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Research Article

Comparative Analysis of the Kinematics Solution Based on the DH Method and Screw Theory

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The premise of analyzing and researching robot technology is to establish a proper mathematical model and then to solve it with kinematics. In this study, a self-developed humanoid hydraulically driven dual-arm robot is taken as the research object, and the DH (Denavit–Hartenberg) parameter method and the rotational exponential formula (POE) are used to solve the kinematics of the robot. The calculation results are verified by simulation. The advantages and disadvantages of the two methods are analyzed. The differences between the two methods are compared. It lays a foundation for other scholars to choose mathematical models when analyzing the mechanism in the future.

1. Introduction

The main methods of kinematics research on tandem robots today are the DH (Denavit–Hartenberg) parameter method and the exponential formula (POE) of screw theory. The DH parameter method was proposed by Denavit and Hartenberg of the University of Chicago in the United States in 1955. It was named in honor of the two scientists [1]. The research on the theory of curl entered a heyday in the nineteenth century. Among them, a comprehensive study and discussion of screw theory was first conducted by Ball, professor of the University of Cambridge in the United Kingdom. And a classic work called “screw theory” was completed [2, 3].

At present, scholars at home and abroad have performed plenty of research studies on the DH method application in the tandem field. They have obtained a batch of research results. Jian et al. [4] used the DH method to solve the coordinate transformation of the serial six-degree-of-freedom robot arm. They established the transformation relationship between the robot arm and image mark. It laid a foundation for the study of accuracy error when the robot arm grabbed the workpiece on a repeated basis. Li et al. [5] used the DH method to establish the motion model of the dual-arm robot. The model was direct and vivid in geometric sense for motion parameters. It laid the foundation for the robot’s motion calibration error analysis. Qiaoling et al. [6] used the DH method to establish a kinematic model of the open-chain pole-climbing robot. It provided a theoretical basis for improving the motion accuracy control of the pole-climbing robot. Wen [7] compared the advantages and disadvantages of the standard DH method and the improved one in the process of establishing multilink coordinate systems. It provided a reference basis for engineers to use different forms of DH methods when establishing mathematical models of tandem robots. Yang et al. [8] used the DH method to express the relationship between speed and position of each part of the wave glider. It broadened the scope of DH method application. Zhao et al. [9] took the photographic robots as research objects which are used in the virtual studio. According to the actual needs of the studio or stage, they proposed a new analysis method for the working end of photographic robot through the improved DH method. Kelemen et al. [10, 11] introduced a new algorithm for the control of kinematically redundant manipulator considering three secondary tasks.

Similarly, scholars at home and abroad have performed a lot of research studies on the application of screw theory in the kinematics of the series mechanism. Wei et al. [12] proposed a serpentine robot kinematics modeling method based on screw theory. It proved that the screw theory modeling method was efficient. Li et al. [13] analyzed the positive kinematics and operability of the tandem manipulator by using screw theory, and they proved that screw theory was feasible. Liu et al. [14] used screw theory to analyze the motion of the manipulator with four freedoms in series. They deduced the singular configuration of the manipulator to avoid singular positions. Zhu [15] solved the problem of kinematic analysis of five-degree-of-freedom tandem welding robots by using screw theory. Yang et al. [16] proposed a method for evaluating and predicting the machining accuracy of multiaxis CNC machine tools based on screw theory. They established the kinematics model of each axis of machine tool and used the distance error formula to derive a model for general machining allowance error. It can greatly simplify the motion error analysis of multiaxis CNC machine tools. Wang et al. [17] used screw theory of the exponential product (POE) formula to solve the positive kinematics problem of the tandem free-floating space robot system. It avoided the introduction of inertia tensor parameters and broadened the application range of the theory. Niu et al. [18] established the positive kinematics model of the minimally invasive abdominal cavity surgery robot by using screw theory. They designed a master-slave control algorithm and conducted experiments. The maximum spatial position error is less than 1 mm. Qu et al. [19] proposed a unified model of open-chain robot output accuracy based on the Lie group and Lie algebra considering joint clearance and structural parameters. They used a six-degree-of-freedom open-chain manipulator to verify and analyze the model and to prove the model was effective. Chen and Zhou [20] analyzed the kinematics of a serial modular redundant robot based on screw theory; they simplified the kinematic modeling process of modular robots with arbitrary configurations and degrees of freedom.

In summary, the premise for analyzing and researching robotics is to establish a proper mathematical model. For the DH method and screw theory, which is more concise and specific for modeling its motion characteristics (controlling the mapping of joints to end motions), what are their respective advantages? This study takes the self-developed high-load humanoid 7-DOF hydraulically driven dual-arm robot as the research object and uses the DH parameter method and the exponential formula of spin theory to establish a mathematical model of the robot. In this way, the kinematics model and calculation process are fully compared, and their respective advantages are demonstrated.

2. Introduction of the Humanoid Hydraulically Driven Dual-Arm Robot

With the funding of the National Key R&D Program, the team independently developed a large-load seven-degree-of-freedom hydraulically driven dual-arm robot. It simulates the joints of human arms. The overall structure of the human arm is composed of three joints of the shoulder, elbow, and wrist with seven degrees of freedom. Among them, there are three shoulder joints, three wrist joints, and one elbow joint, respectively. By considering the arrangement and motion range of human arm joints, the hydraulic two-arm robot can be designed in a more flexible way, and it can avoid obstacles in the space. Therefore, the robot also has seven axes. Among them, the first axis, the second axis, the fourth axis, the sixth axis, and the seventh axis are made of telescopic hydraulic cylinders. In order to achieve the mobility of human arm joints, the connecting part adopts a four-link mechanism. The third and fifth axes use swing hydraulic cylinders to increase the compactness of the structure. According to the layout and parameters of each joint, a three-dimensional modeling of the dual-arm robot is performed to complete the detailed design of the robot structure, as shown in Figure 1. At the same time, it is also designed to wear a remote operation device with two arms to achieve flexible control of human body intention construction, as shown in Figure 2.

3. Motion Analysis Based on the DH Parameter Method

3.1. DH Parameter Method Modeling and Calculation. The heavy-loading humanoid 7-DOF hydraulically driven dual-arm robot is symmetrically arranged for its arms. Therefore, only one mechanical arm needs to be analyzed. A robotic arm is equivalent to rods and joints with the same function. According to the DH coordinate system construction rule, the establishment of the robotic arm coordinate system is as follows: the joint of the robot arm to the box is used as the initial base coordinate system $X_0Y_0Z_0$. The coordinate system on the n joints is $X_nY_nZ_n$. The origin of coordinate of link n is located at the intersection of the joint n-axis and joint $n+1$ axis. The Z-axis of link n coincides with

Figure 1: Modeling diagram of the humanoid 7-DOF hydraulically driven dual-arm robot.

the axis of joint $n+1$. The X-axis of link n is at the common normal of the joint n-axis and the joint $n+1$ axis. The X-axis is directed from joint n-axis to the joint $n+1$ axis. The Y-axis of the link n is determined by the right-hand rule [21].

According to the provisions of the previous paragraph, the coordinate system is established for each joint of the robot arm. In order to simplify the establishment of the coordinate system, the robotic arm is straightened and modeled as the robotic arm DH coordinate system (Figure 3). The DH parameters of the right drill arm of the top plate are obtained from the design dimensions of the robotic arm, as shown in Table 1. a_n is the length of the link between the joints. θ_n is the rotation angle of each joint.

According to the introduction to robotics [21], the parameters in Table 1 are brought into formula (1) to obtain the joint transformation matrix A_n. In formula (1), Rot (Z_{n-1}, θ_n) is the angle of X_{n-1} rotating around the Z_{n-1}-axis by the angle θ_n. And Trans $(0, 0, d_n)$ is the translation of d_n along the Z_{n-1}-axis. Trans $(a_n, 0, 0)$ is the translation of an along the X_n-axis, and Rot (X_n, α_n) is the rotation of the α_n angle around the X_n-axis.

$$A_n = \underset{(1)}{\underline{\mathrm{Rot}(Z_{n-1}, \theta_n)}}\ \underset{(2)}{\underline{\mathrm{Trans}(0, 0, d_n)}}\ \underset{(3)}{\underline{\mathrm{Trans}(a_n, 0, 0)}} \cdot \underset{(4)}{\underline{\mathrm{Rot}(X_n, \alpha_n)}}. \tag{1}$$

According to formula (1),

$$\begin{aligned}
A_1 &= \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & a_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & a_2 + a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_4 &= \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_5 &= \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & a_4 + a_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_6 &= \begin{bmatrix} \cos\theta_6 & 0 & \sin\theta_6 & 0 \\ \sin\theta_6 & 0 & -\cos\theta_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
A_7 &= \begin{bmatrix} \cos\theta_7 & -\sin\theta_7 & 0 & a_6\cos\theta_7 \\ \sin\theta_7 & \cos\theta_7 & 0 & a_6\sin\theta_7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{2}$$

The formula is gained:

$$T_7 = A_1A_2A_3A_4A_5A_6A_7 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & q_1 \\ r_{21} & r_{22} & r_{23} & q_2 \\ r_{31} & r_{32} & r_{33} & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

For details, see Appendix A.

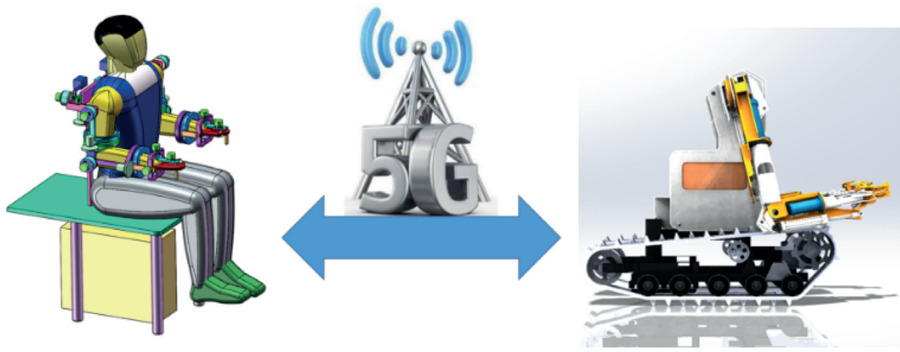

Figure 2: Remote operation control diagram.

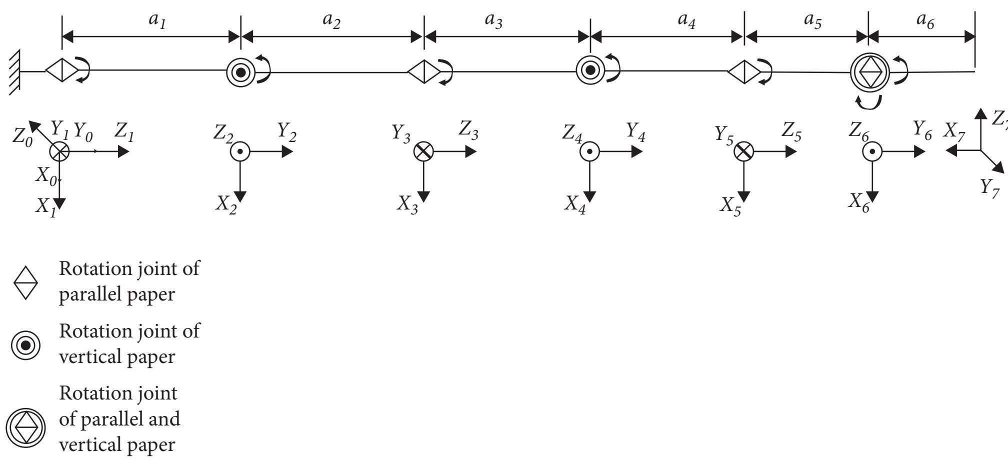

Figure 3: DH robot arm coordinate system.

Table 1: DH robot arm coordinate system parameters.

Link	Variables	d_n	a_n	α_n
1	θ_1	a_1	0	90°
2	θ_2	0	0	90°
3	θ_3	$a_2 + a_3$	0	−90°
4	θ_4	0	0	−90°
5	θ_5	$a_4 + a_5$	0	90°
6	θ_6	0	0	90°
7	θ_7	0	a_6	0

3.2. DH Parameter Method Motion Simulation. MATLAB is a powerful commercial mathematical software, which is mainly used in data analysis, numerical calculation, and data visualization. It is currently widely used in scientific research at home and abroad. By using the numerical calculation function and data visualization function in MATLAB, it is possible to efficiently and quickly solve the cloud map of the robot's workspace [22]. The mechanical arm structure parameters in Table 2 are substituted into the positive kinematics solution equation (2). The equation is calculated by the DH parameter method of the robot arm to draw the working range of the robot arm, as shown in Figure 4. In Figure 4, a is the three-dimensional point cloud of the range of motion of the robotic arm, b is the two-dimensional point cloud of the range of motion in the X and Z directions, c is the two-dimensional point cloud of the range of motion in the Z and Y directions, and d is the two-dimensional point cloud of the range of motion in the Z and Y directions.

4. POE Kinematics Analysis Based on Screw Theory

4.1. The Basis of Screw Theory. If the base coordinate system of the rigid body is A and the dynamic coordinate system is B, then the set of position and orientation transformations of the rigid body on A can be expressed as

Table 2: DH robot arm parameters.

Link	Length (mm)	Joint variables	Range ( )
a_1	940	θ_1	$-75^\circ \sim +75^\circ$
a_2	490	θ_2	$+60^\circ \sim +180^\circ$
a_3	510	θ_3	$-20^\circ \sim +20^\circ$
a_4	240	θ_4	$-135^\circ \sim -50^\circ$
a_5	390	θ_5	$-40^\circ \sim +40^\circ$
a_6	310	θ_6	$+30^\circ \sim +120^\circ$
		θ_7	$-30^\circ \sim +30^\circ$

Note: in order to reflect the generality of the study, the rotation range of each joint in Table 2 is smaller than the actual rotation range.

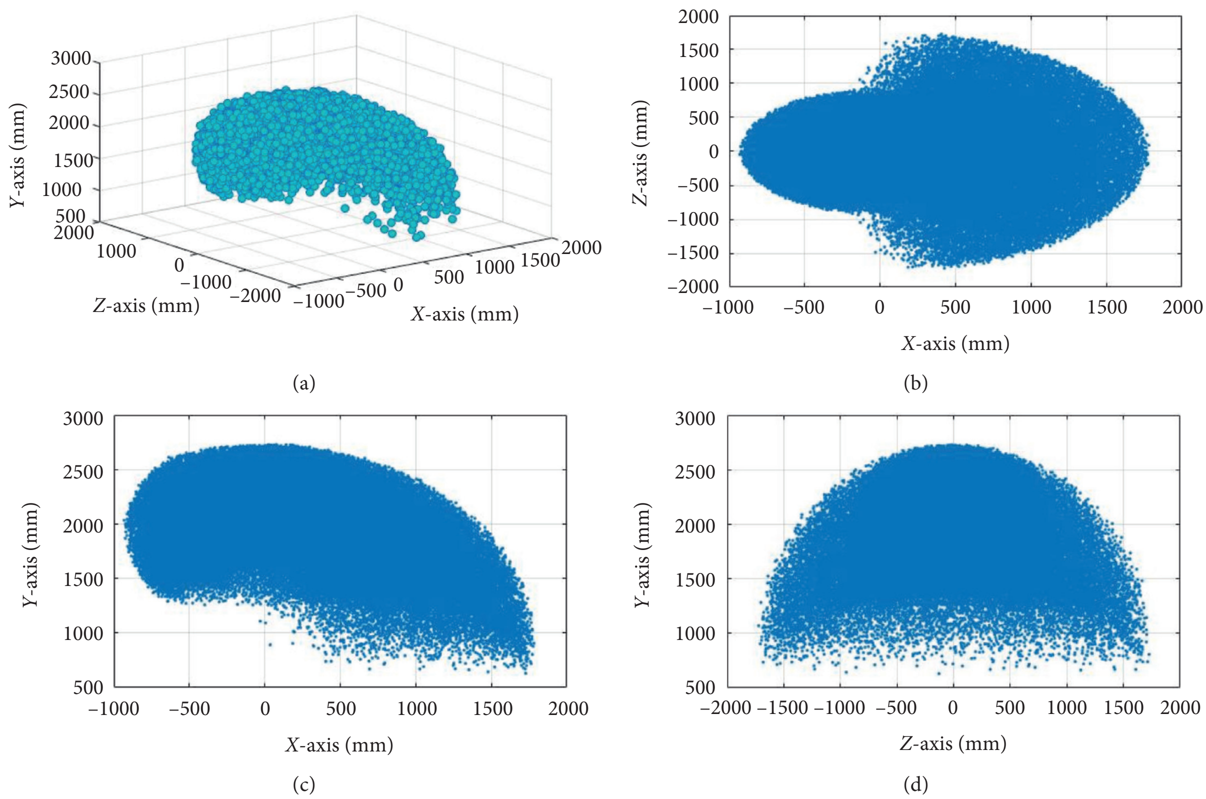

Figure 4: DH workspace cloud illustration.

$$\mathrm{SE}(3) = \left\{ \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} : R \in \mathrm{SO}(3),\ P \in R^3 \right\}. \qquad (4)$$

Here, $\mathrm{SE}(3)$ is the position and orientation transformation matrix of the B coordinate system with respect to the A coordinate system; $R \in \mathrm{SO}(3)$ is the orientation transformation matrix of the B coordinate system with respect to the A coordinate system; and $P \in R^3$ is the position vector of the B coordinate system with respect to the A coordinate system.

When the rigid body rotates around the rotation axis $\omega = (\omega_X \ \ \omega_Y \ \ \omega_Z)^{\mathrm{T}} \in R^3$, as shown in Figure 1, we can know from the Chasles theorem [11] that it can be realized by a composite motion. The motion is that rotating around the axis $\omega = (\omega_X \ \ \omega_Y \ \ \omega_Z)^{\mathrm{T}} \in R^3$ for an angle θ and moving for a distance v linearly parallel to the axis ω. The position and orientation transformation is expressed by the matrix index as

$$g = e^{\widehat{\xi}\theta} \in \mathrm{SE}(3). \qquad (5)$$

In the formula, $\widehat{\xi} = \begin{bmatrix} \widehat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathrm{se}(3)$, $\omega, v \in R^3$ represents the motion screw; $\widehat{\omega} = \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix}$ is a Lie algebra of $\mathrm{SO}(3)$, which is called antialgorithm matrix; $v = -\omega \times q$ is the unit vector in the direction of movement; $q \in R^3$ is a random point on the rotation axis; $\mathrm{se}(3)$ is a Lie algebra of $\mathrm{SE}(3)$; $e^{\widehat{\xi}\theta}$ is the matrix index corresponding to $\widehat{\xi}\theta$. The matrix index is often calculated by using the Rodriguez formula [23].

$$e^{\widehat{\omega}\theta} = I + \widehat{\omega}\sin\theta + \widehat{\omega}(1-\cos\theta). \qquad (6)$$

where I is the unit matrix of 3×3.

According to formula (6), formula (7) can be expanded into the following form:

$$g = e^{\widehat{\xi}\theta} = \begin{cases} \begin{matrix} e^{\widehat{\omega}\theta} & \left(I + e^{\widehat{\omega}\theta}\right)(\omega\times\nu) + \omega\omega^{\mathrm{T}}\nu\theta \\ 0 & 1 \end{matrix}, & \omega \neq 0, \\ \begin{bmatrix} I & \nu\theta \\ 0 & 1 \end{bmatrix}, & \omega = 0. \end{cases} \qquad (7)$$

As described above, the index of the motion screw can represent the relative motion of the rigid body. It is that after the screw motion on the rigid body, the position and orientation transformation of the moving coordinate system B relative to the base coordinate system A is converted to

$$g_{ab}(\theta) = e^{\widehat{\xi}\theta} g_{ab}(0). \qquad (8)$$

where $g_{ab}(0)$ is the rigid body' position and orientation change of B and A in the initial pose. $g_{ab}(\theta)$ is the final position and orientation change of B relative to A.

4.2. POE Modeling and Operation of Screw Theory. When establishing the rotation coordinate system of each joint of screw theory, it is necessary to select one part of the robot arm as reference and one part as the end of the movement. To this end, we set the joint of the robot arm to the box as the inertial coordinate system. The end of the movement of the arm is the tool coordinate system. Therefore, the reference configuration of the manipulator in its natural posture is shown in Figure 5.

It can be known from Figure 6 that the starting joint of the robot arm is the inertial coordinate system S, and the end is the tool coordinate system T. When it is zero, the initial configuration of each joint is

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & a_2 + a_3 \\ 0 & 1 & 0 & a_1 + a_4 + a_5 + a_6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (9)$$

From Figure 5, establish the spin motion coordinates of each joint.

$$\begin{cases} \omega_1 = [\,0\ \ 1\ \ 0\,], \\ \omega_2 = [\,0\ \ 0\ \ 1\,], \\ \omega_3 = [\,1\ \ 0\ \ 0\,], \\ \omega_4 = [\,0\ \ 0\ \ 1\,], \\ \omega_5 = [\,0\ \ 1\ \ 0\,], \\ \omega_6 = [\,1\ \ 0\ \ 0\,], \\ \omega_7 = [\,0\ \ 0\ \ 1\,]. \end{cases} \qquad (10)$$

According to the reference configuration of the link in Figure 6, the points of each joint axis can be obtained:

$$\begin{cases} q_1 = [\,0\ \ 0\ \ 0\,], \\ q_2 = [\,0\ \ a_1\ \ 1\,], \\ q_3 = [\,a_2\ \ a_1\ \ 0\,], \\ q_4 = [\,a_2 + a_3\ \ a_1\ \ 1\,], \\ q_5 = [\,a_2 + a_3\ \ a_1 + a_4\ \ 0\,], \\ q_6 = [\,a_2 + a_3\ \ a_1 + a_4 + a_5\ \ 0\,], \\ q_7 = [\,a_2 + a_3\ \ a_1 + a_4 + a_5\ \ 0\,]. \end{cases} \qquad (11)$$

From $\xi_i = \begin{bmatrix} \omega_i \\ q_i \times \omega_i \end{bmatrix}$ [24], we get the unit spin motion of each joint:

$$\begin{cases} \xi_1 = [\,0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\,]^{\mathrm{T}}, \\ \xi_2 = [\,0\ \ 1\ \ 0\ \ a_1\ \ 0\ \ 0\,]^{\mathrm{T}}, \\ \xi_3 = [\,0\ \ 1\ \ 0\ \ 0\ \ 0\ \ -a_1\,]^{\mathrm{T}}, \\ \xi_4 = [\,0\ \ 1\ \ 0\ \ a_1\ \ -(a_2 + a_3)\ \ 0\,]^{\mathrm{T}}, \\ \xi_5 = [\,0\ \ 1\ \ 0\ \ 0\ \ 0\ \ a_2 + a_3\,]^{\mathrm{T}}, \\ \xi_6 = [\,0\ \ 1\ \ 0\ \ 0\ \ 0\ \ -(a_1 + a_4 + a_5)\,]^{\mathrm{T}}, \\ \xi_6 = [\,0\ \ 1\ \ 0\ \ a_1 + a_4 + a_5\ \ -(a_2 + a_3)\ \ 0\,]^{\mathrm{T}}. \end{cases} \qquad (12)$$

From the exponential product formula (7) [24], we can derive

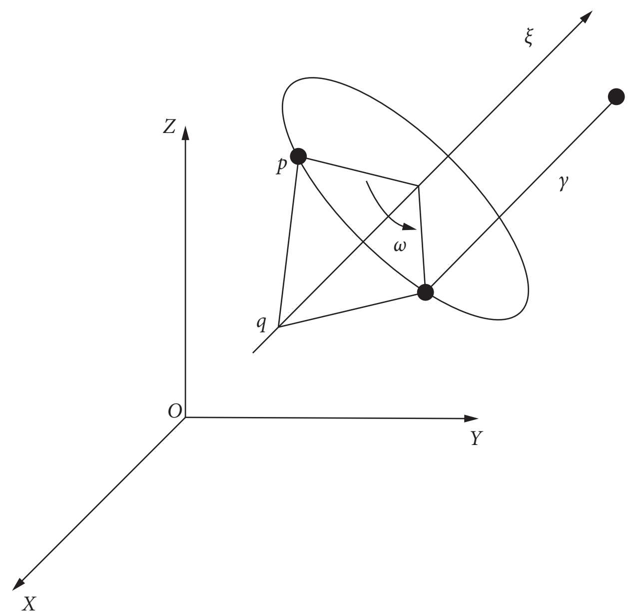

Figure 5: Rotational motion of a rigid body.

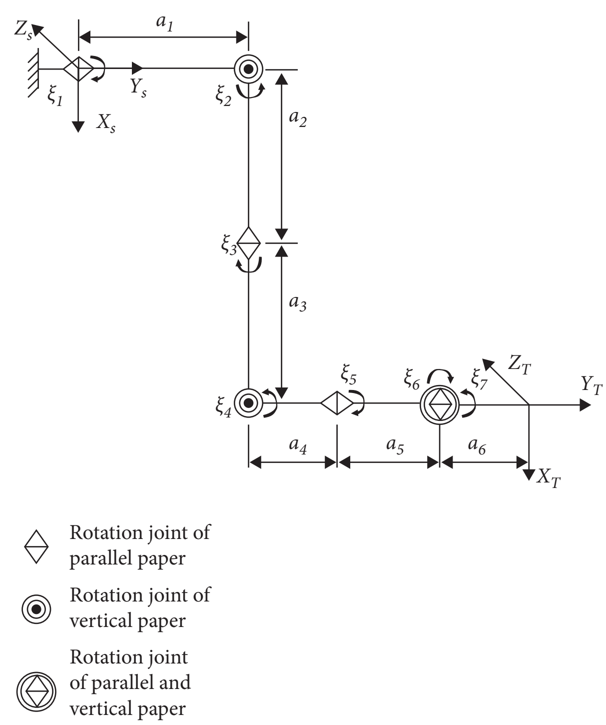

Figure 6: Screw robot arm link reference configuration.

$$
\begin{aligned}
e^{\hat{\xi}_1\theta_1} &= \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_2\theta_2} &= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_1\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_1\left(1-\cos\theta_2\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_3\theta_3} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_3 & -\sin\theta_3 & a_1\left(1-\cos\theta_3\right) \\ 0 & \sin\theta_3 & \cos\theta_3 & -a_1\sin\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_4\theta_4} &= \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & \left(a_2+a_3\right)\left(1-\cos\theta_4\right)+a_1\sin\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & a_1\left(1-\cos\theta_4\right)-\left(a_2+a_3\right)\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_5\theta_5} &= \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & \left(a_2+a_3\right)\left(1-\cos\theta_5\right) \\ 0 & 1 & 0 & 0 \\ -\sin\theta_5 & 0 & \cos\theta_5 & \left(a_2+a_3\right)\sin\theta_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_6\theta_6} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_6 & -\sin\theta_6 & \left(a_1+a_4+a_5\right)\left(1-\cos\theta_6\right) \\ 0 & \sin\theta_6 & \cos\theta_6 & -\left(a_1+a_4+a_5\right)\sin\theta_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
e^{\hat{\xi}_7\theta_7} &= \begin{bmatrix} \cos\theta_7 & -\sin\theta_7 & 0 & \left(a_2+a_3\right)\left(1-\cos\theta_7\right)+\left(a_1+a_4+a_5\right)\sin\theta_7 \\ \sin\theta_7 & \cos\theta_7 & 0 & \left(a_1+a_4+a_5\right)\left(1-\cos\theta_7\right)-\left(a_2+a_3\right)\sin\theta_7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\end{aligned}
\tag{13}
$$

Then, the positive solution of the entire kinematics is

$$
g_{ST}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} e^{\hat{\xi}_3\theta_3} e^{\hat{\xi}_4\theta_4} e^{\hat{\xi}_5\theta_5} e^{\hat{\xi}_6\theta_6} e^{\hat{\xi}_7\theta_7} g_{ST}(0) = \begin{bmatrix} R(\theta) & P(\theta) \\ 0 & 1 \end{bmatrix}. \tag{14}
$$

If $R(\theta) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$, $P(\theta) = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$. For details, see Appendix B.

4.3. POE Kinematics Simulation of Spin Theory. Substitute the structural parameters of the mechanical arm in Table 3 into the positive kinematics solution equation (14), which is calculated by the POE of the mechanical arm rotation theory above. Then, draw the working range of the mechanical arm, as shown in Figure 7. In Figure 7, *a* is the three-dimensional point cloud of the range of motion of the robotic arm, *b* is the two-dimensional point cloud of the range of motion in the *X*

Table 3: Screw robot arm parameters.

Link	Length (mm)	Joint variables	Parameter ( )
a_1	940	θ_1	$-75^\circ \sim +75^\circ$
a_2	490	θ_2	$-30^\circ \sim +90^\circ$
a_3	510	θ_3	$-20^\circ \sim +20^\circ$
a_4	240	θ_4	$+40^\circ \sim +120^\circ$
a_5	390	θ_5	$-40^\circ \sim +40^\circ$
a_6	310	θ_6	$-30^\circ \sim +30^\circ$
		θ_7	$-30^\circ \sim +30^\circ$

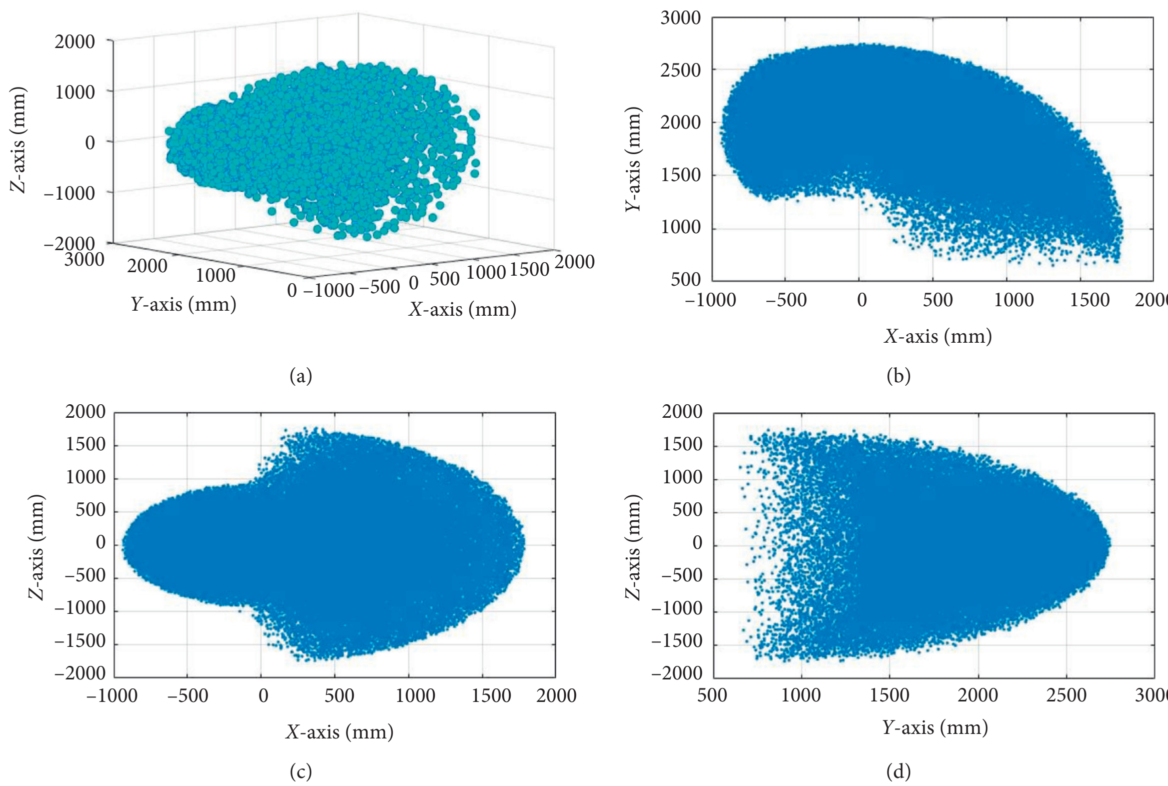

Figure 7: Screw workspace cloud diagram.

and Y directions, c is the two-dimensional point cloud of the range of motion in the X and Z directions, and d is the two-dimensional point cloud of the range of motion in the Y and Z directions.

5. Comparative Analysis of Advantages and Disadvantages of the Two Modeling Methods

5.1. Advantages and Disadvantages of the DH Parameter Method for Motion Analysis

5.1.1. Advantages. The DH parameter method is simpler than the POE of screw theory, and its calculation process is simple. At the same time, the method is highly versatile and does not require the structural order and complexity of the robot.

5.1.2. Disadvantages. The modeling process is relatively complicated. There are several degrees of freedom to establish several coordinates. The latter coordinate must be able to be transformed to the previous one. In addition, the attitude of the mechanism will also make modeling more difficult. This study chooses the initial posture as a straight line. While seeking the best modeling posture, a problem appears. It is that when the initial posture changes, the range of motion of each joint will change accordingly. This is the reason for the inconsistency in the movement ranges of Tables 2 and 3. One of the biggest disadvantages is that except for the standard Stanford robot, the DH parameter method cannot truly represent the initial coordinate position of each joint of the robot. It can be seen from d_n in Table 1 that joints 2 and 3 are at the same area. Therefore, it is easy to produce singularities.

5.2. *The Pros and Cons of POE Kinematics Analysis with Screw Theory*

5.2.1. Advantages. The POE of screw theory only needs two coordinate systems, namely, the inertial coordinate system and the tool coordinate system. In this way, the motion of the rigid body can be described as a whole in the inertial coordinate system. Then, a complete and obvious geometric description can be provided. Compared with the establishment of multiple coordinate systems of the DH parameter method, it is more global and complete. The state of motion also overcomes the singularity produced by the local coordinate system of the DH parameter method; establishing a model in any attitude does not affect the ease of modeling, which greatly simplifies the analysis of complex mechanisms.

5.2.2. Disadvantages. The POE of screw theory is a bit more difficult to get started than the DH parameter method, and the calculation process is cumbersome.

5.3. Comparison of the Expressions and Calculation Processes of the Two Modeling Methods. Compared with the DH parameter method (the coordinate system diagram), screw theory is more concise, considering its reference configuration diagram and the mathematical symbols; the key of the DH parameter method is the establishment of the coordinate parameter table. The key of screw theory is that the calculation process should not be wrong; the DH parameter method is more concise than screw theory with respect to the calculation symbols and calculation results; as for the motion-space-simulation cloud image, the DH parameter method generates the image according to the direction of the end coordinate system, while screw theory generates on the basis of the initial coordinate direction. The DH parameter method has fewer mathematical operations in formula (2) than screw theory in formula (14). The DH parameter method uses the formula (2) to simulate the three-dimensional space cloud image with the same time as the screw theory formula (14).

6. Conclusion

This study first introduces the current research status of the DH parameter method and screw theory index formula in the field of series and parallel mechanisms in recent years. Then, a large load humanoid 7-DOF hydraulically driven double-arm robot with a large load is introduced. Second, the DH parameter method and screw theory exponential formula are used to model the research object, calculate the solution, and simulate the results. Finally, the advantages and disadvantages of the two methods are analyzed, respectively. The expressions and calculation processes of the two modeling methods are compared. It provides a basis for the modeling method study in the future.

Appendix

A. Transformation Matrix of the DH Parameter Method

$$
\begin{aligned}
r_{11} &= s\theta_7 (s\theta_5 (c\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4) - c\theta_7 (s\theta_6 (s\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) + c\theta_1 c\theta_4 s\theta_2) + c\theta_5 (c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3)) \\
&\quad - c\theta_6 (c\theta_5 (c\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 c\theta_2 s\theta_3))), \\
r_{12} &= s\theta_7 (s\theta_6 (s\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) + c\theta_1 c\theta_4 s\theta_2) - c\theta_6 (c\theta_5 (c\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3))) \\
&\quad + c\theta_7 (s\theta_5 (c\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4) + c\theta_5 (c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3)), \\
r_{13} &= s\theta_6 (c\theta_5 (c\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) - c\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_3 s\theta_1 - c\theta_1 c\theta_2 s\theta_3)) \\
&\quad + c\theta_6 (s\theta_4 (s\theta_1 s\theta_3 + c\theta_1 c\theta_2 c\theta_3) + c\theta_1 c\theta_4 s\theta_2), \\
r_{21} &= c\theta_7 (s\theta_6 (s\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) - c\theta_4 s\theta_1 s\theta_2) - c\theta_6 (c\theta_5 (c\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) + s\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_1 c\theta_3 + c\theta_2 s\theta_1 s\theta_3))) \\
&\quad - s\theta_7 (s\theta_5 (c\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) + s\theta_1 s\theta_2 s\theta_4) + c\theta_5 (c\theta_1 c\theta_3 + c\theta_2 s\theta_1 s\theta_3)), \\
r_{22} &= -s\theta_7 (s\theta_6 (s\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) - c\theta_4 s\theta_1 s\theta_2) - c\theta_6 (c\theta_5 (c\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) + s\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_1 c\theta_3 + c\theta_2 s\theta_1 s\theta_3))) \\
&\quad - c\theta_7 (s\theta_5 (c\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) + s\theta_1 s\theta_2 s\theta_4) + c\theta_5 (c\theta_1 c\theta_3 + c\theta_2 s\theta_1 s\theta_3)), \\
r_{23} &= -s\theta_6 (c\theta_5 (c\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 s\theta_1) + s\theta_1 s\theta_2 s\theta_4) - s\theta_5 (c\theta_1 c\theta_3 - c\theta_2 s\theta_1 s\theta_3)) \\
&\quad - c\theta_6 (s\theta_4 (c\theta_1 s\theta_3 - c\theta_2 c\theta_3 c\theta_1) - c\theta_4 s\theta_1 s\theta_2), \\
r_{31} &= s\theta_7 (s\theta_5 (c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2) - c\theta_5 s\theta_2 s\theta_3) \\
&\quad + c\theta_7 (c\theta_6 (c\theta_5 (c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2) + s\theta_2 s\theta_3 s\theta_5) + s\theta_6 (c\theta_2 c\theta_4 - c\theta_3 s\theta_2 s\theta_4)), \\
r_{32} &= c\theta_7 (s\theta_5 (c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2) - c\theta_5 s\theta_2 s\theta_3) \\
&\quad - s\theta_7 (c\theta_6 (c\theta_5 (c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2) + s\theta_2 s\theta_3 s\theta_5) + s\theta_6 (c\theta_2 c\theta_4 - c\theta_3 s\theta_2 s\theta_4)), \\
r_{33} &= s\theta_6 (c\theta_5 (c\theta_2 s\theta_4 + c\theta_3 c\theta_4 s\theta_2) + s\theta_2 s\theta_3 s\theta_5) - c\theta_6 (c\theta_2 c\theta_4 - c\theta_3 s\theta_2 s\theta_4),
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
q_1 = {} & s\theta_5 (a_2 + a_3)(c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3) - (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2)(s\theta_4 (a_2 + a_3) + a_1 (c\theta_4 - 1)) - (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 c\theta_2) \\
& + c\theta_1 c\theta_2 c\theta_4) - s\theta_5 c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)((a_2 + a_3)(c\theta_7 - 1) - s\theta_7 (a_1 + a_4 + a_5)) \\
& - (c\theta_6 (c\theta_4 s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4)(c\theta_6 - 1)(a_1 + a_2 + a_3) - s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) \\
& + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))((c\theta_7 - 1)(a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3)) \\
& - (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4)(c\theta_6 - 1)(a_1 + a_2 + a_3) - s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) \\
& + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)) \\
& (a_1 + a_2 + a_3) + a_1 c\theta_1 s\theta_2 - a_1 s\theta_1 s\theta_3 + c\theta_1 c\theta_2 (a_1 s\theta_4 - (a_2 + a_3)(c\theta_4 - 1)) - (a_2 + a_3)(s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) \\
& + c\theta_1 c\theta_2 c\theta_4)(c\theta_5 - 1) + a_1 c\theta_1 s\theta_2 (c\theta_3 - 1), \\
q_2 = {} & s\theta_2 (a_1 s\theta_4 - (a_2 + a_3)(c\theta_4 - 1)) - (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 s\theta_3 s\theta_5)((a_2 + a_3)(c\theta_7 - 1) - s\theta_7 (a_1 + a_4 + a_5)) \\
& - (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3) - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4)) \\
& \cdot ((c\theta_7 - 1)(a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3)) - a_1 (c\theta_2 - 1) \\
& - s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3)(a_1 + a_2 + a_3) - (a_2 + a_3)(c\theta_5 - 1)(c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) \\
& + (c\theta_6 - 1)(s\theta_2 s\theta_4 - c\theta_2 c\theta_4)(a_1 + a_2 + a_3) - a_1 c\theta_2 (c\theta_3 - 1) \\
& - c\theta_2 c\theta_3 (s\theta_4 (a_2 + a_3) + a_1 (c\theta_4 - 1)) - c\theta_2 s\theta_3 s\theta_5 (a_2 + a_3), \\
q_3 = {} & s\theta_5 (a_2 + a_3)(c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) - ((c\theta_7 - 1)(a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3)) \\
& \cdot (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) -) c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) \\
& + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4) - (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2)(s\theta_4 (a_2 + a_3) + a_1 (c\theta_4 - 1)) \\
& - (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) - s\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) \\
& ((a_2 + a_3)(c\theta_7 - 1) - s\theta_7 (a_1 + a_4 + a_5)) - (c\theta_6 - 1)(c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4)(a_1 + a_2 + a_3) \\
& - s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3))(a_1 + a_2 + a_3) \\
& - a_1 s\theta_1 s\theta_2 - c\theta_2 s\theta_1 (a_1 s\theta_4 - (a_2 + a_3)(c\theta_4 - 1)) - (a_2 + a_3)(c\theta_5 - 1)(s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_3 s\theta_4) \\
& - a_1 s\theta_1 s\theta_2 (c\theta_3 - 1) - a_1 c\theta_1 s\theta_3.
\end{aligned}
\tag{A.2}
$$

B. Transformation Matrix of Screw Theory

$$
\begin{aligned}
r_{11} = {} & s\theta_7 (c\theta_6 (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4) + s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))) \\
& + c\theta_7 (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)), \\
r_{12} = {} & c\theta_7 (c\theta_6 (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4) + s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))) \\
& + s\theta_7 (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)), \\
r_{13} = {} & c\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 c\theta_4) + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)) - s\theta_6 (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4), \\
r_{21} = {} & c\theta_7 (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 s\theta_3 s\theta_5) + s\theta_7 (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3) - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4)), \\
r_{22} = {} & c\theta_7 (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3) - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4)) - s\theta_7 (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 s\theta_3 s\theta_5), \\
r_{23} = {} & c\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 c\theta_5 s\theta_3) + s\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4), \\
r_{31} = {} & s\theta_7 (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4)) \\
& + c\theta_7 (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) - s\theta_5 (c\theta_1 c\theta_3 + s\theta_1 s\theta_2 s\theta_3)), \\
r_{32} = {} & c\theta_7 (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4)) \\
& - s\theta_7 (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) - s\theta_5 (c\theta_1 c\theta_3 + s\theta_1 s\theta_2 s\theta_3)), \\
r_{33} = {} & c\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) \\
& - s\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 c\theta_1 s\theta_2),
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
q_1 = {} & (c\theta_7 (c\theta_6 c\theta_4 s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4) + s\theta_6 (s\theta_5 s\theta_4 s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 + c\theta_1 c\theta_2 s\theta_4) \\
& + s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4 + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)) \\
& - s\theta_7 (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 + c\theta_1 c\theta_2 c\theta_4 - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)))) \\
& \cdot (a_1 + a_4 + a_5 + a_6) - (c\theta_6 (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 - c\theta_1 c\theta_2 s\theta_4 + s\theta_6 (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 + c\theta_1 c\theta_2 c\theta_4) + c\theta_5) \\
& \cdot (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))) ((c\theta_7 - 1)) (a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3))) \\
& - (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) s\theta_4 (a_2 + a_3) + a_1 ((c\theta_4 - 1) + (a_2 + a_3) \\
& \cdot (s\theta_7 (c\theta_6 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 - c\theta_1 c\theta_2 s\theta_4)) + s\theta_6) \\
& \cdot (s\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2 + c\theta_1 c\theta_2 c\theta_4 + c\theta_5) (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))) \\
& + c\theta_7 (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4 - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3))) \\
& - (c\theta_5 (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) - s\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3)) \\
& \cdot ((a_2 + a_3) (c\theta_7 - 1) - s\theta_7 (a_1 + a_4 + a_5)) + s\theta_5 (a_2 + a_3) (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3) \\
& - (c\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) - c\theta_1 c\theta_2 s\theta_4) - (c\theta_6 - 1) (a_1 + a_2 + a_3) + c\theta_5 (c\theta_3 s\theta_1 + c\theta_1 s\theta_2 s\theta_3) (a_1 + a_2 + a_3) \\
& + a_1 c\theta_1 s\theta_2 - a_1 s\theta_1 s\theta_3 + c\theta_1 c\theta_2 \\
& \cdot (a_1 s\theta_4 - (a_2 + a_3)(c\theta_4 - 1)) - (a_2 + a_3) (s\theta_4 (s\theta_1 s\theta_3 - c\theta_1 c\theta_3 s\theta_2) + c\theta_1 c\theta_2 c\theta_4) \\
& \cdot (c\theta_5 - 1) + a_1 c\theta_1 s\theta_2 (c\theta_3 - 1), \\
q_2 = {} & s\theta_2 (a_1 s\theta_4 - (a_2 + a_3)) (c\theta_4 - 1) + (a_2 + a_3) (c\theta_7 (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 s\theta_3 s\theta_5) \\
& + s\theta_7 (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4 - c\theta_2 c\theta_5 s\theta_3)))) \\
& - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4) - (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4 + c\theta_2 s\theta_3 s\theta_5)) ((a_2 + a_3) (c\theta_7 - 1)) \\
& - s\theta_7 (a_1 + a_4 + a_5) - (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3)) \\
& - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4) (c\theta_7 - 1) (a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3) - a_1 (c\theta_2 - 1)) \\
& - (s\theta_7 (c\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + c\theta_2 s\theta_3 s\theta_5)) - c\theta_7 (s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4)) - c\theta_2 c\theta_5 s\theta_3) \\
& - c\theta_6 (s\theta_2 s\theta_4 - c\theta_2 c\theta_3 c\theta_4) (a_1 + a_4 + a_5 + a_6) - s\theta_6 (s\theta_5 (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) - c\theta_2 c\theta_5 s\theta_3) \\
& \cdot (a_1 + a_2 + a_3) - (a_2 + a_3) (c\theta_5 - 1) (c\theta_4 s\theta_2 + c\theta_2 c\theta_3 s\theta_4) + (c\theta_6 - 1) (s\theta_2 s\theta_4 + c\theta_2 c\theta_3 c\theta_4) (a_1 + a_2 + a_3) - a_1 c\theta_2 (c\theta_3 - 1) \\
& - c\theta_2 c\theta_3 (s\theta_4 (a_2 + a_3) + a_1 (c\theta_4 - 1)) - c\theta_2 s\theta_3 s\theta_5 (a_2 + a_3), \\
q_3 = {} & (a_2 + a_3) (s\theta_7 (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) \\
& + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4) + c\theta_7 (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1)) - s\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)))) \\
& - (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1 - (s\theta_5 (c\theta_1 c\theta_3 - -s\theta_1 s\theta_2 s\theta_3) (a_2 + a_3)))) \\
& \cdot (c\theta_7 - 1) - s\theta_7 (a_1 + a_4 + a_5) - ((c\theta_7 - 1) (a_1 + a_4 + a_5) + s\theta_7 (a_2 + a_3)) \\
& \cdot (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) \\
& + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4) - (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2))) \\
& \cdot (s\theta_4 (a_2 + a_3) + a_1 (c\theta_4 - 1)) - (s\theta_7 (c\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) - s\theta_5) (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) \\
& - c\theta_7 (s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) -) c\theta_2 c\theta_4 s\theta_1 + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) + c\theta_6 (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4))) \\
& \cdot (a_1 + a_4 + a_5 + a_6) + s\theta_5 (a_2 + a_3) (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3) - (c\theta_6 - 1) \\
& \cdot (c\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) + c\theta_2 s\theta_1 s\theta_4) (a_1 + a_2 + a_3) - a_1 c\theta_1 s\theta_3 \\
& - s\theta_6 (s\theta_5 (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) + c\theta_5 (c\theta_1 c\theta_3 - s\theta_1 s\theta_2 s\theta_3)) (a_1 + a_2 + a_3) \\
& - a_1 s\theta_1 s\theta_2 - c\theta_2 s\theta_1 (a_1 s\theta_4 - (a_2 + a_3)) (c\theta_4 - 1) - (a_2 + a_3) (c\theta_5 - 1) \\
& \cdot (s\theta_4 (c\theta_1 s\theta_3 + c\theta_3 s\theta_1 s\theta_2) - c\theta_2 c\theta_4 s\theta_1) - a_1 s\theta_1 s\theta_2 (c\theta_3 - 1).
\end{aligned} \tag{B.2}
$$

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] J. Y. Han, T. Yang, and J. J. Yu, *Advanced Mechanisms*, China Machine Press, Beijing, China, 2nd edition, 2015.

[2] J. Dai, "Historical relation between mechanisms and screw theory and the development of finite displacement screws," *Journal of Mechanical Engineering*, vol. 51, no. 13, pp. 13–26, 2015.

[3] J. S. Dai, *Geometrical Foundations and Screw Algebra for Mechanisms and Robotics*, Higher Education Press, Tianjin, China, 2014.

[4] B. L. Jian, C. S. Tsai, Y. C. Kuo, and Y. S. Guo, "An image vision and automatic calibration system for universal robots," *Journal of Low Frequency Noise, Vibration and Active Control*, Article ID 146134841987492, 2019.

[5] G. Li, F. Zhang, Y. Fu, and S. Wang, "Kinematic calibration of serial robot using dual quaternions," *Industrial Robot: The International Journal of Robotics Research and Application*, vol. 46, no. 2, pp. 247–258, 2019.

[6] D. Qiaoling, L. Yan, and L. Sinan, "Design of a micro pole-climbing robot," *International Journal of Advanced Robotic Systems*, vol. 16, no. 3, Article ID 172988141985281, 2019.

[7] H. Wen, "Analysis of D-H modeling method for series robots," *Design and Research*, vol. 269, no. 4, pp. 117-118, 2019.

[8] K. Yang, N. B. Lu, H. S. Sui et al., "Dynamic simulation analysis on wave glider based on D-H approach," *Journal of Harbin Engineering University*, vol. 41, no. 1, pp. 145–152, 2020.

[9] R. H. Zhao and L. Li, "Modeling and MATLAB simulation of 6-DOF manipulator," *Journal of Baoji University of Arts and Sciences (Natural Science)*, vol. 39, no. 3, pp. 69–74, 2019.

[10] M. Kelemen, L. Virgala, T. Liptak et al., "A novel approach for a inverse kinematics solution of a redundant manipulator," *Applied Sciences*, vol. 8, no. 11, 2018.

[11] P. Bozek, M. A. Al Akkad, P. Blist'an et al., "Navigation control and stability investigation of a mobile robot based on a hexacopter equipped with an integrated manipulator," *International Journal of Advanced Robotic Systems*, vol. 14, no. 6, Article ID 1729881417738103, 2017.

[12] W. Wei, Y. J. Li, Z. P. Liao et al., "Kinematics modeling of snake robot based on screw theory," *Journal of South China University of Technology (Natural Science Edition)*, vol. 47, no. 2, pp. 7–14, 2019.

[13] W. W. Li, G. B. Zhou, Z. L. Chen et al., "Forward kinematics and maneuverability analysis of humanoid arm based on screw theory," *Automation & Information Engineering*, vol. 39, no. 5, pp. 6–9, 2018.

[14] Q. S. Liu, J. Yuan, and J. H. Qian, "Analysis on the singular configuration of 4-DOF holding manipulator based on the Screw Theory," vol. 45, no. 1, , pp. 31–35, Journal of Hebei University of Technology, 2016.

[15] H. J. Zhu, "Kinematic analysis of 5-DOF welding robot based on screw theory," *Machinery Manufacturing and Automation*, vol. 48, no. 1, pp. 156–158, 2019.

[16] B. Yang, G. Zhang, Y. Ran, and H. Yu, "Kinematic modeling and machining precision analysis of multi-axis CNC machine tools based on screw theory," *Mechanism and Machine Theory*, vol. 140, pp. 538–552, 2019.

[17] Y. Wang, X. Liang, K. Gong, and Y. Liao, "Kinematical research of free-floating space-robot system at position level based on screw theory," *International Journal of Aerospace Engineering*, vol. 2019, Article ID 6857106, 13 pages, 2019.

[18] G. J. Niu, C. C. Qu, B. Fan, and Y. L. Fu, "Master-slave control of the celiac minimally invasive surgical robot," *ROBOT*, vol. 41, no. 4, pp. 551–560, 2019.

[19] S. Qu, R. Li, S. Bai, and S. Liang, "Unified model for the output accuracy of open-chain manipulators that considers joint clearance and structural parameters," *Journal of Mechanical Science and Technology*, vol. 32, no. 10, pp. 4925–4931, 2018.

[20] W. H. Chen and J. Zhou, "Kinematic analysis and simulation for modular manipulators based on screw theory," *Journal of Beijing University of Aeronautics and Astronautics*, vol. 37, no. 7, pp. 814–818, 2005.

[21] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, Pearson Education International, Beijing, China, 3rd edition, 2005.

[22] R. Li and W. Bin, "On the application of computer simulation in theoretical physics teaching," *The Science Education Article Collects (Early Issue)*, vol. 10, p. 178, 2009.

[23] J. Gallardo-Alvarado, *Kinematic Analysis of Parallel Manipulators by Algebraic Screw Theory*, Springer International Publishing, Berlin, Germany, 2016.

[24] M. Chaslaes, "Note on the general properties of the system of two similar body between them and in any manner places in space; and on the finished moving or infinitely petis of free solid body," *Bull Math, Ferussac*, vol. 14, pp. 321–326, 1930.