Research Article

Circuit Implementation Synchronization between Two Modified Fractional-Order Lorenz Chaotic Systems via a Linear Resistor and Fractional-Order Capacitor in Parallel Coupling

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In this study, a modified fractional-order Lorenz chaotic system is proposed, and the chaotic attractors are obtained. Meanwhile, we construct one electronic circuit to realize the modified fractional-order Lorenz chaotic system. Most importantly, using a linear resistor and a fractional-order capacitor in parallel coupling, we suggested one chaos synchronization scheme for this modified fractional-order Lorenz chaotic system. The electronic circuit of chaos synchronization for modified fractional-order Lorenz chaotic has been given. The simulation results verify that synchronization scheme is viable.

1. Introduction

In the last twenty years, many fractional-order systems [1–9] have been used to discuss the dynamics, wave stability, initials, and boundary effect. Many real-world physical systems can be more accurately described by fractional-order differential equations (FODE) [3–6, 10, 11], e.g., diffusion-wave, super diffusion, heat conduction, dielectric polarization, viscoelasticity, and electromagnetic waves. The complex behavior such as chaos has been observed in many physical fractional-order systems, e.g., the fractional-order brushless DC motor chaotic system [6], the fractional-order Lorenz chaotic system [7], the fractional-order Chua's circuit [8], the fractional-order Duffing chaotic system [9], the fractional-order multistable locally active memristor [12], the fractional-order gyroscopes system [10], and the fractional-order microelectromechanical chaotic system [11]. Meanwhile, synchronization of chaotic systems has attracted extensive attention in recently years, and many synchronization strategies [10, 11, 13–15] have been

widely used in information processing, image encryption, network safety, secure communication, and machine learning.

On the other hand, many chaotic systems and synchronization strategies have been realized by electronic circuits [16–18]. So, synchronization between chaotic systems can be transferred to the synchronization between chaotic electronic circuits. Referring to synchronization between chaotic electronic circuits, the linear resistor coupling between two electronic circuits can realize the linear state variable coupling between chaotic systems, and the linear capacitive coupling or linear inductor coupling between two electronic circuits can realize the first derivative of state variable linear coupling. Up to now, many scholars [19–23] have proposed some synchronization approaches on integer-order chaotic electronic circuits by linear resistor coupling or linear capacitive coupling or a linear inductor coupling. However, to the best of our knowledge, there are little results on synchronization fractional-order chaotic electronic circuits coupled by a linear resistor or linear capacitive or linear inductor. Motivated by the

abovementioned discussion, we have studied the modified fractional-order Lorenz chaotic system based on the integer-order Lorenz chaotic system. Furthermore, using a linear resistor and a fractional-order capacitor in parallel coupling, the chaos synchronization of modified fractional-order Lorenz chaotic electronic circuits has been achieved, while the linear resistor, capacitor, or inductor coupling is usually used in the existing works.

The rest of this study is as follows. In Section 2, a modified fractional-order Lorenz chaotic system is proposed, and its chaotic electronic circuits is designed. In Section 3, chaos synchronization between two modified fractional-order Lorenz chaotic systems is realized via a linear resistor and a fractional-order capacitor in parallel coupling, and the circuit experiment is obtained. Conclusions are drawn in Section 4.

2. A Modified Fractional-Order Lorenz Chaotic System and Its Circuit Realization

A modified Lorenz chaotic system [24] is described as follows:

$$\begin{cases} \frac{dy_1}{dt} = 40(y_2 - y_1), \\ \frac{dy_2}{dt} = 10y_1 + 25y_2 - y_1y_3, \\ \frac{dy_3}{dt} = y_1y_2 - 3y_3. \end{cases} \quad (1)$$

Based on this integer-order Lorenz chaotic system (1), the modified fractional-order Lorenz chaotic system is obtained as follows:

$$\begin{cases} \frac{d^q y_1}{dt} = 40(y_2 - y_1), \\ \frac{d^q y_2}{dt} = 10y_1 + 25y_2 - y_1y_3, \\ \frac{d^q y_3}{dt} = y_1y_2 - 3y_3, \end{cases} \quad (2)$$

where $q = 0.95$ is the fractional order. Herein, the Caputo fractional-order derivatives are used. It is defined as

$$\frac{d^q f(t)}{dt} = \frac{1}{\Gamma(1-q)} \int_0^t \frac{f'(\tau)}{(t-\tau)^q} d\tau, \quad 0 < q < 1. \quad (3)$$

The Lyapunov exponents for this modified fractional-order Lorenz chaotic system (2) can be calculated by the numerical method [25, 26], and the Lyapunov exponents are (2.6245, -0.0461, -5.9568). The positive Lyapunov exponent indicates that chaotic attractors are emerged in system (2). Let $q = 0.95$, and the chaotic attractors of the fractional-order system (2) are shown in Figure 1.

Up to now, circuit implementation of chaotic systems has attracted more and more attentions. Many chaotic circuits have been reported. Now, we discuss the circuit implementation for the fractional-order chaotic system (2). Limited by the output of the operational amplifier, the amplitude of every variable in system (1) must be decreased, and we can choose $y_1 = 10x_1$, $y_2 = 10x_2$, and $y_3 = 10x_3$. So, system (2) can be changed as the following system:

$$\begin{cases} \frac{d^q x_1}{dt} = 40(x_2 - x_1), \\ \frac{d^q x_2}{dt} = 10x_1 + 25x_2 - 10x_1x_3, \\ \frac{d^q x_3}{dt} = 10x_1x_2 - 3x_3. \end{cases} \quad (4)$$

Recently, the guidelines to design circuits for fractional-order chaotic systems have been proposed in many studies [26–29]. To ordinary differential equation (ODE), we can use the linear capacitor C to realize the first-order differential unit (dy_i/dt), $i = 1, 2, 3$. Therefore, compared to the design of the first-order differential circuit unit in ODE, in order to realize the fractional-order differential unit ($d^q x_i/dt$), $i = 1, 2, 3$, we use the fractional-order capacitor F replacing the linear capacitor C in the first-order differential circuit unit in this study. According to [29], the fractional-order capacitor F is shown as Figure 2, and it can realize the operator (d^q/dt). In Figure 2, $C_1 = 3.616 \mu\text{F}$, $C_2 = 4.602 \mu\text{F}$, $C_3 = 1.267 \mu\text{F}$, $R_a = 15.1 \text{ k}\Omega$, $R_b = 1.51 \text{ M}\Omega$, $R_c = 692.9 \text{ M}\Omega$.

Based on the circuitry design method for fractional-order nonlinear systems in [25–29], the circuit diagram to realize the fractional-order nonlinear chaotic system (4) is presented as Figure 3.

In Figure 3, the operational amplifiers are the type of LF353N, the multipliers are the type of AD633, and the power is supplied by $\pm 15 \text{ V}$, and $R_1 = 10 \text{ k}\Omega$, $R_2 = 2.5 \text{ k}\Omega$, $R_3 = 4 \text{ k}\Omega$, and $R_4 = 33.3 \text{ k}\Omega$. Using electronics workbench (EWB), we can obtain the chaotic attractors in the fractional-order chaotic circuit, and the chaotic attractors are shown in Figure 4.

According to the results in Figures 1 and 4, the fractional-order Lorenz chaotic system (4) has been circuit implementation as Figure 3.

3. Synchronization of the Fractional-order Chaotic System (4) via a Linear Resistor and Fractional-Order Capacitor in Parallel Coupling

In this section, synchronization of the chaotic system (4) is discussed. Let system (4) as the driving system. The response system is as follows:

FIGURE 1: The chaotic attractors of the fractional-order system (2) for $q_1 = q_2 = q_3 = 0.95$. (a) The y_1y_2 phase diagram. (b) The y_1y_3 phase diagram.

FIGURE 2: Circuit unit of the fractional-order capacitor F for $q = 0.95$.

FIGURE 3: The circuit diagram to realize the fractional-order system (4) for $q = 0.95$.

FIGURE 4: The circuit experiment phase portraits of the fractional-order chaotic system (4) for $q = 0.95$. (a) The x_1x_2 phase diagram. (b) The x_1x_3 phase diagram.

$$\begin{cases} \frac{d^q y_1}{dt} = 40(y_2 - y_1), \\ \frac{d^q y_2}{dt} = 10y_1 + 25y_2 - 10y_1y_3, \\ \frac{d^q y_3}{dt} = 10y_1y_2 - 3y_3. \end{cases} \quad (5)$$

According to Section 2, circuit implementation of the fractional-order chaotic system (5) is the same as Figure 3. Now, we discuss how to realize the chaos synchronization between the drive system (4) and response system (5) via

circuit implementation. In this study, the state variable x_2 in the driving system (4) and the state variable y_2 in the response system (5) are coupled via linear resistor R_k and fractional-order capacitor F in parallel, and its circuit diagram is shown in Figure 5. The green box in Figure 5 is the coupling part, and resistor $R_5 = 100 \text{ k}\Omega$.

According to Figure 5, one can obtain the following:

$$u = K_R(x_2 - y_2) + \left(\frac{d^q x_2}{dt} - \frac{d^q y_2}{dt} \right). \quad (6)$$

Here, variable $K_R = 100 \text{ k}\Omega / R_k$. Now, according to Figure 5, the coupled driving system and coupled response system are shown as follows:

$$\begin{cases} \frac{d^q x_1}{dt} = 40(x_2 - x_1), \\ \frac{d^q x_2}{dt} = 10x_1 + 25x_2 - 10x_1x_3 - K_R(x_2 - y_2) - \left(\frac{d^q x_2}{dt} - \frac{d^q y_2}{dt} \right), \\ \frac{d^q x_3}{dt} = 10x_1x_2 - 3x_3, \end{cases} \quad (7)$$

$$\begin{cases} \frac{d^q y_1}{dt} = 40(y_2 - y_1), \\ \frac{d^q y_2}{dt} = 10y_1 + 25y_2 - 10y_1y_3 + K_R(x_2 - y_2) + \left(\frac{d^q x_2}{dt} - \frac{d^q y_2}{dt} \right), \\ \frac{d^q y_3}{dt} = 10y_1y_2 - 3y_3. \end{cases} \quad (8)$$

FIGURE 5: The circuit realization of chaos synchronization between the drive system (4) and response system (5).

Next, we discuss how to realize chaos synchronization between the drive system (7) and response system (8) via linear resistor R_k or variable $K_R = 100 \text{ k}\Omega/R_k$. Let synchronization errors $e_i = x_i - y_i$ ($i = 1, 2, 3$), and we can obtain the error system as follows:

$$\begin{cases} \frac{d^q e_1}{dt} = 40(e_2 - e_1), \\ \frac{d^q e_2}{dt} = \frac{[10e_1 + (25 - 2K_R)e_2 + 10e_1e_3 - 10e_1x_3 - 10e_3x_1]}{3}, \\ \frac{d^q e_3}{dt} = -3e_3 - 10e_1e_2 + 10e_1x_2 + 10e_2x_1. \end{cases} \quad (9)$$

According to the error system (9), $e_i = 0$ ($i = 1, 2, 3$) is the equilibrium point of the error system (9). If the equilibrium point $e_i = 0$ ($i = 1, 2, 3$) is asymptotic stability, then chaotic synchronization between the driving system (7) and response system (8) can be achieved. This result indicates that the synchronization of the fractional-order chaotic system (4) can be realized via the linear resistor R_k and fractional-order capacitor F in parallel coupling. On the other hand, if all the Lyapunov exponents (LEs) in system (9) are negative, then chaos synchronization is achieved. In this study, the Lyapunov exponents (LEs) are used to check the chaos synchronization.

Now, MATLAB is used to study the QR decomposition to analyze the synchronization problem of variable parameter K_R . The Jacobi matrix of the error system (9) is as follows:

$$J = \begin{bmatrix} -40 & 40 & 0 \\ \frac{(10 + 10e_3 - 10x_3)}{3} & \frac{(25 - 2K_R)}{3} & \frac{(10e_1 - 10x_1)}{3} \\ -10e_2 + 10x_2 & -10e_1 + 10x_1 & 0 \end{bmatrix}. \quad (10)$$

FIGURE 6: The maximum CLEs of error system (9) with variable $K_R = 100 \text{ k}\Omega/R_k$.

The maximum Lyapunov exponents (CLEs) [24, 25] related to R_k is shown in Figure 6. It can be seen that the maximum CLEs are negative if $K_R = 0.05 \sim 20$. So, choosing linear resistor $R_k = (20 \sim 0.05) \times 100 \text{ k}\Omega$, the chaos synchronization between the drive system (7) and response system (8) can be obtained.

Choose $K_R = 1$, then $R_k = 100 \text{ k}\Omega$. We can yield that all the CLEs of the error system (9) are $\lambda_1 = -2.4781$, $\lambda_2 = -3.2007$, and $\lambda_3 = -5.7004$. So, the chaos synchronization between the drive system (7) and response system (8) can be achieved. Take the initial driving signals $x = (2, 2, 2)$ and the initial response signals $y = (-2, -5, 5)$, and the synchronization results are shown in Figure 7.

Choose $K_R = 10$ and then $R_k = 10 \text{ k}\Omega$. According to Figure 6, the maximum CLEs of the error system (9) is negative. So, the chaos synchronization between the drive system (7) and response system (8) can be achieved. Taking the initial driving

FIGURE 7: The synchronization process on $K_R = 1$. FIGURE 8: The synchronization process on $K_R = 10$.

signals $x = (2, 2, 2)$ and the initial response signals $y = (-2, -5, 5)$, the synchronization results are shown in Figure 8.

In summary, the chaos synchronization of the fractional-order chaotic system (4) can be realized via the linear resistor R_k and fractional-order capacitor F in parallel coupling, and the

maximum CLEs of the error system (9) with the linear resistor R_k are obtained. Furthermore, we find that chaotic synchronization cannot be reached via state variable x_1 and y_1 coupling or via state variable x_3 and y_3 coupling. In addition, we find that chaotic synchronization can be arrived via only linear resistor coupling or via only fractional-order capacitor coupling.

4. Conclusions

In this study, based on one modified Lorenz chaotic system, a modified fractional-order Lorenz chaotic system is suggested. We find that chaotic attractors are emerged in this modified fractional-order Lorenz chaotic system for $q = 0.95$. Furthermore, we discussed the circuit implementation for this fractional-order chaotic system, and the circuit diagram to realize the fractional-order nonlinear chaotic system (4) is presented. More importantly, a synchronization scheme is suggested to realize the chaos synchronization on this modified fractional-order Lorenz chaotic system for $q = 0.95$ via a linear resistor R_k and a fractional-order capacitor F in parallel coupling, which has not been used in the existing study. The electronic circuits for chaos synchronization of the modified fractional-order Lorenz chaotic system have been given. The simulation results verify that synchronization of the chaotic electronic circuit can be achieved. In the following study, whether inductive coupling can achieve synchronization is worth discussing.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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