Research Article

Replacement Analysis of Mutually Exclusive Projects of Unequal Lives Using Kelly Specific Real Option Criterion

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This paper presents a model to address the uncertainty inherent in replacement problems, whereby a firm must select between mutually exclusive projects of unequal lifespans by applying the Kelly criterion (which is not well known to the engineering economics community) within a binomial lattice option-pricing environment. Assuming that only the interest rate, among many factors, is uncertain, Brown and Davis performed an economic analysis of this problem by employing a real option-pricing method and argued that their model yields results opposite to those yielded by the traditional approach. However, the results yielded by the model proposed herein are consistent with those by the traditional approach, unlike Brown and Davis’s model. The conclusion is that since the investment time horizon is infinite, a firm rationale pertaining to the selection of the best method for the investment problem of such types does not exist.

1. Introduction

The Kelly criterion, proposed by John Kelly [1] in 1956, is a formula pertaining to money management system that allows investors to maximize the expected value of a logarithmic return of their wealth, provided that they invest the optimal fraction of their investment capital over a long duration. Since its introduction, the Kelly criterion has been primarily investigated in financial investment and gambling fields [2]. A comprehensive analysis into the real-world success afforded by the Kelly criterion is provided in Poundstone’s well-known book [3], “Fortune’s Formula.” Poundstone describes Thorp earning a significant amount of money by applying the Kelly criterion to a stock market. Furthermore, MacLean, Thorp, and Ziemba [2] analyzed the extraordinary achievement of the legendary investor W. Buffett and claims that Buffett invests like a fully Kelly bettor. From a theoretical perspective, Brieman [4] developed fundamental mathematical properties and the expected log criterion rigorously. Thorp [5] proved that the Kelly criterion generates a significantly better return on investment than any other investment strategy and investigated the mathematical characteristics of the criterion. McEnally [6] presented a few properties pertaining to the geometric mean strategy and used simple numerical examples to explain them; the examples were based on Latane’s independent study, which is equivalent to the Kelly criterion.

From the perspective of portfolio theory, Thorp [5, 7] applied the Kelly criterion to portfolio selection and contrasted it with the Markowitz mean-variance efficiency. He argued that Kelly weightings do not necessarily imply mean-variance efficiency. Markowitz [8] argues that a log-optimal portfolio is a limiting mean-variance portfolio. Based on Latane’s study instead of the Kelly criterion, Roll [9] investigated the relationship among the Kelly capital growth model, mean-variance, and capital asset pricing analysis (CAPM). He compared the Kelly criterion model to a CAPM and reported a similar correspondence between the two models. For example, if the covariance between an asset’s expected return and the average return on all assets in a portfolio is zero, then both models provide an expected return equivalent to the risk-free interest rate. Otherwise, they provide an expected return that exceeds the risk-free interest rate.

Publications relevant to this study are scarce. Zhang [10] discussed the relationship between the optimal geometric
mean return of stocks and their option value based on the
Kelly criterion, in which it was assumed that a definite
betting strategy that guarantees winning does not exist.
Furthermore, Zhang proved that the optimal geometric
return, along with the optimal fraction required for in-
vestment, depends on the objective probabilities. However,
the ratio of the optimal fraction required for investment does
not rely on probabilities in a situation in which no arbitrage
opportunities exist and, furthermore, is a single-step bi-
nomial lattice option-pricing (BLOP) model that can
appropriately represent the investment. Zhu [11] discussed the
Kelly criterion within the option-pricing framework. He
demonstrated that a pricing interval for options with a
specified strike price exists such that having a nonnegative
expected return in a single-period economy, as a replace-
ment for the original investment, enables the investment
performance to be assessed easily based on the Kelly cri-
terion. He further argued that his assertion is valid only
when the option price is outside the interval.

DeFigueiredo and Barr [12] discussed an online reputa-
tion system that provides insurance against trade fraud by
leveraging existing relationships between players. In the
discussion, the Kelly criterion was used to determine the
minimum value for the ratio of the option value, along with
its strike price, within the specified BLOP environment.
Sæen [13] investigated the effect of estimation errors in the
Kelly criterion on a portfolio’s performance. He adopted a
multistep BLOP model and demonstrated its relationship
with the Kelly criterion. However, based on literature review,
studies pertaining to the application of the Kelly criterion to
the economic analysis of real investment projects (including
replacement problems) do not exist. Wu and Chung [14]
proposed a method for option trading to identify a profitable
option portfolio by bidding the optimal fraction of the Kelly
criterion. Compared with prior studies, they argued that
their model is a novel approach for option trading which
included the money management of position sizing. Based
on the following phrase from Levitt’s paper, “there are many
parallels between trading in financial markets and sports
betting. In both settings, investors with heterogeneous be-
iefs and information seek to profit through trading, as
uncertainty is resolved over time,” Johnstone [15] clarified
and extended the formal connections between investment
and betting within conventional binomial asset pricing
models. He argued that any investment position in a binary
asset can be replicated by a conventional bet made at odds
within the market price of the asset. According to him, the
relevant betting odds are based on the risk-neutral proba-
bility rather than actual probabilities. This current study is
motivated by Johnstone’s ideas.

Using the Kelly criterion and BLOP model, mutually
exclusive projects of unequal lives with an implicit as-
sumption of continued project replication over a long
(infinite) duration are considered. In this study, a single and
stationary replacement problem is focused: no technological
change occurs, and the operating cost is constant over an
analysis time horizon. However, for real option methods
and the method proposed herein, it is assumed that the interest
rate is relaxed to be uncertain.

The BLOP model yields results similar to those by the
Black-Scholes model [16], in which assumptions (such as a
geometric Brownian motion and the occurrence of infinites-
imal trading) similar to those of the Kelly criterion are
applied [3, 10, 11]. Therefore, it is both conceivable and
meaningful to discuss the application of the Kelly criterion
to the economic analysis of replacement problems within the
BLOP environment. Brown and Davis [17] argued that
conventional techniques can result in errors in a stochastic
investment environment; hence, it is decided to use the real
option method, which provides results opposite to those of
standard techniques. Because it is assumed that the in-
vestment time horizon is infinite, no firm rationale exists for
selecting the best method to solve the replacement problem.
Therefore, the authors herein present and compare the re-
results obtained from three different approaches: (i) con-
ventional analysis techniques, (ii) Brown and Davis’s real
options method, and (iii) the method proposed herein.
However, Eschenbach et al. [18] reported that if techno-
ological change is slow and/or the uncertainty in forecasts is
significant, then the conventional analysis technique should
be employed. If their suggestion is valid, then Brown and
Davis’s real option method should be reconsidered to solve
the abovementioned problems.

The remainder of this paper is organized as follows:
Section 2 presents the Kelly criterion within the context of
the applicable BLOP environment. Section 3 presents a brief
numerical example to demonstrate the validity of the eco-
nomic analysis of replacement problems. Finally, Section 4
provides the concluding remarks.

2. Development of the Kelly Criterion within
BLOP Environment Context

As it is believed that, among the many option-pricing
models, the characteristics of the BLOP model match well
with those of the Kelly criterion, this section presents a
discussion of the economic analysis of replacement prob-
lems from the viewpoint of the Kelly criterion within the
BLOP environment context.

2.1. Relationship between Kelly Criterion and BLOP Model.
As reported by Zhang [10], the ratio of the optimal fractions
of stocks (and their options), when assumed to generate a
binary outcome, is independent of the objective probability
distribution of the return. This statement implies that a few
options can be used to replace the underlying asset without
changing the optimal geometric mean return, as well as to
determine the objective probability distribution of the return
thereof. This argument is mathematically expressed as follows:

\[
\frac{f_{opt}^1}{f_{opt}^2} = \frac{e_1^0}{e_2^0} = \frac{e_1^0}{e_2^0}
\]

where \( f_{opt} \) is optimal faction of capital on hand to invest in a
stock; \( f_{opt}^1 \) is optimal fraction of capital on hand to invest in
an option; \( e_1^0 \) is excess return from a winning game in a stock
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investment; \(e_1^w\) is excess return from a winning game in an option investment; \(e_2^l\) is excess return from a losing game in a stock investment; and \(e_3^l\) is excess return from a losing game in an option investment.

Equation (1) shows that the Kelly criterion can be used to replicate any investment position in the BLOP environment [19]. When replicating an investment position, the betting odds are based on subjective probabilities equivalent to risk-neutral probabilities in option-pricing theories. The risk-neutral probabilities are both artificial and subjective and cannot be observed in the real world. These probabilities are used to derive betting odds and find arbitrage-free asset prices without referring to objective probabilities. This enables one to manage the options on Kelly bets, which can be valued using the BLOP model. In other words, investing in an option is equivalent to investing in its underlying asset (based on the Kelly criterion).

Before discussing the relationship between the Kelly criterion and the BLOP model, several important mathematical formulas used to evaluate an option value using the model are presented at first, along with the basic concept of the Kelly criterion. Equation (2) shows the well-known option-pricing formula with a risk-neutral probability.

\[
C = \frac{p'C_u + (1 - p')C_d}{1 + r_f}
\]  

where \(C\) is the call option value; \(p'\) is the risk-neutral probability; \(C_u\) is the call option value when stock price increases; \(C_d\) is the call option value when stock price decreases, and \(r_f\) is the risk-free interest rate.

The risk-neutral probability in Equation (2), denoted by \(p\), is generally expressed as follows:

\[
p' = \frac{S(1 + r_f) - S_d}{S_u - S_d} = \frac{(1 + r_f) - d}{u - d},
\]

where \(S_u\) is the stock price when a favorable outcome is yielded, \(S_d\) is the stock price when an unfavorable outcome is yielded, \(u\) is the upward movement rate of stock price, and \(d\) is the downward movement rate of stock price.

Using Equation (3), a risk-free rate of return can be obtained; however, the upward and downward movement rates must be calculated. Equation (4) is used to calculate the upward movement rate. The downward movement rate is the reciprocal of the upward movement rate, expressed as follows:

\[
u = e^{\sigma \sqrt{\delta_i}},
\]

where \(\sigma\) is the volatility of stock return and \(\delta_i\) is the time duration of a step measured in years.

In the following discussion, the basic concept of the Kelly criterion [1, 2, 12] is introduced. Assuming a game/investment with an initial capital amount of \(X_0\) that is played over a long duration with a fraction of capital on hand, denoted by \(f\), where \(0 \leq f \leq 1\), the accumulated capital after \(N\) bets is expressed as

\[
X_N = X_0(1 + f)^g(1 - f)^l,
\]

where \(S\) is the number of wins and \(L\) is the number of losses [20]. In this context, gamblers/investors never enter a complete ruin situation, which is analogous to \(P_r(X_N = 0) = 0\). The primary objective of the Kelly criterion is to identify the optimal fraction (bet size) of \(f^*\) by dividing both sides of Equation (5) by \(X_0\) and taking its logarithm as follows:

\[
G(f) = \ln\left(\frac{X_N}{X_0}\right)^{(1/N)} = \frac{S}{N} \ln(1 + f) + \frac{L}{N} \ln(1 - f),
\]

where \(N = S + L\). Here, \(G(f)\) yields the exponential rate of increase per play. It is noteworthy that

\[
\frac{N}{\ln\left(\frac{X_N}{X_0}\right)^{(1/N)}} = xN.
\]

The expression of \(G(f)\) is the basic expression for the Kelly criterion. In fact, Kelly maximized the expected value of the growth rate coefficient, \(g(f)\), that is,

\[
g(f) = E\left[\frac{1}{N} \ln\left(\frac{X_N}{X_0}\right)\right] = \lim_{N \to \infty} \left\{\frac{S}{N} \ln(1 + f) + \frac{L}{N} \ln(1 - f)\right\} = p \ln(1 + f) + q \ln(1 - f).
\]

Variable \(p\) is an objective winning probability that can be observed in a marketplace, and \(q = 1 - p\).

Taking the derivative of Equation (8) with respect to \(f\), the optimal fraction (bet size) of \(f^*\) that maximizes \(g(f)\) can be obtained as follows:

\[
g'(f) = \frac{p}{1 + f} - \frac{q}{1 - f} = \frac{p - q - f(p + q)}{(1 + f)(1 - f)} = 0.
\]

Setting Equation (9) equal to zero and rearranging it in terms of \(f\) yield the optimal fraction of the current capital that should be bet on each round of play to maximize the growth rate of Equation (8). Therefore,

\[
f^* = p - q.
\]

Furthermore,

\[
g'(f) = \frac{-p}{(1 + f)^2} - \frac{q}{(1 - f)^2}.
\]

As such, \(p\) and \(q \geq 0\) and \(g'(f) < 0\), such that \(g(f)\) is a strictly concave function for \(f \in [0, 1]\). Because \(g(f)\) is concave over \([0, 1]\) and \(f^*\) satisfies \(g''(f) = 0\), \(g(f)\) has a subsequent global maximum at \(f^*\), and

\[
g_{\text{max}} = p \ln(1 + p - q) + q \ln(1 - p + q)
\]

\[
= \ln 2 + p \ln p + q \ln q.
\]

Thus far, the Kelly criterion in terms of even payoff events has been discussed. However, this criterion can be extended to uneven payoff events. Suppose that investors win \(b\) units for every unit wager. Furthermore, suppose that,
on each bet, \( bp - q > 0 \), which implies that the events are favorable for investors. Therefore, the expected value of the growth rate coefficient of \( g(f) \) for uneven payoff events can be expressed as

\[
g(f) = E \ln \left( \frac{X^N}{X_0} \right) = p \ln(1 + bf) + q \ln(1 - f). \tag{13}
\]

By adopting a procedure similar to that described above, the optimal fraction of the current capital to invest can be obtained as follows:

\[
f^* = \frac{p - q}{b}. \tag{14}
\]

More information regarding the Kelly criterion is available in the publication by Maclean [2].

### 2.2. Odds System for Replicating Investment with Bets.

To derive a gross payoff \( a \) equivalent to the amount of capital to bet, which is associated with risk-neutral probabilities and odds, as defined with respect to the number of successes and losses (failures), the following is considered. Because \( N = S + L \), a favorable odd \( O_f \), an unfavorable odd \( O_u \), and a winning probability \( p' \) are defined as follows:

\[
\begin{align*}
O_f &= \frac{S}{L} \\
O_u &= \frac{L}{S} \\
p' &= \frac{S}{S + L}
\end{align*}
\]

Since the odds are described based on risk-neutral probabilities, the winning probability in Equation (16) is regarded as a risk-neutral probability in the BLOP model. Because the odds are specified as the number of \( O_f \) in Equation (15), the probability can be represented as a ratio of \( O_f \) as follows:

\[
p' = \frac{O_f}{(O_f + 1)} = \frac{1}{(O_a + 1)}. \tag{16}
\]

The reciprocal of Equation (16) represents the extent to which a gross payoff \( a \) is applicable to the winning game, and it is expressed as

\[
a = \frac{1}{p'} = \frac{(O_f + 1)}{O_f} = 1 + \frac{O_f}{1 + O_f} = 1 + O_a. \tag{17}
\]

Subsequently, by referring to Figure 1, it is considered that an investment project whose current value is \( S \) dollars is replicated with bets that constitute a portfolio. As shown in Figure 1, investors can simultaneously invest in risky assets (e.g., a stock) and risk-free assets (e.g., a government bond). If they fail to invest in risky assets, then they will be left with the amount required to invest in the risk-free asset. This event occurs when the price of risky assets decreases. Otherwise, they will be left with an amount equal to the amount required to invest in the risky asset. In this regard, the portfolio of interest involves allocating \( f_u^* = S - \left( \frac{S}{S_0} + r_f \right) \) in the risky asset and \( f_d^* = (S_0 + r_f) \) in the risk-free asset, where \( S_0 \) represents the price of the risky asset when an unfavorable outcome is yielded [21]. The value of \( f^* \) is expressed as an absolute monetary value instead of a ratio.

For the investment, the total amount of payoffs is determined by the product of the optimal fraction of investment capital and \( a \) earned in one dollar. Because \( a \) is the present value at \( t = 0 \), this value should be compounded by \( (1 + r_f) \) to qualify as \( a \) for one unit of time elapsed. Therefore, the total amount of payoffs corresponding to the occurrence of the favorable outcome \( TP_f \) is expressed as follows [7]:

\[
TP_f = \alpha_a \cdot f_u^* = \left\{ \frac{S_u - S_d}{(1 + r_f)S - S_d} \left( 1 + r_f^* \right) \right\}
\]

\[
\cdot \left\{ \frac{(1 + r_f)S - S_d}{(1 + r_f)} \right\} = S_u - S_d. \tag{18}
\]

However, when the unfavorable outcome \( TP_u \) occurs, it is expressed as follows:

\[
TP_u = \alpha_d \cdot f_d^* = \left\{ \frac{(1 + r_f)}{1 - p'} \right\} \cdot \left\{ \frac{S_d}{(1 + r_f)} \right\}
\]

\[
= \left\{ \frac{S_u - S_d}{S_u - S(1 + r_f)} \left( 1 + r_f \right) \right\} \cdot \left\{ \frac{S_d}{(1 + r_f)} \right\} \tag{19}
\]

\[
= \frac{(S_u - S_d)S_d}{S_u - S(1 + r_f)}.
\]

The calculated present value of the average of the two values yielded by Equations (18) and (19) yields the value of the original investment at \( t = 0 \), as shown below

\[
\left\{ p' (S_u - S_d) + (1 - p') \left( (S_u - S_d)S_d/Su - S(1 + r_f) \right) \right\} = S. \tag{20}
\]

The result of Equation (20) indicates that the portfolio with bets completely replicates the original investment whose value at \( t = 0 \) is \( S \). To validate this result, a brief numerical example is provided, in which \( S = 50, S_u = 70, \) and \( r_f = 10\% \). To calculate the total amount of payoffs, the risk-neutral probability in the example using Equation (3) should be determined firstly as follows:

\[
p' = \frac{50(1 + 0.1) - 30}{70 - 30} = (1 + 0.1) - 0.6 = 0.625. \tag{21}
\]

In this example, the odds for and against are expressed, respectively, as follows:
The present value of the total amount of payoffs earned from the two events is calculated as follows:
\[
\frac{(0.625)(1.76)(22.7272) + (1 - 0.625)(2.933)(27.2727)}{(1 + 0.1)} = 50.
\] (24)

In fact, the same result can be obtained directly using (20) as follows:

\[
\frac{[(0.625)(70 - 30) + (1 - 0.625)(((70 - 30)/70 - 50)(1 + 0.1))]}{(1 + 0.1)} = 50.
\] (25)

The final value \(v_f\) and the average rate of return \(\tau\) on investment are expressed as follows, respectively:

\[
v_f = X_0e^{g_{\max}},
\] (30)

and

\[
\tau = e^{g_{\max}} - 1.
\] (31)

Equation (28) implies that investors should invest \(f^* = p\) and \(1 - f^* = 1 - p\) of their investment capital in the up and downstate events, respectively, disregarding the odds for each item. As shown by the equation, the value of \(f^*\) is restricted by an objective probability of \(p\), which is between zero and one. The values of \(r_f\) and \(S\) are simply observed in the market, and the variance of stock prices can be estimated easily because a significant amount of stock prices can be obtained in the stock market. Hence, the risk-neutral probability of \(p^*\) in Equation (29) is determined without difficulty. However, the investor must postulate two quantities at \(t = 0\): (1) the objective probability of \(p\) that an upstate event can occur and (2) the initial investment capital.

The example described in the previous section is considered to validate the procedure discussed in this section. Suppose that the objective probability of the upstate event occurring is 0.7 and the initial investment capital is equal to the current stock price of \(S = 50\). Therefore, the expected maximum exponential growth rate, final value, and average rate of return on investment are expressed as

\[
g_{\max} = (0.7)\ln(0.7)\frac{1 + 0.1}{0.625} + (1 - 0.7)\ln\left(1 - \frac{1 + 0.1}{1 - 0.625}\right)
\] (32)

\[
\cdot\left\{(1 - 0.7)\frac{1 + 0.1}{1 - 0.625}\right\} = 0.1077,
\] and
Replacement problems with infinite planning horizons are typically encountered in companies. In this section, the approach proposed herein is applied to Brown and Davis’s example [17] and the results obtained are compared with those of Brown and Davis’s. In this example, Brown and Davis consider two projects that have different lifespans and interest rates, where these variables serve as sources of uncertainty which affect the problem. Next, the manner by which Brown and Davis solved this problem is discussed. Projects A and B were defined with estimated cash flows, as shown in Table 1, with different timelines, that is, three and five years, respectively.

3.1. Traditional Approach. An initial interest rate of 7% was used for the traditional economic analysis. First, the net present values (NPV) of the two projects based on a single life cycle should be calculated as follows: \( NPV_A^1 (7\%) = 11.08 \) and \( NPV_B^1 (7\%) = 17.40 \). Next, assuming that the project will be repeatedly replaced with an identical one over the infinite planning horizon, their NPV, over the planning horizon, denoted by \( NPV^{\infty}_{A/B} \), must be determined. These values are determined by their single life cycle \( NPV_s \) (obtained above) as follows: \( NPV^{\infty}_A = (7\%) = 60.30 \) and \( NPV^{\infty}_B = (7\%) = 60.63 \). Because \( NPV^{\infty}_A < NPV^{\infty}_B \), it is economically desirable for a firm to undertake project B rather than project A.

3.2. Brown and Davis’s Real Option Approach. Next, the uncertainty over the interest rate is considered and its effect on the final decision based on the real option-pricing concept is investigated. In the example, it is assumed that the initial value of 7% will be maintained over the first three years and then change to either 6% or 8% with an equal objective probability. An objective probability is defined as the probability that the interest rate decreases to 6%, resulting in an increase in the project value. Once it changes to either level, it will remain therein indefinitely. Dixit and Pindyck [16] extensively discussed the problems and assumptions similar to those in this example and its derivatives, including those of Brown and Davis [17], which have been created and applied in a number of investment decision problems. Most of these problems were solved by applying real option valuation techniques [12] directly or by considering the optionality embedded in cash flows. The latter case employs only a real options concept to solve this problem. In this case, an example from the perspective of the BLOP model will be discussed.

Table 2 shows \( NPV^1_A, NPV^1_B, NPV^{\infty}_{A/B}, \) and \( NPV^{\infty}_A \) with the interest rate varying from 1% to 10%. The table reveals the dominating project over a certain range of interest rates. As shown in the table, \( NPV^{\infty}_{A/B} (6\%) = 87.29, NPV^{\infty}_A (6\%) = 87.29, NPV^{\infty}_A (8\%) = 39.51, \) and \( NPV^{\infty}_B (8\%) = 40.63. \) If the interest rate changes to 6%, then the next optimal alternative is to select project A. However, if the interest rate changes to 8%, then project B becomes the optimal alternative.

According to Brown and Davis’s real option-pricing approach, if project A is selected at first, then NPV from this decision is the sum of \( NPV_s \) from project A over the next three years and the optimal choice in the three years, which is expressed by the first one of the two values below. However, if project B is selected first, then its NPV is expressed as the second option as follows:

\[
NPV_{A/B}^{\text{max}}(\text{uncertainty}) = NPV_{A/B}^{\text{1}}(7\%) + (0.5)\left[\frac{NPV^{\infty}_{A/B} (6\%) + NPV^{\infty}_{A/B} (8\%)}{(1 + 0.07)^3}\right],
\]

\[
= 11.08 + (0.5)\frac{[88.04 + 40.63]}{(1 + 0.07)^3} = 63.60,
\]

\[
NPV_{A/B}^{\text{max}}(\text{uncertainty}) = NPV_{A/B}^{\text{1}}(7\%) + (0.5)\left[\frac{NPV^{\infty}_{A/B} (6\%) + NPV^{\infty}_{A/B} (8\%)}{(1 + 0.07)^3}\right],
\]

\[
= 17.40 + (0.5)\frac{[88.04 + 40.63]}{(1 + 0.07)^3} = 63.27.
\]
Table 2: NPVs for single life cycle and infinite planning horizon for A and B.

<table>
<thead>
<tr>
<th>MARR (%)</th>
<th>NPV (A, 3)</th>
<th>NPV (B, 5)</th>
<th>NPV (A, ∞)</th>
<th>NPV (B, ∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.18</td>
<td>49.53</td>
<td>1060.04</td>
<td>1020.60</td>
</tr>
<tr>
<td>2</td>
<td>27.50</td>
<td>46.34</td>
<td>746.75</td>
<td>460.61</td>
</tr>
<tr>
<td>3</td>
<td>23.96</td>
<td>37.64</td>
<td>282.35</td>
<td>273.95</td>
</tr>
<tr>
<td>4</td>
<td>20.26</td>
<td>31.26</td>
<td>185.17</td>
<td>180.62</td>
</tr>
<tr>
<td>5</td>
<td>17.28</td>
<td>26.98</td>
<td>126.88</td>
<td>124.62</td>
</tr>
<tr>
<td>6</td>
<td>14.12</td>
<td>22.06</td>
<td>88.04</td>
<td>87.29</td>
</tr>
<tr>
<td>7</td>
<td>11.08</td>
<td>17.40</td>
<td>60.30</td>
<td>60.63</td>
</tr>
<tr>
<td>8</td>
<td>8.15</td>
<td>12.98</td>
<td>40.63</td>
<td>40.63</td>
</tr>
<tr>
<td>9</td>
<td>5.32</td>
<td>8.78</td>
<td>25.08</td>
<td>25.08</td>
</tr>
<tr>
<td>10</td>
<td>2.59</td>
<td>4.79</td>
<td>16.64</td>
<td>16.64</td>
</tr>
</tbody>
</table>

Since $\text{NPV}_A^{\text{max}}$ (uncertainty) > $\text{NPV}_B^{\text{max}}$ (uncertainty), it is recommended that the firm undertakes project A instead of project B. Comparing this with the previous decision, it is recognized that the decision is changed from project B to project A, causing an increase in the firm’s NPV from 60.63 to 63.60.

3.3. Kelly Criterion Approach within the BLOP Environment.

In this section, the same problem from the perspective of Kelly within the context of the applicable BLOP environment is discussed. For the discussion, a binomial lattice model for projects A and B is developed at first, as shown in Figure 2, using the information provided in Table 2. To complete the figure, it may be considered such that NPVs, provided in the table with an interest rate of 7% be the current project value and those with interest rates of 6% and 8% be the up and down states, respectively.

Similar to Brown and Davis’s approach, the two cases in the example will be discussed subsequently. The first case in which an identical project will be repeated over an infinite planning horizon in the context of the applicable BLOP environment via the Kelly criterion is considered. Using Equations (29) and (30), the final values of the two projects can be determined. However, before resolving the equation, the risk-neutral probability, denoted by $p$, for each project using Equations (3) should be obtained. Subsequently, the two risk-neutral probabilities are determined to be $p_A = 0.4290$ and $p_B = 0.4286$ when the objective probability is 50%. As such, $g_A^{\text{max}}$ and $g_B^{\text{max}}$ for the two projects are 0.2131 and 0.3486, respectively. Substituting these values into Equation (30) yields the final values of the two projects: $V_A = 74.6218$ (illustrated in the following) and $V_B = 85.9177$.

Based on the final values, it can be inferred that project B is superior to project A; hence, the firm should undertake project B instead of project A. This is congruent with the decision in the conventional approach but in contrast to the decision based on Brown and Davis’s real option approach. However, it is noteworthy that this phenomenon cannot be generalized to all problems. Table 3 shows the final values of the two projects with different objective probabilities, assuming that the risk-neutral probability is maintained over the entire range of objective probabilities. This assumption is reasonably valid because a 1% difference adjacent to a base interest rate, such as 7% in the example, does not significantly affect the risk-neutral probability. As shown in the table, project B dominates project A over the entire range of objective probabilities. When the objective probability is 50%, the procedure to calculate the final values of projects A and B is as follows:

1. The risk-neutral probability for project A is expressed as $p_A = (60.30 - 39.51/88.04 - 39.51) = 0.4290$ and $p_B = (60.30 - 40.63/87.29 - 40.63) = 0.4286$.

2. The expected maximum exponential growth rate of project A is expressed as $g_A^{\text{max}} = (0.5) \ln (0.5(1.07^3/0.4290)) + (0.5) \ln (0.5(1.07^3/0.4286)) = 0.2131$ and $g_B^{\text{max}} = (0.5) \ln (0.5(1.07^3/0.4286)) + (0.5) \ln (0.5(1.07^3/1 - 0.4286)) = 0.3486$.

3. The final value of project A is expressed as $V_A = e^{0.2131}(60.30) = 74.6218$ and $V_B = e^{0.3486}(60.63) = 85.9177$.

Next, the second case is considered where either project A or project B is selected first. As illustrated in Brown and Davis’s real option approach, the decision concerning the next optimal project (after the first cycle of planning horizons for the first selected project has been determined) is made based on information regarding the movement of the interest rate. Recapturing their illustration here is convenient and useful for understanding the following discussion. Because $\text{NPV}_A^{\text{max}} (6%) > \text{NPV}_B^{\text{max}} (6%) = 88.04 > \text{NPV}_A^{\text{max}} (8%) = 39.51 > \text{NPV}_B^{\text{max}} (8%) = 40.63$, project A with $\text{NPV}_A^{\text{max}} (6%) = 88.04$ and project B with $\text{NPV}_B^{\text{max}} (8%) = 40.63$ must be selected to calculate the final project value, each with an objective probability of 50% at interest rates of 6% and 8%, respectively. These selections with NPVs of the two projects allow us to construct another binomial lattice model for each option, as shown in Figure 3. Table 4 shows the final values of the two projects in a case in which project A or project B is selected first. As illustrated in Brown and Davis’s real option approach, the decision concerning the next optimal project (after the first cycle of planning horizons for the first selected project has been determined) is made based on information regarding the movement of the interest rate. Recapturing their illustration here is convenient and useful for understanding the following discussion. Because $\text{NPV}_A^{\text{max}} (6%) > \text{NPV}_B^{\text{max}} (6%) = 88.04 > \text{NPV}_A^{\text{max}} (8%) = 39.51 > \text{NPV}_B^{\text{max}} (8%) = 40.63$, project A with $\text{NPV}_A^{\text{max}} (6%) = 88.04$ and project B with $\text{NPV}_B^{\text{max}} (8%) = 40.63$ must be selected to calculate the final project value, each with an objective probability of 50% at interest rates of 6% and 8%, respectively. These selections with NPVs of the two projects allow us to construct another binomial lattice model for each option, as shown in Figure 3. Table 4 shows the final values of the two projects in a case in which project A or project B is selected first.
Figure 3: Binomial lattice model with the first selected project.

Table 3: NPV_max for single life cycle and infinite planning horizon for A and B.

<table>
<thead>
<tr>
<th>p (%)</th>
<th>( g_{\text{max}}^A )</th>
<th>( g_{\text{max}}^B )</th>
<th>( V_A )</th>
<th>( V_B )</th>
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<tr>
<td>5</td>
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<tr>
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<tr>
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<td>1.1266</td>
<td>162.3976</td>
<td>187.0166</td>
</tr>
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</table>

(3) The final values of projects A and B are expressed as 
\[
V_A = e^{0.2176 \times (60.30)} = 74.9583 \quad \text{and} \quad V_B = e^{0.3507 \times (60.63)} = 86.0983.
\]

Because \( V_A = 74.9583 < V_B = 86.0983 \), it is recommended that the firm undertakes project B. In this case, project B dominates project A over the entire range of the objective probabilities. This is in contrast to the decision made using Brown and Davis’s real option approach but is congruent with the more conventional approach. As expected, the final values in the second case were greater, even when the improved amount was not much greater than that of the first case.

4. Conclusions

Herein, a replacement problem with an infinite planning horizon based on three different analysis techniques has been discussed: (1) a conventional analysis technique, (2) a real option-pricing technique, and (3) the Kelly criterion technique in the context of an applicable BLOP environment. This study applied each of these techniques to a specific example obtained from Brown and Davis’s study. It is discovered that the third technique yielded the same result as the first but yielded a different result when compared with that of the second technique.

The finding above may apply to the specific example. However, because \( \text{NPV}_{A}^{\infty} = 60.30 < \text{NPV}_{B}^{\infty} \) (uncertainty) = 60.63 \( < \text{NPV}_{B}^{\max} \), these values were prioritized in the three techniques. It is believed that the third technique dominated over the other two primarily because of the objective winning probability. As shown in equation (28), the objective winning probability equals the optimal betting ratio \( f^* \). Therefore, the approach to adequately determine the objective winning probability in advance must be determined. This probability may be obtained based on well-documented data, management’s subjective judgment, and so forth. Once the objective winning probability is determined, the risk-neutral probability is vital to the method proposed herein.

The major disadvantage of applying the method proposed herein is that it hindered the understanding of the concepts of real option-pricing theories and the Kelly criterion. Since the beginning of 2000s, the real option theory has been extensively investigated in both theoretical and practical aspects. However, it has not been applied widely as an investment project analysis in firms worldwide. For example, 4.6% of Korean firms, 0.5% of Japanese firms, and 26.6% of Fortune 500 and FEI companies use real option-pricing theories [22]. As described earlier, the Kelly criterion has been investigated and applied primarily in gambling games and finance, whereas almost no research regarding the Kelly criterion has been conducted from a real investment perspective. Therefore, it is believed that the proposed model in this study will only be adopted after a significant amount of time. In addition, depending on the variety of conditions and assumptions, many different replacement
problems may occur. The problem addressed in this study is extremely simple and hence easy to solve. In fact, for the proposed model to be practically useful, further investigations involving more sophisticated and complex replacement problems are to be conducted. As such, they will be performed in future studies.

Data Availability

Data are available upon request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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