Research Article

Adaptive Fixed-Time Tracking Consensus Control for Multiagent Nonlinear Pure-Feedback Systems with Performance Constraints

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This paper investigates adaptive fixed-time tracking consensus control problems for multiagent nonlinear pure-feedback systems with performance constraints. Compared with existing results of first/second/high-order multiple agent systems, the studied systems have more complex nonlinear dynamics with each agent being modeled as a high-order pure-feedback form. The mean value theorem is introduced to address the problem of nonaffine structure in nonlinear pure-feedback systems. Meanwhile, radial basis function neural networks (RBFNNs) are employed to approximate unknown functions. Furthermore, a constraint variable is used to guarantee that all local tracking errors are within the prescribed boundaries. It is shown that, by utilizing the proposed consensus control protocol, each tracking consensus error can converge into a neighborhood around zero within designed fixed time, the tracking consensus performance can be ensured during the whole process, and all signals in the investigated systems are bounded. Finally, two simulations are performed and the results demonstrate the effectiveness of the proposed control strategy.

1. Introduction

In the past decade, the finite-time consensus control has attracted considerable attention due to its unique advantages in faster convergence rate and higher precision [1] and its wide applications in unmanned aerial vehicles, autonomous underwater vehicles, surface vessels, and so forth [2–5].

Finite-time control was first proposed in 1965, and many related research results emerged in the subsequent decades. A variety of finite-time control protocols [6–11] have developed to address the different agents' dynamics in multiagent systems (MASs). For example, in [12, 13], finite-time consensus control for first- and second-order MASs was proposed, respectively, by using the principle of homogeneity [14]. In [15], a finite-time control for MASs with double integrator dynamics was proposed by utilizing power integrator technique. Furthermore, two classes of finite-time control protocols for MASs composed of first-order and second-order integrator agents were developed by combining the homogeneous domination method with adding a power integrator method. Thereafter, to solve the consensus control problems of high-order uncertain MASs, several

finite-time control strategies were proposed in [16–18]. Even so, there is a limitation for the finite-time control results that the convergence time is strongly dependent on the initial states of the systems; in other words, once the initial states are far away from the equilibrium point, the convergence time will increase as a result. As a matter of fact, it is much more reasonable for a predictable convergence time.

In 2012, to address the limitation of finite-time control, Polyakov [19] proposed fixed-time stability control, which can allow the convergence time of systems have nothing to do with its initial states. Subsequently, many related research results [20–29] for MASs have been carried out. It can be found that most of the existing fixed-time consensus control results are focused on MASs with first-order or second-order, and only a few focus on high-order MASs. Although, in [24–27], the fixed-time control strategies have considered high-order MASs; they are not general enough, since their results apply only to strict-feedback nonlinear systems. However, compared with lower-triangular systems and strict-feedback nonlinear systems, the pure-feedback nonlinear systems [30–32] are more general systems. On the other hand, in many practical industrial systems, there are

some requirements for performance constraint. For example, unknown dead-zone inputs [33], output-constrained [34], preassigned transient performance [35], and other measures are taken to prevent system damage. In [36], the authors investigated the adaptive tracking control problem for a class of nonlinear time-delay systems in the presence of input and tracking error constraints.

Motivated by the above observations, in this paper, we focus on addressing the problem of adaptive fixed-time tracking consensus control for a class of pure-feedback uncertain nonlinear MASs with performance constraint. In the design, to solve the problem of nonaffine structure in nonlinear pure-feedback systems, we employ mean value theorem. Meanwhile, RBF neural networks are used to compensate for uncertain nonlinear terms and some functions that are difficult to calculate induced from the controller design procedure. Furthermore, a predefined boundary function is utilized to cope up with the requirement of the prescribed performance. Finally, based on Lyapunov stability theorem, fixed-time control theory, and graph theory, a novel fixed-time consensus control strategy is designed for the considered pure-feedback nonlinear MASs, which can guarantee that all the local tracking errors are within the range of the predefined boundary function and can converge to the neighborhood of the origin in fixed time and all signals in MASs are bounded.

The main contributions of this paper can be summarized as follows:

- (1) To our best knowledge, this paper is the first attempt to solve the problem of adaptive fixed-time consensus control for MASs with nonlinear pure-feedback. Compared with some existing results [16–18, 24–27] on high-order MASs with nonlinear strict-feedback, the investigated pure-feedback nonlinear MASs are more general.
- (2) By introducing RBF neural networks, the uncertain nonlinear items in each agent's dynamics can be approximated infinitely. Therefore, the designed controller in this paper is more versatile and robust.
- (3) Based on backstepping algorithm and Lyapunov theorem, a novel fixed-time consensus control protocol design method is proposed. Meanwhile, the tracking performance is governed by the predefined boundary function.

The rest of this paper is arranged as follows. Section 2 gives problem description and preliminaries. Section 3 presents the detailed design process of fixed-time controller. In Section 4, the stability analysis will be given. In Section 5, the validity of the proposed control scheme is proven by two simulations. Finally, the conclusion of this work is summarized in Section 6.

2. Problem Description and Preliminaries

2.1. Problem Statement. The considered nonlinear MASs consist of M followers, labeled as agents 1 to M , and a leader, labeled as an agent 0 under a directed communication graph

topology. The dynamic models of M followers can be described as the form of the following pure-feedback nonlinear systems:

$$\begin{cases} \dot{x}_{i,k}(t) = f_{i,k}(\bar{x}_{i,k}(t), x_{i,k+1}(t)) + d_{i,k}(\bar{x}_{i,k}, t), \\ \dot{x}_{i,n_i}(t) = f_{i,n_i}(\bar{x}_{i,n_i}(t), u_i(t)) + d_{i,n_i}(\bar{x}_{i,n_i}, t), \\ y_i(t) = x_{i,1}(t), \end{cases} \quad (1)$$

where $i = 1, \dots, M$, $k = 1, \dots, n_i - 1$, $\bar{x}_{i,k} = [x_{i,1}(t), \dots, x_{i,k}(t)]^T \in \mathbb{R}^k$ with $k = 1, \dots, n_i$, $u_i(t) \in \mathbb{R}$, and $y_i(t) \in \mathbb{R}$ are system state variables, system input, and system output, respectively; $f_{i,k}(\cdot)$ are unknown smooth nonaffine functions; $d_{i,k}(\cdot)$ are unknown but bounded disturbances.

We utilize the mean value theorem to solve the nonaffine structures in pure-feedback systems:

$$\begin{aligned} f_{i,k}(\bar{x}_{i,k}, x_{i,k+1}) &= f_{i,k}(\bar{x}_{i,k}, x_{i,k0}) + g_{i,k} \cdot (x_{i,k+1} - x_{i,k0}), \\ f_{i,n_i}(\bar{x}_{i,n_i}, u_i) &= f_{i,n_i}(\bar{x}_{i,n_i}, x_{i,n_i,0}) + g_{i,n_i} \cdot (u_i - x_{i,n_i,0}), \end{aligned} \quad (2)$$

where $g_{i,k} := g_{i,k}(\bar{x}_{i,k}, x_{i,\mu_k}) = \partial f_{i,k}(\bar{x}_{i,k}, x_{i,\mu_k}) / \partial x_{i,\mu_k}$ with $k = 1, 2, \dots, n_i$, $x_{i,n_i+1} = u_i$, $x_{i,\mu_k} = \mu_k x_{i,k+1} + (1 - \mu_k) x_{i,k0}$ with $0 < \mu_k < 1$, and $x_{i,k0}$ are known at the given time t_0 .

Therefore, system (1) can be rewritten as

$$\begin{cases} \dot{x}_{i,k} = f_{i,k}(\bar{x}_{i,k}(t), x_{i,k0}) + g_{i,k} \cdot (x_{i,k+1} - x_{i,k0}) + d_{i,k}(\bar{x}_{i,k}, t), \\ \dot{x}_{i,n_i} = f_{i,n_i}(\bar{x}_{i,n_i}(t), x_{i,n_i,0}) + g_{i,n_i} \cdot (u_i - x_{i,n_i,0}) + d_{i,n_i}(\bar{x}_{i,n_i}, t), \\ y_i = x_{i,1}. \end{cases} \quad (3)$$

Assumption 1 (see [37]). $g_{i,k}(\cdot)$ are unknown nonlinear functions and bounded, and there exists a positive constant b_i such that $b_i \leq |g_{i,k}(\cdot)|$.

For ease of description, we defined vector functions $\bar{y}_{0,i} = [y_0, y_0^{(1)}, \dots, y_0^{(i)}]^T$, where y_0 is the leader's output and $y_0^{(i)}$ is its i^{th} derivative.

Assumption 2. Suppose that the leader's vector functions $\bar{y}_{0,i}$ are known smooth continuous functions and bounded. $\bar{y}_{0,i} \in \Omega_{0,i} \subset \mathbb{R}^{i+1}$ with $i = 1, \dots, n_i$, where $\Omega_{0,i}$ being known compact sets and y_0 is an n_i -order differentiable function.

2.2. Graph Theory. The communication between agents can be represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{N_1, N_2, \dots, N_M\}$ denotes the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges. An edge $(j, i) \in \mathcal{E}$ means that agent i can obtain information from agent j , but not vice versa, where j and i are the parent node and child node, respectively. $\mathcal{N}_i = \{j | (j, i) \in \mathcal{E}\}$ is the set of neighbors of a node i , which is the set of nodes with edges incoming to node i . $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{M \times M}$ is the related adjacency matrix, if $(j, i) \in \mathcal{E}$, $a_{i,j} > 0$; otherwise, $a_{i,j} = 0$. Self-edges are not allowed, i.e., $a_{i,i} = 0$. The Laplacian matrix for graph $\mathcal{G}$ is denoted by $\mathcal{L} = (l_{i,j})_{N \times N}$ with $l_{i,i} = \sum_{j=1, j \neq i}^N |a_{i,j}|$ and $l_{i,j} = -a_{i,j}$ for $i \neq j$, and $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{M \times M}$ where

$\mathcal{D} = \text{diag}[d_1, \dots, d_M]$; $d_i = \sum_{j=1, j \neq i}^M a_{ij}$ is the diagonal element of the degree matrix $\mathcal{D}$. A directed graph $\mathcal{G}$ has a directed spanning tree if there exists at least one agent that has directed paths to all other agents.

If including a leader agent N_0 , the communication topology for the $M + 1$ agents is described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \mathcal{V} \cup \{N_0\}$ and $\mathcal{E} = \mathcal{V} \times \mathcal{V}$. The connection matrix between the followers and the leader can be expressed as $H = \text{diag}\{h_1, \dots, h_N\}$ where $h_i > 0$ if and only if N_i can directly obtain the leader's information; otherwise, $h_i = 0$.

2.3. RBF Neural Networks. RBF neural network [38] has powerful unknown function approximation ability. In this paper, we approximate uncertain nonlinear items in each agent's dynamics via the following RBF neural network:

$$\hat{f}(Z) = W^T S(Z), \quad (4)$$

where the input vector $Z \in \Omega_Z \subset R^q$, $W = [w_1, w_2, \dots, w_l]^T \in R^l$ is the weight vector; $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T \in R^l$ is the radial basis function vector with neural network node number $l > 0$. The basis function $S(Z)$ is always chosen as the following form of Gaussian function:

$$S_i(Z) = \exp\left[-\frac{(Z - \mu_i)^T (Z - \mu_i)}{\eta_i^2}\right], \quad i = 1, 2, \dots, l, \quad (5)$$

where η_i is the width of the Gaussian function, and $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{iq}]^T$ is the center of the receptive field. In theory, as long as a sufficient number l of nodes are selected, RBF neural network can approximate any continuous function $f(Z)$ in a compact set $\Omega_Z \subset R^q$ with arbitrary precision ε .

$$f(Z) = W^{*T} S(Z) + \delta(Z), \quad \forall Z \in \Omega_Z \subset R^q, \quad (6)$$

where $\delta(Z)$ is the approximation error and satisfies $|\delta(Z)| \leq \varepsilon$, and W^* is the ideal weight vector, which is chosen as the value of W that minimizes ε for all $Z \in \Omega_Z$, i.e.,

$$W^* = \arg \min_{W \in R^l} \sup_{Z \in \Omega_Z} \{ |f(Z) - \hat{f}(Z)| \}. \quad (7)$$

Remark 1. In this paper, we choose $\theta_{i,j} = \max\{\|W_{i,j}^*\|/b_i, i = 1, 2, \dots, M, j = 1, 2, \dots, n_i\}$ with $\tilde{\theta}_{i,j} = \theta_{i,j} - \hat{\theta}_{i,j}$, where $\hat{\theta}_{i,j}$ are the estimates of the unknown constants $\theta_{i,j}$, b_i are positive design parameters and related to Assumption 1, and $\|\cdot\|$ is the norm.

Assumption 3 (see [39]). There exist unknown constants $Q_{i,j}$ such that $|\tilde{\theta}_{i,j}| \leq Q_{i,j} < \infty, i = 1, 2, \dots, M, j = 1, 2, \dots, n_i$.

Lemma 1 (see [40]). Consider the Gaussian function (5). $\|S(Z)\|$ has an upper bound such that $\|S(Z)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2p^{2k}k^2/\eta^2} := s$, where $p = (1/2)(\min_{i \neq j} \|\mu_i - \mu_j\|)$.

2.4. Fixed-Time Control. Consider the following nonlinear system,

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0, f(0) = 0, \quad (8)$$

where $x(t) \in R^n$ is a system variable, and $f(x): R^n \rightarrow R^n$ is a nonlinear function which may be discontinuous; the solutions of system (8) are understood in the sense of Filippov [41]. For system (8), the following definitions and lemmas are introduced.

Definition 1 (see [1]). The equilibrium $x = 0$ of system (8) is finite-time convergent if there are an open neighborhood $U \subseteq D$ of the origin and a positive definite function $T(x_0): U \setminus \{0\} \rightarrow (0, \infty)$, such that $\lim_{t \rightarrow T(x_0)} x(t, x_0) = 0$ and $x(t, x_0) = 0, \forall t > T(x_0)$ for any initial condition $x_0 \in U$, where $T(x_0)$ is the settling time function. The equilibrium $x = 0$ of system (8) is finite-time stable if it is Lyapunov stable and finite-time convergent. Moreover, when $U = D = R^n$, then the equilibrium $x = 0$ is globally finite-time stable.

Definition 2 (see [19]). The equilibrium of system (8) is said to be fixed-time stable if it is globally finite-time stable and the settling time function $T(x_0)$ is uniformly bounded for any initial states, that is, $\exists T_{\max}$ such that $T(x_0) \leq T_{\max}, \forall x_0 \in R^n$.

Lemma 2 (see [39]). Considering system (8), suppose that there exists a continuous differentiable positive definite function $V(x): R^n \rightarrow R$ such that the following inequality holds:

$$\dot{V}(x) \leq -\phi_1 V^\alpha(x) - \phi_2 V^\beta(x) + \tau, \quad (9)$$

where $\phi_1 > 0, \phi_2 > 0, \alpha \in (0, 1), \beta \in (1, \infty), \tau \in (0, \infty)$, and $\varpi \in (0, 1)$. Then, the equilibrium of system (8) is fixed-time stable and the convergence time function T satisfies the following inequality:

$$T \leq T_{\max} := \frac{1}{\phi_1 \varpi (1 - \alpha)} + \frac{1}{\phi_2 \varpi (\beta - 1)}. \quad (10)$$

The residual set of the solution of system (8) is given by

$$x \in \left\{ V(x) \leq \min \left\{ \left(\frac{\tau}{(1 - \varpi)\phi_1} \right)^{1/\alpha}, \left(\frac{\tau}{(1 - \varpi)\phi_2} \right)^{1/\beta} \right\} \right\}. \quad (11)$$

Lemma 3 (see [42]). For $x_i \in R, i = 1, 2, \dots, n, 0 < r_1 \leq 1, r_2 > 1$, the following inequalities hold:

$$\begin{aligned} \left(\sum_{i=1}^n |x_i| \right)^{r_1} &\leq \sum_{i=1}^n |x_i|^{r_1}, \\ \left(\sum_{i=1}^n |x_i| \right)^{r_2} &\leq n^{r_2-1} \sum_{i=1}^n |x_i|^{r_2}. \end{aligned} \quad (12)$$

Lemma 4 (see [40]) (see [43]). For $x \in R$ and any positive constant ξ , the following inequality holds:

$$0 \leq |x| < \xi + \frac{x^2}{\sqrt{x^2 + \xi^2}}. \quad (13)$$

$$\frac{a}{\sqrt{a+c}} \geq \frac{b}{\sqrt{b+c}}. \quad (14)$$

Lemma 5. For $a \geq b > 0$, $b, a \in \mathbb{R}$ and any positive constant c , then the following inequality is satisfied:

Proof

$$\frac{a}{\sqrt{a+c}} - \frac{b}{\sqrt{b+c}} = \frac{a+c}{\sqrt{a+c}} - \frac{b+c}{\sqrt{b+c}} - \frac{c}{\sqrt{a+c}} + \frac{c}{\sqrt{b+c}} = (\sqrt{a+c} - \sqrt{b+c}) + \left(\frac{c}{\sqrt{b+c}} - \frac{c}{\sqrt{a+c}} \right) \geq 0. \quad (15)$$

2.5. Tracking Error Constraint. In this paper, we will investigate the output feedback tracking control problem with prescribed performance. The local tracking error for the i th follower is defined as

$$e_i = \sum_{j=1}^M a_{ij}(y_i - y_j) + h_i(y_i - y_0). \quad (16)$$

To meet the requirement of output performance constraint, a positive decreasing smooth function as the following is selected:

$$F_b = \xi_0 e^{-at} + \xi_\infty, \quad (17)$$

where $\xi_0 > 0$, $\xi_\infty > 0$ are proper parameters, a is a positive constant, and $|e_i(0)| < F_b(0)$.

Defining the following state transformation as the error constraint variable of the i th follower,

$$z_{i,1}(t) = \frac{e_i(t)}{\sigma_1 F_b - e_i} q(e_i(t)) + \frac{e_i(t)}{\sigma_2 F_b + e_i} (1 - q(e_i(t))), \quad (18)$$

where σ_1 and σ_2 are proper design parameters. F_b satisfies the condition given in (17) and $q(e_i(t))$ satisfies

$$q(e_i(t)) = \begin{cases} 1, & e_i(t) \geq 0, \\ 0, & e_i(t) < 0. \end{cases} \quad (19)$$

The time derivative of $z_{i,1}$ is

$$\dot{z}_{i,1} = \frac{1}{(\sigma_1 F_b - e_i)^2} [\dot{e}_i(\sigma_1 F_b - e_i) - e_i(\sigma_1 \dot{F}_b - \dot{e}_i)] q + \frac{1}{(\sigma_2 F_b + e_i)^2} [\dot{e}_i(\sigma_2 F_b + e_i) - e_i(\sigma_2 \dot{F}_b + \dot{e}_i)] \cdot (1 - q). \quad (20)$$

For ease of writing, we obtain

$$\dot{z}_{i,1} = F_b \Phi_{F_i} \dot{e}_i - \dot{F}_b \Phi_{F_i} e_i, \quad (21)$$

where

$$\Phi_{F_i} = \frac{\sigma_1 q}{(\sigma_1 F_b - e_i)^2} + \frac{\sigma_2 (1 - q)}{(\sigma_2 F_b + e_i)^2}. \quad (22)$$

Remark 2. From the definition of $z_{i,1}$ and $q(e_i(t))$, it is obvious that when tracking error approaches the boundary F_b , $z_{i,1}$ increases as a result. Then, by choosing proper parameters, the tracking error will be forced within prescribed boundary, i.e., $-\sigma_2 F_b(t) < e_i(t) < \sigma_1 F_b(t)$.

3. Fixed-Time Controller Design

In this section, an adaptive RBF neural network fixed-time controller is designed for MASs with uncertain nonlinear pure-feedback in the framework of backstepping method. The coordinate transformation is as follows:

$$z_{i,k} = x_{i,k} - v_{i,k-1}, \quad (23)$$

where $i = 1, 2, \dots, M$, $k = 2, 3, \dots, n_i$, and $v_{i,k}$ represents the virtual controller of the k th subsystem of the i th follower, and $z_{i,1}$ is given in (18).

In this paper, we employ RBF neural network to approximate uncertain functions $\hat{f}_{i,k}(Z_{i,k})$,

$$\hat{f}_{i,k}(Z_{i,k}) = W_{i,k}^{*T} S_{i,k}(Z_{i,k}) + \delta_{i,k}(Z_{i,k}). \quad (24)$$

From Young's inequality, complete square formula, and Lemma 3, the following inequalities hold:

$$z_{i,k} \delta_{i,k}(Z_{i,k}) \leq b_i \zeta_{i,k_3} z_{i,k}^2 + \frac{\varepsilon_{i,k}^2}{4b_i \zeta_{i,k_3}}, \quad (25)$$

$$z_{i,k} W_{i,k}^{*T} S_{i,k}(Z_{i,k}) \leq \frac{b_i \theta_{i,k} S_{i,k}^T(Z_{i,k}) S_{i,k}(Z_{i,k}) z_{i,k}^2}{2\eta_{i,k}^2} + \frac{\eta_{i,k}^2}{2}, \quad (26)$$

$$-b_i \zeta_{i,k_1} 2^{(1+\alpha)/2} \left(\left(\frac{1}{2} z_{i,k}^2 \right)^{(1+\alpha)/2} + \left(\frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\alpha)/2} \right) \leq -b_i \zeta_{i,k_1} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_{i,k}^2 + \frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\alpha)/2}, \quad (27)$$

$$-b_i \zeta_{i,k_2} 2^{(1+\beta)/2} \left(\left(\frac{1}{2} z_{i,k}^2 \right)^{(1+\beta)/2} + \left(\frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\beta)/2} \right) \leq -2b_i \zeta_{i,k_2} \left(\frac{1}{2} z_{i,k}^2 + \frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\beta)/2}, \quad (28)$$

where $Z_{i,k} = [x_{i,1}, \dots, x_{i,k}, \hat{\theta}_{i,1}, \dots, \hat{\theta}_{i,k}, h_i \bar{y}_{0,k}^T, a_{i1} y_{1,1}, \dots, a_{iM} y_{M,1}, \dots, a_{i1} y_{1,k}, \dots, a_{iM} y_{M,k}, F_b, \dot{F}_b, \dots, F_b^{(k)}]$ with $i = 1, \dots, M$, $k = 1, \dots, n_i$ is the input vector, and b_i , ζ_{i,k_3} , ζ_{i,k_2} , ζ_{i,k_1} , $\varepsilon_{i,k}$, $\eta_{i,k}$, and γ_i are positive parameters.

$$V_{i,1} = \frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i}. \quad (29)$$

From (3), (16), and (18)–(23), we can obtain the time derivative of $V_{i,1}$:

Step 1. Choosing the Lyapunov function as

$$\dot{V}_{i,1} = (d_i + h_i) z_{i,1} F_b \Phi_{F_i} (f_{i,1} + g_{i,1} z_{i,2} + g_{i,1} v_{i,1} - g_{i,1} x_{i,10} + d_{i,1}) - z_{i,1} F_b \Phi_{F_i} \left(\sum_{j=1}^M a_{ij} \dot{x}_{j,1} + h_i \dot{y}_0 \right) - z_{i,1} \dot{F}_b \Phi_{F_i} e_i - \frac{b_i \tilde{\theta}_{i,1} \hat{\theta}_{i,1}}{\gamma_i}. \quad (30)$$

The virtual controller $v_{i,1}$ is defined as

$$v_{i,1} = \frac{1}{(d_i + h_i) F_b \Phi_{F_i}} \left(-\frac{\hat{\theta}_{i,1}}{2\eta_{i,1}^2} S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1} - \zeta_{i,11} S_{z_{i,1},1} - \zeta_{i,12} S_{z_{i,1},2} - \zeta_{i,13} z_{i,1} + x_{i,10} \right), \quad (31)$$

where $\zeta_{i,11}$, $\zeta_{i,12}$, $\zeta_{i,13}$, and $\eta_{i,1}$ are positive design parameters. $S_{z_{i,1},1}$ and $S_{z_{i,1},2}$ are defined by the following equalities:

$$S_{z_{i,k},1} = \begin{cases} \frac{(z_{i,k}^2)^{(1+\alpha)/2}}{z_{i,k}}, & |z_{i,k}| \geq \varepsilon_{k0}, \\ \sum_{j=1}^{n_i} a_j (z_{i,k}^2)^j (\varepsilon_{k0}^2)^{-j+(1+\alpha)/2}, & |z_{i,k}| < \varepsilon_{k0}, \end{cases} \quad (32)$$

$$S_{z_{i,k},2} = \begin{cases} \frac{(z_{i,k}^2)^{(1+\beta)/2}}{z_{i,k}}, & |z_{i,k}| \geq \varepsilon_{k0}, \\ \sum_{j=1}^{n_i} a_j (z_{i,k}^2)^j (\varepsilon_{k0}^2)^{-j+(1+\beta)/2}, & |z_{i,k}| < \varepsilon_{k0}, \end{cases} \quad (33)$$

where $k = 1, \dots, n_i$, ε_{k0} is the positive design parameter, and coefficients a_j with $j = 1, \dots, n_i$ are calculated using the following equation:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & \cdots & n_i - 1 & n_i \\ 0 & 2 \times 1 & \cdots & (n_i - 1)(n_i - 2) & n_i(n_i - 1) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \prod_{j=0}^{n_i-2} (n_i - 1 - j) & \prod_{j=0}^{n_i-2} (n_i - j) \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{n_i} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n_i} \end{bmatrix}, \quad (34)$$

where $b_1 = 1$, $b_2 = 3/4$, $b_3 = (3/4)((3/4) - 1)$, ..., and $b_n = \prod_{j=0}^{n-2} ((3/4) - j)$.

Remark 3. The above process is to eliminate the problem of fixed-time controller's singularity. In this paper, we just

consider the situation of $|z_{i,k}| \geq \varepsilon_{k0}$, since when $|z_{i,k}| < \varepsilon_{k0}$, there we just add an additional term and it clearly does not affect the final conclusion.

Substituting (31) into (30) yields

$$\begin{aligned} \dot{V}_{i,1} \leq & (d_i + h_i)z_{i,1}F_b\Phi_{F_i}g_{i,1}z_{i,2} + z_{i,1}\hat{f}_{i,1}(Z_{i,1}) - \frac{b_i\hat{\theta}_{i,1}}{2\eta_{i,1}^2}S_{i,1}^T(Z_{i,1})S_{i,1}(Z_{i,1})z_{i,1}^2 - b_i\zeta_{i,11}z_{i,1}S_{z_{i,1},1} - b_i\zeta_{i,12}z_{i,1}S_{z_{i,1},2} \\ & - b_i\zeta_{i,13}z_{i,1}^2 - \frac{b_i\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}}{\gamma_i}, \end{aligned} \quad (35)$$

where $\hat{f}_{i,1}(Z_{i,1}) = (d_i + h_i)F_b\Phi_{F_i}(f_{i,1} + d_{i,1}) - F_b\Phi_{F_i}(\sum_{j=1}^M a_{ij}\dot{x}_{j,1} + h_i\dot{y}_0) - \hat{F}_b\Phi_{F_i}e_i$.

From (24)–(26), we have

$$\dot{V}_{i,1} \leq \frac{b_i\tilde{\theta}_{i,1}}{\gamma_i} \left(\frac{\gamma_i}{2\eta_{i,1}^2}S_{i,1}^T(Z_{i,1})S_{i,1}(Z_{i,1})z_{i,1}^2 - \dot{\hat{\theta}}_{i,1} \right) + (d_i + h_i)z_{i,1}F_b\Phi_{F_i}g_{i,1}z_{i,2} + \sigma_{i,1} - b_i\zeta_{i,11}z_{i,1}S_{z_{i,1},1} - b_i\zeta_{i,12}z_{i,1}S_{z_{i,1},2}, \quad (36)$$

where $\sigma_{i,1} = (1/2)\eta_{i,1}^2 \pm (\varepsilon_{i,1}^2/4b_i\zeta_{i,13})$.

The adaptive law $\dot{\hat{\theta}}_{i,1}$ is defined as

$$\dot{\hat{\theta}}_{i,1} = \frac{\gamma_i}{2\eta_{i,1}^2}S_{i,1}^T(Z_{i,1})S_{i,1}(Z_{i,1})z_{i,1}^2 - \lambda_i\hat{\theta}_{i,1}, \quad (37)$$

where λ_i with $i = 1, \dots, M$ is the positive parameter.

From Assumption 3 and (37), we have

$$\begin{aligned} \dot{V}_{i,1} \leq & -b_i \zeta_{i,11} z_{i,1} S_{z_{i,1},1} - b_i \zeta_{i,12} z_{i,1} S_{z_{i,1},2} - b_i \zeta_{i,11} 2^{(1+\alpha)/2} \left(\frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - b_i \zeta_{i,12} 2^{(1+\beta)/2} \left(\frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ & + (d_i + h_i) z_{i,1} F_b \Phi_{F_i} g_{i,1} z_{i,2} + C_{i,1}, \end{aligned} \quad (38)$$

where $C_{i,1} = \sigma_{i,1} + \beta_{i,1} + (\lambda_i b_i / 2\gamma_i) \theta_{i,1}^2$, $\beta_{i,1} = \zeta_{i,11} 2^{(1+\alpha)/2} (b_i Q_{i,1}^2 / 2\gamma_i)^{(1+\alpha)/2} + \zeta_{i,12} 2^{(1+\beta)/2} (b_i Q_{i,1}^2 / 2\gamma_i)^{(1+\beta)/2}$, and $(\lambda_i b_i \tilde{\theta}_{i,1} / \gamma_i) \leq (\lambda_i b_i / \gamma_i) ((\theta_{i,1}^2 / 2) - (\tilde{\theta}_{i,1}^2 / 2)) \leq (\lambda_i b_i / 2\gamma_i) \theta_{i,1}^2$.

According to (27), (28), (32), and (33), and when $|z_{i,1}| \geq \varepsilon_{10}$,

$$\dot{V}_{i,1} \leq -b_i \zeta_{i,11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \zeta_{i,12} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\beta)/2} + (d_i + h_i) z_{i,1} F_b \Phi_{F_i} g_{i,1} z_{i,2} + C_{i,1}. \quad (39)$$

Step 2. From (3) and (23), we can obtain

$$\begin{aligned} \dot{z}_{i,2} &= \dot{x}_{i,2} - \dot{v}_{i,1} \\ &= f_{i,2}(\bar{x}_{i,2}, x_{i,20}) + g_{i,2}(x_{i,3} - x_{i,20}) \\ &\quad + d_{i,2} - \dot{v}_{i,1}, \end{aligned} \quad (40)$$

where $\dot{v}_{i,1} = (\partial v_{i,1} / \partial x_{i,1}) \dot{x}_{i,1} + \sum_{j=1}^M a_{ij} ((\partial v_{i,1} / \partial x_{j,j}) x_{j,1} + (\partial v_{i,1} / \partial x_{j,2}) \dot{x}_{j,2}) + h_i ((\partial v_{i,1} / \partial x_{0,1}) \dot{x}_{0,1} + (\partial v_{i,1} / \partial x_{0,2}) \dot{x}_{0,2}) + (\partial v_{i,1} / \partial \hat{\theta}_{i,1}) \dot{\hat{\theta}}_{i,1} + (\partial v_{i,1} / \partial F_b) \dot{F}_b + (\partial v_{i,1} / \partial \vec{F}_b) \vec{F}_b$.

Construct Lyapunov function as

$$V_{i,2} = V_{i,1} + \frac{1}{2} z_{i,2}^2 + \frac{b_i \tilde{\theta}_{i,2}^2}{2\gamma_i}. \quad (41)$$

The derivative of $V_{i,2}$ is written as

$$\begin{aligned} \dot{V}_{i,2} \leq & -b_i \zeta_{i,11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\alpha)/2} + z_{i,2} \hat{f}_{i,2}(Z_{i,2}) - 2b_i \zeta_{i,12} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ & + z_{i,2} g_{i,2} z_{i,3} + z_{i,2} g_{i,2} v_{i,2} - z_{i,2} g_{i,2} x_{i,20} - z_{i,2} \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} - \frac{b_i \tilde{\theta}_{i,2} \dot{\hat{\theta}}_{i,2}}{\gamma_i} + z_{i,2} M_{i,1}(Z_{i,2}) + C_{i,1}, \end{aligned} \quad (42)$$

where $\hat{f}_{i,2}(Z_{i,2}) = f_{i,2}(\bar{x}_{i,2}, x_{i,20}) + d_{i,2} + (d_i + h_i) z_{i,1} F_b \Phi_{F_i} g_{i,1} - (\partial v_{i,1} / \partial x_{i,1}) \dot{x}_{i,1} - \sum_{j=1}^M a_{ij} ((\partial v_{i,1} / \partial x_{j,j}) \dot{x}_{j,1} + (\partial v_{i,1} / \partial x_{j,2}) \dot{x}_{j,2}) - h_i ((\partial v_{i,1} / \partial x_{0,1}) \dot{x}_{0,1} + (\partial v_{i,1} / \partial x_{0,2}) \dot{x}_{0,2}) - M_{i,1}(Z_{i,2})$.

Remark 4. $M_{i,1}(Z_{i,2})$ is a smooth function that is used to compensate $(\partial v_{i,1} / \partial \hat{\theta}_{i,1}) \dot{\hat{\theta}}_{i,1}$.

The virtual controller $v_{i,2}$ is defined as

$$v_{i,2} = -\frac{\hat{\theta}_{i,2}}{2\eta_{i,2}} S_{i,2}^T(Z_{i,2}) S_{i,2}(Z_{i,2}) z_{i,2} + x_{i,20} - \zeta_{i,21} S_{z_{i,2},1} - \zeta_{i,22} S_{z_{i,2},2} - \zeta_{i,23} z_{i,2}, \quad (43)$$

where $\zeta_{i,21}$, $\zeta_{i,22}$, $\zeta_{i,23}$, and $\eta_{i,2}$ are positive design parameters.

According to (24)–(26) and substituting (43) into (42), we have

$$\begin{aligned} \dot{V}_{i,2} \leq & \frac{b_i \tilde{\theta}_{i,2}}{\gamma_i} \left(\frac{\gamma_i}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2}) S_{i,2}(Z_{i,2}) z_{i,2}^2 - \dot{\hat{\theta}}_{i,2} \right) + z_{i,2} g_{i,2} z_{i,3} + \sigma_{i,2} - b_i \zeta_{i,21} z_{i,2} S_{z_{i,2},1} - b_i \zeta_{i,22} z_{i,2} S_{z_{i,2},2} + C_{i,1} \\ & - b_i \zeta_{i,11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \zeta_{i,12} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\beta)/2} + z_{i,2} \left(M_{i,1}(Z_{i,2}) - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} \right), \end{aligned} \quad (44)$$

where $\sigma_{i,2} = (1/2)\eta_{i,2}^2 + (\varepsilon_{i,2}^2/4b_i\zeta_{i,23})$.

Then, the adaptive law $\hat{\theta}_{i,2}$ is defined as

$$\dot{\hat{\theta}}_{i,2} = \frac{\gamma_i}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2}) S_{i,2}(Z_{i,2}) z_{i,2}^2 - \lambda_i \hat{\theta}_{i,2}. \quad (45)$$

From Assumption 3 and (45), we have

$$\begin{aligned} \dot{V}_{i,2} \leq & -b_i \zeta_{i,21} z_{i,2} S_{z_{i,2},1} - b_i \zeta_{i,22} z_{i,2} S_{z_{i,2},2} - b_i \zeta_{i,21} 2^{(1+\alpha)/2} \left(\frac{b_i \tilde{\theta}_{i,2}^2}{2\gamma_i} \right)^{(1+\alpha)/2} \\ & - b_i \zeta_{i,22} 2^{(1+\beta)/2} \left(\frac{b_i \tilde{\theta}_{i,2}^2}{2\gamma_i} \right)^{(1+\beta)/2} - b_i \zeta_{i,11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\alpha)/2} \\ & - 2b_i \zeta_{i,12} \left(\frac{1}{2} z_{i,1}^2 + \frac{b_i \tilde{\theta}_{i,1}^2}{2\gamma_i} \right)^{(1+\beta)/2} + \sum_{j=1}^2 C_{i,j} + z_{i,2} g_{i,2} z_{i,3} + z_{i,2} \left(M_{i,1}(Z_{i,2}) - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} \right), \end{aligned} \quad (46)$$

where $C_{i,2} = \sigma_{i,2} + \beta_{i,2} + (\lambda_i b_i / 2\gamma_i) \theta_{i,2}^2$, $\beta_{i,2} = b_i \zeta_{i,21} 2^{(1+\alpha)/2} (b_i Q_{i,2}^2 / 2\gamma_i)^{(1+\alpha)/2} + b_i \zeta_{i,22} 2^{(1+\beta)/2} (b_i Q_{i,2}^2 / 2\gamma_i)^{(1+\beta)/2}$, and $(\lambda_i b_i \tilde{\theta}_{i,2} \hat{\theta}_{i,2} / \gamma_i) \leq (\lambda_i b_i / \gamma_i) ((\theta_{i,2}^2 / 2) - (\tilde{\theta}_{i,2}^2 / 2)) \leq (\lambda_i b_i / 2\gamma_i) \theta_{i,2}^2$.

According to (27), (28), (32), and (33) and when $|z_{i,2}| \geq \varepsilon_{20}$,

$$\begin{aligned} \dot{V}_{i,2} \leq & -b_i 2^{(1+\alpha)/2} \sum_{j=1}^2 \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^2 \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} + z_{i,2} \left(M_{i,1}(Z_{i,2}) - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} \right) \\ & + \sum_{j=1}^2 C_{i,j} + z_{i,2} g_{i,2} z_{i,3}. \end{aligned} \quad (47)$$

From Lemmas 1, 4, and 5 and (37), we have

$$\begin{aligned} -z_{i,2} \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \dot{\hat{\theta}}_{i,1} & \leq -z_{i,2} \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \left(\frac{\gamma_i}{2\eta_{i,1}^2} S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2 - \lambda_i \hat{\theta}_{i,1} \right) \\ & \leq \frac{z_{i,2}^2 (\partial v_{i,1} / \partial \hat{\theta}_{i,1})^2 ((\gamma_i / 2\eta_{i,1}^2) S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2)^2}{\sqrt{z_{i,2}^2 (\partial v_{i,1} / \partial \hat{\theta}_{i,1})^2 ((\gamma_i / 2\eta_{i,1}^2) S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2)^2 + \kappa^2}} + z_{i,2} \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \lambda_i \hat{\theta}_{i,1} \\ & \leq \frac{z_{i,2}^2 (\partial v_{i,1} / \partial \hat{\theta}_{i,1})^2 ((\gamma_i / 2\eta_{i,1}^2) S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2)^2}{\sqrt{z_{i,2}^2 (\partial v_{i,1} / \partial \hat{\theta}_{i,1})^2 ((\gamma_i / 2\eta_{i,1}^2) S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2)^2 + \kappa^2}} + z_{i,2} \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \lambda_i \hat{\theta}_{i,1}. \end{aligned} \quad (48)$$

Therefore, $M_{i,1}(Z_{i,2})$ can be defined as

$$M_{i,1}(Z_{i,2}) = -\frac{z_{i,2}(\partial v_{i,1}/\partial \hat{\theta}_{i,1})^2((\gamma_i/2\eta_{i,1}^2)S^2 z_{i,1}^2)^2}{\sqrt{z_{i,2}^2(\partial v_{i,1}/\partial \hat{\theta}_{i,1})^2((\gamma_i/2\eta_{i,1}^2)S^2 z_{i,1}^2)^2 + \kappa^2}} - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \lambda_i \hat{\theta}_{i,1}, \quad (49)$$

with the result that

$$z_{i,2} \left(M_{i,1}(Z_{i,2}) - \frac{\partial v_{i,1}}{\partial \hat{\theta}_{i,1}} \hat{\theta}_{i,1} \right) \leq 0. \quad (50)$$

Substituting (50) into (47), we can obtain

$$\dot{V}_{i,2} \leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^2 \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^2 \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} + \sum_{j=1}^2 C_j + z_{i,2} g_{i,2} z_{i,3}. \quad (51)$$

Step k . ($3 \leq k \leq n_i - 1$): from (3) and (23), we have

$$\begin{aligned} \dot{z}_{i,k} &= \dot{x}_{i,k} - \dot{v}_{i,k-1} \\ &= f_{i,k}(\bar{x}_{i,k}, x_{i,k0}) + g_{i,k}(x_{i,k+1} - x_{i,k0}) \\ &\quad + d_{i,k} - \dot{v}_{i,k-1}, \end{aligned} \quad (52)$$

where $\dot{v}_{i,k-1} = \sum_{j=1}^{k-1} (\partial v_{i,k-1}/\partial x_{i,j}) \dot{x}_{i,j} + h_i \sum_{j=1}^k (\partial v_{i,k-1}/\partial x_{0,j}) \dot{x}_{0,j} + \sum_{j=1}^M a_{ij} \sum_{q=1}^k ((\partial v_{i,k-1}/\partial x_{j,q}) \dot{x}_{j,q}) + \sum_{j=1}^{k-1} (\partial v_{i,k-1}/\partial \hat{\theta}_{i,j}) \dot{\hat{\theta}}_{i,j} + \sum_{j=0}^{k-1} (\partial v_{i,k-1}/\partial F_b^{(j)}) F_b^{(j+1)}$.

Construct Lyapunov function as

$$V_{i,k} = V_{i,k-1} + \frac{1}{2} z_{i,k}^2 + \frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i}. \quad (53)$$

The derivative of $V_{i,k}$ is

$$\begin{aligned} \dot{V}_{i,k} &\leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{k-1} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{k-1} \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ &\quad + z_{i,k} \hat{f}_{i,k}(Z_{i,k}) + z_{i,k} g_{i,k} z_{i,k+1} + z_{i,k} g_{i,k} v_{i,k} - z_{i,k} g_{i,k} x_{i,k0} - z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} - \frac{b_i \tilde{\theta}_{i,k} \dot{\hat{\theta}}_{i,k}}{\gamma_i} + z_{i,k} M_{i,k-1}(Z_{i,k}) + \sum_{j=1}^{k-1} C_{i,j}, \end{aligned} \quad (54)$$

where $\hat{f}_{i,k}(Z_{i,k}) = f_{i,k}(\bar{x}_{i,k}, x_{i,k0}) + d_{i,k} + z_{i,k-1} g_{i,k-1} - \sum_{j=1}^{k-1} (\partial v_{i,k-1}/\partial x_{i,j}) \dot{x}_{i,j} - \sum_{j=1}^M a_{ij} \sum_{q=1}^k ((\partial v_{i,k-1}/\partial x_{j,q}) \dot{x}_{j,q}) - h_i \sum_{j=1}^k (\partial v_{i,k-1}/\partial x_{0,j}) \dot{x}_{0,j} - \sum_{j=0}^{k-1} (\partial v_{i,k-1}/\partial F_b^{(j)}) F_b^{(j+1)} - M_{i,k-1}(Z_{i,k})$.
The virtual controller $v_{i,k}$ is defined as

$$\begin{aligned} v_{i,k} &= -\frac{\hat{\theta}_{i,k}}{2\eta_{i,k}^2} S_{i,k}^T(Z_{i,k}) S_{i,k}(Z_{i,k}) z_{i,k} + x_{i,k0} \\ &\quad - \zeta_{i,k1} S_{z_{i,k},1} - \zeta_{i,k2} S_{z_{i,k},2} - \zeta_{i,k3} z_{i,k}, \end{aligned} \quad (55)$$

where $\zeta_{i,k1}$, $\zeta_{i,k2}$, $\zeta_{i,k3}$, and $\eta_{i,k}$ are positive design parameters.

According to (24)–(26) and substituting (55) into (54), we have

$$\begin{aligned} \dot{V}_{i,k} \leq & \frac{b_i \tilde{\theta}_{i,k}}{\gamma_i} \left(\frac{\gamma_i}{2\eta_{i,k}^2} S_{i,k}^T(Z_{i,k}) S_{i,k} (Z_{i,k})^2 - \hat{\theta}_{i,k} \right) + z_{i,k} g_{i,k} z_{i,k+1} + \sigma_{i,k} - b_i \zeta_{i,k1} z_{i,k} S_{z_{i,k},1} - b_i \zeta_{i,k2} z_{i,k} S_{z_{i,k},2} + \sum_{j=1}^{k-1} C_{i,j} \\ & + z_{i,k} \left(M_{i,k-1}(Z_{i,k}) - \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right) - b_i 2^{(1+\alpha)/2} \sum_{j=1}^{k-1} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{k-1} \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2}, \end{aligned} \quad (56)$$

where $\sigma_{i,k} = (1/2)\eta_{i,k}^2 + (\varepsilon_{i,k}^2/4b_i\zeta_{i,k3})$.

Then, the adaptive law $\dot{\hat{\theta}}_{i,k}$ is defined as

$$\dot{\hat{\theta}}_{i,k} = \frac{\gamma_i}{2\eta_{i,k}^2} S_{i,k}^T(Z_{i,k}) S_{i,k} (Z_{i,k})^2 - \lambda_i \hat{\theta}_{i,k}. \quad (57)$$

Substituting (57) into (56), we can obtain

$$\begin{aligned} \dot{V}_{i,k} \leq & -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{k-1} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{k-1} \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} - b_i \zeta_{i,k1} z_{i,k} S_{z_{i,k},1} - b_i \zeta_{i,k2} z_{i,k} S_{z_{i,k},2} \\ & - b_i \zeta_{i,k1} 2^{(1+\alpha)/2} \left(\frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - b_i \zeta_{i,k2} 2^{(1+\beta)/2} \left(\frac{b_i \tilde{\theta}_{i,k}^2}{2\gamma_i} \right)^{(1+\beta)/2} + z_{i,k} \left(M_{i,k-1}(Z_{i,k}) - \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right) + z_{i,k} g_{i,k} z_{i,k+1} + \sum_{j=1}^k C_{i,j}, \end{aligned} \quad (58)$$

where $C_{i,k} = \sigma_{i,k} + \beta_{i,k} + (\lambda_i b_i / 2\gamma_i) \theta_{i,k}^2$, $\beta_{i,k} = b_i \zeta_{i,k1} 2^{(1+\alpha)/2} (b_i Q_{i,k}^2 / 2\gamma_i)^{(1+\alpha)/2} + b_i \zeta_{i,k2} 2^{(1+\beta)/2} (b_i Q_{i,k}^2 / 2\gamma_i)^{(1+\beta)/2}$, and $(\lambda_i b_i \tilde{\theta}_{i,k} \hat{\theta}_{i,k} / \gamma_i) \leq (\lambda_i b_i / \gamma_i) ((\theta_{i,k}^2 / 2) - (\tilde{\theta}_{i,k}^2 / 2)) \leq (\lambda_i b_i / 2\gamma_i) \theta_{i,k}^2$.

According to (27), (28), (32), and (33) and when $|z_{i,k}| \geq \varepsilon_{k0}$,

$$\begin{aligned} \dot{V}_{i,k} \leq & -b_i 2^{(1+\alpha)/2} \sum_{j=1}^k \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^k \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} + \sum_{j=1}^k C_j + z_{i,k} g_{i,k} z_{i,k+1} \\ & + z_{i,k} \left(M_{i,k-1}(Z_{i,k}) - \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right). \end{aligned} \quad (59)$$

From Lemmas 1, 4, and 5 and (57), we have

$$\begin{aligned}
-z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} &\leq -z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \left(\frac{\gamma_i}{2\eta_{i,j}^2} S_{i,j}^T(Z_{i,j}) S_{i,j}(Z_{i,j}) z_{i,j}^2 - \lambda_i \hat{\theta}_{i,j} \right) \\
&\leq \sum_{j=1}^{k-1} \frac{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S_{i,j}^T(Z_{i,j}) S_{i,j}(Z_{i,j}) z_{i,j}^2)^2}{\sqrt{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S_{i,j}^T(Z_{i,j}) S_{i,j}(Z_{i,j}) z_{i,j}^2)^2 + \kappa^2}} + \lambda_i z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \hat{\theta}_{i,j} \\
&\leq \sum_{j=1}^{k-1} \frac{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2}{\sqrt{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2 + \kappa^2}} + \lambda_i z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \hat{\theta}_{i,j}.
\end{aligned} \tag{60}$$

Therefore, $M_{i,k-1}(Z_{i,k})$ is defined as

$$M_{i,k-1}(Z_{i,k}) = - \sum_{j=1}^{k-1} \frac{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2}{\sqrt{z_{i,k}^2 (\partial v_{i,k-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2 + \kappa^2}} - \lambda_i z_{i,k} \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \hat{\theta}_{i,j}, \tag{61}$$

with the result that

Substituting (61) into (59) yields

$$z_{i,k} \left(M_{i,k-1}(Z_{i,k}) - \sum_{j=1}^{k-1} \frac{\partial v_{i,k-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right) \leq 0. \tag{62}$$

$$\dot{V}_{i,k} \leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^k \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^k \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} + \sum_{j=1}^k C_j + z_{i,k} g_{i,k} z_{i,k+1}. \tag{63}$$

Step n_i . From (3) and (23), we have

Construct Lyapunov function as

$$\begin{aligned}
\dot{z}_{i,n_i} &= \dot{x}_{i,n_i} - \dot{v}_{i,n_i-1} = f_{i,n_i}(\bar{x}_{i,n_i}, x_{i,n_i0}) + g_{i,n_i}(u_i - x_{i,n_i0}) \\
&\quad + d_{i,n_i} - \dot{v}_{i,n_i-1},
\end{aligned} \tag{64}$$

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2} z_{i,n_i}^2 + \frac{b_i \tilde{\theta}_{i,n_i}^2}{2\gamma_i}. \tag{65}$$

The derivative of V_{i,n_i} is

$$\begin{aligned}
\text{where } \dot{v}_{i,n_i-1} &= \sum_{j=1}^{n_i-1} (\partial v_{i,n_i-1} / \partial x_{i,j}) \dot{x}_{i,j} + h_i \sum_{j=1}^{n_i} (\partial v_{i,n_i-1} / \partial x_{0,j}) \dot{x}_{0,j} \\
&\quad + \sum_{j=1}^M a_{ij} \sum_{q=1}^{n_i} ((\partial v_{i,n_i-1} / \partial x_{j,q}) \dot{x}_{j,q}) + \sum_{j=1}^{n_i-1} (\partial v_{i,n_i-1} / \partial \hat{\theta}_{i,j}) \dot{\hat{\theta}}_{i,j} \\
&\quad + \sum_{j=0}^{n_i-1} (\partial v_{i,n_i-1} / \partial F_b^{(j)}) F_b^{(j+1)}.
\end{aligned}$$

$$\dot{V}_{i,n_i} \leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{n_i-1} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{n_i-1} \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} \tag{66}$$

$$+ z_{i,n_i} \hat{f}_{i,n_i}(Z_{i,n_i}) + z_{i,n_i} g_{i,n_i} u_i - z_{i,n_i} g_{i,n_i} x_{i,n_i0} - z_{i,n_i} \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} - \frac{b_i \tilde{\theta}_{i,n_i} \dot{\hat{\theta}}_{i,n_i}}{\gamma_i} + z_{i,n_i} M_{i,n_i-1}(Z_{i,n_i}) + \sum_{j=1}^{n_i-1} C_{i,j},$$

where

$$\begin{aligned}\hat{f}_{i,n_i}(Z_{i,n_i}) &= f_{i,n_i}(\bar{x}_{i,n_i}, x_{i,n_i,0}) + d_{i,n_i} + z_{i,n_i-1}g_{i,n_i-1} - \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial x_{i,j}} \dot{x}_{i,j} - \sum_{j=1}^M a_{ij} \sum_{q=1}^{n_i} \left(\frac{\partial v_{i,n_i-1}}{\partial x_{j,q}} \dot{x}_{j,q} \right) \\ &\quad - h_i \sum_{j=1}^{n_i} \frac{\partial v_{i,n_i-1}}{\partial x_{0,j}} \dot{x}_{0,j} - \sum_{j=0}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial F_b^{(j)}} F_b^{(j+1)} - M_{i,n_i-1}(Z_{i,n_i}).\end{aligned}\quad (67)$$

The actual controller is defined as

$$\begin{aligned}u_i &= -\zeta_{i,n_i,1} S_{z_{i,n_i,1}} - \zeta_{i,n_i,2} S_{z_{i,n_i,2}} - \frac{\hat{\theta}_{i,n_i}}{2\eta_{i,n_i}} S_{i,n_i}^T(Z_{i,n_i}) S_{i,n_i}(Z_{i,n_i}) z_{i,n_i} \\ &\quad - \zeta_{i,n_i,3} z_{i,n_i} + x_{i,n_i,0},\end{aligned}\quad (68)$$

where $\zeta_{i,n_i,1}$, $\zeta_{i,n_i,2}$, $\zeta_{i,n_i,3}$, and η_{i,n_i} are positive design parameters.

The adaptive law $\hat{\theta}_{i,n_i}$ is defined as

$$\dot{\hat{\theta}}_{i,n_i} = \frac{\gamma_i}{2\eta_{i,n_i}} S_{i,n_i}^T(Z_{i,n_i}) S_{i,n_i}(Z_{i,n_i}) z_{i,n_i}^2 - \lambda_i \hat{\theta}_{i,n_i}. \quad (69)$$

Substituting (69) and (68) into (66), we can obtain

$$\begin{aligned}\dot{V}_{i,n_i} &\leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{n_i-1} \zeta_{i,j,1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{n_i-1} \zeta_{i,j,2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ &\quad - b_i \zeta_{i,n_i,1} z_{i,n_i} S_{z_{i,n_i,1}} - b_i \zeta_{i,n_i,2} z_{i,n_i} S_{z_{i,n_i,2}} \\ &\quad - b_i \zeta_{i,n_i,1} 2^{(1+\alpha)/2} \left(\frac{b_i \tilde{\theta}_{i,n_i}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - b_i \zeta_{i,n_i,2} 2^{(1+\beta)/2} \left(\frac{b_i \tilde{\theta}_{i,n_i}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ &\quad + z_{i,n_i} \left(M_{i,n_i-1}(Z_{i,n_i}) - \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right) + \sum_{j=1}^{n_i} C_{i,j},\end{aligned}\quad (70)$$

where $C_{i,n_i} = \sigma_{i,n_i} + \beta_{i,n_i} + (\lambda_i b_i / 2\gamma_i) \theta_{i,n_i}^2$, $\beta_{i,n_i} = b_i \zeta_{i,n_i,1} 2^{(1+\alpha)/2} (b_i Q_{i,n_i}^2 / 2\gamma_i)^{(1+\alpha)/2} + b_i \zeta_{i,n_i,2} 2^{(1+\beta)/2} (b_i Q_{i,n_i}^2 / 2\gamma_i)^{(1+\beta)/2}$, $\sigma_{i,n_i} = (1/2) \eta_{i,n_i}^2 + (\varepsilon_{i,n_i}^2 / 4b_i \zeta_{i,n_i,3})$, and $(\lambda_i b_i \tilde{\theta}_{i,n_i} \hat{\theta}_{i,n_i} / \gamma_i) \leq (\lambda_i b_i / \gamma_i) ((\theta_{i,n_i}^2 / 2) - (\tilde{\theta}_{i,n_i}^2 / 2)) \leq (\lambda_i b_i / 2\gamma_i) \theta_{i,n_i}^2$.

According to (27), (28), (32), and (33) and when $|z_{i,n_i}| \geq \varepsilon_{n_i,0}$,

$$\begin{aligned}\dot{V}_{i,n_i} &\leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{n_i} \zeta_{i,j,1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{n_i} \zeta_{i,j,2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} \\ &\quad + z_{i,n_i} \left(M_{i,n_i-1}(Z_{i,n_i}) - \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \right) \\ &\quad + \sum_{j=1}^{n_i} C_{i,j}.\end{aligned}\quad (71)$$

Same to the process of (60), the $M_{i,n_i-1}(Z_{i,n_i})$ can be defined as

$$M_{i,n_i-1}(Z_{i,n_i}) = - \sum_{j=1}^{n_i-1} \frac{z_{i,n_i}^2 (\partial v_{i,n_i-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2}{\sqrt{z_{i,k}^2 (\partial v_{i,n_i-1} / \partial \hat{\theta}_{i,j})^2 ((\gamma_i / 2\eta_{i,j}^2) S^2 z_{i,j}^2)^2 + \kappa^2}} - \lambda_i z_{i,n_i} \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j}, \quad (72)$$

with the result that

$$z_{i,n_i} \left(M_{i,n_i-1}(Z_{i,n_i}) - \sum_{j=1}^{n_i-1} \frac{\partial v_{i,n_i-1}}{\partial \theta_{i,j}} \dot{\theta}_{i,j} \right) \leq 0. \quad (73)$$

Substituting (73) into (71) yields

$$\dot{V}_{i,n_i} \leq -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{n_i} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} - 2b_i \sum_{j=1}^{n_i} c_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} + \tau_i, \quad (74)$$

where $\tau_i = \sum_{j=1}^{n_i} C_{i,j}$.

Setting $\phi_{i,1} = \min b_i 2^{(1+\alpha)/2} \{\zeta_{i,11}, \zeta_{i,21}, \dots, \zeta_{i,n_{i,1}1}\}$ and $\phi_{i,2} = \min 2b_i \{\zeta_{i,12}, \zeta_{i,22}, \dots, \zeta_{i,n_{i,2}2}\}$ and from Lemma 3, we have

$$\begin{aligned} -b_i 2^{(1+\alpha)/2} \sum_{j=1}^{n_i} \zeta_{i,j1} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\alpha)/2} &\leq -\phi_{i,1} \left(\sum_{j=1}^{n_i} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right) \right)^{(1+\alpha)/2}, \\ -2b_i \sum_{j=1}^{n_i} \zeta_{i,j2} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right)^{(1+\beta)/2} &\leq -n_i^{(1-\beta)/2} \phi_{i,2} \left(\sum_{j=1}^{n_i} \left(\frac{1}{2} z_{i,j}^2 + \frac{b_i \tilde{\theta}_{i,j}^2}{2\gamma_i} \right) \right)^{(1+\beta)/2}. \end{aligned} \quad (75)$$

Then, (74) can be rewritten as

$$\dot{V}_{i,n_i} \leq -\phi_{i,1} V_{i,n_i}^{(1+\alpha)/2} - n_i^{(1-\beta)/2} \phi_{i,2} V_{i,n_i}^{(1+\beta)/2} + \tau_i. \quad (76)$$

Up to this point, the design of fixed-time controller is finished.

4. Stability Analysis

Theorem 1. Consider pure-feedback nonlinear multiagent systems (1) satisfying Assumptions 1–3. If given initial values $-\sigma_2 F_b(0) < e_i(0) < \sigma_1 F_b(0)$, then by utilizing virtual controllers (31), (43), and (52), actual controller (68) and adaptive laws (37), (45), (57), and (69) can ensure that the local consensus errors converge to a small neighborhood of the origin within a fixed time. Besides, the local tracking errors can meet the prescribed performance range, i.e., $-\sigma_2 F_b(t) < e_i(t) < \sigma_1 F_b(t)$.

Proof. Choose the total Lyapunov function V as follows:

$$V = \sum_{i=1}^M V_{i,n_i}. \quad (77)$$

According to the process above and (76), the derivative of V can be written as

$$\dot{V} \leq -\sum_{i=1}^M \phi_{i,1} V_{i,n_i}^{(1+\alpha)/2} - \sum_{i=1}^M n_i^{(1-\beta)/2} \phi_{i,2} V_{i,n_i}^{(1+\beta)/2} + \sum_{i=1}^M \tau_i. \quad (78)$$

Setting $\mu_1 = \min\{\phi_{1,1}, \phi_{2,1}, \dots, \phi_{M,1}\}$, $\tau = \sum_{i=1}^M \tau_i$, and $\mu_2 = \min\{n_1^{(1-\beta)/2} M^{(1-\beta)/2} \phi_{1,2}, \dots, n_M^{(1-\beta)/2} M^{(1-\beta)/2} \phi_{M,2}\}$ and from Lemma 3, we have

$$-\sum_{i=1}^M \phi_{i,1} V_{i,n_i}^{(1+\alpha)/2} \leq -\mu_1 \left(\sum_{i=1}^M V_{i,n_i} \right)^{(1+\alpha)/2}, \quad (79)$$

$$-\sum_{i=1}^M n_i^{(1-\beta)/2} \phi_{i,2} V_{i,n_i}^{(1+\beta)/2} \leq -\mu_2 \left(\sum_{i=1}^M V_{i,n_i} \right)^{(1+\beta)/2}, \quad (80)$$

where $\mu_1 > 0$, $\mu_2 > 0$, $\alpha \in (0, 1]$, and $\beta \in (1, \infty)$.

Furthermore, substituting (79) and (80) into (78), we have

$$\dot{V} \leq -\mu_1 V^{(1+\alpha)/2} - \mu_2 V^{(1+\beta)/2} + \tau. \quad (81)$$

According to Lemma 2, choose the appropriate parameters to make (81) satisfy the following conditions.

Case 1. If $V > (\tau / ((1 - \omega)\mu_1))^{2/(1+\alpha)}$, $\omega \in (0, 1)$, then (81) can be written as

$$\dot{V} \leq -\omega\mu_1 V^{(1+\alpha)/2} - \mu_2 V^{(1+\beta)/2}. \quad (82)$$

The solution of system (1) converges on the compact set,

$$x \in \left\{ V(x) \leq \left(\frac{\tau}{(1 - \omega)\mu_1} \right)^{2/(1+\alpha)} \right\}. \quad (83)$$

The fixed convergence time is

$$T \leq T_{\max} := \frac{2}{\omega\mu_1(1-\alpha)} + \frac{2}{\mu_2(\beta-1)}. \quad (84)$$

Case 2. If $V > (\tau/((1-\omega)\mu_2))^{2/(1+\beta)}$, $\omega \in (0, 1)$, then (81) can be written as

$$\dot{V} \leq -\mu_1 V^{(1+\alpha)/2} - \omega\mu_2 V^{(1+\beta)/2}. \quad (85)$$

The solution of system (1) converges on the compact set,

$$x \in \left\{ V(x) \leq \left(\frac{\tau}{(1-\omega)\mu_2} \right)^{2/(1+\beta)} \right\}. \quad (86)$$

The fixed convergence time is

$$T \leq T_{\max} := \frac{2}{\mu_1(1-\alpha)} + \frac{2}{\omega\mu_2(\beta-1)}. \quad (87)$$

Combining Cases 1 and 2, the solution of system (1) converges on

$$x \in \left\{ V(x) \leq \min \left\{ \left(\frac{\tau}{(1-\omega)\mu_1} \right)^{2/(1+\alpha)}, \left(\frac{\tau}{(1-\omega)\mu_2} \right)^{2/(1+\beta)} \right\} \right\}. \quad (88)$$

The fixed convergence time is

$$T_s \leq T_{\max} := \frac{2}{\omega\mu_1(1-\alpha)} + \frac{2}{\omega\mu_2(\beta-1)}. \quad (89)$$

It can be seen from (82) and (85) that V is bounded, with the result that $z_{i,j}$ and $\tilde{\theta}_{i,j}$ are bounded, where $i = 1, \dots, M$, $j = 1, \dots, n_i$, as $\hat{\theta}_{i,j} = \theta_{i,j} - \tilde{\theta}_{i,j}$ is also bounded. Because $e_i = \sum_{j=1}^M a_{ij}(y_i - y_j) + h_i(y_i - y_0)$ and e_i and y_0 are bounded, y_i are bounded. Because $z_{i,2} = x_{i,2} - v_{i,1}$, $x_{i,2}$ are bounded. Similarly, $v_{i,k-1}$ and $x_{i,k}$, $k = 1, \dots, n_i$ are bounded. Therefore, all signals in the overall closed-loop system (1) are bounded.

Remark 5. In this paper, the adaptive fixed-time tracking consensus problem has been solved for pure-feedback nonlinear MASs. Compared with existing results [16–18, 24–27] on high-order MASs with nonlinear strict-feedback, our studied system is more general. Furthermore, the introduction of tracking error constraint extends the application scope of existing fixed-time control methods.

Remark 6. In fact, when the structure of the system is too complex, the design of the controller is very difficult. In this paper, the RBF neural network is used to approximate the

uncertain nonlinear function, so the precise system dynamics structure is not needed when designing the controller.

5. Simulation Results

In this section, two simulation examples are employed to verify the controller designed as described in the above paragraphs.

5.1. Example 1: Numerical Example. Consider the second-order nonlinear MASs consisting of four followers and one leader, and the dynamic of each agent N_i is given as the following pure-feedback form:

$$\begin{cases} \dot{x}_{i,1} = \frac{1 - e^{-x_{i,1}}}{1 + e^{-x_{i,2}}} + x_{i,2}^3 + x_{i,2}e^{-1-x_{i,1}} + d_{i,1}(\bar{x}_{i,1}, t) \\ \dot{x}_{i,2} = x_{i,1}^2 x_{i,2} + (2u_i + u_i^3)(x_{i,1}^2 + x_{i,2}^2) + d_{i,2}(\bar{x}_{i,2}, t) \\ y_i = x_{i,1}, \end{cases} \quad (90)$$

where $x_{i,1}$ and $x_{i,2}$, with $i = 1, \dots, 4$, are the state variables, u_i is the system input, and y_i is the system output. $d_{i,1}(\bar{x}_{i,1}, t) = 0.7x_{i,1}^2 \cos(1.5t)$ and $d_{i,2}(\bar{x}_{i,2}, t) = 0.5(x_{i,1}^2 + x_{i,2}^2)\sin^3(t)$ are the external disturbance terms.

The directed network topology for the simulation is shown in Figure 1. We choose $a_{ij} = 1$ on $j \in N_i$, $a_{ij} = 0$; otherwise, $h_1 = h_2 = h_3 = 0$, and $h_4 = 1$. The leader's output $y_0(t)$ is $0.5 \sin(1.5t) + \cos(0.5t)$ and the initial states and adaptive laws of the four followers are set to $x_1(0) = [0.5, 0.5]^T$, $x_2(0) = [0.1, 0.1]^T$, $x_3(0) = [-0.3, -0.3]^T$, and $x_4(0) = [0.3, 0.3]^T$, where $x_j(0) = [x_{j,1}(0), x_{j,2}(0)]^T$ with $i = 1, \dots, 4$, and $\hat{\theta}_i = [\hat{\theta}_{i,1}, \hat{\theta}_{i,2}] = [0, 0]^T$ with $i = 1, \dots, 4$. To display the tracking errors between the leader and the followers, we use $e_i = |y_i(t) - y_0(t)|$, where $i = 1, \dots, 4$, and the design parameters are chosen as follows: $\alpha = 1/3$, $\beta = 2$, $\zeta_{i,11} = \zeta_{i,12} = \zeta_{i,21} = \zeta_{i,22} = 0.2$, $\zeta_{i,13} = \zeta_{i,23} = 10$, $b_i = 5$, $\gamma_i = 5$, $\eta_{i,1} = \eta_{i,2} = 0.25$, $\varepsilon_{10} = \varepsilon_{20} = 0.001$, $\lambda_i = 0.04$, $x_{i,10} = 0.2$, and $x_{i,20} = 0.5$, where $i = 1, \dots, 4$. It is worth noting that $\zeta_{i,13} = \zeta_{i,23}$ should be at least ten times that of $\zeta_{i,11} = \zeta_{i,12} = \zeta_{i,21} = \zeta_{i,22}$ and b_i setting of 5 is much better than setting it to 2. The performance boundary function's design parameters ξ_0 , a , ξ_∞ , σ_1 , and σ_2 are chosen as 0.9, 1, 0.17, 2, and 2, respectively.

Then, the virtual/actual control signals and the adaptive laws are, respectively, designed as follows:

FIGURE 1: Network topology for a group of one leader and four followers F1 to F4.

$$\begin{aligned}
 v_{i,1} &= \frac{1}{(d_i + h_i)F_b\Phi_{F_i}} - \left(\frac{\hat{\theta}_{i,1}}{2\eta_{i,1}^2} S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1} - \zeta_{i,11} S_{z_{i,1},1} - \zeta_{i,12} S_{z_{i,1},2} - \zeta_{i,13} z_{i,1} + x_{i,10} \right), \\
 u_i &= -\zeta_{i,21} S_{z_{i,2},1} - \zeta_{i,22} S_{z_{i,2},2} - \frac{\hat{\theta}_{i,2}}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2}) S_{i,2}(Z_{i,2}) z_{i,2} - \zeta_{i,23} z_{i,2} + x_{i,20}, \\
 \dot{\hat{\theta}}_{i,1} &= \frac{\gamma_i}{2\eta_{i,1}^2} S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1}) z_{i,1}^2 - \lambda_i \hat{\theta}_{i,1}, \\
 \dot{\hat{\theta}}_{i,2} &= \frac{\gamma_i}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2}) S_{i,2}(Z_{i,2}) z_{i,2}^2 - \lambda_i \hat{\theta}_{i,2},
 \end{aligned} \tag{91}$$

where $i = 1, \dots, 4$.

In this simulation example, we set the width of the RBF neural network as four. $W_{i,1}S_{i,1}(Z_{i,1})$ and $W_{i,2}S_{i,2}(Z_{i,2})$ contain seven and five nodes, respectively. The center of the Gaussian function is set as

$$\begin{aligned}
 \xi_{i,1} &= \begin{bmatrix} -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \end{bmatrix}, \\
 \xi_{i,2} &= \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \end{bmatrix}.
 \end{aligned} \tag{92}$$

In Figure 2, the tracking curves of the output signals y_i are presented. It can be seen that, by utilizing the proposed fixed-time control strategy, the followers can fast track the

leader. In Figure 3, the errors between the followers and the leader are shown. It can be seen that all local tracking errors are within the prescribed boundaries. In Figure 4, the responses of adaptive laws in MASs are given, which shows that adaptive signals $\hat{\theta}_{i,j}$ ($i = 1, \dots, 4, j = 1, 2$) are all bounded for $t \geq 0$. In Figure 5, the responses of control inputs in MASs are given, which shows that control input signals u_i ($i = 1, \dots, 4$) are all bounded for $t \geq 0$. All the above simulation results illustrate the validity of the proposed method.

5.2. Example 2: Physical Example. Consider the following multiple electromechanical systems consisting of two followers and one leader. The dynamic of each electromechanical system is described as

$$\begin{aligned}
 M\ddot{q} + B\dot{q} + N \sin q + \Delta_1(\dot{q}, q, I) &= I \\
 \dot{L}\dot{I} &= V_e - RI - K_B\dot{q} + \Delta_1(\dot{q}, q, I),
 \end{aligned} \tag{93}$$

where q is the angular motor position, V_e is the input control voltage, and I is the motor armature, $M = (J/K_\tau) + (mL_0^2/3K_\tau) + (M_0L_0^2/K_\tau) + (2M_0R_0^2/5K_\tau)$, $B = (B_0/K_\tau)$, and $N = (mL_0g/2K_\tau) + (M_0L_0g/K_\tau)$, $\Delta_1(\dot{q}, q)$, and $\Delta_2(\dot{q}, q)$ denote the model errors. J denotes the rotor inertia, K_τ is the coefficient to characterize the electromechanical conversion of armature current to torque, L_0 is the link length, m means the link mass, M_0 stands for the load mass, B_0 is the coefficient of viscous friction at the joint, R_0 is the radius of the load, g is the gravity coefficient, R is the armature resistance, L is the armature inductance, and K_B is the back-emf coefficient.

FIGURE 2: Response curves of the output signals y_i and y_0 . FIGURE 3: Response curves of the tracking consensus error e_i .

Let $x_{i,1} = q$, $x_{i,2} = \dot{q}$, $x_{i,3} = I$, $y_i = x_{i,1}$, and $u_i = V_e$. Then, the electromechanical system can be rewritten as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2}, \\ \dot{x}_{i,2} = e_{21}x_{i,2}x_{i,3}^2 + e_{22}\sin x_{i,1} + e_{23}x_{i,2} - e_{23}x_{i,2}^2x_{i,3}^3, \\ \dot{x}_{i,3} = e_{31}u_i + e_{32}x_{i,2} + e_{33}x_{i,3}^2 - e_{33}x_{i,2}^2\sin x_{i,3}, \\ y_i = x_{i,1}, \end{cases} \quad (94)$$

where $e_{21} = 1/M$, $e_{22} = -N/M$, $e_{23} = -B/M$, $e_{31} = 1/L$, $e_{32} = -K_B/L$, and $e_{33} = -R/L$.

Assume that there are three followers and one leader in MASs, and the communications are $h_1 = 1$, $h_2 = h_3 = 0$, $a_{2,1} = a_{3,1} = 1$, and $a_{1,2} = a_{1,3} = a_{2,3} = a_{3,2} = 0$. The parameters are chosen as follows: $M = 0.0642$, $N = 1.1408$, $B = 0.0181$, $L = 0.025$, $K_B = 0.9$, and $R = 5.0$. The leader's output $y_0(t)$ is $\sin(0.5t) + 0.5\sin(t)$. The predefined performance function' parameters ξ_0 , a , ξ_∞ , σ_1 , and σ_2 are chosen as 0.9, 7, 0.17, 1, and 1, respectively. The purpose of

FIGURE 4: Adaptive laws in MASs.

FIGURE 5: Control inputs in MASs.

FIGURE 6: Response curves of the tracking errors between this paper and [44].

FIGURE 8: Response curves of the output signals y_i and y_0 .

FIGURE 7: Response curves of the tracking errors between this paper and [25].

this example is to design a fixed-time controller to make each follower's output y_i track the leader's output signal y_0 within fixed time.

FIGURE 9: Response curves of the tracking consensus error e_i .

Then, the virtual/actual control signals and the adaptive laws are, respectively, designed as follows:

FIGURE 10: Adaptive laws in MASSs.

$$\begin{aligned}
 v_{i,1} &= \frac{1}{(d_i + h_i)F_b\Phi_{F_i}} \left(-\zeta_{i,11}S_{z_{i,1},1} - \zeta_{i,12}S_{z_{i,1},2} - \zeta_{i,13}z_{i,1} \right), \\
 v_{i,2} &= -\frac{\hat{\theta}_{i,2}}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2})S_{i,2}(Z_{i,2})z_{i,2} + x_{i,20} - \zeta_{i,21}S_{z_{i,2},1} - \zeta_{i,22}S_{z_{i,2},2} - \zeta_{i,23}z_{i,2}, \\
 u_i &= -\zeta_{i,31}S_{z_{i,3},1} - \zeta_{i,33}S_{z_{i,3},3} - \frac{\hat{\theta}_{i,3}}{3\eta_{i,3}^2} S_{i,3}^T(Z_{i,3})S_{i,3}(Z_{i,3})z_{i,3} - \zeta_{i,33}z_{i,3} + x_{i,30}, \\
 \dot{\hat{\theta}}_{i,2} &= \frac{\gamma_i}{2\eta_{i,2}^2} S_{i,2}^T(Z_{i,2})S_{i,2}(Z_{i,2})z_{i,2}^2 - \lambda_i \hat{\theta}_{i,2}, \\
 \dot{\hat{\theta}}_{i,3} &= \frac{\gamma_i}{2\eta_{i,3}^2} S_{i,3}^T(Z_{i,3})S_{i,3}(Z_{i,3})z_{i,3}^2 - \lambda_i \hat{\theta}_{i,3},
 \end{aligned} \tag{95}$$

FIGURE 11: Control inputs in MASs.

where $i = 1, 2, 3$.

To verify the effectiveness of our designed fixed-time consensus control protocol, we compared the tracking performances in system (94) of two existing control strategies that are fixed-time controller in [25] and traditional finite-time controller in [44]. For fair comparison, we choose the same design parameters as follows: $\zeta_{i,11} = \zeta_{i,12} = 10$, $\zeta_{i,21} = \zeta_{i,22} = 10$, $\zeta_{i,31} = \zeta_{i,32} = 10$, $\zeta_{i,23} = \zeta_{i,33} = 60$, $x_{i,10} = x_{i,20} = x_{i,30} = 0.1$, and $b_i = 10$. The initial conditions are $x_1(0) = [0.15, 0.15, 0.15]^T$, $x_2(0) = [0.2, 0.2, 0.2]^T$, and $x_3(0) = [0.25, 0.25, 0.25]^T$, where $x_i(0) = [x_{i,1}(0), x_{i,2}(0), x_{i,3}(0)]$, and $[\hat{\theta}_{i,2}(0), \hat{\theta}_{i,3}(0)] = [0.3, 0.3]^T$ with $i = 1, 2, 3$. The design of RBF neural network is the same as in [44].

Remark 7. Since the control protocols in [25, 44] are not designed for MASs, we choose the performance of the agent 1 that tracks directly the leader's output to compare, and the comparative results are shown in Figures 6 and 7.

In Figure 8, the tracking curves of the output signals y_i are presented. It can be seen that, by using the proposed controller in this paper, the followers can quickly follow the leader's output. Figure 9 shows that each local error is bounded by prescribed functions. In Figures 10 and 11, the responses of adaptive laws and control inputs in MASs are given, respectively. In Figures 6 and 7, the tracking error curves between this paper and [25, 44] are presented. It can be seen that the tracking performance by using our

controller is better than theirs, since the proposed scheme has higher tracking performance with higher accuracy, and the comparison results further illustrate the effectiveness and superiority of the proposed method in this paper.

6. Conclusion

In this paper, the adaptive fixed-time tracking consensus problem has been investigated for multiple uncertain nonlinear pure-feedback systems with performance constraint already. Mean value theorem is employed to address the problem of nonaffine structure in pure-feedback nonlinear systems, and RBF neural networks are introduced to approximate unknown nonlinear functions. Based on Lyapunov stability theory, it is proved that the proposed control scheme can ensure all the signals in MASs are bounded, and each local tracking error can converge to a small neighborhood of zero within fixed time. Furthermore, the performance of tracking error is guaranteed to be bounded by the prescribed boundary function. The effectiveness of the work has been proved by stability analysis along with simulation results.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Pinwei Li put forward research topics; designed the research plan; drafted the manuscript; and revised the manuscript. Jiyang Dai was involved in guidance support; obtaining research funding; and final review of the manuscript. Jin Ying checked the English grammar and guided the simulation programming.

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