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Research Article

Padé-Approximation-Based Preview Repetitive Control for Continuous-Time Linear Systems

Jia-Yu Zhao[1], **Zhao-Hong Wang**[1], **Jian-De Yan**[2], **and Yong-Hong Lan**[1]

[1]*School of Automation and Electronic Information, Xiangtan University, Xiangtan 411105, Hunan, China*
[2]*School of Electrical and Information Engineering, Hunan Institute of Engineering, Xiangtan 411105, Hunan, China*

Correspondence should be addressed to Jia-Yu Zhao; 806988027@qq.com

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This paper concerns a Padé-approximation-based preview repetitive control (PRC) scheme for continuous-time linear systems. Firstly, the state space representation of the repetitive controller is transformed into a nondelayed one by Padé approximation. Then, an augmented dynamic system is constructed by using the nominal state equation with the error system and the state equation of a repetitive controller. Next, by using optimal control theory, a Padé-approximation-based PRC law is obtained. It consists of state feedback, error integral compensation, output integral of repetitive controller, and preview compensation. Finally, the effectiveness of the method is verified by a numerical simulation.

1. Introduction

Preview control (PC) is an extended feedforward control method. By making full use of the known future reference signal or disturbance signal, the performance of the closed-loop system can be improved. PC has received considerable attention since it was introduced by Tomizuka in 1970s. For more than five decades, the most extensive research into preview control has focused on the linear quadratic optimal control problem with preview compensation [1–5], especially for discrete-time systems. Subsequently, linear matrix inequality (LMI) technique has been extensively used to handle the PC problems for uncertain discrete-time systems [6]. Combining LMI-based PC with other control schemes, some new concepts are proposed, such as adaptive PC [7], fault tolerant PC [8], H_∞ PC [9], observer-based PC [10], and distributed PC [11]. However, in continuous-time systems, the methods that can be used to deal with PC issue are relatively limited, and differential operation is widely used instead of differential operation [12–16].

On the contrary, repetitive control (RC) is an effective technique to improve the performance of tracking periodic reference or suppressing periodic disturbance. It was originated in 1980s by Inoue et al. [17] and then developed by Hara et al. [18] and Doh et al. [19]. Over the last a couple of decades, a great deal of research has been devoted to the theory and applications of the RC. A summary of RC works can be referred to in [20], and recent literatures of RC can be found in [21–24].

The preview repetitive control strategy(PRC), which combines preview control and repetitive control, can significantly improve the control performance of the closed-loop system, especially the tracking of periodic signals. Through the difference operator, the relationship between the system and the future signal and the relationship between the repetitive controller and the tracking error are established. Then, preview information can be fused with the plant object to form an augmented autonomous discrete system [25–27]. Recently, benefiting from LMI technology, PRC law can combine with other control schemes, such as robust sliding-mode PRC law [28], generalized-discrete-time optimal PRC method [29], and robust guaranteed-cost PRC scheme [30]. In addition, Li [31] proposed an observer-based PRC strategy for uncertain discrete-time systems using two-dimensional model approach. In [32], the design method of adaptive fuzzy finite time control for switched pure feedback nonlinear systems with given performance based on the observer is discussed. The problem of output feedback control for discrete-time systems with two quantized signals in measurement output and control input was discussed in [33].

It is worth pointing out that the above studies about PRC only focus on discrete-time system and less on continuous-time systems. In [34], the optimal PRC for continuous-time linear system was obtained and the result was applied to the tracking problem of permanent magnet synchronous motor drive system. However, due to the time-delay part of RC, Padé-approximation technique can be used to reduce the complexity of constructing augmented error system. This observation drives our current research.

This paper focuses on the Padé-approximation-based PRC problem of linear continuous-time systems. Main contributions are summarized as follows. (1) The state space representation of the repetitive controller is transformed into a delay-free state space representation by Padé approximation. (2) An augmented dynamic system is obtained by processing the repetitive controller and tracking error signal. (3) Based on the optimal control theory, the regulator problem of the augmented system is solved, from which the optimal PRC law for the original system can be derived.

This paper is organized as follows. Section 2 presents the problem formulation and the basic assumptions. The Padé-approximation-based PRC law is derived in Section 3. Section 4 provides a numerical simulation. Finally, some conclusions are drawn in Section 5.

2. Problem Statement

Consider the following linear system:

$$\begin{cases} \dot{x}_p(t) = Ax_p(t) + Bu(t), \\ y(t) = Cx_p(t) + Du(t), \end{cases} \tag{1}$$

where $x_p(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^q$ is the control input, and $y(t) \in \mathbb{R}^m$ is the output of the plant. A, B, C, and D are constant matrices with appropriate dimensions. For simplicity, this paper only considers the single-input single-output plant $(q = m = 1)$.

Figure 1 shows the basic configuration of the repetitive control system, where $G(s)$ is the controlled plant, $r(t)$ is a periodic reference input with period L, and $C_R(s)$ is a repetitive controller. The output of the repetitive controller $v(t)$ is

$$\begin{cases} v(t) = v(t-L) + e(t), \\ v(t) = 0, \quad t \in [-L, 0], \end{cases} \tag{2}$$

where

$$e(t) = r(t) - y(t) \tag{3}$$

is the tracking error.

The following assumptions will be needed throughout the paper.

Assumption 1. The reference input $r(t) \in \mathbb{R}^m$ to be tracked by $y(t)$ is a periodic reference signal and the period is L. Furthermore, $r(t)$ is piecewise differentiable. At some nondifferential points t, we take the left derivative $\dot{r}(t-0)$ or the right derivative $\dot{r}(t+0)$ instead of $\dot{r}(t)$.

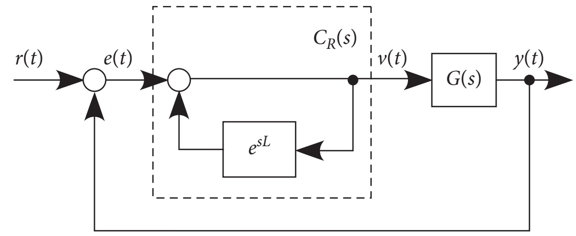

Figure 1: Repetitive control system.

Assumption 2. The periodic reference input $r(t)$ is previewable in the sense that the future value of $r(\sigma)\,(t \le \sigma \le t + l_r)$ is available at each instant of time t. The value of l_r is presented as the preview length of the reference input $r(t)$.

Assumption 3. System (1) is finite spectrum controllability. Namely, the pair (A, B) is stabilizable, and the following condition holds:

$$\operatorname{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + m. \tag{4}$$

Assumption 4. (C, A) is detectable.

Remark 1. The overall block diagram in Figure 2 consists of four blocks: (i) repetitive controller block, (ii) preview control compensator block, (iii) state feedback controller block, and (iv) error integral compensator block. The repetitive controller block generates the periodic signal of period L and is used to improve the learning performance between repetition periods. The preview control compensator block is used to improve the tracking performance by using the future value of the desired reference input signal. The state feedback controller block is used to improve the stability of the system in each period. The error integral compensator block is used to reduce static error.

The objective of this brief is to develop a Padé-approximation-based PRC law as described in Figure 2 such that, in the steady state, the output vector $y(t)$ of system (1) tracks the desired output $r(t)$ without static error, namely,

$$\lim_{t \longrightarrow \infty} e(t) = 0. \tag{5}$$

Note that, in Figure 1,

$$C_R(s) = \frac{V(s)}{E(s)} = \frac{1}{1 - e^{-sL}}, \tag{6}$$

where $V(s)$ and $E(s)$ are Laplace transforms of $v(t)$ and $e(t)$, respectively.

From Padé-approximation formula [35, 36],

$$e^{-sL} = \frac{1 - (1/2)Ls + (5/44)(Ls)^2 - (1/66)(Ls)^3 + \cdots}{1 + (1/2)Ls + (5/44)(Ls)^2 + (1/66)(Ls)^3 + \cdots}, \tag{7}$$

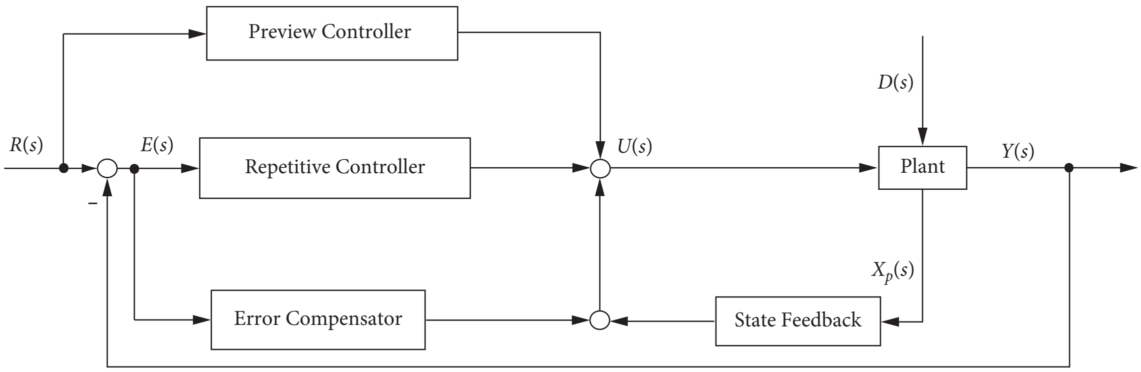

FIGURE 2: Block diagram of the closed-loop system under Padé-approximation-based PRC.

for simplicity, let

$$e^{-sL} \approx \frac{1-(1/2)Ls}{1+(1/2)Ls}. \tag{8}$$

Substituting (8) into the formula of (6) yields

$$sV(s) = \frac{1}{L}E(s) + \frac{s}{2}E(s). \tag{9}$$

Through the inverse Laplace transform of formula (9), we obtain

$$\dot{v}(t) = \frac{1}{L}e(t) + \frac{1}{2}\dot{e}(t). \tag{10}$$

Remark 2. Different from (2), there is no time delay in the state space model of the RC system represented by (10), so an augmented error system without time delay can be obtained.

3. Design Controller of Preview Repetitive Control

In this section, by using the optimal control theory, we will design a Padé-approximation-based preview repetitive controller for plant (1).

Introducing the $\overline{n}\times 1$ augmented vector $z(t)$,

$$z(t) = \begin{bmatrix} \dot{x}_p(t) \\ e(t) \\ v(t) \end{bmatrix}, \tag{11}$$

where $\overline{n} = n+2m$, from (1), (2), and (10), we get the following augmented dynamic system:

$$\dot{z}(t) = \overline{A}z(t) + \overline{B}\dot{u}(t) + \overline{D}_1\dot{r}(t), \tag{12}$$

where $\overline{A}, \overline{B}$, and $\overline{D}_1$ are constant matrices defined by

$$\begin{cases} \overline{A} = \begin{bmatrix} A & 0 & 0 \\ -C & 0 & 0 \\ -\dfrac{C}{2} & \dfrac{1}{L}I_m & 0 \end{bmatrix} \in \mathbb{R}^{\overline{n}\times n}, \\ \overline{B} = \begin{bmatrix} B \\ -D \\ -\dfrac{D}{2} \end{bmatrix} \in \mathbb{R}^{\overline{n}\times m}, \\ \overline{D}_1 = \begin{bmatrix} 0 \\ I_m \\ \dfrac{I_m}{2} \end{bmatrix} \in \mathbb{R}^{\overline{n}\times m}. \end{cases} \tag{13}$$

System (12) is called an augmented dynamic system of (1). To obtain an optimal controller, we wish to find a controller $u(t)$ that minimizes

$$J = \lim_{t_a\longrightarrow\infty} \frac{1}{2}\int_{t_0}^{t_a}\left(z^T(t)Q_z z(t) + \dot{u}^T(t)R\dot{u}(t)\right)\mathrm{d}t, \tag{14}$$

subject to the state equation (12), where $Q_z \in \mathbb{R}^{\overline{n}\times\overline{n}}$ and $R \in \mathbb{R}^{q\times q}$ are both positive definite matrices, $Q_z = \mathrm{diag}\left[\,Q_x \;\; Q_e \;\; Q_v\,\right]$, $Q_x \in \mathbb{R}^{n\times n}$, $Q_e \in \mathbb{R}^{m\times m}$, and $Q_v \in \mathbb{R}^{m\times m}$.

Therefore, the problem of PRC design based on Padé approximation can be transformed into solving the optimal control input $u(t)$ of system (12) under the performance index (14). To this end, we have to ensure the existence conditions of stabilizability of $(\overline{A}, \overline{B})$ and the detectability of $(Q_z^{(1/2)}, \overline{A})$.

Lemma 1. *$(\overline{A}, \overline{B})$ is stabilizable if and only if Assumption 3 holds.*

Proof. Let C_+ be the closed right-half of the complex plane. According to PBH rank test, $(\overline{A}, \overline{B})$ is stabilizable if and only if the matrix $(\overline{A} - sI_{\overline{n}}, \overline{B})$ is of full row rank for any $s \in C_+$. Then, according to the structure of the matrices $\overline{A}$ and $\overline{B}$, we obtain

$$\operatorname{rank}\begin{bmatrix} \overline{A} - sI_{\overline{n}} & \overline{B} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} A - sI_n & 0 & 0 & B \\ -C & -sI_m & 0 & -D \\ -\dfrac{C}{2} & -\dfrac{1}{L}I_m & -sI_m & -\dfrac{D}{2} \end{bmatrix}. \quad (15)$$

When $s = 0$, we have

$$\operatorname{rank}\begin{bmatrix} \overline{A} - sI_{\overline{n}} & \overline{B} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} A & 0 & 0 & B \\ -C & 0 & 0 & -D \\ -\dfrac{C}{2} & -\dfrac{1}{L}I_m & 0 & -\dfrac{D}{2} \end{bmatrix}. \quad (16)$$

Because $(1/L)I_m$ is reversible, the matrix $[\overline{A} - sI_{\overline{n}}, \overline{B}]$ is of full row rank if and only if

$$\operatorname{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + m. \quad (17)$$

When $s > 0$, by elementary transformation, we obtain

$$\begin{aligned} &\operatorname{rank}\begin{bmatrix} \overline{A} - sI_{\overline{n}} & \overline{B} \end{bmatrix} \\ &\longrightarrow \operatorname{rank}\begin{bmatrix} A - sI_n & 0 & 0 & B \\ 0 & -sI_m & 0 & -D \\ 0 & 0 & -sI_m & -\dfrac{D}{2} \end{bmatrix} \\ &\longrightarrow \operatorname{rank}\begin{bmatrix} A - sI_n & 0 & 0 & B \\ 0 & -sI_m & 0 & 0 \\ 0 & 0 & -sI_m & 0 \end{bmatrix}. \end{aligned} \quad (18)$$

Because $-sI_m$ is reversible, the matrix $[\overline{A} - sI_{\overline{n}}, \overline{B}]$ is of full row rank if and only if $[A - sI_{\overline{n}}, B]$ is of full row rank (namely, the pair (A, B) is stabilizable). The proof is completed. □

Lemma 2. *If Q_z is a positive definite matrix, then $(Q_z^{(1/2)}, \overline{A})$ is detectable.*

Proof. According to PBH rank test, $(Q_z^{(1/2)}, \overline{A})$ is detectable if and only if $\Phi = \begin{bmatrix} \overline{A} - sI_{\overline{n}} \\ Q_z^{(1/2)} \end{bmatrix}$ has full column rank for any $s \in C_+$, where C_+ is the closed right-half of the complex plane. Since Q_z is a positive definite matrix, obviously, $Q_z^{(1/2)}$ is nonsingular. Thus, it is evident that Φ is of full column rank. The proof is completed. □

Lemma 3. *For a quadratic form $F(X) = X^T \widetilde{A} X$, where $X \in \mathbb{R}^{n\times 1}$ and $\widetilde{A} \in \mathbb{R}^{n\times n}$ are symmetric matrices which do not depend on X, then*

$$\frac{\partial F}{\partial X} = X^T\left(\widetilde{A} + \widetilde{A}^T\right) = 2X^T\widetilde{A} = 2\widetilde{A}X. \quad (19)$$

The following theorems give the main results of this paper.

Theorem 1. *Let $u(t) = 0$, $x(t) = 0$, and $r(t) = 0$, for $t \le 0$. Under the performance index (14) and Assumptions 1–4, the Padé-approximation-based optimal preview repetitive controller $u(t)$ of system (1) is given by*

$$u(t) = F_x x_p(t) + F_e \int_0^t e(s)\mathrm{d}s + F_v \int_0^t v(s)\mathrm{d}s + f_1(t), \quad (20)$$

where

$$f_1(t) = -R^{-1}\overline{B}^T \int_0^{l_r} e^{\overline{A}_c \delta} K \overline{D}_1 r(t+\delta)\mathrm{d}\delta. \quad (21)$$

F_x, F_e, and F_v are the feedback gains given by

$$\begin{aligned} F_x &= -R^{-1}\overline{B}^T K_x, \\ F_e &= -R^{-1}\overline{B}^T K_e, \\ F_v &= -R^{-1}\overline{B}^T K_v. \end{aligned} \quad (22)$$

$K = \begin{bmatrix} K_x & K_e & K_v \end{bmatrix} \in \mathbb{R}^{\overline{n}\times\overline{n}}$ is the unique semidefinite solution of the algebraic Riccati equation (ARE):

$$\overline{A}^T K + K\overline{A} - K\overline{B}R^{-1}\overline{B}^T K + Q_z = 0. \quad (23)$$

In addition, $\overline{A}_c$ in (21) is a stable matrix defined by

$$\overline{A}_c = \overline{A}^T - K\overline{B}R^{-1}\overline{B}^T. \quad (24)$$

Proof. According to the theory of linear quadratic regulation, when system (12) is under performance index (14), the Hamilton function is

$$H = \frac{1}{2}z^T(t)Q_z z(t) + \frac{1}{2}\dot{u}^T(t)R\dot{u}(t) + \lambda^T(t)\left(\overline{A}z(t) + \overline{B}\dot{u}(t) + \overline{D}_1\dot{r}(t)\right), \tag{25}$$

where $\lambda(t)$ is the adjoint vector; by Lemma 3, we can obtain

$$\begin{aligned} \frac{\partial H}{\partial \dot{u}(t)} &= R\dot{u}(t) + \overline{B}^T\lambda, \\ \frac{\partial H}{\partial z(t)} &= Q_z z(t) + \overline{A}^T\lambda. \end{aligned} \tag{26}$$

Then,

$$\frac{\partial^2 H}{\partial \dot{u}^2(t)} = R > 0. \tag{27}$$

Based on the optimal control theory [34], the following regular equations are obtained:

$$\begin{cases} \dfrac{\partial H}{\partial \dot{u}(t)} = 0, \\ \dot{\lambda} = -\dfrac{\partial H}{\partial z(t)}. \end{cases} \tag{28}$$

That is,

$$\begin{cases} \dot{u}(t) = -R^{-1}\overline{B}^T\lambda, \\ \dot{\lambda} = -Q_z z(t) - \overline{A}^T\lambda. \end{cases} \tag{29}$$

Combining the first equation of (29) into (12) yields

$$\dot{z}(t) = \overline{A}z(t) - \overline{B}R^{-1}\overline{B}^T\lambda + \overline{D}_1\dot{r}(t). \tag{30}$$

Set

$$\lambda = K(t)z(t) + g(t), \tag{31}$$

where $K(t) \in \mathbb{R}^{\overline{n}\times\overline{n}}$ and $g(t) \in \mathbb{R}^{\overline{n}\times 1}$ satisfy the boundary conditions:

$$\begin{aligned} K(t_a) &= 0, \\ g(t_a) &= 0. \end{aligned} \tag{32}$$

The above conditions can be rewritten as

$$\begin{cases} \dot{z}(t) = \overline{A}z(t) - \overline{B}R^{-1}\overline{B}^T\lambda + \overline{D}_1\dot{r}(t), \\ \dot{u}(t) = -R^{-1}\overline{B}^T\lambda, \\ \dot{\lambda} = -Q_z z(t) - \overline{A}^T\lambda, \\ \lambda = K(t)z(t) + g(t), \\ K(t_\alpha) = 0, \\ g(t_\alpha) = 0. \end{cases} \tag{33}$$

Some simple calculations yield

$$\dot{K}(t) = -Q_z - \overline{A}^T K(t) + K(t)\overline{B}R^{-1}\overline{B}^T K(t) - K(t)\overline{A}, \tag{34}$$

$$\dot{g}(t) = -\left[\overline{A}^T - K(t)\overline{B}R^{-1}\overline{B}^T\right]g(t) - K(t)\overline{D}_1\dot{r}(t). \tag{35}$$

Then, set

$$\dot{g}(t) = -\overline{A}_c(\tau)g(\tau), \tag{36}$$

where $\Phi(t,\tau)$ is the state-transition matrix of (35) which satisfies the initial condition $\Phi(t,t) = I$ and $\overline{A}_c(\tau) = \overline{A}^T - K(\tau)\overline{B}R^{-1}\overline{B}^T$. Then, for $\tau \in [t, t_a]$, the solution of (35) is given by

$$g(t_a) = \Phi(t_a,t)g(t) + \int_t^{t_a}\Phi(t_a,\tau)m(\tau)\mathrm{d}\tau, \tag{37}$$

where

$$m(\tau) = -K(\tau)\overline{D}_1\dot{r}(\tau). \tag{38}$$

Under the boundary condition $g(t_a) = 0$, it follows from (37) that

$$g(t) = -\int_t^{t_a\wedge(t+l_r)}\Phi^{-1}(t_a,t)\Phi(t_a,\tau)m(\tau)\mathrm{d}\tau, \tag{39}$$

where $t_a\wedge(t+l_r)\colon = \min\{t_a, t+l_r\}$.

Now, we let $t_a \longrightarrow \infty$. Then, it is well known that, under Lemmas 1 and 2 that the matrix pair $(\overline{A}, \overline{B})$ is stabilizable and $(Q_z^{(1/2)}, \overline{A})$ is detectable, the solution $K(t)$ of (34) converges to one constant matrix K, which is the unique nonnegative definite solution of the ARE (23). Moreover, $\overline{A}_c(\tau)$ converges to $\overline{A}_c$, which is stable, and $\Phi(t,\tau)$ reduces to $e^{-\overline{A}_c(t-\tau)}$ as $t_a \longrightarrow \infty$. Therefore,

$$\begin{aligned} g(t) &= -\int_t^{t+l_r} e^{\overline{A}_c(\tau-t)}m(\tau)\mathrm{d}\tau \\ &= -\int_0^{l_r} e^{\overline{A}_c\sigma}m(t+\sigma)\mathrm{d}\sigma. \end{aligned} \tag{40}$$

Furthermore, substituting (38) into (40), we obtain

$$\begin{aligned} g(t) &= -\int_0^{l_r} e^{\overline{A}_c\sigma}m(t+\sigma)\mathrm{d}\sigma \\ &= \int_0^{l_r} e^{\overline{A}_c\sigma}K\overline{D}_1\dot{r}(t+\sigma)\mathrm{d}\sigma. \end{aligned} \tag{41}$$

Combining (29), (31), and (41), the optimal control input $\dot{u}(t)$ is given by

$$\begin{aligned} \dot{u}(t) = &-R^{-1}\overline{B}^T K_x\dot{x}_p(t) \\ &- R^{-1}\overline{B}^T K_e e(t) - R^{-1}\overline{B}^T K_v v(t) \\ &- R^{-1}\overline{B}^T\int_0^{l_r} e^{\overline{A}_c\sigma}K\overline{D}_1\dot{r}(t+\sigma)\mathrm{d}\sigma. \end{aligned} \tag{42}$$

Integrating both sides of the above formula over $[-L, t]\,(L > l_r)$ yields

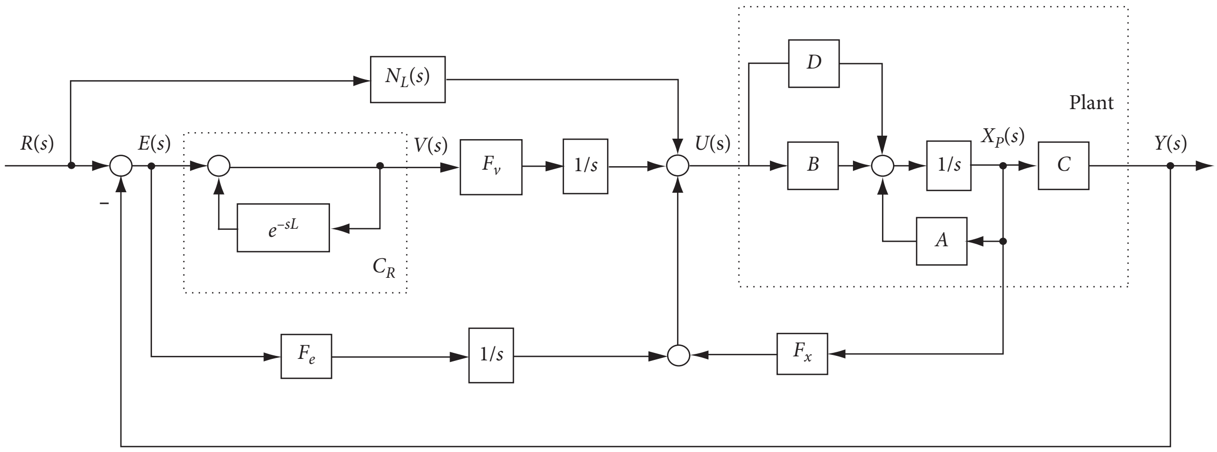

Figure 3: Configuration of the Padé-approximation-based PRC system.

$$\begin{aligned} u(t) &= u(t-L) - R^{-1}\overline{B}^T K_x \int_{-L}^{t} \dot{x}_p(s)\mathrm{d}s \\ &\quad - R^{-1}\overline{B}^T K_e \int_{-L}^{t} e(s)\mathrm{d}s - R^{-1}\overline{B}^T K_v \int_{-L}^{t} v(s)\mathrm{d}s \\ &\quad - R^{-1}\overline{B}^T \int_{-L}^{t} \left[\int_0^{l_r} e^{\overline{A}_c\sigma} K\overline{D}_1 \dot{r}(s+\sigma)\mathrm{d}\sigma \right]\mathrm{d}s. \end{aligned} \tag{43}$$

Note that $x(t) = 0$, $u(t) = 0$, and $r(t) = 0$, for $t \le 0$. The above formula can be rewritten as

$$\begin{aligned} u(t) &= -R^{-1}\overline{B}^T K_x x_p(t) \\ &\quad - R^{-1}\overline{B}^T K_e \int_{-L}^{t} e(s)\mathrm{d}s - R^{-1}\overline{B}^T K_v \int_{-L}^{t} v(s)\mathrm{d}s \\ &\quad - R^{-1}\overline{B}^T \int_{-L}^{t} \left[\int_0^{l_r} e^{\overline{A}_c\sigma} K\overline{D}_1 \dot{r}(s+\sigma)\mathrm{d}\sigma \right]\mathrm{d}s. \end{aligned} \tag{44}$$

For the last part of equation (44), exchange the integration sequence and integrate in $[-L, t]$. We can obtain Theorem 1.

The proof is completed.

The structure of the closed-loop system is shown in Figure 3. In fact, taking the two sides' Laplace transforms of (20), we have

$$U(s) = F_x X_p(s) + \frac{F_e}{s} E(s) + \frac{F_v}{s} V(s) + F_1(s), \tag{45}$$

where $F_1(s)$ is computed as

$$\begin{aligned} F_1(s) &= -R^{-1}\overline{B}^T \int_{-\infty}^{+\infty} e^{-st} \int_0^{l_r} e^{\overline{A}_c\tau} K\overline{D}_1 r(t+\tau)\mathrm{d}\tau\mathrm{d}t \\ &= -R^{-1}\overline{B}^T \int_0^{l_r} e^{\overline{A}_c\tau} K\overline{D}_1 \mathrm{d}\tau \int_{-\infty}^{+\infty} e^{-st} r(t+\tau)\mathrm{d}t \\ &= -R^{-1}\overline{B}^T \int_0^{l_r} e^{\tau(\overline{A}_c + sI_{\overline{n}})} K\overline{D}_1 \mathrm{d}\tau R(s) \\ &= N_L(s) R(s). \end{aligned} \tag{46}$$

Here, $N_L(s)$ is given by

$$N_L(s) = R^{-1}\overline{B}^T \left(\overline{A}_c + sI_{\overline{n}}\right)^{-1} \left(I_{\overline{n}} - e^{l_r(\overline{A}_c + sI_{\overline{n}})} \right) K\overline{D}_1. \tag{47}$$

Then, $U(s)$ can be rewritten as

$$U(s) = F_x X_p(s) + \frac{F_e}{s} E(s) + \frac{F_v}{s} V(s) + N_L(s) R(s). \tag{48}$$

Therefore, from (48), we see that the configuration of the closed-loop system is as shown in Figure 3. □

4. Simulation Results

In this section, to demonstrate the effectiveness of the proposed Padé-approximation-based PRC law design, the same scenario in [37] was considered to demonstrate the validity of our method. The numerical simulation example is

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -2 & 3 \\ 4 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 2 & 0 \end{bmatrix} x(t) + u(t). \end{cases} \tag{49}$$

We employ the Padé-approximation-based PRC law (20) to track the following reference input:

$$r(t) = \sin\frac{2\pi}{10} + 0.5\sin\frac{4\pi}{10} + 0.5\sin\frac{6\pi}{10}. \tag{50}$$

The weight matrices Q_z, R in performance index (14) are chosen as

$$Q_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 500 \end{bmatrix}, \quad R = 1. \tag{51}$$

Let the initial condition be $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $x(t) = 0$, $u(t) = 0$, and $r(t) = 0\,(-10 \le t \le 0)$. The preview lengths are given by $l_r = 0$, $l_r = 0.05$, and $l_r = 0.1$.

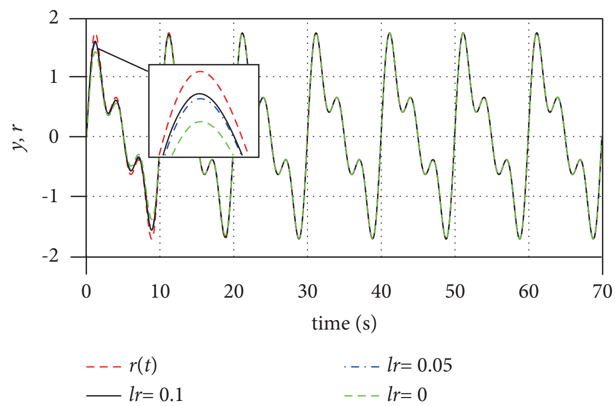

Figure 4: System output responses with different preview lengths.

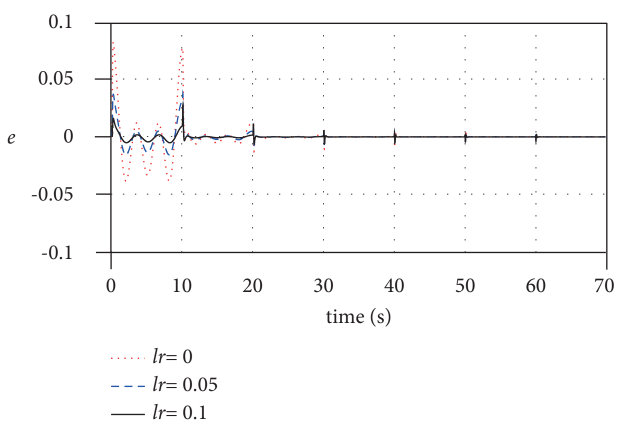

Figure 5: Tracking errors with different preview lengths.

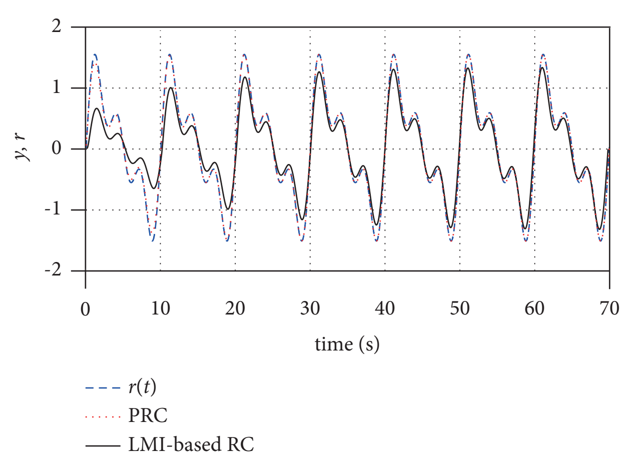

Figure 6: System output responses with different methods.

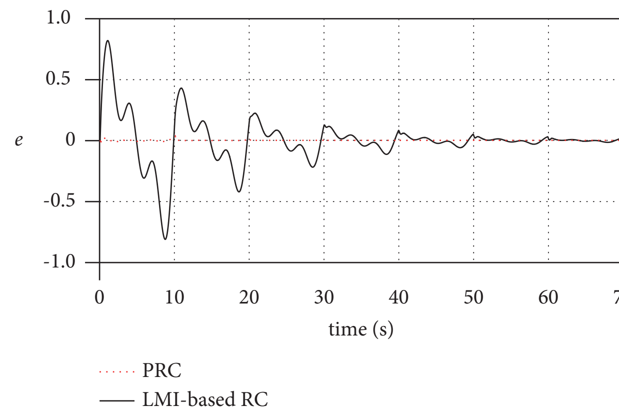

Figure 7: Tracking errors with different methods.

Figures 4 and 5 show the situation of output responses and tracking errors under different preview lengths. Obviously, the Padé-approximation-based RC with preview compensation can effectively reduce the static error and peak overshoot in the process of tracking reference signal (50). With the increase of preview lengths, the peak overshoot and static error will be reduced. However, when the preview length reaches a certain degree, it has little effect on the output response. When $l_r = 0.1$, the control system achieves the best tracking performance.

In Figures 6 and 7, with $l_r = 0.1$, compare the results of the proposed Padé-based approximation PRC law with those of the LMI-based RC law designed in [37, 38]. It can be seen that the controller proposed in this paper can obviously improve the response speed and tracking accuracy of the system.

From the above simulation results, it is clear to see that the designed Padé-approximation-based PRC law provides the expected tracking performance, which illustrates the significance of the proposed controller design method.

5. Conclusions

In this study, the state space representation of the repetitive controller was transformed into a nondelayed one by Padé approximation. Then, an augmented dynamic system was constructed by using the nominal state equation with the error system and the state equation of the repetitive controller. Finally, by optimal control theory, a Padé-approximation-based PRC law was obtained. The simulation results showed the superiority of the control method. It is worth mentioning that the proposed Padé-approximation-based PRC can be further designed for linear time-delay systems, which is our future research work.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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