Research Article

Antidisturbance Control for Helicopter Stochastic Systems

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In this paper, an antidisturbance controller is presented for helicopter stochastic systems under disturbances. To enhance the antidisturbance abilities, the nonlinear disturbance observer method is applied to reject the time-varying disturbances. Then, the antidisturbance nonlinear controller is designed by combining the backstepping control scheme. And the stochastic theory is used to guarantee that the closed-loop system is asymptotically bounded in mean square while the proposed control method is shown via some traditional nonlinear control techniques, which still show some common issues such as “dimension explosion” or others. The result of this paper can be regarded as a typical case of the nonlinear control method to help and promote the generation of advanced methods.

1. Introduction

For actual systems, the nonlinearities of the model are probably one of the most noticeable characteristics, which are issued from the physical laws, material science, mathematical derivation, and so on. Hence, nonlinear systems are typical research models in various control fields and can be used to describe the dynamics of the system states. In order to handle nonlinear control problems, many classical control methods were proposed based on the nonlinear control theory [1–3], such as the backstepping control method, feedback linearization technique, and nonlinear disturbance observer-based control scheme. Based on these nonlinear control theories, a number of advanced control techniques were proposed for various control systems. In [4], a class of nonlinear systems was studied via the event-triggered robust adaptive fuzzy control method. In [5], the adaptive neural control scheme was adopted for the nonlinear multiple output systems under the time-varying output constraints. Combining the nonlinear disturbance observer control with sliding-mode fuzzy neural network methods, a new control scheme was presented to deal with the nonlinear systems with disturbances in [6]. In [7, 8], the issue of adaptive fuzzy tracking control was discussed for a class of strict-feedback

nonlinear systems. In [9], the periodic event-triggered control method was used to design the controller for nonlinear networked control systems. In [10], the robust adaptive control problem was investigated for state-constrained nonlinear systems under the input saturation and unknown control direction. The observer-based H_∞ control was used for discrete-time T-S fuzzy systems in [11]. In [12], the switched-observer-based adaptive output-feedback control scheme was designed for pure-feedback switched nonlinear systems with unknown gains. The model-based adaptive event-triggered control method was discussed for nonlinear continuous-time systems in [13]. From the above discussion, the nonlinear control methods and theories are effective for disposing many troublesome nonlinear control problems. Therefore, the nonlinear control system theories are useful approaches to solve the issues of flight control for helicopters.

The flight control is a crucial issue of helicopters. An excellent flight controller guarantees the well flight performances of flying helicopters and prevents the crash, breakdown, and so on. In many books involving helicopters [14, 15], linear control methods were used for designing the helicopter flight controller and obtained expected control performances while, in fact, helicopter models are

complicated nonlinear systems, which are constructed according to the relationships of the components of helicopters. In some outstanding results, the helicopter flight controllers are designed by using the linear models, which are evolved from the nonlinear helicopter models under the linearization techniques. In [16], the tracking control issue was discussed for small-scale unmanned helicopter linear systems. In [17], the flight controller was designed for helicopters via the probabilistic robust linear parameter-varying control method using the iterative scenario approach. In [18], the model reference resilient control methods were designed for linear helicopter systems with time-varying disturbances. In [19], tracking control issues were discussed for the linear helicopter systems under time-varying disturbances and input stochastic perturbation. However, the dynamics of the helicopter states are complicated with many nonlinear characteristics. With development of the helicopter flight control techniques, the nonlinear flight controllers are designed for various helicopter systems. In [20], the adaptive trajectory tracking control approach was proposed for model-scaled nonlinear helicopter systems. The trajectory tracking control problems were discussed for small-scale nonlinear unmanned helicopters under model uncertainties in [21]. In [22], the composite block backstepping trajectory tracking control scheme was presented for disturbed nonlinear unmanned helicopter systems. In [23], the fixed-time autonomous shipboard landing control issues were studied for nonlinear helicopter systems under external disturbances. The finite-time control issue was discussed for small-scale nonlinear unmanned helicopter systems with disturbances in [24]. In [25], the sliding-mode control and extended disturbance observer methods were used to solve the tracking control issue of unmanned nonlinear helicopter systems. The dynamic decoupling control optimization methods were adopted for small-scale nonlinear unmanned helicopter systems in [26]. While helicopters fly in full noise surroundings, various random disturbances exist in helicopter control system models. In [27], the antidisturbance control was proposed for the attitude and altitude nonlinear helicopter systems under random disturbances. The random disturbances are considered for flight control, which improves the control performances and precision of flying helicopters.

In this paper, in order to enhance the control precision and robustness of the helicopter models, the random disturbances are considered to construct the flight controller. The helicopter stochastic systems are better to describe the flying dynamics states of the helicopter. While with the random disturbances introducing in the models, many advanced control methods, such as dynamic surface control, fuzzy control, and other intelligent control methods, cannot be used to design the flight controller directly because many control variables will lose

some good characteristics, such as the continuity and derivability, under the random disturbances. Therefore, in this manuscript, the backstepping control method and nonlinear disturbance observer control scheme are used to establish the nonlinear flight controller. Due to the characteristics of underactuation, strong nonlinearity, and high-order of helicopter stochastic system models, the phenomenon of “dimension explosion” is inevitable for nonlinear controller designed, and the controller solved steps are presented in this paper. Our results present an approach to design the strong robust flight controller.

The organization of this paper is standard. The problem statement is stated in Section 2. The position loop control issues are discussed in Section 3. The attitude loop control problems are studied in Section 4. The stability analysis of the main result is given in Section 5. The conclusions are presented in Section 6.

2. Problem Statement

Consider the following helicopter dynamic system:

$$\begin{aligned} dp &= v dt, \\ dv &= \left(-ge_3 + \frac{1}{m} Rf \right) dt + \delta_1 dt + G_1 d\beta_1, \\ dR &= R w^\times dt, \\ Jdw &= (-w^\times Jw + \tau) dt + \delta_2 dt + G_2 d\beta_2, \end{aligned} \quad (1)$$

where p and v denote the position and velocity in inertial frame, respectively, g is the gravitational acceleration, $e_3 = (0 \ 0 \ 1)^T$ is a unit vector, and R is the rotation matrix from body frame to inertial frame and defined by

$$\begin{aligned} R &= \begin{pmatrix} C\theta C\psi & S\theta S\phi C\psi - C\phi S\psi & C\phi S\theta C\psi + S\phi S\psi \\ C\theta S\psi & S\phi S\theta S\psi + C\phi C\psi & C\phi S\theta S\psi - S\phi C\psi \\ -S\theta & S\phi C\theta & C\phi C\theta \end{pmatrix} \\ &= \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} = (R_1 \ R_2 \ R_3), \end{aligned} \quad (2)$$

where $C \cdot$ and $S \cdot$ denote $\cos(\cdot)$ and $\sin(\cdot)$ with respective variables. $\phi(t)$, $\theta(t)$, and $\psi(t)$ are roll, pitch, and yaw angles in body frame, respectively. $w = (p(t) \ q(t) \ r(t))$ is the angular velocity. p , q , and r are the roll, pitch, and yaw angular rates, respectively. J is the inertia matrix and denoted by $J = \text{diag}\{J_{xx} \ J_{yy} \ J_{zz}\}$. f and τ are the sum of the external forces and moments. Consider the characteristics of these forces and moments, which are denoted as follows:

$$f = (0 \ 0 \ T_m)^T, \quad \tau_u = \begin{pmatrix} C_m + T_m L_z - H_z \\ C_m + T_m L_z 00 \\ -T_m L_y T_m L_x H_x \end{pmatrix} \begin{pmatrix} a \\ b \\ T_t \end{pmatrix} + \begin{pmatrix} -T_m L_y \\ -T_m L_x \\ -Q_m \end{pmatrix} = A(T_m)\rho + B(T_m), \quad (3)$$

with $Q_m = C_k T_m^{1.5} + D_k$. C_m , L_z , H_z , L_y , L_x , H_x , and D_k are known constants. β_1 and β_2 are independent one-dimensional standard Wiener processes, $(d\beta_1/dt) = \xi_1$ and $(d\beta_2/dt) = \xi_2$, where ξ_1 and ξ_2 are independent one-dimensional stochastic noises. $G_1 \xi_1$ and $G_2 \xi_2$ are the stochastic accelerated velocity and stochastic angular acceleration generated by the stochastic force and stochastic moment, G_1 and G_2 are known and bounded weight parameters, and δ_1 and δ_2 are disturbances with bounded derivatives, which are $\|\dot{\delta}_1\| \leq \mu_1$ and $\|\dot{\delta}_2\| \leq \mu_2$.

The following definitions and lemmas are crucial important to further analyze the main results of this paper. In order to state those definitions and lemmas, consider the following system:

$$dx = f(t, x)dt + g(t, x)d\beta, \quad (4)$$

where x is the system state, $f \in \mathcal{L}_1(\mathbb{R}_+; \mathbb{R}^n)$ and $g \in \mathcal{L}_2(\mathbb{R}_+; \mathbb{R}^{n \times m})$ are known functions, and β is the one-dimensional standard Wiener process.

For $V(t, x) \in \mathcal{C}^{2,1}(\mathbb{R}; \mathbb{R}^n \times \mathbb{R}_+)$, the infinitesimal generator along (4) is defined by

$$\begin{aligned} \mathcal{L}V(t, x) &= V_t(t, x) + V_x(t, x)f(t, x) \\ &+ \frac{1}{2} \text{Tr} \left[g^T(t, x) V_{xx}(t, x) g(t, x) \right], \end{aligned} \quad (5)$$

where Tr is the trace of a matrix, and

$$\begin{aligned} V_t(t, x) &= \frac{\partial V(t, x)}{\partial t}, \\ V_x(t, x) &= \left(\frac{\partial V(t, x)}{\partial \vartheta_1}, \dots, \frac{\partial V(t, x)}{\partial \vartheta_n} \right), \\ V_{xx}(t, x) &= \left(\frac{\partial^2 V(t, x)}{\partial \vartheta_i \partial \vartheta_j} \right)_{n \times n}. \end{aligned} \quad (6)$$

Definition 1 (see [28]). Let $p > 0$. System (4) is said to be asymptotically bounded in p -th moment if there is a positive constant H such that

$$\lim_{t \rightarrow \infty} \sup E|x(t; x_0)|^p \leq H, \quad (7)$$

for all $(t_0, x_0) \in \mathbb{R}_+ \times \mathbb{R}^n$. When $p = 2$, we say system (4) is asymptotically bounded in mean square.

Lemma 1. For system (4), assume that there exists a function $V \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$, positive constants $k_i, k'_i, p_i, p'_i, c, d_c$, such that

$$\sum_{i=1}^n k_i |x_i|^{p_i} \leq V(x(t), t) \leq \sum_{i=1}^n k'_i |x_i|^{p'_i}, \quad (8)$$

$$\mathcal{L}V(x(t), t) \leq -\lambda V(x(t), t) + d_c. \quad (9)$$

Then, there exists a unique strong solution $x(t) = x(t; x_0, t_0)$ of system (4) for each $x(t_0) = x_0 \in \mathbb{R}^n$, and system (4) is p -th moment exponentially practically stable, where $p = \min\{p_1, \dots, p_n\}$.

Lemma 2. For any vectors $x, y \in \mathbb{R}^n$ and any scalars $\epsilon > 0, p > 1$, there holds $x^T y \leq (\epsilon^p/p)|x|^p + (1/q\epsilon^q)|y|^q$, where $q = (p/(p-1))$.

3. Position Loop Control

In this section, the DOBC method and backstepping method are used to construct the flight controller for helicopters under stochastic disturbances. We first build disturbance observers to estimate the common disturbances δ_1 and δ_2 .

3.1. Disturbance Observer Designed for Position Control.

In what follows, the disturbance observer is designed to estimate the disturbance δ_1 ,

$$\hat{\delta}_1 = \sigma_1 + \rho_1(v), \quad (10)$$

$$d\sigma_1 = -\frac{\partial \rho_1(v)}{\partial v} \left((-ge_3 + \frac{1}{m} Rf) dt + \hat{\delta}_1 dt \right) + \sigma_{11} dt,$$

where σ_1 is an auxiliary variable, σ_{11} is an auxiliary function to be designed, and $\rho_1(v)$ is a nonlinear function to be designed.

Estimate error is defined as $\tilde{\delta}_1 = \hat{\delta}_1 - \delta_1$. Then, the dynamic of the estimate error is given by

$$d\tilde{\delta}_1 = -\frac{\partial \rho_1(v)}{\partial v} (\tilde{\delta}_1 dt + G_1 d\beta_1) - \dot{\delta}_1 dt + \sigma_{11} dt. \quad (11)$$

The disturbance observer gain is selected as $(\partial \rho_1(v)/\partial v) = L_1$, with $L_1 = -\text{diag}\{l_1 \ l_1 \ l_1\}$, where $l_1 > 0$.

Choose the Lyapunov function as follows:

$$V_{01} = \frac{1}{4} \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2. \quad (12)$$

Then, the infinitesimal generator of V_{01} along with (11) is shown as follows:

$$\mathcal{L}V_{01} = -L_1 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 - \tilde{\delta}_1^T \tilde{\delta}_1 \tilde{\delta}_1^T \dot{\delta}_1 + \tilde{\delta}_1^T \tilde{\delta}_1 \tilde{\delta}_1^T \sigma_{11} + \frac{1}{2} \text{Tr} \left(G_1^T L_1 \left(2\tilde{\delta}_1 \tilde{\delta}_1^T + \tilde{\delta}_1^T \tilde{\delta}_1 I \right) L_1 G_1 \right). \quad (13)$$

From Lemma 2, there exist parameters $\varepsilon_{01} > 0$ and $\varepsilon_{02} > 0$ such that

$$-\tilde{\delta}_1^T \tilde{\delta}_1 \tilde{\delta}_1^T \dot{\delta}_1 \leq \frac{3\varepsilon_{01}^{(4/3)}}{4} \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 + \frac{1}{4\varepsilon_{01}^4} \|\dot{\delta}_1\|^4, \quad (14)$$

$$\frac{1}{2} \text{Tr} \left(G_1^T L_1 \left(2\tilde{\delta}_1 \tilde{\delta}_1^T + \tilde{\delta}_1^T \tilde{\delta}_1 I \right) L_1 G_1 \right) \leq \frac{3}{2} \|\tilde{\delta}_1\|^2 \|L_1 G_1\|_F^2 \leq \frac{3\varepsilon_{02}^2}{4} \|L_1 G_1\|_F^4 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 + \frac{3}{4\varepsilon_{02}^2}.$$

Design σ_{11} as

$$\sigma_{11} = - \left(\frac{3\varepsilon_{01}^{(4/3)}}{4} + \frac{3\varepsilon_{02}^2}{4} \|L_1 G_1\|_F^4 \right) \tilde{\delta}_1 + \sigma_{12}, \quad (15)$$

where σ_{12} is introduced in the following steps. Hence, from (15) and $\|\tilde{\delta}_1\| \leq \mu_1$, we have

$$\begin{aligned} \mathcal{L}V_{01} &\leq -L_1 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 + \frac{1}{4\varepsilon_{01}^4} \|\mu_1\|^4 + \frac{3}{4\varepsilon_{02}^2} + \tilde{\delta}_1^T \tilde{\delta}_1 \tilde{\delta}_1^T \sigma_{12} \\ &= -L_1 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 + \tilde{\delta}_1^T \tilde{\delta}_1 \tilde{\delta}_1^T \sigma_{12} + H_{01}, \end{aligned} \quad (16)$$

where

$$H_{01} = \frac{1}{4\varepsilon_{01}^4} \|\mu_1\|^4 + \frac{3}{4\varepsilon_{02}^2}. \quad (17)$$

3.2. Position Loop Controller Designed. Consider the position loop system as follows:

$$\begin{aligned} dp &= v dt, \\ dv &= \left(-ge_3 + \frac{1}{m} Rf \right) dt + \delta_1 dt + G_1 d\beta_1. \end{aligned} \quad (18)$$

The predefined position is $p_r = (x_r, y_r, z_r)$, define the tracking error as $e_p = p - p_r$. The dynamic of the error system is given by

$$\begin{aligned} de_p &= v dt - dp_r = e_v dt, \\ de_v &= \left(-ge_3 + \frac{T_m}{m} R_{3e} + \frac{T_m}{m} R_{3c} - \ddot{p}_r + \tilde{\delta}_1 + \hat{\delta}_1 \right) dt + G_1 d\beta_1. \end{aligned} \quad (19)$$

Define the controller $u_1 = (T_m/m)R_{3c}$ and $R_{3c} = (R_{13c} \ R_{23c} \ R_{33c})^T$, and

$$u_1 = ge_3 + \ddot{p}_r - \hat{\delta}_1 - k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v), \quad (20)$$

where $k_1 > 0$, $k_2 > 0$, $n_1 > 0$, and $n_2 > 0$ are parameters to be designed. At this point, R_{3c} and T_m can be constructed as

$$R_{3c} = \frac{m}{T_m} u_1, \quad (21)$$

$$T_m = m \left\| ge_3 + \ddot{p}_r - \hat{\delta}_1 - k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v) \right\|.$$

Then, the closed-loop position loop system is written as

$$de_p = e_v dt, \quad (22)$$

$$de_v = \left(-k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v) + \frac{T_m}{m} R_{3e} + \tilde{\delta}_1 \right) dt + G_1 d\beta_1. \quad (23)$$

Remark 1. In fact, the helicopter should not be overturn under controller (21), for the continuity of $\cos \phi \cos \theta$. So that, we guarantee $\cos \phi \cos \theta > 0$ to ensure that the helicopter will not be overturn. Meanwhile, the defined signal R_{33c} should also satisfy $R_{33c} > 0$. Thus, the parameters k_1 and k_2 are designed to satisfy $g + \ddot{z}_r + \delta_1 e - k_1 - k_2 > 0$. In fact,

the acceleration of disturbance in z axis is far less than acceleration of gravity.

Consider $\|R_3\| = 1$, $R_{3e} = R_3 - R_{3c}$, $\bar{R}_3 = \begin{pmatrix} R_{13} \\ R_{23} \end{pmatrix}$, and $\bar{R}_{3c} = \begin{pmatrix} R_{13c} \\ R_{23c} \end{pmatrix}$. Thus,

$$\begin{aligned} R_3 &= \begin{pmatrix} R_{13} \\ R_{23} \\ R_{33} \end{pmatrix} = \begin{pmatrix} R_{13} \\ R_{23} \\ \sqrt{1 - R_{13}^2 - R_{23}^2} \end{pmatrix}, \\ R_{3e} &= \begin{pmatrix} R_{13} \\ R_{23} \\ R_{33} \end{pmatrix} - \begin{pmatrix} R_{13c} \\ R_{23c} \\ R_{33c} \end{pmatrix} = \begin{pmatrix} R_{13e} \\ R_{23e} \\ \sqrt{1 - R_{13}^2 - R_{23}^2} - \sqrt{1 - R_{13c}^2 - R_{23c}^2} \end{pmatrix}. \end{aligned} \quad (24)$$

Due to $R_{33} > 0$ and $R_{33c} > 0$, we have

$$\sqrt{1 - R_{13}^2 - R_{23}^2} - \sqrt{1 - R_{13c}^2 - R_{23c}^2} = \frac{\alpha_1 \bar{R}_{3e}}{\sqrt{1 - R_{13}^2 - R_{23}^2} + \sqrt{1 - R_{13c}^2 - R_{23c}^2}}, \quad (25)$$

where $\alpha_1 = -(R_{13} + R_{13c}, R_{23} + R_{23c})$. Hence,

$$R_{3e} = \begin{pmatrix} \bar{R}_{3e} \\ \frac{\alpha_1 \bar{R}_{3e}}{\sqrt{1 - R_{13}^2 - R_{23}^2} + \sqrt{1 - R_{13c}^2 - R_{23c}^2}} \end{pmatrix} = \begin{pmatrix} I_2 \\ \alpha_1 \end{pmatrix} \bar{R}_{3e} = \alpha \bar{R}_{3e} = \alpha (\bar{R}_3 - \bar{R}_{3c}), \quad (26)$$

where $\alpha = \begin{pmatrix} I_2 \\ (\alpha_1 / (\sqrt{1 - R_{13}^2 - R_{23}^2} + \sqrt{1 - R_{13c}^2 - R_{23c}^2})) \end{pmatrix}$.

The velocity tracking error equation (23) can be rewritten as

$$de_v = \left(-k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v) + \frac{T_m}{m} \alpha \bar{R}_{3e} + \tilde{\delta}_1 \right) dt + G_1 d\beta_1. \quad (27)$$

Choose Lyapunov function $V_{11}(t)$ as follows:

$$V_{11}(t) = k_1 \int_0^{n_1 e_p + n_2 e_v} \tanh(\mu) d\mu + k_2 \int_0^{n_2 e_v} \tanh(\mu) d\mu + \frac{n_1}{2} e_v^T e_v. \quad (28)$$

Then, the infinitesimal generator of V_{11} is represented as

$$\begin{aligned}
\mathcal{L}V_{11}(t) = & -n_2 \left(k_1 \tanh(n_1 e_p + n_2 e_v) + k_2 \tanh(n_2 e_v) \right)^T \\
& \times \left(k_1 \tanh(n_1 e_p + n_2 e_v) + k_2 \tanh(n_2 e_v) \right) - n_1 k_2 e_v^T \tanh(n_2 e_v) \\
& + F(t)^T \left(\frac{T_m}{m} \alpha \bar{R}_{3e} + \tilde{\delta}_1 \right) + \frac{1}{2} \text{Tr} \left(\begin{pmatrix} 0 \\ G_1 \end{pmatrix}^T \begin{pmatrix} \frac{\partial^2 V_{11}}{\partial e_p^2} & \frac{\partial^2 V_{11}}{\partial e_p \partial e_v} \\ \frac{\partial^2 V_{11}}{\partial e_p \partial e_v} & \frac{\partial^2 V_{11}}{\partial e_v^2} \end{pmatrix} \begin{pmatrix} 0 \\ G_1 \end{pmatrix} \right), \quad (29)
\end{aligned}$$

where

$$\begin{aligned}
F(t) = & n_1 e_v + n_2 k_1 \tanh(n_1 e_p + n_2 e_v) + n_2 k_2 \tanh(n_2 e_v), \\
\frac{\partial^2 V_{11}}{\partial e_p^2} = & \text{diag} \{ n_1^2 k_1 (1 + \tanh^2(n_1 e_{p_1} + n_2 e_{v_1})), n_1^2 k_1 (1 + \tanh^2(n_1 e_{p_2} + n_2 e_{v_2})), n_1^2 k_1 (1 + \tanh^2(n_1 e_{p_3} + n_2 e_{v_3})) \}, \\
\frac{\partial^2 V_{11}}{\partial e_p \partial e_v} = & \text{diag} \{ n_1 n_2 k_1 (1 + \tanh^2(n_1 e_{p_1} + n_2 e_{v_1})), n_1 n_2 k_1 (1 + \tanh^2(n_1 e_{p_2} + n_2 e_{v_2})), n_1 n_2 k_1 (1 + \tanh^2(n_1 e_{p_3} + n_2 e_{v_3})) \}, \\
\frac{\partial^2 V_{11}}{\partial e_v^2} = & \text{diag} \{ n_2^2 k_1 (1 + \tanh^2(n_1 e_{p_1} + n_2 e_{v_1})), n_2^2 k_2 (1 + \tanh^2(n_2 e_{v_1})), \\
& n_2^2 k_1 (1 + \tanh^2(n_1 e_{p_2} + n_2 e_{v_2})), n_2^2 k_2 (1 + \tanh^2(n_2 e_{v_2})), \\
& n_2^2 k_1 (1 + \tanh^2(n_1 e_{p_3} + n_2 e_{v_3})), n_2^2 k_2 (1 + \tanh^2(n_2 e_{v_3})) \} + \text{diag} \{ 1, 1, 1 \}. \quad (30)
\end{aligned}$$

For the function $\|\tanh(\cdot)\| \leq 1$ and the bounded parameter G_1 , there exists a parameter $H_{11} > 0$, such that

$$\frac{1}{2} \text{Tr} \left(\begin{pmatrix} 0 \\ G_1 \end{pmatrix}^T \begin{pmatrix} \frac{\partial^2 V_{11}}{\partial e_p^2} & \frac{\partial^2 V_{11}}{\partial e_p \partial e_v} \\ \frac{\partial^2 V_{11}}{\partial e_p \partial e_v} & \frac{\partial^2 V_{11}}{\partial e_v^2} \end{pmatrix} \begin{pmatrix} 0 \\ G_1 \end{pmatrix} \right) \leq H_{11}. \quad (31)$$

Notice that $-n_1 k_2 e_v^T \tanh(n_2 e_v) \leq -(n_1/n_2) k_2 \tanh(n_2 e_v)^T \tanh(n_2 e_v)$. Then, we have

$$\begin{aligned}
\mathcal{L}V_{11}(t) \leq & -n_2 \left(k_1 \tanh(n_1 e_p + n_2 e_v) + k_2 \tanh(n_2 e_v) \right)^T \left(k_1 \tanh(n_1 e_p + n_2 e_v) + k_2 \tanh(n_2 e_v) \right) \\
& - \frac{n_1}{n_2} k_2 \tanh(n_2 e_v)^T \tanh(n_2 e_v) + F(t)^T \left(\frac{T_m}{m} \alpha \bar{R}_{3e} + \tilde{\delta}_1 \right) + H_{11} \\
= & - \begin{pmatrix} \tanh(n_1 e_p + n_2 e_v) \\ \tanh(n_2 e_v) \end{pmatrix} \begin{pmatrix} n_2 k_1^2 & n_2 k_1 k_2 \\ n_2 k_1 k_2 & n_2 k_2^2 + \frac{n_1}{n_2} k_2 \end{pmatrix} \begin{pmatrix} \tanh(n_1 e_p + n_2 e_v) \\ \tanh(n_2 e_v) \end{pmatrix} \\
& + F(t)^T \tilde{\delta}_1 + F(t)^T \frac{T_m}{m} \alpha \bar{R}_{3e} + H_{11}. \quad (32)
\end{aligned}$$

In fact, according to Lemma 2, there exist parameters $\varepsilon_{11} > 0$ and $\varepsilon_{12} > 0$, such that

$$F(t)^T \tilde{\delta}_1 \leq \frac{\varepsilon_{11} \tilde{\delta}_1^T \tilde{\delta}_1^T}{3} \|F(t)\|^2 F(t) + \frac{2}{3\varepsilon_{11}^{(1/2)}}, \quad (33)$$

$$F(t)^T \frac{T_m}{m} \alpha \bar{R}_{3e} \leq \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \alpha^T F(t) + \frac{2}{3\varepsilon_{12}^{(1/2)}}.$$

Let $k_3 = \min \left\{ \text{eig} \begin{pmatrix} n_2 k_1^2 & n_2 k_1 k_2 \\ n_2 k_1 k_2 & n_2 k_2^2 + (n_1/n_2) k_2 \end{pmatrix} \right\}$, we obtain

$$\begin{aligned} \mathcal{L}V_{11}(t) \leq & -k_3 \begin{pmatrix} \tanh(n_1 e_p + n_2 e_v) \\ \tanh(n_2 e_v) \end{pmatrix}^T \begin{pmatrix} \tanh(n_1 e_p + n_2 e_v) \\ \tanh(n_2 e_v) \end{pmatrix} + \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \alpha^T F(t) \\ & + \frac{\varepsilon_{11} \tilde{\delta}_1^T \tilde{\delta}_1^T}{3} \|F(t)\|^2 F(t) + \frac{2}{3\varepsilon_{11}^{(1/2)}} + \frac{2}{3\varepsilon_{12}^{(1/2)}} + H_{11}. \end{aligned} \quad (34)$$

Since $h(x) = (\tanh(x)/x)$ is a continuous function, for any $\bar{x} > 0$, $0 \leq x \leq \bar{x}$, there exists a positive number $\beta(\bar{x})$ such that $\beta\|x\| \leq \|\tanh(x)\| \leq \|x\|$. Thus,

$$\begin{aligned} -\tanh(n_1 e_p + n_2 e_v)^T \tanh(n_1 e_p + n_2 e_v) & \leq -\beta^2(n_1 e_p + n_2 e_v)^T (n_1 e_p + n_2 e_v), \\ -\tanh(n_2 e_v)^T \tanh(n_2 e_v) & \leq -\beta^2 n_2^2 e_v^T e_v. \end{aligned} \quad (35)$$

Define Lyapunov function as follows:

Based on (16) and (34), we obtain

$$V_1(t) = V_{01}(t) + V_{11}(t). \quad (36)$$

$$\begin{aligned} \mathcal{L}V_1(t) \leq & -k_3 \beta^2 (n_1 e_p + n_2 e_v)^T (n_1 e_p + n_2 e_v) - k_3 \beta^2 n_2^2 e_v^T e_v \\ & + \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \alpha^T F(t) + \frac{\varepsilon_{11} \tilde{\delta}_1^T \tilde{\delta}_1^T}{3} \|F(t)\|^2 F(t) \\ & + \frac{2}{3\varepsilon_{11}^{(1/2)}} + \frac{2}{3\varepsilon_{12}^{(1/2)}} + H_{11} - L_1 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 + \tilde{\delta}_1^T \tilde{\delta}_1 \sigma_{12} + H_{01}. \end{aligned} \quad (37)$$

Hence, σ_{12} is designed as

$$\sigma_{12} = -\frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t). \quad (38)$$

Then,

$$\mathcal{L}V_1(t) \leq -l_1 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 - k_4 e_p^T e_p - k_4 e_v^T e_v + \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \alpha^T F(t) + H_1, \quad (39)$$

where $k_4 = \min \left\{ \text{eig} \left\{ \begin{pmatrix} k_3 \beta^2 n_1^2 & k_3 \beta^2 n_1 n_2 \\ k_3 \beta^2 n_1 n_2 & 2k_3 \beta^2 n_2^2 \end{pmatrix} \right\} \right\} > 0$ and $H_1 = H_{11} + H_{01} + (2/3\varepsilon_{11}^{(1/2)}) + (2/3\varepsilon_{12}^{(1/2)})$.

Moreover, the error dynamic of disturbance $\tilde{\delta}_1$ can be rewritten as

$$d\tilde{\delta}_1 = - \left(L_1 + \left(\frac{3\varepsilon_{01}^{(4/3)}}{4} + \frac{3\varepsilon_{02}^2}{4} \|L_1 G_1\|_F^4 \right) \right) \tilde{\delta}_1 dt - L_1 G_1 d\beta_1 - \dot{\delta}_1 dt - \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t) dt. \quad (40)$$

4. Attitude Loop Control

In this section, the DOBC method and backstepping method are used to construct the flight controller for helicopters under stochastic disturbances. We first build disturbance observers to estimate the common disturbances δ_1 and δ_2 .

4.1. Disturbance Observer Designed for Attitude Control. In order to estimate the disturbance δ_2 , we design the following disturbance observers:

$$\begin{aligned} \hat{\delta}_2 &= \sigma_2 + J\rho_2(w), \\ d\sigma_2 &= -\frac{\partial \rho_2(w)}{\partial w} ((-w^\times Jw + \tau)dt + \hat{\delta}_2 dt) + \sigma_{02} dt, \end{aligned} \quad (41)$$

where σ_2 is an auxiliary variable, σ_{02} is an auxiliary function to be designed, and $\rho_2(v)$ is a nonlinear function to be designed.

Define estimate error as $\tilde{\delta}_2 = \hat{\delta}_2 - \delta_2$. Then, the dynamic of the estimate error can be shown as

$$d\tilde{\delta}_2 = -\frac{\partial \rho_2(w)}{\partial w} (\tilde{\delta}_2 dt + G_2 d\beta_2) - \dot{\delta}_2 dt + \sigma_{02} dt. \quad (42)$$

The disturbance observer gain is selected as $(\partial \rho_2(w)/\partial w) = L_2$, with $L_2 = -\text{diag}\{l_2 \ l_2 \ l_2\}$, where $l_2 > 0$.

Choose Lyapunov function as follows:

$$V_{02} = \frac{1}{4} \left(\tilde{\delta}_2^T \tilde{\delta}_2 \right)^2. \quad (43)$$

Similarly with V_{01} , for V_{02} , there exist $\varepsilon_{03} > 0$ and $\varepsilon_{04} > 0$, such that

$$\mathcal{L}V_{02} \leq -l_2 \left(\tilde{\delta}_2^T \tilde{\delta}_2 \right)^2 + H_{02}, \quad (44)$$

where

$$H_{02} = \frac{1}{4\varepsilon_{03}^4} \|\mu_2\|^4 + \frac{3}{4\varepsilon_{04}^2}. \quad (45)$$

Design the auxiliary variable σ_{02} as follows:

$$\sigma_{02} = - \left(\frac{3\varepsilon_{03}^{(4/3)}}{4} + \frac{3\varepsilon_{04}^2}{4} \|L_2 G_2\|_F^4 \right) \tilde{\delta}_2. \quad (46)$$

The error dynamics of disturbance (42) can be rewritten as

$$d\tilde{\delta}_2 = - \left(l_2 + \frac{3\varepsilon_{03}^{(4/3)}}{4} + \frac{3\varepsilon_{04}^2}{4} \|L_2 G_2\|_F^4 \right) \tilde{\delta}_2 dt - \dot{\delta}_2 dt - L_2 G_2 d\beta_2. \quad (47)$$

4.2. Attitude Loop Controller Designed. The predefined tracking signals are position and yaw angle, which are denoted by p_r and ψ_r . After designing the position loop control, the reasonable attitude tracking signals are obtained, which are given as $\bar{R}_{3c}$ and ψ_r . Moreover, defining $\gamma_r = (\bar{R}_3^T \ \psi)^T$ and $\gamma = (\theta \ \phi \ \psi)^T$, we can confirm that $\|\partial \gamma_r / \partial \gamma\| = \cos \theta \neq 0$, for $|\theta| < (\pi/2)$. Hence, the mapping that from γ to γ_r is a local topological homeomorphism, which means that the dynamic of γ_r can represent the dynamic of γ for $|\theta| < (\pi/2)$. The error dynamic of γ_r is given by

$$d\bar{R}_{3e} = \hat{R}_3 w_1 dt - d\bar{R}_{3c}, \quad (48)$$

$$d\psi = \left(\frac{\sin \phi}{\cos \theta} q + \frac{\cos \phi}{\cos \theta} r \right) dt - d\psi_r, \quad (49)$$

where $w_1 = (p \ q)^T$ and $\hat{R}_3 = \begin{pmatrix} -R_{12} & R_{11} \\ -R_{22} & R_{21} \end{pmatrix}$.

For (48), dR_{3c} can be written as

$$dR_{3c} = d \left(\frac{u_1}{\sqrt{u_1^T u_1}} \right) = \frac{1}{\sqrt{u_1^T u_1}} \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) du_1. \quad (50)$$

For du_1 , we have

$$du_1 = d\tilde{p}_r - d\hat{\delta}_1 - k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v). \quad (51)$$

From (10), $d\hat{\delta}_1$ is written as follows:

$$d\hat{\delta}_1 = -\bar{l}_1 \tilde{\delta}_1 dt - L_1 G_1 d\beta_1 - \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t) dt, \quad (52)$$

where $\bar{l}_1 = l_1 + (3\varepsilon_{01}^{(4/3)}/4) + (3\varepsilon_{02}^2/4) \|L_1 G_1\|_F^4$.

Hence, we get

$$du_1 = \Delta_{01}(t) dt + \Delta_{02}(t) \tilde{\delta}_1 dt + \Delta_{03}(t) d\beta_1, \quad (53)$$

for $x = (x_1, x_2, x_3)$, $\text{Tanh}^2(x) = \text{diag}\{\tanh^2(x_1), \tanh^2(x_2)y, \tanh^2(x_3)\}$, where

$$\begin{aligned}\Delta_{01}(t) &= \ddot{p}_r - k_1 n_1 (I + \text{Tanh}^2(n_1 e_p + n_2 e_v)) e_v - ((k_2 n_2 (I + \text{Tanh}^2(n_2 e_v)) + k_1 n_2 (I + \text{Tanh}^2(n_1 e_p + n_2 e_v))) \\ &\quad \cdot (-k_1 \tanh(n_1 e_p + n_2 e_v) - k_2 \tanh(n_2 e_v) + \frac{T_m}{m} \alpha \bar{R}_{3e}) + \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t), \\ \Delta_{02}(t) &= \bar{l}_1 I - (k_2 n_2 (I + \text{Tanh}^2(n_2 e_v)) + k_1 n_2 (I + \text{Tanh}^2(n_1 e_p + n_2 e_v))), \\ \Delta_{03}(t) &= L_1 G_1 - (k_2 n_2 (I + \text{Tanh}^2(n_2 e_v)) + k_1 n_2 (I + \text{Tanh}^2(n_1 e_p + n_2 e_v))) G_1.\end{aligned}\tag{54}$$

From (50),

$$\begin{aligned}dR_{3c} &= \frac{1}{\|u_1\|} \left(\Delta_{101}(t) + \Delta_{102}(t) e_v + \Delta_{103}(t) \Delta_{104}(t) + \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t) \right) dt \\ &\quad + \frac{1}{\|u_1\|} \Delta_{12}(t) \tilde{\delta}_1 dt + \frac{1}{\|u_1\|} \Delta_{13}(t) d\beta_1,\end{aligned}\tag{55}$$

where

$$\begin{aligned}\Delta_{101}(t) &= \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) \ddot{p}_r, \\ \Delta_{102}(t) &= -k_2 \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) (I + \text{Tanh}^2(e_v)), \\ \Delta_{103}(t) &= - \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) (k_2 (I + \text{Tanh}^2(e_v)) + k_1 (I + \text{Tanh}^2(e_p + e_v))), \\ \Delta_{104}(t) &= \left(-k_1 \tanh(e_p + e_v) - k_2 \tanh(e_v) + \frac{T_m}{m} \alpha \bar{R}_{3e} \right), \\ \Delta_{12}(t) &= \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) (\bar{l}_1 I - (k_2 (I + \text{Tanh}^2(e_v)) + k_1 (I + \text{Tanh}^2(e_p + e_v)))), \\ \Delta_{13}(t) &= \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) (L_1 G_1 - (k_2 (I + \text{Tanh}^2(e_v)) + k_1 (I + \text{Tanh}^2(e_p + e_v))) G_1).\end{aligned}\tag{56}$$

For $\|u_1 u_1^T\|_F \leq \|u_1\|_F \|u_1^T\|_F = \|u_1\|^2$, one has

$$\left\| \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) \right\|_F \leq \|I_3\|_F + \frac{\|u_1 u_1^T\|_F}{\|u_1\|^2} \leq \sqrt{3} + 1. \quad (57)$$

Then, we confirm that

$$\begin{aligned} \|\Delta_{101}(t)\| &\leq (\sqrt{3} + 1) \|\ddot{p}_r\|, \\ \|\Delta_{102}(t)\|_F &\leq k_2 (\sqrt{3} + 1) \|I + \tanh^2(e_\nu)\|_F \leq k_2 (3\sqrt{2} + \sqrt{6}), \\ \|\Delta_{103}(t)\|_F &\leq (\sqrt{3} + 1) \|k_2(I + \tanh^2(e_\nu)) + k_1(I + \tanh^2(e_p + e_\nu))\|_F \leq (6 + 2\sqrt{3})(k_1 + k_2), \\ \|\Delta_{12}(t)\| &\leq (\sqrt{3} + 1) (\bar{l}_1 + 2\sqrt{3}(k_1 + k_2)), \\ \|\Delta_{13}(t)\| &\leq (\sqrt{3} + 1) (\|L_1 G_1\| + 2\sqrt{3}(k_1 + k_2) \|G_1\|). \end{aligned} \quad (58)$$

Thus, (48) can be rewritten as

$$\begin{aligned} d\bar{R}_{3e} &= \hat{R}_3 e_{w_1} dt + \hat{R}_3 w_{1c} dt - E_0 \frac{1}{\|u_1\|} \left(\Delta_{101}(t) + \Delta_{102}(t) e_\nu + \Delta_{103}(t) \Delta_{104}(t) + \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t) \right) dt \\ &\quad - E_0 \frac{1}{\|u_1\|} \Delta_{12}(t) \tilde{\delta}_1 dt - E_0 \frac{1}{\|u_1\|} \Delta_{13}(t) d\beta_1, \end{aligned} \quad (59)$$

where $E_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$.

Choose Lyapunov function $V_{21}(t)$ as

$$V_{21}(t) = \frac{1}{4} (\bar{R}_{3e}^T \bar{R}_{3e})^2. \quad (60)$$

The infinitesimal generator of V_{21} along with (60) is shown as

$$\begin{aligned} \mathcal{L}V_{21}(t) &= \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \hat{R}_3 e_{w_1} + \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \hat{R}_3 w_{1c} - \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{101}(t) \\ &\quad - \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{102}(t) e_\nu - \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{103}(t) \Delta_{104}(t) \\ &\quad - \frac{\varepsilon_{11}}{3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \|F(t)\|^2 F(t) - \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{12}(t) \tilde{\delta}_1 \\ &\quad + \frac{1}{2\|u_1\|^2} \text{Tr} \left(\Delta_{13}^T(t) E_0^T \left(2\bar{R}_{3e} \bar{R}_{3e}^T + \bar{R}_{3e}^T \bar{R}_{3e} I \right) E_0 \Delta_{13}(t) \right). \end{aligned} \quad (61)$$

From Lemma 2, there exist parameters $\varepsilon_{21} > 0$, $\varepsilon_{22} > 0$, $\varepsilon_{231} > 0$, $\varepsilon_{232} > 0$, $\varepsilon_{24} > 0$, and $\varepsilon_{25} > 0$, such that

$$\begin{aligned}
& \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \hat{R}_3 e_{w_1} \leq \frac{3}{4\varepsilon_{21}^{(1/3)}} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{\varepsilon_{21}}{4} \|\hat{R}_3\|^4 (e_{w_1}^T e_{w_1})^2, \\
& -\bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{101}(t) \leq \frac{3\varepsilon_{22}^{(1/3)}}{4\|u_1\|^{(4/3)}} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{1}{4\varepsilon_{22}} \|E_0\|^4 \|\Delta_{101}(t)\|^4, \\
& -\bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{102}(t) e_\nu \leq \frac{\varepsilon_{231}}{2} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{\varepsilon_{232}}{2\|u_1\|^4} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 (e_\nu^T e_\nu)^2 + \frac{\|E_0\|^4 \|\Delta_{102}(t)\|^4}{8\varepsilon_{232}\varepsilon_{231}}, \\
& -\bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{12}(t) \tilde{\delta}_1 \leq \frac{3\varepsilon_{24}^{(1/3)}}{4\|u_1\|^{(4/3)}} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{1}{4\varepsilon_{24}} \|E_0\|_F^4 \|\Delta_{12}(t)\|^4 \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2, \\
& \frac{1}{2\|u_1\|^2} \text{Tr} \left(\Delta_{13}^T(t) E_0^T \left(2\bar{R}_{3e} \bar{R}_{3e}^T + \bar{R}_{3e}^T \bar{R}_{3e} I \right) E_0 \Delta_{13}(t) \right) \leq \frac{3\varepsilon_{25}}{4\|u_1\|^2} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{1}{2\varepsilon_{25}} \|E_0 \Delta_{13}(t)\|_F^4.
\end{aligned} \tag{62}$$

Furthermore, for $T_m = m\|u_1\|$, there exist parameters $\varepsilon_{26} > 0$ and $\varepsilon_{27} > 0$, such that

$$\begin{aligned}
& -\bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \Delta_{103}(t) \Delta_{104}(t) \\
& \leq \frac{3\varepsilon_{26}^{(1/3)}}{4\|u_1\|^{(4/3)}} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{1}{4\varepsilon_{26}} \|E_0\|_F^4 \|\Delta_{103}(t)\|^4 \left(\|k_1 \tanh(e_p + e_\nu)\| + k_2 \|\tanh(e_\nu)\| \right)^4 \\
& \quad + \frac{3\varepsilon_{27}^{(1/3)}}{4} \|E_0\|_F^{(4/3)} \|\Delta_{103}(t)\|_F^{(4/3)} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{1}{4\varepsilon_{27}} \|\alpha\|_F^4 \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2.
\end{aligned} \tag{63}$$

Then, $\mathcal{L}V_{21}(t)$ satisfies

$$\begin{aligned}
\mathcal{L}V_{21}(t) & \leq \left(H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}} \|\alpha\|_F^4 \right) \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \hat{R}_3 w_{1c} + H_{23} \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 \\
& \quad - \frac{\varepsilon_{11}}{3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \|F(t)\|^2 F(t) + \frac{\varepsilon_{232}}{2\|u_1\|^4} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 (e_\nu^T e_\nu)^2 + \frac{\varepsilon_{21}}{4} \|\hat{R}_3\|^4 (e_{w_1}^T e_{w_1})^2 + H_{21},
\end{aligned} \tag{64}$$

where

$$\begin{aligned}
H_{21} &= \frac{1}{2\varepsilon_{25}} \|E_0\|_F^4 \left((\sqrt{3} + 1) (\|L_1 G_1\| + 2\sqrt{3} (k_1 + k_2) \|G_1\|) \right)_F^4 + \frac{1}{4\varepsilon_{22}} \|E_0\|_F^4 (\sqrt{3} + 1) \|\ddot{p}_r\|^4 \\
&\quad + \frac{1}{4\varepsilon_{26}} \|E_0\|_F^2 (6 + 2\sqrt{3}) (k_1 + k_2)^4 (k_1 + k_2)^4 + \frac{\|E_0\|_F^4 \|k_2 (3\sqrt{2} + \sqrt{6})\|^4}{8\varepsilon_{232}\varepsilon_{231}}, \\
H_{22} &= \frac{1}{4\varepsilon_{23}} \|E_0\|_F^4 (k_2 (3\sqrt{2} + \sqrt{6}))^2, \\
H_{23} &= \frac{1}{4\varepsilon_{24}} \left((\sqrt{3} + 1) (\|\tilde{l}_1\|_F + 2\sqrt{3} (k_1 + k_2)) \right)^4, \\
H_{24} &= 3\varepsilon_{22}^{(1/3)} + 3\varepsilon_{24}^{(1/3)} + 3\varepsilon_{26}^{(1/3)}, \\
H_{25} &= \frac{3}{4\varepsilon_{21}^{(1/3)}} + \frac{3\varepsilon_{27}^{(1/3)}}{4} \|E_0\|_F^{(4/3)} (6 + 2\sqrt{3}) (k_1 + k_2)^{(4/3)} + \frac{\varepsilon_{231}}{2}.
\end{aligned} \tag{65}$$

Define $V_2(t) = V_1 + V_{21}$. Based on the above discussion and according to (39) and (64), we have

$$\begin{aligned}
\mathcal{L}V_2(t) &\leq -(l_1 - H_{23}) \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 - k_4 e_p^T e_p - k_4 e_v^T e_v + \frac{\varepsilon_{21}}{4} \|\hat{R}_3\|^4 (e_{w_1}^T e_{w_1})^2 + \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \hat{R}_3 w_{1c} \\
&\quad + \left(H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}} \|\alpha\|_F^4 \right) \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T \alpha^T F(t) \\
&\quad + \frac{\varepsilon_{232}}{2\|u_1\|^4} \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 (e_v^T e_v)^2 - \frac{\varepsilon_{11}}{3} \bar{R}_{3e}^T \bar{R}_{3e} \bar{R}_{3e}^T E_0 \frac{1}{\|u_1\|} \|F(t)\|^2 F(t) + H_2,
\end{aligned} \tag{66}$$

where $H_2 = H_1 + H_{21}$. Hence, we design the virtual control law w_{1c} as

$$\begin{aligned}
w_{1c} &= - \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}} \|\alpha\|_F^4 \right) \hat{R}_3^{-1} \bar{R}_{3e} - \frac{\varepsilon_{12} T_m^3 \|\alpha\|^2 \|F(t)\|^2}{3m^3} \hat{R}_3^{-1} \alpha^T F(t) \\
&\quad - \frac{\varepsilon_{232}}{2\|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} + \frac{\varepsilon_{11} \|F(t)\|^2}{3\|u_1\|} \hat{R}_3^{-1} E_0 F(t),
\end{aligned} \tag{67}$$

where $k_5 > 0$. Then, we obtain

$$\mathcal{L}V_2(t) \leq -(L_1 - H_{23}) \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 - k_4 e_p^T e_p - k_4 e_v^T e_v - k_5 \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 + \frac{\varepsilon_{21}}{4} \|\hat{R}_3\|^4 (e_{w_1}^T e_{w_1})^2 + H_2. \tag{68}$$

From (61), the dynamic of $\bar{R}_{3e}$ can be rewritten as

$$d\bar{R}_{3e} = \bar{R}_{3e1}(t)dt + \bar{R}_{3e2}(t)\tilde{\delta}_1 dt + \bar{R}_{3e3}(t)d\beta_1, \quad (69) \quad \text{where}$$

$$\begin{aligned} \bar{R}_{3e1}(t) = & \hat{R}_3 e_{w_1} - \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \bar{R}_{3e} - \frac{\varepsilon_{12}T_m^3\|\alpha\|^2\|F(t)\|^2}{3m^3} \hat{R}_3^{-1} \alpha^T F(t) \\ & - \frac{\varepsilon_{232}}{2\|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} - \frac{E_0}{\|u_1\|} (\Delta_{101}(t) + \Delta_{102}(t)e_v + \Delta_{103}(t)\Delta_{104}(t)), \end{aligned} \quad (70)$$

$$\bar{R}_{3e2}(t) = \frac{E_0}{\|u_1\|} \Delta_{12}(t),$$

$$\bar{R}_{3e3}(t) = -\frac{E_0}{\|u_1\|} \Delta_{13}(t).$$

For (37), we have

$$de_\psi = \left(E_{11}w_1 + \frac{\cos\phi}{\cos\theta}e_r + \frac{\cos\phi}{\cos\theta}r_c - \dot{\psi}_r \right) dt, \quad (71)$$

where $E_{11} = (0 \ \sin\phi/\cos\theta)$ and r_c is the virtual control law to be designed.

Choose Lyapunov function V_{31} as

$$V_{31} = \frac{1}{4}e_\psi^4. \quad (72)$$

The derivative of $V_{31}(t)$ along with (71) is derived as follows:

$$\dot{V}_{31}(t) = e_\psi^3 \left(E_{11}w_1 + \frac{\cos\phi}{\cos\theta}e_r + \frac{\cos\phi}{\cos\theta}r_c - \dot{\psi}_r \right). \quad (73)$$

Then, the virtual control law r_c is designed as

$$r_c = -\frac{\cos\theta}{\cos\phi} (E_{11}w_{1c} - \dot{\psi}_r + k_3e_\psi). \quad (74)$$

At this point, the dynamic of e_ψ can be written as

$$de_\psi = \left(-k_6e_\psi + E_{11}e_{w_1} + \frac{\cos\phi}{\cos\theta}e_r \right) dt = (-k_6e_\psi + E_1e_w) dt, \quad (75)$$

where $E_1 = (0 \ (\sin\phi/\cos\theta) \ (\cos\phi/\cos\theta))$ and $e_w = (p, q, r)^T - (p_c, q_c, r_c)^T$. Thus, based on Lemma 2, there exists $\varepsilon_{28} > 0$, such that

$$\dot{V}_{31}(t) = e_\psi^3 (-k_6e_\psi + E_1e_w) = -k_6e_\psi^4 + e_\psi^3 E_1e_w \leq -\left(k_6 - \frac{3\varepsilon_{28}^{(1/3)}}{4} \right) e_\psi^4 + \frac{1}{4\varepsilon_{28}} \|E_1\|^4 (e_w^T e_w)^2. \quad (76)$$

Define $V_3(t) = V_2(t) + V_{31}(t)$; combining (68) and (76) yields

$$\begin{aligned} \mathcal{L}V_3(t) \leq & -(l_1 - H_{23}) \left(\tilde{\delta}_1^T \tilde{\delta}_1 \right)^2 - k_4 e_p^T e_p - k_4 e_v^T e_v - k_5 \left(\bar{R}_{3e}^T \bar{R}_{3e} \right) + \frac{\varepsilon_{21}}{4} \|\hat{R}_3\|^4 \|E_0\|^4 (e_w^T e_w)^2 + H_2 \\ & - \left(k_6 - \frac{3\varepsilon_{28}^{(1/3)}}{4} \right) e_\psi^4 + \frac{1}{4\varepsilon_{28}} \|E_1\|^4 (e_w^T e_w)^2. \end{aligned} \quad (77)$$

In what follows, consider the dynamic of angular velocity:

$$Jdw = (-w^\times Jw + A(T_m)\rho + B(T_m) + \delta_2)dt + G_2 d\beta_2. \quad (78)$$

Defining $\rho = A(T_m)^{-1}(u_2 - B(T_m) + w^*Jw - \tilde{\delta}_2)$, the error dynamic of e_w is

$$Jde_w = (u_2 + \tilde{\delta}_2)dt + G_2d\beta_2 - dw_c, \quad (79)$$

where dw_c is obtained in Appendix. Then, according to (79),

$$Jde_w = (u_2 + \tilde{\delta}_2)dt + G_2d\beta_2 - W_{c1}(t)dt - W_{c2}(t)\tilde{\delta}_1dt - W_{c20}(t, \tilde{\delta}_1)dt - W_{c3}(t)d\beta_1, \quad (80)$$

which implies that

$$de_w = (J^{-1}u_2 - J^{-1}W_{c1}(t) - J^{-1}W_{c2}(t)\tilde{\delta}_1 - J^{-1}W_{c20}(t, \tilde{\delta}_1) + J^{-1}\tilde{\delta}_2)dt - J^{-1}W_{c3}(t)d\beta_1 + J^{-1}G_2d\beta_2. \quad (81)$$

Choose Lyapunov function as follows:

$$V_{41}(t) = \frac{1}{4}(e_w^T e_w)^2. \quad (82)$$

The infinitesimal generator of V_{41} along with (81) is obtained as follows:

$$\begin{aligned} \mathcal{L}V_{41}(t) &= e_w^T e_w e_w^T J^{-1} u_2 - e_w^T e_w e_w^T J^{-1} W_{c1}(t) - e_w^T e_w e_w^T J^{-1} W_{c2}(t) \tilde{\delta}_1 - e_w^T e_w e_w^T J^{-1} W_{c20}(t, \tilde{\delta}_1) \\ &\quad + e_w^T e_w e_w^T J^{-1} \tilde{\delta}_2 + \frac{1}{2} \text{Tr}(W_{c3}^T(t) J^{-T} (2e_w e_w^T + e_w^T e_w I) J^{-1} W_{c3}(t)) \\ &\quad + \frac{1}{2} \text{Tr}(G_2^T J^{-T} (2e_w e_w^T + e_w^T e_w I) J^{-1} G_2). \end{aligned} \quad (83)$$

From Lemma 2, there exist parameters $\varepsilon_{31} > 0$, $\varepsilon_{32} > 0$, $\varepsilon_{33} > 0$, $\varepsilon_{34} > 0$, and $\varepsilon_{35} > 0$ such that

$$\begin{aligned} -e_w^T e_w e_w^T J^{-1} W_{c2}(t) \tilde{\delta}_1 &\leq \frac{3}{4\varepsilon_{31}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} \|W_{c2}(t)\|_F^{(4/3)} (e_w^T e_w)^2 + \frac{\varepsilon_{31}}{4} (\tilde{\delta}_1^T \tilde{\delta}_1)^2, \\ e_w^T e_w e_w^T J^{-1} \tilde{\delta}_2 &\leq \frac{3}{4\varepsilon_{32}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} (e_w^T e_w)^2 + \frac{\varepsilon_{32}}{4} (\tilde{\delta}_2^T \tilde{\delta}_2)^2, \\ \frac{1}{2} \text{Tr}(W_{c3}^T(t) J^{-T} (2e_w e_w^T + e_w^T e_w I) J^{-1} W_{c3}(t)) &\leq \frac{3\varepsilon_{33}}{4} \|J^{-1}\|_F^4 \|W_{c3}(t)\|_F^4 (e_w^T e_w)^2 + \frac{3}{4\varepsilon_{33}}, \\ \frac{1}{2} \text{Tr}(G_2^T J^{-T} (2e_w e_w^T + e_w^T e_w I) J^{-1} G_2) &\leq \frac{3\varepsilon_{34}}{4} \|J^{-1}\|_F^4 \|G_2\|_F^4 (e_w^T e_w)^2 + \frac{3}{4\varepsilon_{34}}, \\ -e_w^T e_w e_w^T J^{-1} W_{c20}(t, \tilde{\delta}_1) &\leq \frac{3}{4\varepsilon_{35}^{(1/3)}} \Pi(e_w^T e_w)^2 + \frac{\varepsilon_{35}}{2} (\tilde{\delta}_1^T \tilde{\delta}_1)^2, \end{aligned} \quad (84)$$

where $\Pi = \|J^{-1}\| \|W_{c21}(t)\|^{(4/3)} \|\Delta_{502}(t)\|^{(4/3)} \|\alpha_{31}\|^{(4/3)} \|F(t)\|^{(4/3)} + \|J^{-1}\| \|W_{c21}(t)\|^{(4/3)} \|\Delta_{512}(t)\|^{(4/3)} \|\alpha_{32}\|^{(4/3)} \|F(t)\|^{(4/3)}$. Hence, based on (83), we get

$$\begin{aligned}
\mathcal{L}V_{41}(t) \leq & e_w^T e_w e_w^T J^{-1} u_2 - e_w^T e_w e_w^T J^{-1} W_{c1}(t) + \frac{3}{4\epsilon_{31}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} \|W_{c2}(t)\|_F^{(4/3)} (e_w^T e_w)^2 + \frac{\epsilon_{31}}{4} (\tilde{\delta}_1^T \tilde{\delta}_1)^2 \\
& + \frac{3}{4\epsilon_{32}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} (e_w^T e_w)^2 + \frac{\epsilon_{32}}{4} (\tilde{\delta}_2^T \tilde{\delta}_2)^2 + \frac{3\epsilon_{33}}{4} \|J^{-1}\|_F^4 \|W_{c3}(t)\|^4 (e_w^T e_w)^2 + \frac{3}{4\epsilon_{33}} \\
& + \frac{3\epsilon_{34}}{4} \|J^{-1}\|_F^4 \|G_2\|^4 (e_w^T e_w)^2 + \frac{3}{4\epsilon_{34}} + \frac{3}{4\epsilon_{35}^{(1/3)}} \Pi (e_w^T e_w)^2 + \frac{\epsilon_{35}}{4} (\tilde{\delta}_1^T \tilde{\delta}_1)^2.
\end{aligned} \tag{85}$$

Design the controller u_2 as

$$\begin{aligned}
u_2 = & - \left(k_7 + \frac{3}{4\epsilon_{31}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} \|W_{c2}(t)\|_F^{(4/3)} + \frac{3}{4\epsilon_{32}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} + \frac{3\epsilon_{33}}{4} \|J^{-1}\|_F^4 \|W_{c3}(t)\|^4 \right. \\
& \left. + \frac{3\epsilon_{34}}{4} \|J^{-1}\|_F^4 \|G_2\|^4 + \frac{3}{4\epsilon_{35}^{(1/3)}} \Pi \right) e_w + W_{c1}(t) + u_3,
\end{aligned} \tag{86}$$

where u_3 is designed in the following steps.

$\mathcal{L}V_{41}(t)$ can be written by

$$\mathcal{L}V_{41}(t) \leq -k_7 (e_w^T e_w)^2 + \left(\frac{\epsilon_{31}}{4} + \frac{\epsilon_{35}}{4} \right) (\tilde{\delta}_1^T \tilde{\delta}_1)^2 + \frac{\epsilon_{32}}{4} (\tilde{\delta}_2^T \tilde{\delta}_2)^2 + e_w^T e_w e_w^T J^{-1} u_2 + \frac{3}{4\epsilon_{33}} + \frac{3}{4\epsilon_{34}}. \tag{87}$$

Define Lyapunov function as follows:

$$V(t) = V_{02}(t) + V_3(t) + V_{41}. \tag{88}$$

Combining (44), (77), and (87), the infinitesimal generator of (88) is represented as

$$\begin{aligned}
\mathcal{L}V(t) \leq & - \left(l_1 - H_{23} - \frac{\epsilon_{31}}{4} - \frac{\epsilon_{35}}{4} \right) (\tilde{\delta}_1^T \tilde{\delta}_1)^2 - \left(l_2 - \frac{\epsilon_{32}}{4} \right) (\tilde{\delta}_2^T \tilde{\delta}_2)^2 - k_4 e_p^T e_p - k_4 e_v^T e_v \\
& - k_5 \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 - \left(k_6 - \frac{3\epsilon_{28}^{(1/3)}}{4} \right) e_\psi^4 - k_7 (e_w^T e_w)^2 + e_w^T e_w e_w^T J^{-1} u_3 \\
& + \frac{\epsilon_{21}}{4} \|\hat{R}_3\|_F^4 \|E_0\|^4 (e_{w_1}^T e_{w_1})^2 + \frac{1}{4\epsilon_{28}} \|E_1\|^4 (e_w^T e_w)^2 + H_3,
\end{aligned} \tag{89}$$

where

$$u_3 = - \left(\frac{\epsilon_{21}}{4} \|\hat{R}_3\|_F^4 \|E_0\|^4 + \frac{1}{4\epsilon_{28}} \|E_1\|^4 \right) J e_w. \tag{91}$$

$$H_3 = H_2 + \frac{3}{4\epsilon_{33}} + \frac{3}{4\epsilon_{34}} + H_{02}. \tag{90}$$

Then, from (89),

Recalling that $e_{w_1}^T e_{w_1} \leq e_w^T e_w$, u_3 is designed as

$$\begin{aligned}
\mathcal{L}V(t) \leq & - \left(l_1 - H_{23} - \frac{\epsilon_{31}}{4} - \frac{\epsilon_{35}}{4} \right) (\tilde{\delta}_1^T \tilde{\delta}_1)^2 - \left(l_2 - \frac{\epsilon_{32}}{4} \right) (\tilde{\delta}_2^T \tilde{\delta}_2)^2 - k_4 e_p^T e_p \\
& - k_4 e_v^T e_v - k_5 \left(\bar{R}_{3e}^T \bar{R}_{3e} \right)^2 - \left(k_6 - \frac{3\epsilon_{28}^{(1/3)}}{4} \right) e_\psi^4 - k_7 (e_w^T e_w)^2 + H_3.
\end{aligned} \tag{92}$$

Based on (86) and (91), (81) can be written as

$$\begin{aligned}
 Jde_w = & - \left(k_7 + \frac{3}{4\varepsilon_{31}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} \|W_{c2}(t)\|_F^{(4/3)} + \frac{3}{4\varepsilon_{32}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} + \frac{3\varepsilon_{33}}{4} \|J^{-1}\|_F^4 \|W_{c3}(t)\|^4 + \frac{3\varepsilon_{34}}{4} \|J^{-1}\|_F^4 \|G_2\|^4 \right. \\
 & \left. + \frac{3}{4\varepsilon_{35}^{(1/3)}} \Pi \right) e_w dt - \left(\frac{\varepsilon_{21}}{4} \|\hat{R}_3\|_F^4 \|E_0\|^4 + \frac{1}{4\varepsilon_{28}} \|E_1\|^4 \right) J e_w dt - W_{c20}(t, \tilde{\delta}_1) dt + \tilde{\delta}_2 dt \\
 & - W_{c3}(t) d\beta_1 + G_2 d\beta_2.
 \end{aligned} \tag{93}$$

By using (40), (41), (22), (27), (68), (75), and (93), we get

$$\begin{aligned}
 d\tilde{\delta}_1 = & - \left(L_1 + \left(\frac{3\varepsilon_{01}^{(4/3)}}{4} + \frac{3\varepsilon_{02}^2}{4} \|L_1 G_1\|_F^4 \right) \right) \tilde{\delta}_1 dt - L_1 G_1 d\beta_1 - \dot{\delta}_1 dt - \frac{\varepsilon_{11}}{3} \|F(t)\|^2 F(t)^T dt, \\
 d\tilde{\delta}_2 = & - \left(l_2 + \frac{3\varepsilon_{03}^{(4/3)}}{4} + \frac{3\varepsilon_{04}^2}{4} \|L_2 G_2\|_F^4 \right) \tilde{\delta}_2 dt - \dot{\delta}_2 dt - L_2 G_2 d\beta_2, \\
 de_p = & e_v dt, \\
 de_v = & \left(-k_1 \tanh(e_p + e_v) - k_2 \tanh(e_v) + \frac{T_m}{m} \alpha \bar{R}_{3e} + \tilde{\delta}_1 \right) dt + G_1 d\beta_1, \\
 de_\psi = & (-k_6 e_\psi + E_1 e_w) dt, \\
 d\bar{R}_{3e} = & \bar{R}_{3e1}(t) dt + \bar{R}_{3e2}(t) \tilde{\delta}_1 + \bar{R}_{3e3}(t) d\beta_1, \\
 Jde_w = & - \left(k_7 + \frac{3}{4\varepsilon_{31}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} \|W_{c2}(t)\|_F^{(4/3)} + \frac{3}{4\varepsilon_{32}^{(1/3)}} \|J^{-1}\|_F^{(4/3)} + \frac{3\varepsilon_{33}}{4} \|J^{-1}\|_F^4 \|W_{c3}(t)\|^4 + \frac{3\varepsilon_{34}}{4} \|J^{-1}\|_F^4 \|G_2\|^4 \right. \\
 & \left. + \frac{3}{4\varepsilon_{35}^{(1/3)}} \Pi \right) e_w dt - \left(\frac{\varepsilon_{21}}{4} \|\hat{R}_3\|_F^4 \|E_0\|^4 + \frac{1}{4\varepsilon_{28}} \|E_1\|^4 \right) J e_w dt - W_{c20}(t, \tilde{\delta}_1) dt + \tilde{\delta}_2 dt \\
 & - W_{c3}(t) d\beta_1 + G_2 d\beta_2.
 \end{aligned} \tag{94}$$

Remark 2. In this paper, the dynamics of helicopter are modeled as the stochastic systems due to the disturbances including the random disturbances. Many control variables of the helicopter stochastic systems are discontinuous. Hence, some advanced control methods cannot be used to design the flight controller, directly, such as the dynamic surface control, fuzzy control, and other control methods. In order to construct the antidisturbance flight controller for the helicopter stochastic systems, the disturbance observer method and backstepping control scheme are applied in our paper.

Remark 3. The control method proposed in this paper shows some weaknesses, such as the complicated controller structure and tedious mathematical derivation

while the helicopter stochastic system models are first discussed in this paper and the nonlinear control method is feasible in theory. In the future, we will construct the more advanced antidisturbance flight controller with the simple structure based on the proposed control scheme of this paper.

Remark 4. The structure block diagram of the anti-disturbance control scheme is shown in Figure 1. The noises from inside and outside of the helicopter are considered to construct the helicopter models and divided as to different parts: the random disturbances and other times-varying disturbances. Then, the stochastic control theory is applied to analyze the stability of the closed-loop systems. And the stochastic antidisturbance flight controller is constructed.

FIGURE 1: The structure block diagram of the antidisturbance control scheme.

5. Stability Analysis

In this section, the asymptotic boundedness in mean square of the composite closed-loop system (94) is guaranteed under the controllers u_1 and u_2 .

Theorem 1. *Considering helicopter system (1) with disturbances, design the disturbance observers as (10) and (41) and the controllers as (20) and (86). The composite closed-loop system (94) is asymptotically bounded in mean square. If there*

exist parameters $k_1 > 0, k_2 > 0, k_3 > 0, k_4 > 0, k_5 > 0, k_6 > 0, k_7 > 0, k_8 > 0, k_9 > 0, l_1 > 0, l_2 > 0, \varepsilon_{01} > 0, \varepsilon_{02} > 0, \varepsilon_{03} > 0, \varepsilon_{04} > 0, \varepsilon_{11} > 0, \varepsilon_{12} > 0, \varepsilon_{21} > 0, \varepsilon_{22} > 0, \varepsilon_{23} > 0, \varepsilon_{231} > 0, \varepsilon_{232} > 0, \varepsilon_{24} > 0, \varepsilon_{25} > 0, \varepsilon_{26} > 0, \varepsilon_{27} > 0, \varepsilon_{28} > 0, \varepsilon_{31} > 0, \varepsilon_{32} > 0, \varepsilon_{33} > 0, \varepsilon_{34} > 0$, and $\varepsilon_{35} > 0$, such that $l_1 - H_{23} - (\varepsilon_{31}/4) - (\varepsilon_{35}/4) > 0, l_2 - (\varepsilon_{32}/4) > 0, k_4 - H_{22} > 0, k_6 - (3\varepsilon_{28}^{(1/3)}/4) > 0$ hold.

Proof. Consider the Lyapunov function $V(t)$ as follows:

$$\begin{aligned} V(t) &= V_{02}(t) + V_3(t) + V_{41}(t) \\ &= \frac{1}{4}(\tilde{\delta}_1^T \tilde{\delta}_1)^2 + \frac{1}{4}(\tilde{\delta}_2^T \tilde{\delta}_2)^2 + k_1 \int_0^{e_p + e_v} \tanh(\mu) d\mu + k_2 \int_0^{e_v} \tanh(\mu) d\mu \\ &\quad + \frac{1}{2}e_v^T e_v + \frac{1}{4}(\bar{R}_{3e}^T \bar{R}_{3e})^2 + \frac{1}{4}e_\psi^4 + \frac{1}{4}(e_w^T e_w)^2. \end{aligned} \quad (95)$$

Since $h(x) = (\tanh(x)/x)$ is a continuous function, for any $\bar{x} > 0$ and $0 \leq x \leq \bar{x}$, there exists a positive number $\beta_1 = \beta(\bar{x})$, such that $\beta_1 \|x\| \leq \|\tanh(x)\| \leq \|x\|$. Hence, we have

$$k_1 \int_0^{e_p + e_v} \tanh(\mu) d\mu + k_2 \int_0^{e_v} \tanh(\mu) d\mu \geq k_8 e_p^T e_p + k_8 e_v^T e_v, \quad (96)$$

where $k_8 = \max \left\{ \text{eig} \left\{ \begin{pmatrix} (1/2)k_1\beta_1 & (1/2)k_1\beta_1 \\ (1/2)k_1\beta_1 & (1/2)k_1\beta_1 + (1/2)k_2\beta_1 \end{pmatrix} \right\} \right\} > 0$. Moreover,

$$k_1 \int_0^{e_p + e_v} \tanh(\mu) d\mu + k_2 \int_0^{e_v} \tanh(\mu) d\mu \leq k_9 e_p^T e_p + k_9 e_v^T e_v, \quad (97)$$

where $k_9 = \min \left\{ \text{eig} \left\{ \begin{pmatrix} (1/2)k_1 & (1/2)k_1 \\ (1/2)k_1 & (1/2)k_1 + (1/2)k_2 \end{pmatrix} \right\} \right\} > 0$.

Then, we confirm that

$$\begin{aligned} &\frac{1}{4}(\tilde{\delta}_1^T \tilde{\delta}_1)^2 + \frac{1}{4}(\tilde{\delta}_2^T \tilde{\delta}_2)^2 + k_8 e_p^T e_p + \left(k_8 + \frac{1}{2}\right) e_v^T e_v + \frac{1}{4}(\bar{R}_{3e}^T \bar{R}_{3e})^2 + \frac{1}{4}e_\psi^4 + \frac{1}{4}(e_w^T e_w)^2 \\ &\leq V(t) \leq \frac{1}{4}(\tilde{\delta}_1^T \tilde{\delta}_1)^2 + \frac{1}{4}(\tilde{\delta}_2^T \tilde{\delta}_2)^2 + k_9 e_p^T e_p + \left(k_9 + \frac{1}{2}\right) e_v^T e_v + \frac{1}{4}(\bar{R}_{3e}^T \bar{R}_{3e})^2 + \frac{1}{4}e_\psi^4 + \frac{1}{4}(e_w^T e_w)^2, \end{aligned} \quad (98)$$

which satisfies (8) in Lemma 1 with $p = 2$.

From the discussion in Sections 3 and 4, we have

$$\begin{aligned} \mathcal{L}V(t) &\leq -\left(l_1 - H_{23} - \frac{\varepsilon_{31}}{4} - \frac{\varepsilon_{35}}{4}\right)(\tilde{\delta}_1^T \tilde{\delta}_1)^2 - \left(l_2 - \frac{\varepsilon_{32}}{4}\right)(\tilde{\delta}_2^T \tilde{\delta}_2)^2 - k_4 e_p^T e_p \\ &\quad - k_4 e_v^T e_v - k_5 (\bar{R}_{3e}^T \bar{R}_{3e})^2 - \left(k_6 - \frac{3\varepsilon_{28}^{(1/3)}}{4}\right) e_\psi^4 - k_7 (e_w^T e_w)^2 + H_3 \leq -\lambda V(t) + H_3, \end{aligned} \quad (99)$$

where

$$\lambda = \min \left\{ 4 \left(l_1 - H_{23} - \frac{\varepsilon_{31}}{4} - \frac{\varepsilon_{35}}{4} \right), 4 \left(l_2 - \frac{\varepsilon_{32}}{4} \right), \frac{k_4}{k_9}, \frac{k_4}{k_9 + (1/2)}, 4k_5, 4 \left(k_6 - \frac{3\varepsilon_{28}^{(1/3)}}{4} \right), 4k_7 \right\}, \quad (100)$$

which implies that $V(t)$ satisfies (9) in Lemma 1 with $p = 2$.

Consequently, based on the discussion above, the composite closed-loop system (94) is asymptotically bounded in mean square. $\square$

6. Conclusion

This paper studies the tracking control issue for nonlinear helicopter systems under stochastic disturbances and time-varying disturbances. These disturbances are attenuated and rejected via the nonlinear disturbance observer control method and stochastic control theory, and the backstepping control approach is adopted for nonlinear helicopter

systems. The problems of “dimension explosion” are analyzed for complicated nonlinear control systems.

Since the helicopter systems are modeled as the stochastic systems, many advanced nonlinear control methods can be directly used. The phenomenon of “dimension explosion” is inevitable in nonlinear controller constructing process. In the future, we will combine some advanced nonlinear control methods and stochastic control theory and propose the more advanced antidisturbance flight control scheme with simple structure.

Appendix

From (67), dw_{1c} can be written by

$$\begin{aligned} dw_{1c} = & -d \left(\left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \hat{R}_3^{-1} \bar{R}_{3e} \right) \\ & - d \left(\frac{\varepsilon_{12}\|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2}{3} \hat{R}_3^{-1} \alpha^T F(t) \right) \\ & - d \left(\frac{\varepsilon_{232}}{2\|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} \right) + d \left(\frac{\varepsilon_{11}\|F(t)\|^2}{3\|u_1\|} \hat{R}_3^{-1} E_0 F(t) \right). \end{aligned} \quad (A.1)$$

A. The Calculation of $d \left(\left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \hat{R}_3^{-1} \bar{R}_{3e} \right)$

Consider

$$\begin{aligned} & d \left(\left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \hat{R}_3^{-1} \bar{R}_{3e} \right) \\ & = d \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \hat{R}_3^{-1} \bar{R}_{3e} \\ & \quad + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) d \hat{R}_3^{-1} \bar{R}_{3e} \\ & \quad + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4 \right) \hat{R}_3^{-1} d \bar{R}_{3e}. \end{aligned} \quad (A.2)$$

For du_1 , we deduce

$$d\|u_1\| = -\frac{du_1^T \cdot u_1 + u_1^T du_1}{2\sqrt{u_1^T u_1}} = -\frac{u_1^T}{\|u_1\|} (\Delta_{01}(t)dt + \Delta_{02}(t)\tilde{\delta}_1 dt + \Delta_{03}(t)d\beta_1), \quad (\text{A.3})$$

which means that

$$\begin{aligned} d\frac{H_{24}}{4\|u_1\|^{(4/3)}} &= \frac{H_{24}}{3\|u_1\|^{(7/3)}} d\|u_1\| = \frac{H_{24}u_1^T}{3\|u_1\|^{(10/3)}} (\Delta_{01}(t)dt + \Delta_{02}(t)\tilde{\delta}_1 dt + \Delta_{03}(t)d\beta_1), \\ d\frac{3\varepsilon_{25}}{4\|u_1\|^2} &= \frac{3\varepsilon_{25}}{2\|u_1\|^3} d\|u_1\| = \frac{3\varepsilon_{25}u_1^T}{2\|u_1\|^4} (\Delta_{01}(t)dt + \Delta_{02}(t)\tilde{\delta}_1 dt + \Delta_{03}(t)d\beta_1). \end{aligned} \quad (\text{A.4})$$

For $\|\alpha\|_F$, one has

$$\alpha^T \alpha = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-R_{13} - R_{13c}}{R_{33} + R_{33c}} & \frac{-R_{23} - R_{23c}}{R_{33} + R_{33c}} \\ \frac{-R_{13} - R_{13c}}{R_{33} + R_{33c}} & \frac{-R_{23} - R_{23c}}{R_{33} + R_{33c}} \end{pmatrix} = \begin{pmatrix} 1 + \left(\frac{R_{13} + R_{13c}}{R_{33} + R_{33c}}\right)^2 & \frac{(R_{13} + R_{13c})(R_{23} + R_{23c})}{(R_{33} + R_{33c})^2} \\ \frac{(R_{13} + R_{13c})(R_{23} + R_{23c})}{(R_{33} + R_{33c})^2} & 1 + \left(\frac{R_{23} + R_{23c}}{R_{33} + R_{33c}}\right)^2 \end{pmatrix}. \quad (\text{A.5})$$

Hence,

$$\begin{aligned} \|\alpha\|_F &= (\text{Tr}(\alpha\alpha^T))^{(1/2)} = (\text{Tr}(\alpha^T \alpha))^{(1/2)} = 2 + \frac{(R_{13} + R_{13c})^2 + (R_{23} + R_{23c})^2}{(R_{33} + R_{33c})^2}, \\ d\|\alpha\|_F &= \Delta_{21}(t)dR_3 + \Delta_{21}(t)\frac{1}{\sqrt{u_1^T u_1}} \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) \Delta_{01}(t)dt + \Delta_{21}(t)\frac{1}{\sqrt{u_1^T u_1}} \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) \Delta_{02}(t)\tilde{\delta}_1 dt \\ &\quad + \Delta_{21}(t)\frac{1}{\sqrt{u_1^T u_1}} \left(I_3 - \frac{u_1 u_1^T}{u_1^T u_1} \right) \Delta_{03}(t)d\beta_1, \end{aligned} \quad (\text{A.6})$$

where

$$\Delta_{21}(t) = \left(\frac{2(R_{13} + R_{13c})}{(R_{33} + R_{33c})^2} \frac{2(R_{23} + R_{23c})}{(R_{33} + R_{33c})^2} \frac{2(R_{13} + R_{13c})^2 + (R_{23} + R_{23c})^2}{(R_{33} + R_{33c})^2} \right). \quad (\text{A.7})$$

From the above discussion,

$$d\left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}} \|\alpha\|_F^4 \right) = \Delta_{31}(t)dt + \Delta_{32}(t)\tilde{\delta}_1 dt + \Delta_{33}(t)d\beta_1, \quad (\text{A.8})$$

where

$$\begin{aligned}
 \Delta_{31}(t) &= \frac{H_{24}u_1^T}{3\|u_1\|^{(10/3)}}\Delta_{01}(t) + \frac{3\varepsilon_{25}u_1^T}{2\|u_1\|^4}\Delta_{01}(t) + \frac{1}{\varepsilon_{27}}\|\alpha\|_F^3\Delta_{22}(t), \\
 \Delta_{32}(t) &= \frac{H_{24}u_1^T}{3\|u_1\|^{(10/3)}}\Delta_{02}(t) + \frac{3\varepsilon_{25}u_1^T}{2\|u_1\|^4}\Delta_{02}(t) + \frac{1}{\varepsilon_{27}}\|\alpha\|_F^3\Delta_{23}(t), \\
 \Delta_{33}(t) &= \frac{H_{24}u_1^T}{3\|u_1\|^{(10/3)}}\Delta_{03}(t) + \frac{3\varepsilon_{25}u_1^T}{2\|u_1\|^4}\Delta_{03}(t) + \frac{1}{\varepsilon_{27}}\|\alpha\|_F^3\Delta_{24}(t).
 \end{aligned} \tag{A.9}$$

Note that

$$\begin{aligned}
 \widehat{R}_3^{-1} &= \frac{1}{\|\widehat{R}_3\|} \begin{pmatrix} R_{22} & -R_{11} \\ R_{21} & -R_{12} \end{pmatrix} = \frac{1}{\cos\phi \cos\theta} \begin{pmatrix} R_{22} & -R_{11} \\ R_{21} & -R_{12} \end{pmatrix} \\
 &= \frac{1}{R_{33}} \begin{pmatrix} R_{22} & -R_{11} \\ R_{21} & -R_{12} \end{pmatrix},
 \end{aligned} \tag{A.10}$$

In fact, $dR = R \times w dt$, which is

$$\begin{pmatrix} dR_{11} & dR_{12} & dR_{13} \\ dR_{21} & dR_{22} & dR_{23} \\ dR_{31} & dR_{32} & dR_{33} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}. \tag{A.12}$$

Thus, $d\widehat{R}_3^{-1}$ can be obtained for the controller u_2 . Moreover, we can easily get that

then we have

$$d\widehat{R}_3^{-1} = \frac{-dR_{33}}{R_{33}^2} \begin{pmatrix} R_{22} & -R_{11} \\ R_{21} & -R_{12} \end{pmatrix} + \frac{1}{R_{33}} \begin{pmatrix} dR_{22} & -dR_{11} \\ dR_{21} & -dR_{12} \end{pmatrix}. \tag{A.11}$$

$$d\left(\left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4\right)\widehat{R}_3^{-1}\overline{R}_{3e}\right) = \Delta_{41}(t)dt + \Delta_{42}(t)\tilde{\delta}_1 dt + \Delta_{43}(t)d\beta_1, \tag{A.13}$$

where

$$\begin{aligned}
 \Delta_{41}(t) &= \widehat{R}_3^{-1}\overline{R}_{3e}\Delta_{31}(t) + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4\right)d\widehat{R}_3^{-1}\overline{R}_{3e} \\
 &\quad + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4\right)\widehat{R}_3^{-1}\overline{R}_{3e1}(t), \\
 \Delta_{42}(t) &= \widehat{R}_3^{-1}\overline{R}_{3e}\Delta_{32}(t) + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4\right)\widehat{R}_3^{-1}\overline{R}_{3e2}(t), \\
 \Delta_{43}(t) &= \widehat{R}_3^{-1}\overline{R}_{3e}\Delta_{33}(t) + \left(k_5 + H_{25} + \frac{H_{24}}{4\|u_1\|^{(4/3)}} + \frac{3\varepsilon_{25}}{4\|u_1\|^2} + \frac{1}{4\varepsilon_{27}}\|\alpha\|_F^4\right)\widehat{R}_3^{-1}\overline{R}_{3e3}(t).
 \end{aligned} \tag{A.14}$$

B. The Calculation of $d((\varepsilon_{12} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 / 3) \widehat{R}_3^{-1} \alpha^T F(t))$

Consider

$$\begin{aligned}
 & d\left(\frac{\varepsilon_{12} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2}{3} \widehat{R}_3^{-1} \alpha^T F(t)\right) \\
 &= \varepsilon_{12} \|u_1\|^2 d\|u_1\| \|\alpha\|^2 \|F(t)\|^2 \widehat{R}_3^{-1} \alpha^T F(t) + \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\| d\|\alpha\| \|F(t)\|^2 \widehat{R}_3^{-1} \alpha^T F(t) \\
 &+ \frac{2\varepsilon_{12}}{3} \|u_1\| \|\alpha\|^2 \|F(t)\| d\|F(t)\| \widehat{R}_3^{-1} \alpha^T F(t) + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 d\widehat{R}_3^{-1} \alpha^T F(t) \\
 &+ \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \widehat{R}_3^{-1} d\alpha^T F(t) + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \widehat{R}_3^{-1} \alpha^T dF(t).
 \end{aligned} \tag{B.1}$$

Hence,

$$\begin{aligned}
 & \varepsilon_{12} \|u_1\|^2 d\|u_1\| \|\alpha\|^2 \|F(t)\|^2 \widehat{R}_3^{-1} \alpha^T F(t) \\
 &= \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\| \|F(t)\|^2 \widehat{R}_3^{-1} \alpha^T F(t) \left(\Delta_{22}(t) dt + \Delta_{23}(t) \widetilde{\delta}_1 dt + \Delta_{24}(t) d\beta_1 \right).
 \end{aligned} \tag{B.2}$$

For $(2\varepsilon_{12}/3) \|u_1\|^3 \|\alpha\|^2 \|F(t)\| d\|F(t)\| \widehat{R}_3^{-1} \alpha^T F(t)$, we first calculate the $dF(t)$:

$$dF(t) = \Delta_{401}(t) dt + \Delta_{402}(t) \widetilde{\delta}_1 dt + \Delta_{403}(t) d\beta_1, \tag{B.3}$$

where

$$\begin{aligned}
 \Delta_{401}(t) &= ((k_1 + k_2 + 1)I + k_1 \tanh^2(e_p + e_v) + k_2 (\tanh^2(e_v))) (-k_1 \tanh(e_p + e_v) - k_2 \tanh(e_v) + \frac{T_m}{m} \alpha \bar{R}_{3e}) \\
 &+ k_1 (I + \tanh^2(e_p + e_v)) e_v, \\
 \Delta_{402}(t) &= ((k_1 + k_2 + 1)I + k_1 \tanh^2(e_p + e_v) + k_2 (\tanh^2(e_v))), \\
 \Delta_{403}(t) &= ((k_1 + k_2 + 1)I + k_1 \tanh^2(e_p + e_v) + k_2 (\tanh^2(e_v))) G_1.
 \end{aligned} \tag{B.4}$$

Thus, we obtain

$$d\|F(t)\| = \frac{dF(t)^T \cdot F(t) + F(t)^T dF(t)}{2\sqrt{F(t)^T F(t)}} = -\frac{F(t)^T}{\|F(t)\|} (\Delta_{401}(t) dt + \Delta_{402}(t) \widetilde{\delta}_1 dt + \Delta_{403}(t) d\beta_1). \tag{B.5}$$

Then, we drive

$$\begin{aligned}
& \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\| d\|F(t)\| \hat{R}_3^{-1} \alpha^T F(t) \\
& = -\frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\| \hat{R}_3^{-1} \alpha^T F(t) \left(\frac{F(t)^T}{\|F(t)\|} (\Delta_{401}(t) dt + \Delta_{402}(t) \tilde{\delta}_1 dt + \Delta_{403}(t) d\beta_1) \right).
\end{aligned} \tag{B.6}$$

In order to calculate $(\varepsilon_{12}/3) \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} d\alpha^T F(t)$, we first give the derivative of $d\alpha$ as follows:

$$\begin{aligned}
d\alpha = & \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-(R_{33} + R_{33c})dR_{13} + (R_{13} + R_{13c})dR_{33}}{(R_{33} + R_{33c})^2} & \frac{-(R_{33} + R_{33c})dR_{23} + (R_{23} + R_{23c})dR_{33}}{(R_{33} + R_{33c})^2} \end{pmatrix} \\
& + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-(R_{33} + R_{33c})dR_{13c} + (R_{13} + R_{13c})dR_{33c}}{(R_{33} + R_{33c})^2} & \frac{-(R_{33} + R_{33c})dR_{23c} + (R_{23} + R_{23c})dR_{33c}}{(R_{33} + R_{33c})^2} \end{pmatrix}.
\end{aligned} \tag{B.7}$$

Due to

$$\begin{aligned}
\frac{-(R_{33} + R_{33c})dR_{13c} + (R_{13} + R_{13c})dR_{33c}}{(R_{33} + R_{33c})^2} & = \Delta_{501}(t) + \Delta_{502}(t) \tilde{\delta}_1 + \Delta_{503}(t) d\beta_1, \\
\frac{-(R_{33} + R_{33c})dR_{23c} + (R_{23} + R_{23c})dR_{33c}}{(R_{33} + R_{33c})^2} & = \Delta_{511}(t) + \Delta_{512}(t) \tilde{\delta}_1 + \Delta_{513}(t) d\beta_1,
\end{aligned} \tag{B.8}$$

where

$$\begin{aligned}
\Delta_{501}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{13} + R_{13c})) \Delta_{01}(t), \\
\Delta_{502}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{13} + R_{13c})) \Delta_{02}(t), \\
\Delta_{503}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{13} + R_{13c})) \Delta_{03}(t), \\
\Delta_{511}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{23} + R_{23c})) \Delta_{01}(t), \\
\Delta_{512}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{23} + R_{23c})) \Delta_{02}(t), \\
\Delta_{513}(t) & = \frac{1}{(R_{33} + R_{33c})^2} (R_{33} + R_{33c} \ 0 \ -(R_{23} + R_{23c})) \Delta_{03}(t).
\end{aligned} \tag{B.9}$$

Then, $d\alpha$ is represented as

$$d\alpha = \Delta_{51}(t) + \Delta_{52}(t, \tilde{\delta}_1) + \Delta_{53}(t)d\beta_1, \quad (\text{B.10})$$

where

$$\alpha_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{-(R_{33} + R_{33c})dR_{13} + (R_{13} + R_{13c})dR_{33}}{(R_{33} + R_{33c})^2} & \frac{-(R_{33} + R_{33c})dR_{23} + (R_{23} + R_{23c})dR_{33}}{(R_{33} + R_{33c})^2} \end{pmatrix},$$

$$\alpha_{31} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$\alpha_{32} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

(B.11)

$$\Delta_{51} = \alpha_2 + \alpha_{31}\Delta_{501}(t) + \alpha_{32}\Delta_{511}(t),$$

$$\Delta_{52}(t, \tilde{\delta}_1) = \Delta_{502}(t)\tilde{\delta}_1\alpha_{31} + \Delta_{512}(t)\tilde{\delta}_1\alpha_{32},$$

$$\Delta_{53} = \alpha_{31}\Delta_{503}(t) + \alpha_{32}\Delta_{513}(t).$$

Hence, we can obtain that

$$\frac{\varepsilon_{12}}{3}\|u_1\|^3\|\alpha\|^2\|F(t)\|^2\hat{R}_3^{-1}d\alpha^T F(t) = \frac{\varepsilon_{12}}{3}\|u_1\|^3\|\alpha\|^2\|F(t)\|^2\hat{R}_3^{-1}(\Delta_{51}(t) + \Delta_{52}(t, \tilde{\delta}_1) + \Delta_{53}(t)d\beta_1)^T F(t). \quad (\text{B.12})$$

For the part of $(\varepsilon_{12}/3)\|u_1\|^3\|\alpha\|^2\|F(t)\|^2\hat{R}_3^{-1}\alpha^T dF(t)$, we have

$$\frac{\varepsilon_{12}}{3}\|u_1\|^3\|\alpha\|^2\|F(t)\|^2\hat{R}_3^{-1}\alpha^T dF(t) = \frac{\varepsilon_{12}}{3}\|u_1\|^3\|\alpha\|^2\|F(t)\|^2\hat{R}_3^{-1}\alpha^T (\Delta_{401}(t)dt + \Delta_{402}(t)\tilde{\delta}_1 dt + \Delta_{403}(t)d\beta_1). \quad (\text{B.13})$$

Hence,

$$d\left(\frac{\varepsilon_{12}\|u_1\|^3\|\alpha\|^2\|F(t)\|^2}{3}\hat{R}_3^{-1}\alpha^T F(t)\right) = \Delta_{61}(t)dt + \Delta_{621}(t)\tilde{\delta}_1 dt + \Delta_{622}(t, \tilde{\delta}_1)dt + \Delta_{63}(t)d\beta_1, \quad (\text{B.14})$$

where

$$\begin{aligned}
\Delta_{61}(t) &= -\varepsilon_{12} \|u_1\| \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) u_1^T \Delta_{01}(t) + \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\| \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) \Delta_{22}(t) \\
&\quad - \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \hat{R}_3^{-1} \alpha^T F(t) F(t)^T \Delta_{401}(t) + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) \\
&\quad + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \Delta_{51}(t)^T F(t) + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T \Delta_{401}(t), \\
\Delta_{621}(t) &= -\varepsilon_{12} \|u_1\| \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) u_1^T \Delta_{02}(t) + \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\| \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) \Delta_{23}(t) \\
&\quad - \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \hat{R}_3^{-1} \alpha^T F(t) F(t)^T \Delta_{402}(t), \\
\Delta_{63}(t) &= -\varepsilon_{12} \|u_1\| \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) u_1^T \Delta_{03}(t) + \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\| \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T F(t) \Delta_{24}(t) \\
&\quad - \frac{2\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \hat{R}_3^{-1} \alpha^T F(t) F(t)^T \Delta_{403}(t) + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \Delta_{53}(t)^T F(t) \\
&\quad + \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \alpha^T \Delta_{403}(t), \\
\Delta_{622}(t, \tilde{\delta}_1) &= \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1} \Delta_{52}(t, \tilde{\delta}_1)^T F(t).
\end{aligned} \tag{B.15}$$

C. The Calculation of $d((\varepsilon_{232}/2 \|u_1\|^4) (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e})$

Consider

$$\begin{aligned}
d\left(\frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e}\right) &= d\frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} + \frac{\varepsilon_{232}}{2 \|u_1\|^4} d(e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} \\
&\quad + \frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 d\hat{R}_3^{-1} \bar{R}_{3e} + \frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} d\bar{R}_{3e}.
\end{aligned} \tag{C.1}$$

For the derivatives of those teams, we have

$$\begin{aligned}
d\frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} &= \frac{4\varepsilon_{232}}{2 \|u_1\|^6} (e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} u_1^T (\Delta_{01}(t) dt + \Delta_{02}(t) \tilde{\delta}_1 dt + \Delta_{03}(t) d\beta_1), \\
\frac{\varepsilon_{232}}{2 \|u_1\|^4} d(e_v^T e_v)^2 \hat{R}_3^{-1} \bar{R}_{3e} &= \frac{4\varepsilon_{232}}{2 \|u_1\|^4} \hat{R}_3^{-1} \bar{R}_{3e} e_v^T e_v^T (-k_1 \tanh(e_p + e_v) - k_2 \tanh(e_v) \\
&\quad + \frac{T_m}{m} \alpha \bar{R}_{3e} + \tilde{\delta}_1 + G_1 d\beta_1), \\
\frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} d\bar{R}_{3e} &= \frac{\varepsilon_{232}}{2 \|u_1\|^4} (e_v^T e_v)^2 \hat{R}_3^{-1} (\bar{R}_{3e1}(t) dt + \bar{R}_{3e2}(t) \tilde{\delta}_1 dt + \bar{R}_{3e3}(t) d\beta_1).
\end{aligned} \tag{C.2}$$

Therefore, one has

$$d\left(\frac{\varepsilon_{232}}{2\|u_1\|^4}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e}\right) = \Delta_{71}(t)dt + \Delta_{72}(t)\tilde{\delta}_1 dt + \Delta_{73}(t)d\beta_1, \quad (C.3)$$

where

$$\begin{aligned} \Delta_{71}(t) &= \frac{4\varepsilon_{232}}{2\|u_1\|^6}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e} u_1^T \Delta_{01}(t) + \frac{4\varepsilon_{232}}{2\|u_1\|^4} \hat{R}_3^{-1} \bar{R}_{3e} e_\nu^T e_\nu e_\nu^T (-k_1 \tanh(e_p + e_\nu) - k_2 \tanh(e_\nu)) \\ &\quad + \frac{T_m}{m} \alpha \bar{R}_{3e} + \frac{\varepsilon_{232}}{2\|u_1\|^4}(e_\nu^T e_\nu)^2 d\hat{R}_3^{-1} \bar{R}_{3e} + \frac{\varepsilon_{232}}{2\|u_1\|^4}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e1}(t), \\ \Delta_{72}(t) &= \frac{4\varepsilon_{232}}{2\|u_1\|^6}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e} u_1^T \Delta_{02}(t) + \frac{4\varepsilon_{232}}{2\|u_1\|^4} \hat{R}_3^{-1} \bar{R}_{3e} e_\nu^T e_\nu e_\nu^T + \frac{\varepsilon_{232}}{2\|u_1\|^4}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e2}(t), \\ \Delta_{73}(t) &= \frac{4\varepsilon_{232}}{2\|u_1\|^6}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e} u_1^T \Delta_{03}(t) + \frac{4\varepsilon_{232}}{2\|u_1\|^4} \hat{R}_3^{-1} \bar{R}_{3e} e_\nu^T e_\nu e_\nu^T G_1 + \frac{\varepsilon_{232}}{2\|u_1\|^4}(e_\nu^T e_\nu)^2 \hat{R}_3^{-1} \bar{R}_{3e3}(t). \end{aligned} \quad (C.4)$$

D. The Calculation of $d((\varepsilon_{11}\|F(t)\|^2/3\|u_1\|)\hat{R}_3^{-1}E_0F(t))$

Consider

$$\begin{aligned} d\left(\frac{\varepsilon_{11}\|F(t)\|^2}{3\|u_1\|}\hat{R}_3^{-1}E_0F(t)\right) &= d\frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2\hat{R}_3^{-1}E_0F(t) + \frac{\varepsilon_{11}}{3\|u_1\|}d\|F(t)\|^2\hat{R}_3^{-1}E_0F(t) \\ &\quad + \frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2d\hat{R}_3^{-1}E_0F(t) + \frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2\hat{R}_3^{-1}E_0dF(t). \end{aligned} \quad (D.1)$$

For the derivatives of those teams, we have

$$\begin{aligned} d\frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2\hat{R}_3^{-1}E_0F(t) &= \frac{\varepsilon_{11}}{3\|u_1\|^3}\|F(t)\|^2\hat{R}_3^{-1}E_0F(t)u_1^T(\Delta_{01}(t)dt + \Delta_{02}(t)\tilde{\delta}_1 dt + \Delta_{03}(t)d\beta_1) \\ \frac{\varepsilon_{11}}{3\|u_1\|}d\|F(t)\|^2\hat{R}_3^{-1}E_0F(t) &= \frac{2\varepsilon_{11}}{3\|u_1\|}\|F(t)\|\hat{R}_3^{-1}E_0F(t)d\|F(t)\|, \\ &= -\frac{2\varepsilon_{11}}{3\|u_1\|}\|F(t)\|\hat{R}_3^{-1}E_0F(t)\frac{F(t)^T}{\|F(t)\|}(\Delta_{401}(t)dt + \Delta_{402}(t)\tilde{\delta}_1 dt + \Delta_{403}(t)d\beta_1), \\ \frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2\hat{R}_3^{-1}E_0dF(t) &= \frac{\varepsilon_{11}}{3\|u_1\|}\|F(t)\|^2\hat{R}_3^{-1}E_0(\Delta_{401}(t)dt + \Delta_{402}(t)\tilde{\delta}_1 dt + \Delta_{403}(t)d\beta_1). \end{aligned} \quad (D.2)$$

Hence, we deduce

$$d\left(\frac{\varepsilon_{11}\|F(t)\|^2}{3\|u_1\|}\hat{R}_3^{-1}E_0F(t)\right) = \Delta_{81}(t)dt + \Delta_{82}(t)\tilde{\delta}_1 dt + \Delta_{83}(t)d\beta_1, \quad (D.3)$$

where

$$\begin{aligned}
 \Delta_{81}(t) &= \frac{\varepsilon_{11}}{3\|u_1\|^3} \|F(t)\|^2 \hat{R}_3^{-1} E_0 F(t) u_1^T \Delta_{01}(t) - \frac{2\varepsilon_{11}}{3\|u_1\|} \hat{R}_3^{-1} E_0 F(t) F(t)^T \Delta_{401}(t) \\
 &\quad + \frac{\varepsilon_{11}}{3\|u_1\|} \|F(t)\|^2 \hat{R}_3^{-1} E_0 F(t) + \frac{\varepsilon_{11}}{3\|u_1\|} \|F(t)\|^2 \hat{R}_3^{-1} E_0 \Delta_{401}(t) \\
 \Delta_{82}(t) &= \frac{\varepsilon_{11}}{3\|u_1\|} \|F(t)\|^2 \hat{R}_3^{-1} E_0 F(t) u_1^T \Delta_{02}(t) - \frac{2\varepsilon_{11}}{3\|u_1\|} \hat{R}_3^{-1} E_0 F(t) F(t)^T \Delta_{402}(t) + \frac{\varepsilon_{11}}{3\|u_1\|^3} \|F(t)\|^2 \hat{R}_3^{-1} E_0 \Delta_{402}(t), \\
 \Delta_{83}(t) &= \frac{\varepsilon_{11}}{3\|u_1\|} \|F(t)\|^2 \hat{R}_3^{-1} E_0 F(t) u_1^T \Delta_{03}(t) - \frac{2\varepsilon_{11}}{3\|u_1\|} \hat{R}_3^{-1} E_0 F(t) F(t)^T \Delta_{403}(t) + \frac{\varepsilon_{11}}{3\|u_1\|^3} \|F(t)\|^2 \hat{R}_3^{-1} E_0 \Delta_{403}(t).
 \end{aligned} \tag{D.4}$$

E. The Calculation of dw_{1c}

dw_{1c} can be obtained as

$$dw_{1c} = W_{1c1}(t)dt + W_{1c2}(t)\tilde{\delta}_1 dt + \Delta_{622}(t, \tilde{\delta}_1)dt + W_{1c3}(t)d\beta_1, \tag{E.1}$$

where

$$\begin{aligned}
 W_{1c1}(t) &= \Delta_{41}(t) + \Delta_{61}(t) + \Delta_{71}(t) + \Delta_{81}(t), \\
 W_{1c2}(t) &= \Delta_{42}(t) + \Delta_{621}(t) + \Delta_{72}(t) + \Delta_{82}(t), \\
 W_{1c3}(t) &= \Delta_{43}(t) + \Delta_{63}(t) + \Delta_{73}(t) + \Delta_{83}(t).
 \end{aligned} \tag{E.2}$$

F. The Calculation of dr_c

For r_c , we have

$$r_c = -\frac{\cos \theta}{\cos \phi} \left(\frac{\sin \phi}{\cos \theta} w_{1c} - d\psi_r + k_3 e_\psi \right) = -\tan \phi (E_2 q_c - d\psi_r + k_3 e_\psi), \tag{F.1}$$

where $E_2 = \begin{pmatrix} 0 & 1 \end{pmatrix}$. Define $\Omega = \begin{pmatrix} \phi & \theta & \psi \end{pmatrix}$, then the derivative of Ω is

$$\dot{\Omega} = \begin{pmatrix} 1 & S\phi T\theta & C\phi T\theta \\ 0 & C\phi & -S\phi \\ 0 & \frac{S\phi}{C\theta} & \frac{C\phi}{C\theta} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}, \tag{F.2}$$

which means that $d\psi$ is available for control designed. Then, dr_c can be denoted by

$$dr_c = W_{rc1}(t)dt + W_{rc2}(t)\tilde{\delta}_1 dt + W_{rc20}(t, \tilde{\delta}_1)dt + W_{rc3}(t)d\beta_1, \tag{F.3}$$

where

$$\begin{aligned}
 W_{rc1}(t) &= -(E_2 w_{1c} - d\psi_r + k_3 e_\psi)(1 + \tan^2 \phi) d\phi \\
 &\quad - \tan \phi E_2 W_{1c1}(t)dt + \tan \phi d\dot{\psi}_r \\
 &\quad - \tan \phi k_6 (-k_6 e_\psi + E_1 e_w), \\
 W_{rc2}(t) &= -\tan \phi E_2 W_{1c2}(t), \\
 W_{rc20}(t, \tilde{\delta}_1) &= -\tan \phi E_2 \Delta_{622}(t, \tilde{\delta}_1), \\
 W_{rc3}(t) &= -\tan \phi E_2 W_{1c3}.
 \end{aligned} \tag{F.4}$$

G. The Calculation of dw_c

The derivative of w_c is obtained as follows:

$$dw_c = \left(\frac{dw_{1c}}{dr_c} \right) = W_{c1}(t)dt + W_{c2}(t)\tilde{\delta}_1 + W_{c20}(t, \tilde{\delta}_1) + W_{c3}(t)d\beta_1. \quad (G.1)$$

Notice that

$$W_{c20}(t, \tilde{\delta}_1) = \begin{pmatrix} \Delta_{622}(t, \tilde{\delta}_1) \\ W_{rc20}(t, \tilde{\delta}_1) \end{pmatrix} = W_{c21}(t)(\Delta_{502}(t)\tilde{\delta}_1\alpha_{31} + \Delta_{512}(t)\tilde{\delta}_1\alpha_{32})^T F(t), \quad (G.2)$$

with

$$W_{c21}(t) = \begin{pmatrix} 1 \\ -\tan \phi E_2 \end{pmatrix} \frac{\varepsilon_{12}}{3} \|u_1\|^3 \|\alpha\|^2 \|F(t)\|^2 \hat{R}_3^{-1}, \quad (G.3)$$

which implies that

$$\|W_{c20}(t, \tilde{\delta}_1)\| \leq \|W_{c21}(t)\| \|\Delta_{502}(t)\| \|\tilde{\delta}_1\| \|\alpha_{31}\| \|F(t)\| + \|W_{c21}(t)\| \|\Delta_{502}(t)\| \|\tilde{\delta}_1\| \|\alpha_{31}\| \|F(t)\|. \quad (G.4)$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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