 0.6 & 0.9 & 0.1 \end{bmatrix},
M_{21} = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix},
G_{21} = 0.5,
A_{22} = \begin{bmatrix} 1.1 & 1.5 & 2.4 \\ 1.5 & 0.7 & 0.6 \\ 0.1 & 1.1 & 0.8 \end{bmatrix},
B_{22} = \begin{bmatrix} 0.6 & 0.2 \\ 0.5 & 0.3 \\ 0.7 & 0.5 \end{bmatrix},
C_{22} = \begin{bmatrix} 0.6 & 0.3 & 0.4 \end{bmatrix}^T,
D_{22} = \begin{bmatrix} 0.7 & 0.6 & 0.7 \end{bmatrix},
M_{22} = \begin{bmatrix} 0.5 & 0.6 \end{bmatrix},
G_{22} = 0.3.
\]

The TPM of the corresponding FMSSPS (1) is selected as

\[
\Pi = \begin{bmatrix} 0.25 & 0.75 \\ 0.65 & 0.35 \end{bmatrix}.
\]

For another MP \( \Phi \) in SCP (7), the TPM is chosen as \( \Phi = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix} \).

Let \( \gamma = 5, \epsilon = 0.002, h_1(\xi(i)) = (15 - x_2(i))/30, \) and \( h_2(\xi(i)) = 1 - h_1(\xi(i)) \). In view of the LMIs of Theorem 1,
the controller gains $K_{qi}$ ($q = 1, 2; f = 1, 2, 3, 4$) can be derived as follows:

$$K_{1,1} = \begin{bmatrix} -0.0008 & -0.0016 & -0.0001 \\ -0.0010 & -0.0011 & 0.0005 \end{bmatrix},$$

$$K_{1,2} = \begin{bmatrix} -0.0011 & -0.0063 & -0.0088 \\ -0.0010 & -0.0061 & -0.0085 \end{bmatrix}$$

$$K_{2,1} = \begin{bmatrix} -0.0007 & 0.0032 & 0.0022 \\ -0.0006 & 0.0017 & 0.0015 \end{bmatrix}$$

$$K_{2,2} = \begin{bmatrix} -0.0009 & 0.0146 & 0.0249 \\ -0.0010 & 0.0148 & 0.0244 \end{bmatrix}$$

To carry on the simulation study, the external disturbance and the initial condition are selected as $\omega(k) = 0.9 \exp(-0.4t)\sin(50t)$ and $x(0) = [-0.1 \ 0.2 \ 0.3]^T$, respectively. The possible mode switching of Markov process $\phi(i)$ and the evolution of states $x_1(i), x_2(i)$, and $x_3(i)$ over 100 realizations are depicted in Figure 1, respectively. The control input over 100 realizations is plotted in Figure 2. Furthermore, the evolution of the output $z(i)$ over 100 realizations is shown in Figure 3.

5. Conclusions

In this work, the $H_{\infty}$ control problem has been discussed for FMSSPSs with the SCP. In order to coordinate the data transmission and save the bandwidth usage, the SCP with a compensator is applied to schedule the information exchange. Furthermore, some sufficient criteria have been forwarded such that the resulting system is SS. Finally, one example is exhibited to show the effectiveness and correctness of the developed results. In addition, some advanced techniques including the sliding mode-based filter will be researched in our following work [28, 29].

Data Availability

No data were utilized to support this work.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Science and Technology Project of China Southern Power Grid Company Ltd. (nos. 080037KK52190037 and GZHKJXM20190108).

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