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Research Article

Joint Pricing and Inventory Control Decisions for Fashion Apparel with Considering Fashion Level and Partial Backlogging

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Apparel inventories have been crawling up for decades. The fashion level of clothing drops over time, resulting in a continuous decline in market value, a rapid reduction in demand, and unsold inventory. Poor inventory management weighs down the apparel industry. Inventory planning decisions are crucial for a fashion apparel retailer. When the good is highly perishable, the retailer may need to backlog demand in order to market the good at a reasonable price and reduce inventory costs. This study formulates a finite horizon inventory model for fashion apparel under deteriorating fashion level and partial backlogging conditions. This model jointly optimizes the selling price, length of inventory-holding period, and length of shortage period by maximizing the average profit. The conditions that characterize the optimal solution are established and an algorithm is proposed to search for the optimal solutions. A numerical example is provided to illustrate the algorithm and the solution procedure. Furthermore, sensitivity analysis of the key system parameters is carried out and some interesting managerial insights are presented. Findings clearly suggest that the presence of shortage has got an affirmative effect on ordering policy of the fashion apparel retailer.

1. Introduction

With the rapid development of the world economy and the advent of the era of globalization, people's lifestyles have undergone great changes. In addition to comfort, they are also pursuing the individuality and fashion sense of clothing. According to the statistics from American Fashion [1], in recent years, the global apparel market has reached 3 trillion US dollars, accounting for about 2% of the global gross domestic product (GDP). Taking the American sports and leisure and lifestyle brand Nike as an example, the company is the world's second largest fashion brand. The company's 2017 revenue reached US $344.6 billion, and its market value was close to US $105 billion. Fast fashion is an industrial practice that is widely applied in fashion retailing. The central idea is to offer a continuous stream of new merchandise to the market which reflects the latest fashion trends and helps capture the hottest designs in the current market. For example, famous international fashion retail brands such as H&M, Top Shop, and Zara have all implemented fast fashion strategies [2]. Fashion clothing sales are influenced not only by price but also by fashion level, which depends on elements such as style, color, and material. Fashion products differ significantly from other types of commercial products. In general, the higher the fashion level of clothing products, the higher their market value. As time goes on, the fashion level of the clothing drops, resulting in a continuous decline in market value; market demand may thus fall off rapidly and result in an accumulation of unsold inventory. In other words, fashion apparel decreases in value over the selling season. The latest data show that excess inventory is a major problem in the fashion industry. For example, H&M announced at the end of March 2018 that the total value of its accumulated unsold apparel was US $4 billion. In 2016, the total inventory of 79 quoted textile and apparel companies in China amounted to US $81 billion [3]. Inventory planning decisions are crucial for supply chain managers in the fashion industry. Having short product lifecycle and highly volatile demand, fashion items require extra attention for the initial ordering before selling season starts, as well as the subsequent replenishment decisions during the selling season [4, 5]. Bowers and Agarwal [6] proposed a dynamic hierarchical model for apparel production planning and scheduling in which they considered all processes from daily scheduling and short-term production planning to long-term decisions to better match the demand with a minimal total cost. Fisher et al. [7] studied replenishment order quantity to minimize the cost of lost sales and obsolete inventory in a two-stage model. They identified the optimal reorder time and quantified the benefit of lead time reduction. Raman and Kim [8] quantified the impact of inventory-holding cost in the fashion supply chain based on data from Northco, a school uniform manufacturer in Northeastern USA. However, the impact of the fashion level of clothing products on pricing and replenishment strategies of apparel retailer has been largely neglected in prior studies [6–8]. Due to the time-varying characteristics of fashion clothing, implementing a joint pricing and replenishment strategy has obvious practical significance for managing inventory replenishment and improving net profits. Chen et al. [3] analyzed the joint optimal dynamic pricing strategy and replenishment cycle for fashion apparel considering the effect of fashion level. The decay of fashion over time leads to a decrease in clothing sales demand, which makes the storage time of products longer and ultimately leads to an increase in inventory costs. In reality, clothing retailers can reduce inventory costs by delaying in meeting demand of consumers, that is, allowing the products to be temporarily out of stock or partly out of stock to delay meeting consumers' needs. When it is economic to backlog demand, the reseller can plan for periods of shortage during which demand can be partially backordered [9]. When the good is highly perishable, the retailer may need to backlog demand in order to market the good at a reasonable price. Apart from the direct effects of fashion level and demand rate on changes in inventory levels, apparel inventory will inevitably suffer from a shortage. In some inventory systems, such as fashionable items, the length of the waiting time for next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate is variable and is dependent on the waiting time for the next replenishment [10]. To enhance the exploratory power and the connection between research and practice, we develop our theoretical framework learning from the study by Chen et al. [3] on inventory management for fashion apparel. Chen et al. [3] used a dynamic pricing model to examine the effect of fashion level on firm performance. We draw upon inventory planning decisions theory as a response to the call for an increased focus on the partial backlogging and contribute to the broader application of inventory planning decisions theory in inventory management for fashion apparel. In addition, this study also contributes to the deterioration items literature by simultaneously considering the fashion level and partial backlogging.

Therefore, this paper aims to focus on the pricing and replenishment strategy problems for the fashion apparel retailer with considering fashion level and partial backlogging. Based on this research background, the following questions arise: (1) How does the fashion level of apparel affect product demand? (2) How can we determine the optimal price, the optimal length of inventory-holding period, and the optimal length of shortage period to maximize the apparel retailer's average profit during the selling period? (3) How the inventory control parameters, such as the deteriorating rate of the fashion level, the backlogging resistance, the shortage cost, and inventory-holding cost, influence apparel retailer's performance? In our inventory model, the shortage is allowed, and the partial backlogging rate is sensitive to the waiting time. Firstly, we prove that, for any given replenishment cycle, the average profit is strictly concave in selling price and, for any given selling price, the average profit function is strictly pseudo-concave in replenishment cycle; that is, there exists a unique and optimal solution to the problem. Next, an algorithm is proposed to obtain the optimal solution of the problem. Finally, the sensitivity analyses of the parameters are conducted to demonstrate the effectiveness of the proposed model. The research results show that when the purchasing cost, the shortage cost, and the holding of fashion apparel increase, the average profit of system will decrease. The optimal average profit and order quantity are highly sensitive to the deterioration rate of fashion level and the backordering resistance.

The rest of this paper is organized as follows. In Section 2, we briefly review the related literature. In Section 3, we outline the notations and assumptions used to address the problem. Section 4 contributes to the formulation of the model. We obtain theoretical results and provide an iterative algorithm in Section 5. Numerical examples, sensitivity analysis, and some managerial insights are provided in Section 6. Finally, Section 7 offers a conclusion and discusses the implications for management.

2. Literature Review

To date, the literature on fashion inventory management has focused on quick response strategies. In the fashion industry, more and more companies, such as Uniqlo, Zara, Topshop, and Primark, are adopting such strategies [11, 12]. Quick response strategies were first proposed in the 1980s in the US fashion industry [13]. Fashion retailers can postpone order placement until close to the beginning of the selling season to obtain more accurate market information about the sales of particular types of fashion items (i.e., style trends). This allows them to adjust their initial inventory forecast, so that supply can better match demand. Other related studies include that by Aviv et al. [14], where they studied the potential benefits of response pricing for seasonal fashion merchandise sellers. Cachon and Swinney [15] discuss the quick response mediated fast fashion program in the apparel industry. Chow et al. [16] study the impacts of imposing a minimum order quantity on a two-echelon supply chain implementing the quick response strategies. Choi [17] studied a sourcing decision with the quick response strategy under an environmental sustainability practice. Chan et al. [18] determined the number of observations to take under quick response for apparel inventory control. Choi [19] constructed a formal analytical fashion supply chain model based on the Bayesian information updating to examine the effects of quick response.

Cosgun et al. [20] addressed the simultaneous determination of markdown prices and optimal initial inventory levels under the cross-price effects in a random demand setting for multiproduct groups. All of these studies proposed using quick response strategies to deal with volatile market demand by reducing order lead time to more accurately determine the appropriate quantity to order.

Inventory planning decisions are crucial for fashion apparel. As such apparel has a short product lifecycle and is associated with highly volatile demand, initial orders before the selling season begins require careful attention [4, 5], and the same applies for replenishment decisions during the selling season. Due to the complexity of fashion inventory management, very few studies have focused on fashion inventory management from a pricing and replenishment policy perspective. Deterioration items exist commonly in the real business environment. As stated in previous literature [21, 22], deterioration refers to spoilage, decay, pilferage, decay, evaporation, obsolescence, or damage such that the items cannot maintain the original value or utility. Most of the physical goods undergo decay or deterioration over time, such as medicine, volatile liquids, fashion items, and others [23]. For the property of the deterioration items, the research on pricing and replenishment strategy for the fashion apparel retailer can be referred to the research on inventory and pricing decisions of perishable products [24–26]. For example, Wu et al. [24] formulated an inventory model with deteriorating items and price-sensitive demand and obtained the retailer's optimal selling price and the length of replenishment cycle. Avinadav et al. [25] used a price- and time-dependent demand function to develop a mathematical model that calculates the optimal price and replenishment period of perishable items. Herbon et al. [26] formulated an inventory model with deteriorating items and price-sensitive demand and obtained the retailer's optimal dynamic selling price and the length of replenishment cycle. In the above-mentioned inventory models, the shortage of products has been neglected. The shortage setting should be considered to face the various practical requirements.

Nevertheless, few research efforts have explicitly considered the inventory decisions of fashion apparel from the perspective of pricing and replenishment cycles. Fisher et al. [27] studied how replenishment order quantity may minimize the cost of lost sales, backorders, and obsolete inventory in a two-stage model. Kogan and Spiegel [28] developed a model with an initial inventory level and addressed the problem of maximizing profits from the production, storage, and sale of fashion goods. In their proposed inventory model, the selling price remains unchanged throughout the sales period. Such a price strategy ignores the fashion level of the apparel, which declines over time. Tsao [29] developed an analytical model of inventory control for deteriorating goods and fashion goods under trade credit and partial backlogging conditions. In that study, two-phase pricing and inventory decisions were considered and the optimal replenishment strategy was identified. The above-mentioned works of literature have not considered the influence of fashion level on pricing and inventory decisions. Chen et al. [3] used a dynamic pricing model to examine the effect of fashion level on firm performance. From the aspect of modeling, the most closely related studies to this paper are those by Tsao [29] and Chen et al. [3]. However, unlike these studies, we incorporate the fashion level and shortage into the optimization model. This paper derives a pricing and replenishment policy for fashion apparel and adopts a price- and time-dependent demand function to model the finite time horizon inventory. The major objective of this policy is to simultaneously determine the optimal price, optimal length of inventory-holding period, and optimal length of shortage period to maximize average profit. We also establish the conditions that characterize the optimal solutions and propose an algorithm to search for optimal solutions. To be specific, a systematic comparison between this paper and other related papers is given in Table 1 to show the novelty and advantage of this paper.

3. The Model

3.1. Problem Formulation. First, we describe the problem as follows. We model an apparel retailer that sells fashionable clothing products. The demand for fashionable clothing is determined by the selling price p and fashion level of the clothing $\omega(t)$ during the selling period. The replenishment problem for a single fashion item is considered. The inventory system evolves as follows: Q units of items arrive at the inventory system at the beginning of each cycle. The inventory level $I(t)$ decreases due to consumers' demand during the replenishment cycle. During the interval $0 \le t \le t_1$, the inventory level decreases due to demand. A shortage occurs due to demand and partial backlogging during the time interval $t_1 \le t \le t_1 + t_2$. The length of replenishment cycle T equals the length of inventory-holding period t_1 plus the length of shortage period t_2. The backlogged demand during the shortage period is S. Thus, the retailer's ordering quantity for one replenishment cycle is $Q = I(0) + S$. The selling price, length of inventory-holding period, and length of shortage period are decision variables. The major objective of this policy is to simultaneously determine the selling price, the length of inventory-holding period, and the length of shortage period to maximize average profit.

3.2. Assumptions. The mathematical model in this paper is developed on the basis of the following assumptions.

Assumption 1. The demand function is nonnegative and explicitly depends separately on both the price and fashion level. In practice, the more fashionable a garment, the higher its market value, increasing consumers' incentive to buy it [3]. That is, the higher the fashion level is, the larger the demand will be, according to the following formula:

$$D(p,t) = \begin{cases} d(p) + \omega(t), & 0 \le t \le t_1, \\ d(p), & t_1 \le t \le t_1 + t_2. \end{cases} \tag{1}$$

Without loss of generality, let the function $d(p)$ be linear; that is, $d(p) = a - bp$. a is the fixed term of demand

Table 1: A comparison of the present work with related previous works.

References	Quick response	Inventory planning decisions	Fashion level	Shortage
[14–19]	Yes	No	No	No
[24–26]	No	Yes	No	No
[3]	No	Yes	Yes	No
[29]	No	Yes	No	Yes
Our paper	No	Yes	Yes	Yes

determined by the market size and b represents the sensitivity of demand with respect to price. The demand function of paper is based on several previous studies of perishable products. In those studies, the demand function was usually assumed to have a linear form. This assumption has been widely adopted in the perishable inventory management literature, such as Li et al. [22], Zhang et al. [23], and Herbon et al. [26].

Assumption 2. The fashion level at time t is given by

$$\omega(t) = \omega_0 e^{-\eta t}. \quad (2)$$

Equation (2) is motivated by the model presented in Chen et al.'s work [3] and is proposed based on the following consideration. The fashion level function is strictly decreasing with time t; $d\omega/dt < 0$. Therefore, we can deduce that the demand is decreasing with time t, so the assumption of the demand function in (1) is reasonable.

Assumption 3. Shortages are allowed. We adopt the notation used in Abad [9] where the unsatisfied demand is backlogged, and the fraction of shortage back-ordered is

$$B(\tau) = k_0 e^{-k_1 \tau}. \quad (3)$$

where $0 < k_0 < 1$ is the backlogging intensity, $\tau > 0$ is the waiting time up to the next replenishment, and $k_1 > 0$ is the backlogging resistance. Note that $B(\tau)$ proportions of the consumers who arrive during the shortage period are willing to wait for the next replenishment resulting in the backlogged demand, whereas the remaining $1 - B(\tau)$ proportions of the consumers are lost, resulting in lost sales [22]. From formula (3), it can be seen that $\partial B(\tau)/\partial\tau < 0$ and $\partial^2 B(\tau)/\partial\tau^2 > 0$, and the partial backlogging decreases rapidly with the increase of waiting time, indicating that customers are becoming impatient, which is consistent with the reality.

4. Model Formulation

From Figure 1, we can see that the inventory level $I(t)$ decreases due to consumers' demand during the time interval $0 \le t \le t_1$. The differential equation that represents inventory status is given by

$$\frac{dI_1(t)}{dt} = -d(p) - \omega(t). \quad (4)$$

With the condition $I(t_1) = 0$, solving formula (4) yields $I_1(t) = \int_t^{t_1} (d(p) + \omega(u))du$. Therefore, the initial inventory level is then given by $I_0 = I_1(0) = \int_0^{t_1} (d(p) + \omega(u))du$. During the second interval $t_1 \le t \le t_1 + t_2$, shortage occurred, and the demand is partially backlogged according to the fraction $B(T - t)$. That is, the inventory level at time t is governed by the following differential equation:

$$\frac{dI_2(t)}{dt} = -d(p)B(t_1 + t_2 - t). \quad (5)$$

With the condition $I(t_1) = 0$, solving (5) yields $I_2(t) = -d(p)\int_{t_1+t_2-t}^{t_2} B(\tau)d\tau$. If we put $t = T$ into the above formula, the maximum amount of demand backlogging will be obtained as follows: $S = d(p)M(t_2)$, where $M(t_2) = \int_0^{t_2} B(\tau)d\tau$. Order quantity per cycle is the total of S and I_0; that is,

$$Q = \int_0^{t_1} (d(p) + \omega(u))du + d(p)M(t_2). \quad (6)$$

Thus, the inventory level during the replenishment cycle is described as follows:

$$I(t) = \begin{cases} \int_t^{t_1} (d(p) + \omega(u))du, & 0 \le t \le t_1, \\ -d(p)\int_{t_1+t_2-t}^{t_2} B(\tau)d\tau, & t_1 \le t \le t_1 + t_2. \end{cases} \quad (7)$$

Now, we can obtain inventory costs and sales revenue per cycle which consist of the following five elements: fixed order cost A, sales revenue $SR = pQ$, purchase cost $PC = cQ$, inventory cost $HC = h\int_0^{\lambda} I_1(t)dt$, and shortage cost $SC = sd(p)\int_0^{t_2} \tau B(\tau)d\tau$, where p is the selling price per unit, c is the purchasing cost per unit, h is inventory-holding cost per unit, and s is the shortage cost per unit.

Therefore, the total profit (denoted by $\Pi(t_1, t_2, p)$) during the time interval $[0, T]$ is given by

$$\begin{aligned} \Pi(t_1, t_2, p) &= SR - PC - HC - SC - OC - A \\ &= (p - c)\left\{\int_0^{t_1} (d(p) + \omega(u))du + d(p)M(t_2)\right\} \\ &\quad - h\int_0^{t_1}\int_t^{t_1} (d(p) + \omega(u))du dt \\ &\quad - s\, d(p)\int_0^{t_2} \tau B(\tau)d\tau. \end{aligned} \quad (8)$$

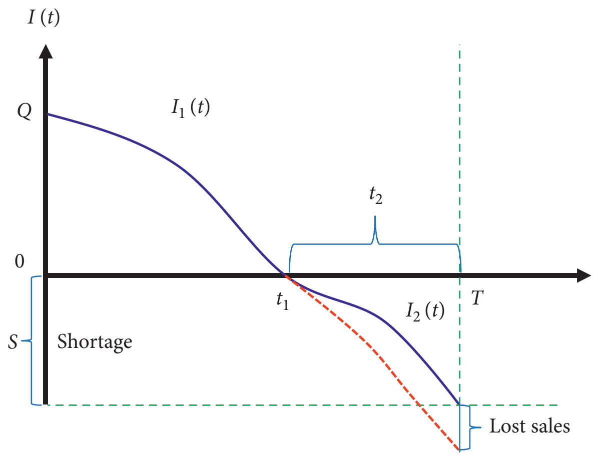

Figure 1: The apparel inventory level.

The average profit functions are given by

$$\pi(t_1, t_2, p) = \frac{\Pi(t_1, t_2, p)}{t_1 + t_2}. \tag{9}$$

5. Results and Algorithm

The retailer's objective is to maximize $\pi(t_1, t_2, p)$ by jointly determining (t_1, t_2, p). In this section, we will explore the mathematical properties of the model to assist in proposing an algorithm to search for the optimal solution.

The necessary conditions for the average profit (9) to be maximized are

$$\frac{\partial \pi(t_1, t_2, p)}{\partial p} = \frac{1}{(t_1 + t_2)} \frac{\partial \Pi}{\partial p} = 0, \tag{10}$$

$$\frac{\partial \pi(t_1, t_2, p)}{\partial t_1} = \frac{(\partial \Pi/\partial t_1)(t_1 + t_2) - \Pi}{(t_1 + t_2)^2} = 0, \tag{11}$$

$$\frac{\partial \pi(t_1, t_2, p)}{\partial t_2} = \frac{(\partial \Pi/\partial t_2)(t_1 + t_2) - \Pi}{(t_1 + t_2)^2} = 0. \tag{12}$$

Proposition 1. *For any given t_1 and t_2, the average profit $\pi(t_1, t_2, p)$ is strictly concave in p, and there exists a unique global maximum point $p^* \in (c, \infty)$ that satisfies the first-order necessary condition $\partial\pi/\partial p = 0$.*

Proof. From equation (8), we get $\partial^2\Pi/\partial p^2 = -2b(t_1 + M(t_2)) < 0$. According to equation (10), we conclude that $\partial^2\pi/\partial p^2 = (1/(t_1 + t_2))(\partial^2\Pi/\partial p^2) < 0$; therefore, π is strictly concave in p. In addition, we have $\lim_{p\longrightarrow+\infty}(\partial\pi/\partial p) < 0$ and $\lim_{p\longrightarrow c}(\partial\pi/\partial p) > 0$. Therefore, there exists a unique maximum point $p^* \in (c, \infty)$ to satisfy $\partial\pi(t_1, t_2, p)/\partial p = 0$.

The length of shortage period t_2 falls in the range of $(0, (p - c)/s]$. For any given selling price p, the objective function (9) is equivalent to the following programming problem:

$$\max \pi(t_1, t_2 | p),$$
$$\text{s.t.} \begin{cases} t_1 \geq 0, \\ \dfrac{p - c}{s - t_2} \geq 0. \end{cases} \tag{13}$$

Note that formula (13) represents fractional programs. A fractional program is defined as the nonlinear program sup $\Phi(x)/\Gamma(x)$, $x \in U = \{x \in X: h(x) \leq 0\}$, where $\Gamma(x) > 0$. If $\Phi(x) > 0$ is concave, $\Gamma(x)$ and $h(x)$ are convex; then the problem is a concave fractional program. For a differentiable and strictly concave fractional program, there is at most one maximum, and a solution to the Kuhn-Tucker conditions is a maximum. We refer readers to Li et al.'s work [22] for the details on the definition and properties of fractional program. □

Proposition 2. *For any given p, the average profit $\pi(t_1, t_2 | p)$ is strictly pseudoconcave in (t_1, t_2).*

Proof. From equation (9), it is easy to get $\partial^2\Pi/\partial t_1^2 < 0$, $\partial^2\Pi/\partial t_1\partial t_2 = 0$, and $\partial^2\Pi/\partial t_2^2 = d(p)[(p - c - st_2)B'(t_2) - sB(t_2)]$. Since $0 \leq B(t_2) \leq 1$ and $B'(t_2) \leq 0$, we get $\partial^2\Pi/\partial t_2^2 < 0$. Let H denote Hessian matrix of Π; then we have $H = (\partial^2\Pi/\partial t_1^2)(\partial^2\Pi/\partial t_2^2) - (\partial^2\Pi/\partial t_1\partial t_2)(\partial^2\Pi/\partial t_1\partial t_2) > 0$. Therefore, H is negative definite, and $\Pi(t_1, t_2, p)$ is strictly concave in (t_1, t_2). Also, the denominator $t_1 + t_2$ is a linear function. Hence, $\pi(t_1, t_2 | p)$ is strictly pseudoconcave in (t_1, t_2).

Next, to simplify the expression of the model, let

$$\Delta = (p - c)\omega(0)g(0) + A + d(p) \left\{(p - c)(g(0) - M(g(0))) + s\int_0^{g(0)} \tau B(\tau)d\tau\right\}, \tag{14}$$

where $g(0) = t_2|_{G(t_1=0, t_2)=0}$. We also define the roots of $\partial\pi(t_1, t_2, p)/\partial t_1 = 0$ and $\partial\pi(t_1, t_2, p)/\partial t_2 = 0$ as t_1^+ and t_2^+, respectively. □

Proposition 3. *For any given $p \in (c, \infty)$, $\pi(t_1, t_2 | p)$ has a global maximum point (t_1^*, t_2^*), which is determined by the following rules:*

(1) If $\Delta \leq 0$, then $(t_1^*, t_2^*) = (0, g(0))$

(2) If $\Delta \geq 0$, then $(t_1^*, t_2^*) = (t_1^+, t_2^+)$

Proof. Let λ_1 and λ_2 be two Lagrange Multipliers. Kuhn-Tucker condition of equation (13) is

$$\begin{cases} \dfrac{\partial \pi(t_1, t_2, p)}{\partial t_1} + \lambda_1 = 0, \\ \partial \pi(t_1, t_2, p) \partial t_2 - \lambda_2 = 0, \\ \lambda_1 t_1 = 0, \\ \lambda_2 \left(\dfrac{(p-c)}{s} - t_1 \right) = 0, \\ \lambda_1, \lambda_2 \geq 0. \end{cases} \tag{15}$$

To solve the above equations, we can consider the three following cases:

Case (i). If $\lambda_2 \neq 0$, then $t_2 = (p-c)/s$ and $\partial\pi/\partial t_2 < 0$, which contradicts the constraint of $\lambda_2 \geq 0$. Therefore, this scenario will not occur.

Case (ii). If $\lambda_1 = 0$ and $\lambda_2 = 0$, then $\partial\pi(t_1, t_2|p)/\partial t_1 = 0$ and $\partial\pi(t_1, t_2|p)/\partial t_2 = 0$.

Case (iii). If $\lambda_1 \neq 0$ and $\lambda_2 = 0$, then $t_1 = 0$, $\partial\pi(t_1, t_2|p)/\partial t_2 = 0$, and $\lambda_1 = -\partial\pi(t_1, t_2|p)/\partial t_1$.

Next, we prove that case (ii) and case (iii) are feasible. Equations (11) and (12) are equivalent to

$$\frac{\partial \Pi}{\partial t_1} - (t_1 + t_2) = \Pi, \tag{16}$$

$$\frac{\partial \Pi}{\partial t_2} - (t_1 + t_2) = \Pi. \tag{17}$$

From equations (16) and (17), we get

$$\frac{\partial \Pi}{\partial t_1} - \frac{\partial \Pi}{\partial t_2} = 0. \tag{18}$$

We define $G(t_1, t_2) = (\partial\Pi/\partial t_1) - (\partial\Pi/\partial t_2)$. Since $\partial G(t_1, t_2)/\partial t_2 > 0$, $G(t_1, t_2)$ increases as t_2 increases. Moreover $\lim_{t_2 \longrightarrow ((p-c)/s)} G(t_1, t_2) = (\partial\Pi/\partial t_1) > 0$ and $\lim_{t_2 \longrightarrow 0^+} G(t_1, t_2) = (c - p - ht_1)\omega(t_1) - ht_1 d(p) < 0$. Thus, there exists a unique $t_2 \in (0, (p-c)/s)$ such that $G(t_1, t_2) = 0$. Therefore, t_2 can be regarded as the function of t_1. Let $t_2 = g(t_1)$ and substitute it into equation (16); then we get $(\partial\Pi/\partial t_1)(t_1 + g(t_1)) - \Pi = 0$. Let $F(t_1) = (\partial\Pi/\partial t_1)(t_1 + g(t_1)) - \Pi$; then we obtain the first derivative of $F(t_1)$ with respect to t_1: $(\mathrm{d}F(t_1)/\mathrm{d}t_1) = (\partial^2\Pi/\partial t_1^2)(t_1 + g(t_1)) < 0$. In addition, $\lim_{t_1 \longrightarrow 0^+}(\partial\Pi/\partial t_1) = (p-c)(d(p) + \omega_0) > 0$ and $\lim_{t_1 \longrightarrow \infty}(\partial\Pi/\partial t_1) < 0$. Thus, there exists a unique $\bar{t}_1 \in [0, +\infty)$ such that $(\partial\Pi/\partial t_1)|_{t_1 = \bar{t}_1} = 0$. Therefore, $F(t_1)$ is strictly decreasing in $t_1 \in [0, \bar{t}_1]$, and we get $\lim_{t_1 \longrightarrow \bar{t}} F(t_1) = -\Pi < 0$. Moreover, $\lim_{\lambda \longrightarrow 0^+} F(t_1) = \Delta$.

(a) If $\Delta \geq 0$, then $\lim_{t_1 \longrightarrow 0^+} F(t_1) > 0$. By Intermediate Value Theorem, there exists a unique $t_1^+ \in [0, \bar{t}_1]$ satisfying equation (16). Thus, Case (ii) is feasible, and $t_1^* = t_1^+$.

(b) If $\Delta \leq 0$, then $\lim_{\lambda \longrightarrow 0^+} F(t_1) < 0$. Therefore, $(\partial\pi/\partial t_1) < 0$, and then $\lambda_1 = -(\partial\pi/\partial t_1) > 0$, which means that Case (iii) is feasible and $t_1^* = 0$. Subsequently, we get t_2^* by substituting t_1^* into equation (18). From Proposition 2, the local maximum point (t_1^*, t_2^*) is the global maximum point.

Not surprisingly, similar to the works of Li et al. [22] and Tsao [29], it is very difficult to solve the nonlinear equations (10)–(12). Based on the preceding propositions, we use a Gauss-Newton iterative method and propose the following algorithm to obtain the optimal solution:

Step 1: Set $i = 1$, and initialize the value of $p = p^{(i)} > c$.

Step 2: Calculate $\Delta(p^{(i)})$. According to Proposition 3, execute one of the following substeps:

(1) If $\Delta(p^{(i)}) \geq 0$, then $t_1^{(i)} = t_1^+$ and $t_2^{(i)} = t_2^+$.

(2) If $\Delta(p^{(i)}) \leq 0$, then $t_1^{(i)} = 0$ and $t_2^{(i)} = g(0)$.

Step 3: Solve $\partial\pi(\lambda^{(i)}, \mu^{(i)}, p)/\partial p = 0$ and mark the solution as $p = p^{(i+1)}$. If the difference between $p^{(i)}$ and $p^{(i+1)}$ is small enough, such as $|p^{(i+1)} - p^{(i)}| < 10^{-6}$, the optimal solution is $(t_1^*, t_2^*, p^*) = (t_1^{(i)}, t_2^{(i)}, p^{(i+1)})$. Otherwise, set $i = i + 1$ and go back to Step 2.

Step 4: We can obtain Q^* using (6) and π^* using (9), respectively. □

6. Numerical Analysis

In this section, we conduct numerical analyses to gain managerial insights. The basic parameter values are chosen mainly based on the work of Chen et al. [3] and satisfy the assumptions mentioned in the Model Formulation section. The parameters are designed as follows: market size $a = 1000$, price sensitivity parameter $b = 20$, purchasing cost per unit $c = 40$, inventory-holding cost per unit $h = 2$, shortage cost per unit $s = 3$, $k_0 = 0.9$, $k_1 = 0.01$, the deteriorating rate of the fashion level $\eta = 0.02$, initial fashion level $\omega_0 = 100$, and fixed cost per order $A = 400$. Then the corresponding optimal solutions are given as follows: the optimal price $p^* = 47.4156$, the optimal length of inventory-holding period $t_1^* = 2.6312$, the optimal length of shortage period $t_2^* = 0.7487$, the optimal order quantity $Q^* = 452.6448$, and the optimal average profit $\pi^*(t_1, t_2, p) = 890.806$. Next, we examine the impacts of the related parameters on the optimal solutions and obtain some useful insights.

6.1. Impact of the Deteriorating Rate of the Fashion Level (η). The optimal selling price p^*, optimal length of inventory-holding period t_1^*, optimal length of shortage period t_2^*, and optimal average profit π^* decrease in η (Table 2). Table 2 shows the effect of deterioration rate η of the fashion level. The smaller the deterioration rate, the slower the lowering of fashion level because the fashion level largely depends on the

Table 2: Impact of the deteriorating rate of the fashion level.

η	p^*	t_1^*	t_2^*	π^*
0.02	47.41	2.63	0.7487	890
0.03	47.40	2.61	0.7487	885
0.04	47.39	2.59	0.7486	884
0.05	47.37	2.57	0.7485	883
0.06	47.36	2.56	0.7482	881
0.07	47.35	2.54	0.7478	879
0.08	47.34	2.52	0.7475	877
0.09	47.32	2.50	0.7470	873

elements of style and material, all of which affect the deterioration rate. Therefore, the larger the deterioration rate, the faster the attenuation of fashion products. The faster attenuation of fashion level leads to the shorter sales cycle. Then the market demand will be correspondingly reduced, resulting in the lower average profit. It can be seen from Table 2 that the change of the deterioration rate has a great influence on average profit but little influence on optimal pricing and replenishment period. The results show that when the deterioration rate of fashion apparel increases, the fashion products retailer should pay attention to reducing the selling price appropriately, reducing the inventory-holding period and out of stock period, and reducing the quantity of orders as much as possible, so as to improve the average profit.

6.2. Impact of the Backlogging Resistance (k_1). As can be seen from Table 3, the optimal selling price p^* and optimal length of inventory-holding period t_1^* increase in k_1, whereas the optimal length of shortage period t_2^* and optimal average profit π^* decrease in k_1. The larger the backlogging resistance is, the more reluctant the customer is to wait. In this case, the inventory-holding time should be appropriately extended, the shortage time should be shortened, and the total order quantity should be correspondingly reduced. It can be seen from Table 3 that the change of the backlogging resistance has a great influence on average profit but little influence on optimal pricing and replenishment period. Therefore, for customer groups with different backlogging resistance, the corresponding pricing and replenishment strategies will also be very different, which needs attention of clothing retailers.

6.3. Impact of the Shortage Cost (s). As can be seen from Table 4, the optimal selling price p^* and optimal length of inventory-holding period t_1^* increase in s, whereas the optimal length of shortage period t_2^* and optimal average profit π^* decrease in s. The optimal average profit is very sensitive to the change of shortage cost. When the shortage cost increases, the clothing retailer should reduce the order quantity in order to obtain higher profit. Furthermore, the results of Tables 2–4 show that the optimal average profit π^* increases with the optimal length of shortage period t_2^*. It suggests that the presence of shortage has got an affirmative effect on fashion apparel retailer ordering policy.

6.4. Sensitivity Analysis of Other Parameters (a, b, c, h). We now study the effects of changes in the values of the parameters a, b, c, and h on the optimal selling price p^*, order quantity Q^*, and average profit π^*. The sensitivity analysis is performed by changing each value of the parameters by −60%, −40%, −20%, 0%, +20%, +40%, and +60%, taking one parameter at a time and keeping the remaining parameter values unchanged. The computational results are shown in Figure 2.

The sensitivity analysis shown in Figure 2 indicates the following observations.

The optimal selling price p^* decreases in the values of parameters b, c, and h. Moreover, p^* is lowly positive sensitive to changes in parameters c and h, whereas p^* is a highly positive sensitive to change in c. It is reasonable that the purchase cost has a great and positive effect upon the optimal price.

When the values of parameters b, c, and h increase, the optimal order quantity Q^* will decrease. Q^* is moderately sensitive to changes in parameters c and h, whereas Q^* is highly sensitive to changes in a and b.

When the values of parameters b, c, and h increase, the optimal average profit π^* decreases. π^* is moderately sensitive to changes in parameters c and h, whereas Q^* is highly sensitive to changes in a and b. These results indicate that increases in cost have a negative effect on the average profit.

TABLE 3: Impact of the backlogging resistance.

k_1	p^*	t_1^*	t_2^*	π^*
0.010	47.41	2.61	0.7487	890
0.015	47.89	2.62	0.7295	886
0.020	48.07	2.64	0.7134	881
0.025	48.40	2.65	0.6952	877
0.030	48.91	2.68	0.6790	873

TABLE 4: Impact of the shortage cost.

s	p^*	t_1^*	t_2^*	π^*
2.0	47.22	2.58	0.8435	901
2.5	47.31	2.61	0.7719	893
3.0	47.41	2.63	0.7487	891
3.5	47.52	2.67	0.7376	886
4.0	47.62	2.68	0.7180	879

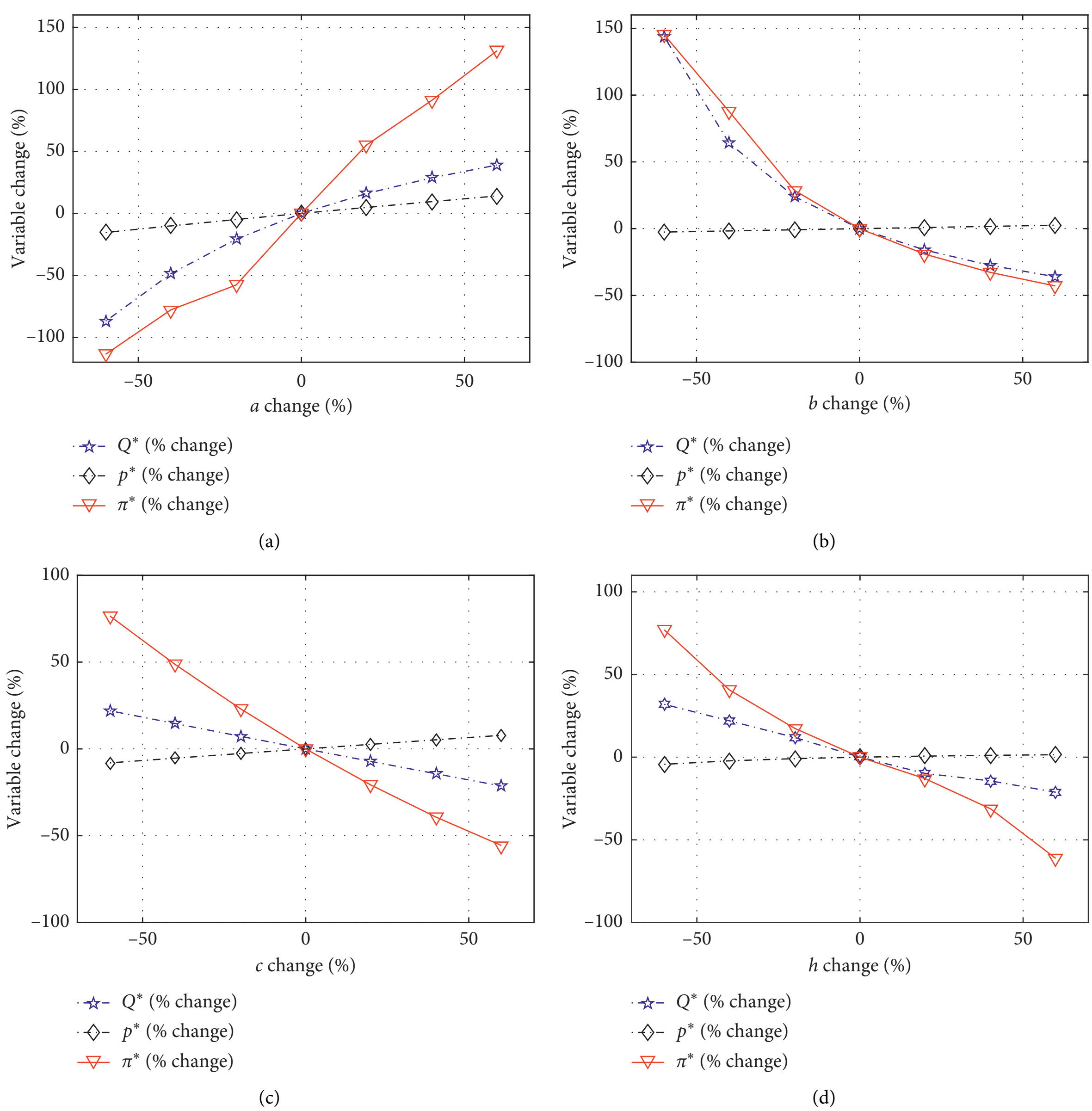

FIGURE 2: Effect of changes in values of parameters a, b, c, and h on the model variables.

7. Conclusions

The fashion level of clothing drops over time, resulting in a continuous decline in market value, a rapid reduction in demand, and unsold inventory. In reality, clothing retailers can reduce inventory costs by delaying in meeting demand of consumers, that is, allowing the products to be temporarily out of stock or partly out of stock to delay meeting consumers' needs. Thus, an inventory model is proposed for deteriorating fashion level apparel under shortage policy. In the model, the shortage is allowed, and the partial backlogging rate is sensitive to the waiting time. Firstly, we prove that, for any given replenishment cycle, the average profit is strictly concave in selling price and, for any given selling price, the average profit function is strictly pseudoconcave in replenishment cycle; that is, there exists a unique and optimal solution to the problem. Next, an algorithm is proposed to obtain the optimal price, optimal length of inventory-holding period, and optimal length of shortage period, so as to maximize the profit. Finally, the sensitivity analyses of the parameters are conducted to demonstrate the effectiveness of the proposed model. The research results show that when the purchasing cost, the shortage cost, and the holding of fashion apparel increase, the average profit of system will decrease. The optimal average profit and order quantity are highly sensitive to the deterioration rate of fashion level and the backordering resistance. The presence of shortage has got an affirmative effect on ordering policy of the fashion apparel retailer.

In this paper, we mainly focus on the impact of fashion level and products' shortage on the operational policy. Although we take the pricing decision into account in the model, the attention on the pricing policy is limited, since it is not affected by fashion level and time. In fact, dynamic pricing is a powerful tool to manage the demand changing over time products. Dynamic pricing is a price discrimination based on the time of purchase by customers. It can reduce the fluctuation of demand to a certain extent and increase the company's income. Therefore, the problem of dynamic pricing strategy and replenishment cycle for fashion apparel by considering the effect of fashion level and products' shortage would be worth exploring in the future. In addition, this paper considers the deterministic situation, thus considering the stochastic situation, such as stochastic demand, which can be another future research direction.

Data Availability

The raw/processed data required to reproduce the findings in this paper cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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