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Research Article

On the Stabilization and Observer Design of Polytopic Perturbed Linear Fractional-Order Systems

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In this paper, the problem of stabilization and observer design of parameter-dependent perturbed fractional-order systems is investigated. Sufficient conditions on the practical Mittag–Leffler and Mittag–Leffler stability are given based on the Lyapunov technique. Firstly, the problem of stabilization using the state feedback is developed. Secondly, under some sufficient hypotheses, an observer design which provides an estimation of the state is constructed. Finally, numerical examples are provided to validate the contributed results.

1. Introduction

Stability and control problems of nonlinear systems have drawn a great deal of attention from multiple fields of science and engineering (see [1, 2] and the references therein). On the other hand, fractional-order nonlinear systems (FONS) have become one of the most important subjects to be investigated on the control theory field. This includes fault estimation [1], observer design [3], stabilization [3, 4], stability [5, 6], finite-time stability (FTS) [7], neural networks [8], and stochastic systems [9].

Lyapunov stability of linear time-varying (LTV) systems and applications to control theory have received significant attention [9, 10]. Parameter-dependent systems [11, 12] are acknowledged as one of important representations of these classes of systems. The use of Lyapunov functions is surely the main tool for solving the stability problem [13, 14]. In order to get less conservative results, parameter-dependent Lyapunov functions have, in the past few years, been employed, and multiple techniques demand that these functions have been suggested for the stability and stabilization [15, 16]. Furthermore, observer design has been a subject of interest, and a variety of methods has been developed for constructing nonlinear observers [17, 18]. On the other hand, dealing with fractional-order systems, few works have been done to investigate parameter-dependent perturbed fractional-order systems (see [19]). Indeed, Abdellatif et al. in [19] have investigated the practical Mittag–Leffler stability (MLS) of nonlinear fractional-order systems depending on a parameter by using the Lyapunov techniques.

The study of linear systems with polytopic uncertainty has received a great deal of attention, and some significant results have been obtained (see [20–24]). In fact, authors in [20] have investigated stability of certain polytopic systems. With regard to fractional-order systems, authors in [21] have presented the problem of stability and stabilization for linear parameter-varying (LPV) polytopic systems with time-varying time delays. With regard to fractional-order systems, authors in [22] have investigated a new polytopic type uncertain state-space model for fractional-order linear systems.

In this work, the problem of stabilization and observer design of parameter-dependent perturbed fractional-order systems has been investigated. Up to now, the problem of stabilization and observer design of parameter-dependent perturbed integer-order systems has been tackled by several researchers. However, few works have been done to investigate such context on fractional-order systems. In this context, this paper presents a complete methodology to solve this problem. On the other hand, to the best of our knowledge, among all the existing works dealing with the parameter-dependent systems, no paper has treated the special class which is described in the present work. That is to say, the polytopic system means that the parameters are continuous functions with respect to time and their sum equal to 1. On comparison, our polytopic system presents a perturbation term which is not the case in the work of [22]. Thus, we think that the contribution of our work is consistent.

In fact, the suggested polytopic fractional nonlinear system in this work is written as follows:

$$\begin{aligned} {}_{C}D^{\alpha}_{t_0,t}w(t) &= A(\omega(t))w(t) + B(\omega(t))v(t) + f(t,\omega(t),w) \\ z &= Cw(t), \end{aligned} \tag{1}$$

where ${}_{C}D^{\alpha}_{t_0,t}$ is the Caputo fractional derivative operator of order α, $w(t) \in \mathbb{R}^n$ is the state of the system, $v(t) \in \mathbb{R}^p$ is the control, $z(t) \in \mathbb{R}^q$ is the output, and the function $f: \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is the perturbation term. Matrices $A(\omega(t))$ and $B(\omega(t))$ belong to the convex envelope $\wp$ defined by

$$\wp = \left\{ \sum_{i=1}^{m} \omega_i(t)F_i, \omega_i(t) \in \mathbb{R}_+ \text{ and } \sum_{i=1}^{m} \omega_i(t) = 1 \right\}, \tag{2}$$

with F_i are constant matrices and $C \in \mathbb{R}^{q\times n}$ is the output matrix.

The paper is organized as follows. Basic results related to the fractional calculus are presented in Section 2. After that, the main results, dealing with the problem of stabilization and observer design of parameter-dependent perturbed fractional-order systems, are shown in Section 3. Illustrative examples with simulation results are described in Section 4 to demonstrate the efficiency of the proposed scheme. Finally, conclusion is presented in Section 5.

2. Problem Statement and Preliminaries

In what follows, some general definitions and basic results regarding the fractional calculus have been shown [25].

Definition 1. Given an interval $[x, y]$ of $\mathbb{R}$, the RLFI of order $\alpha > 0$ of a function $w \in L^1([x, y])$ is defined as

$$I^{\alpha}_{x}w(u) = \frac{1}{\Gamma(\alpha)} \int_{x}^{u} (u-m)^{\alpha-1} w(\tau)\mathrm{d}m, \quad u \in [x, y], \tag{3}$$

where $\Gamma(\alpha) = \int_0^{+\infty} e^{-m} m^{\alpha-1}\mathrm{d}m$.

Definition 2. Given an interval $[d_0, y]$ of $\mathbb{R}$, the Caputo fractional derivative of order α of $p(t)$ is defined by

$${}_{C}D^{\alpha}_{d_0,u}w(u) = I^{k-\alpha}_{d_0} w^{(k)}(u), \quad u \in [d_0, y], \tag{4}$$

where $0 < k-1 < \alpha \le k$.

When $0 < \alpha < 1$, the Caputo fractional derivative of order α of an absolutely continuous function w on $[d_0, y]$ reduces to

$${}_{C}D^{\alpha}_{d_0,u}w(u) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{u} (t-m)^{-\alpha} w'(m)\mathrm{d}m, \quad t \in [d_0, y]. \tag{5}$$

By the next definition, a frequently used function in the resolution of fractional-order systems is presented. This function can be regarded as a generalization of the exponential function.

Lemma 1 (see [26]). *Let $M \in \mathbb{R}^{p\times p}$ be a symmetric, positive definite matrix and $\alpha \in]0,1[$. Then, the following relationship holds:*

$$\frac{1}{2}{}_{C}D^{\alpha}_{d_0,u}\left(w^T(u)Mw(u)\right) \le w^T(u)M\,{}_{C}D^{\alpha}_{d_0,u}w(u). \tag{6}$$

Definition 3. The Mittag–Leffler function with two parameters is defined as follows:

$$E_{m,v}(p) = \sum_{s=0}^{+\infty} \frac{p^s}{\Gamma(km+v)}, \tag{7}$$

where $m > 0$, $v > 0$, and $p \in \mathbb{C}$. When $v = 1$, one has $E_m(p) = E_{m,1}(p)$; furthermore, $E_{1,1}(p) = e^p$.

Considering the nonhomogeneous linear fractional differential equation with the Caputo fractional derivative

$$\begin{aligned} {}_{C}D^{\alpha}_{t_0,t}w(t) &= kw, \quad t \ge d_0, \\ w(d_0) &= w_0, \end{aligned} \tag{8}$$

its solution has the form [27]:

$$w(t, d_0, w_0) = w_0 E_{\alpha}\left(k(t-d_0)^{\alpha}\right). \tag{9}$$

3. Main Results

3.1. Stabilization. In this section, the problem of stabilization of system (1) is treated. We consider system (1) with

$$A(\omega(t)) = 2\omega_1(t)\omega_2(t)A_{12} + \omega_1^2(t)A_1 + \omega_2^2(t)A_2, \tag{10}$$

$$B(\omega(t)) = 2\omega_1(t)\omega_2(t)B_{12} + \omega_1^2(t)B_1 + \omega_2^2(t)B_2. \tag{11}$$

Consider the following conditions:

(H1) $l(t,\omega(t),w_1) - l(t,\omega(t),w_2) \le vw_1 - w_2$ which implies that $l(t,\omega(t),w) \le vw + r(t)$ where $r(t) = l(t,\omega(t),0)$.

(H2) There exist constant matrices $K_{12} \in \mathbb{R}^{p\times n}$, $K_1 \in \mathbb{R}^{p\times n}$, and $K_2 \in \mathbb{R}^{p\times n}$ and a symmetric positive definite matrix P such that

$$\begin{aligned} M(B_1K + A_1) + (B_1K + A_1)^T M &< -d_1 M, \\ M(B_{12}K + A_{12}) + (B_{12}K + A_{12})^T M &< -d_2 M, \\ M(B_2K + A_2) + (B_2K + A_2)^T M &< -d_3 M, \end{aligned} \tag{12}$$

where d_1, d_2, and d_3 are strict positive reals and $K = K_{12} + K_1 + K_2$.

Theorem 1. *Suppose that (H1) and (H2) hold with*

$$\left(-\frac{d}{\lambda_{\min}(M)} + 2\lambda_{\max}(M)\nu + \varepsilon\right) < 0, \quad d = \inf(d_1, d_2, d_3) \text{ and } \varepsilon > 0, \tag{13}$$

where

$$g(t) = \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}\left(\frac{-d/\lambda_{\min}(M) + 2\lambda_{\max}(M)\nu + \varepsilon}{\lambda_{\max}(M)}(t-s)^{\alpha}\right) r^2(s)\mathrm{d}s, \tag{14}$$

which is the bounded function.

Then, system (1) in closed loop with $\nu = Kw$ is practical Mittag–Leffler stable.

Proof. Let $V(t,w) = w^T M w$ be a Lyapunov function candidate. Using Lemma 1, the time fractional Caputo derivative of V along the trajectories of system (1) is given by

$$\begin{aligned}{}^{C}D^{\alpha}_{t_0,t}V(t,w) \le{}& w^T\left[\omega_1^2(t)\left(M(A_1+B_1K) + (A_1+B_1K)^T M\right) + 2\omega_1(t)\omega_2\left(M(A_{12}+B_{12}K) + (A_{12}+B_{12}K)^T M\right)\right.\\ &\left.+\,\omega_2^2(t)\left(M(A_2+B_2K) + (A_2+B_2K)^T M\right)\right]w + 2w^T M f(t,\omega(t),w),\end{aligned} \tag{15}$$

and we have

$$\begin{aligned}{}^{C}D^{\alpha}_{t_0,t}V(t,w) &\le \left(-\frac{d}{\lambda_{\min}(M)} + 2\lambda_{\max}(M)\nu + \varepsilon\right)\|w\|^2 + \frac{(\lambda_{\max}(M)r(t))^2}{\varepsilon}\\ &\le \left(\frac{-d/\lambda_{\min}(M) + 2\lambda_{\max}(M)\nu + \varepsilon}{\lambda_{\max}(M)}\right)V(t,w) + \frac{(\lambda_{\max}(M)r(t))^2}{\varepsilon}\\ &\le -\rho V(t,w) + h(t),\end{aligned} \tag{16}$$

where

$$\begin{aligned}\rho &= \left(\frac{d/\lambda_{\min}(M) - 2\lambda_{\max}(M)\nu - \varepsilon}{\lambda_{\max}(M)}\right),\\ h(t) &= \frac{(\lambda_{\max}(M)r(t))^2}{\varepsilon}.\end{aligned} \tag{17}$$

Following the same proof of Theorem 1 in [3], we get the practical Mittag–Leffler stability of the closed-loop system (1). □

Remark 1. The above theorem gives a fundamental result on the practical Mittag–Leffler stability of the closed-loop system (1). Note that the work introduced in [3] was among the first works which describes the concept of practical stability.

3.2. Observer Design. In this section, the problem of observer design of system (1) is treated. To obtain an estimation of the state (we can reconstitute the state), we shall consider the following observer:

$$\begin{aligned}{}_{C}D^{\alpha}_{t_0,t}\widehat{w}(t) &= A(\omega(t))\widehat{w}(t) + B(\omega(t))\nu(t) + f(t,\omega(t),\widehat{w})\\ &\quad - L(\omega(t))Ce,\\ z &= C\widehat{w}(t),\end{aligned} \tag{18}$$

where $\widehat{w}(t)$ is the state estimate of $w(t)$, $e = \widehat{w} - w$ is the error estimation, and $L(\omega(t)) \in \wp$, where

$$L(\omega(t)) = 2\omega_1(t)\omega_2(t)L_{12} + \omega_1^2(t)L_1 + \omega_2^2(t)L_2. \tag{19}$$

The error system is defined as follows:

$${}^{C}D^{\alpha}_{t_0,t}e(t) = (A(\omega(t)) - L(\omega(t))C)e + \Delta f, \tag{20}$$

where $\Delta f = f(t,\omega(t),\widehat{w}) - f(t,\omega(t),w)$.

Consider the following hypothesis:

(H3) There exist constant matrices $L_1 \in \mathbb{R}^{n\times q}$, $L_{12} \in \mathbb{R}^{n\times q}$, and $L_2 \in \mathbb{R}^{n\times q}$ and a symmetric positive definite matrix P_1 such that

$$\begin{aligned} M_1(-L_1C + A_1) + (-L_1C + A_1)^T M_1 &< -r_1' I, \\ M_1(-L_{12}C + A_{12}) + (-L_{12}C + A_{12})^T M_1 &< -r_2' I, \\ M_1(-L_2C + A_1) + (-L_2C + A_1)^T M_1 &< -r_3' I, \end{aligned} \tag{21}$$

where r_1', r_2', and r_3' are strict positive reals.

Theorem 2. *Suppose that (H1) and (H3) hold with*

$$(-r' + 2\lambda_{\max}(M_1)\nu) < 0, \quad r' = \inf(r_1', r_2', r_3'). \tag{22}$$

Then, system (1) is Mittag–Leffler stable.

Proof. Let $W(t,w) = e^T M_1 e$ be a Lyapunov function candidate. Using Lemma 1, the time fractional Caputo derivative of V along the trajectories of system (20) is given by

$$\begin{aligned} {}^{C}D^{\alpha}_{t_0,t} W(t,e) \le e^T \Big[& \omega_1^2(t)\big(M_1(A_1 - L_1C) + (A_1 - L_1C)^T M_1\big) + 2\omega_1(t)\omega_2(t)\big(M_1(A_{12} - L_{12}C) + (A_{12} - L_{12}C)^T M_1\big) \\ & + \omega_2^2(t)\big(M_1(A_1 - L_2C) + (A_1 - L_2C)^T M_1\big)\Big] e + 2e^T M_1 \Delta f. \end{aligned} \tag{23}$$

We have

$${}^{C}D^{\alpha}_{t_0,t} W(t,e) \le -r' e^2 + 2\lambda_{\max}(M_1)\nu e^2 \le -\tilde{r} e^2, \tag{24}$$

where $\tilde{r} = -r' + 2\lambda_{\max}(M_1)\nu$ which ends the proof. □

4. Numerical Examples

4.1. Example 1. Consider the system

$$\begin{aligned} {}_{C}D^{\alpha}_{t_0,t} x(t) &= A(\omega(t))x(t) + B(\omega(t))v(t) \\ &\quad + \frac{0.1}{1 + \omega_1^2(t) + \omega_2^2(t)}(x_1, \cos(x_2)), \\ y &= Cx(t), \end{aligned} \tag{25}$$

where $A(\omega(t)) = \omega_1^2(t)A_1 + 2\omega_1(t)\omega_2(t)A_{12} + \omega_2^2(t)A_2$ and $B(\omega(t)) = \omega_1^2(t)B_1 + 2\omega_1(t)\omega_2(t)B_{12} + \omega_2^2(t)B_2$, $\omega_1(t) = \sin^2(t)$, and $\omega_2(t) = \cos^2(t)$ with $A_1 = \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$, $A_{12} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -2 & 0 \\ -2 & -3 \end{pmatrix}$, $B_1 = \begin{pmatrix} 0.5 \\ 0.2 \end{pmatrix}$, $B_{12} = \begin{pmatrix} 0.2 \\ 0.1 \end{pmatrix}$, $B_2 = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$.

After solving LMIs (12) using the LMI MATLAB Toolbox, we have

$$\begin{aligned} M &= \begin{pmatrix} 0.0499 & -0.0481 \\ -0.0481 & 0.113 \end{pmatrix}, \\ d &= \inf(d_1, d_2, d_3) = \inf(0.0047, 0.0058, 0.0774) = 0.0047. \end{aligned} \tag{26}$$

Also, we have $K_1 = \begin{pmatrix} -0.7106 & 0.1393 \end{pmatrix}$, $K_{12} = \begin{pmatrix} -0.6106 & 2.5393 \end{pmatrix}$, and $K_2 = \begin{pmatrix} 0.4106 & 0.4607 \end{pmatrix}$.

The constant ε is chosen which is equal to 0.1, and the Lipschitz constant $\nu = 0.1$ satisfies the condition of Theorem 1.

The simulation result of the present numerical example is given as follows.

Remark 2. It is clear from Figure 1 that the states of the system are practical Mittag–Leffler stable. The convergence is towards a ball of center the origin, and this validates the result of Theorem 1, that is to say the practical Mittag–Leffler stability of the system. Based on these schemes, we can say that the result obtained by Theorem 1 is satisfactory.

4.2. Example 2. Consider the system

$$\begin{aligned} {}_{C}D^{\alpha}_{t_0,t} x(t) &= A(\omega(t))x(t) + B(\omega(t))v(t) \\ &\quad + \frac{0.1}{1 + \omega_1^2(t) + \omega_2^2(t)}(x_1, \cos(x_2)), \\ y &= Cx(t), \end{aligned} \tag{27}$$

where $A(\omega(t)) = \omega_1^2(t)A_1 + 2\omega_1(t)\omega_2(t)A_{12} + \omega_2^2(t)A_2$ and $B(\omega(t)) = \omega_1^2(t)B_1 + 2\omega_1(t)\omega_2(t)B_{12} + \omega_2^2(t)B_2$, $\omega_1(t) = \sin^2(t)$, and $\omega_2(t) = \cos^2(t)$,with $A_1 = \begin{pmatrix} -1 & 0 \\ -1 & -2 \end{pmatrix}$, $A_{12} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$, $A_2 = \begin{pmatrix} -2 & 0 \\ -2 & -3 \end{pmatrix}$, $B_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $B_{12} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $B_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$.

After solving LMIs (21) using the LMI MATLAB Toolbox, we have

$$\begin{aligned} M_1 &= \begin{pmatrix} 61.0577 & -14.2617 \\ -14.2617 & 25.4036 \end{pmatrix}, \\ L_1 &= \begin{pmatrix} 0.3562 \\ 0.8842 \end{pmatrix}, \\ L_{12} &= \begin{pmatrix} 1.5269 \\ 6.787 \end{pmatrix}, \\ L_2 &= \begin{pmatrix} -0.2535 \\ 0.6647 \end{pmatrix}, \\ r' &= \inf(r_1', r_2', r_3') = \inf(50.8072, 53.9269, 76.2107) = 50.8072. \end{aligned} \tag{28}$$

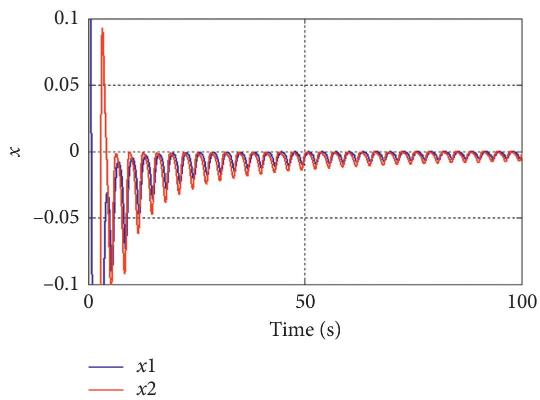

Figure 1: The evolution of the system states x_1 andx_2.

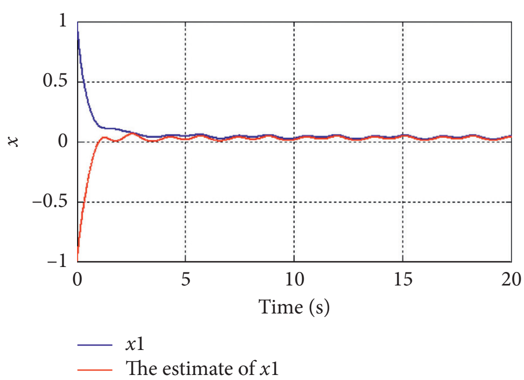

Figure 2: The evolution of the system state x_1 and its estimate $\hat{x}_1$.

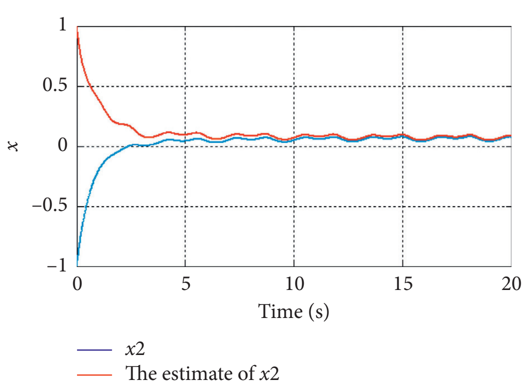

Figure 3: The evolution of the system state x_2 and its estimate $\hat{x}_2$.

The Lipschitz constant $\nu = 0.3$ satisfies the condition of Theorem 2.

The simulation result of the present numerical example is given as follows.

Remark 3. It is clear from Figures 2 and 3 that the estimated states of the systems follow the actual real state which validates the observer design.

Remark 4. Figure 4 shows that the errors $e_1 = \hat{x}_1 - x_1$ and $e_2 = \hat{x}_2 - x_2$ converge towards the origin, and this validates

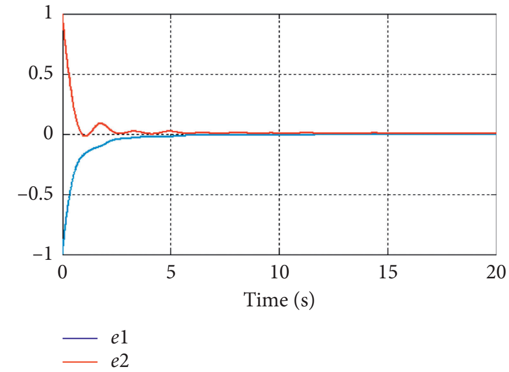

Figure 4: The evolution of the system errors $e_1 = \hat{x}_1 - x_1$ and $e_2 = \hat{x}_2 - x_2$.

the result of Theorem 2, that is to say the Mittag–Leffler stability of the error system. Based on these schemes, we can say that the result obtained by Theorem 2 is satisfactory.

5. Conclusion

In this paper, a complete methodology to solve the problem of stabilization and observer design of parameter-dependent perturbed fractional-order systems has been presented. To validate the theoretical results, a numerical example is studied in the simulation section. The simulation results show that the synchronization of the suggested chaotic fractional systems is satisfactory.

Data Availability

The present work is purely fundamental research based on theoretical analysis. It develops illustrative numerical models by means of simulations.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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