Research Article

Fractional q -Integral Operators for the Product of a q -Polynomial and q -Analogue of the I -Functions and Their Applications

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In this article, we derive four theorems concerning the fractional integral image for the product of the q -analogue of general class of polynomials with the q -analogue of the I -functions. To illustrate our main results, we use q -fractional integrals of Erdélyi–Kober type and generalized Weyl type fractional operators. The study concludes with a variety of results that can be obtained by using the relationship between the Erdélyi–Kober type and the Riemann–Liouville q -fractional integrals, as well as the relationship between the generalized Weyl type and the Weyl type q -fractional integrals.

1. Introduction and Preliminaries

It is commonly accepted that q -calculus is the basic extension of classical fractional calculus. The topic deals with investigations of q -calculus of any order, and it is significant and widespread because of its various applications in areas similar to existing fractional calculus, patterns of the q -transform analysis, q -distinction (differential), q -integral equations, and so on (see [1, 3]). As a result of these applications, many researchers use the q -fractional calculus formula to evaluate various special capacities such as the q -hypergeometric function [4–7], the q -analogue of Fox's H -function [8, 9], the general class of q -polynomials [10], the q -integral inequalities [11–14], and so on.

The q -analogue of Riemann–Liouville fractional integral operator of a function $f(y)$ due to Al-Salam [15] is defined as

$$I_q^\mu f(y) = \frac{1}{\Gamma_q(\mu)} \int_0^y [y - tq]_{\mu-1} f(t) d(t; q), \quad (1)$$

where $\Re(\mu) > 0$ and $0 < |q| < 1$.

The q -analogue of the Erdélyi–Kober type fractional integral operator is given by Agarwal [16] as

$$I_q^{\eta, \mu} f(y) = \frac{y^{-\eta-\mu}}{\Gamma_q(\mu)} \int_0^y [y - tq]_{\mu-1} t^\eta f(t) d(t; q), \quad (2)$$

where $\Re(\mu) > 0$, $0 < |q| < 1$, and η is real or complex.

Al-Salam [15] has also established the basic q -analogue of the Weyl fractional integral operator for arbitrary order μ , which is as follows:

$$K_q^{-\mu} f(y) = \frac{q^{-\mu(\mu-1)/2}}{\Gamma_q(\mu)} \int_y^\infty (t - y)_{\mu-1} f(tq^{1-\mu}) d(t; q), \quad (3)$$

where $\Re(\mu) > 0$ and $K_q^0 f(y) = f(y)$.

In the same article, Al-Salam defined the q -extension of the generalized Weyl fractional integral operator, which is given as

$$K_q^{\eta, \mu} f(y) = \frac{q^{-\eta} y^\eta}{\Gamma_q(\mu)} \int_y^\infty (t - y)_{\mu-1} t^{-\eta-\mu} f(tq^{1-\mu}) d(t; q), \quad (4)$$

where $\Re(\mu) > 0$ and η is an arbitrary complex quantity.

According to Gasper and Rahman [17], the following q -integrals were introduced:

$$\int_0^y f(t) d(t; q) = y(1-q) \sum_{k=0}^{\infty} q^k f(yq^k), \quad (5)$$

$$\int_y^{\infty} f(t) d(t; q) = y(1-q) \sum_{k=1}^{\infty} q^{-k} f(yq^{-k}), \quad (6)$$

$$\int_0^{\infty} f(t) d(t; q) = (1-q) \sum_{k=-\infty}^{\infty} q^k f(q^k). \quad (7)$$

Also,

$$[x-y]_v = x^y \prod_{n=0}^{\infty} \left[\frac{1-(y/x)q^n}{1-(y/x)q^{v+n}} \right]. \quad (8)$$

In connection to result (5), (2) can be stated as follows:

$$I_q^{\eta, \mu} f(y) = \frac{(1-q)}{\Gamma_q(\mu)} \sum_{k=0}^{\infty} q^{k(1+\eta)} [1-q^{k+1}]_{\mu-1} f(yq^k), \quad (9)$$

where $\Re(\mu) > 0$ and η is real or complex.

For $f(y) = y^{v-1}$, then

$$I_q^{\eta, \mu} (y^{\alpha-1}) = \frac{\Gamma_q(\alpha+\eta)}{\Gamma_q(\alpha+\eta+\mu)} y^{\alpha-1}. \quad (10)$$

Now, by using (6) in (4), the q -generalized Weyl fractional integral operator can be written as

$$K_q^{\eta, \mu} f(y) = \frac{q^{-\eta} y^{\eta}}{\Gamma_q(\mu)} \int_y^{\infty} (t-y)_{\mu-1} t^{-\eta-\mu} f(tq^{1-\mu}) d(t; q). \quad (11)$$

For $f(y) = y^{\alpha}$, then we obtain

$$K_q^{\eta, \mu} (y^{\alpha}) = \frac{\Gamma_q(\eta-\alpha)}{\Gamma_q(\eta-\alpha+\mu)} y^{\alpha} q^{-\mu\alpha}. \quad (12)$$

We begin by returning to a form of q -polynomials $f_{n,N}(y; q)$ in terms of a bounded complex sequence $\{S_{n,q}\}_{n=0}^{\infty}$, provided that

$$f_{n,N}(y, q) = \sum_{j=0}^{\lfloor n/N \rfloor} \begin{bmatrix} n \\ Nj \end{bmatrix} S_{j,q} y^j, \quad (n = 0, 1, 2, \dots). \quad (13)$$

Also, q -gamma function is given (cf. Gasper and Rahman [17]) as

$$\Gamma_q(\beta) = \frac{(q; q)_{\infty} (1-q)^{1-\beta}}{(q^{\beta}; q)_{\infty}} = \frac{[1-q]_{\beta-1}}{(1-q)^{\beta-1}}, \quad (14)$$

where $\beta \neq 0, -1, -2, \dots$ and

$$(\beta; q)_{\infty} = \prod_{j=0}^{\infty} (1 - \beta q^j). \quad (15)$$

Further, for real or complex β and $0 < |q| < 1$, the q -shifted factorial is defined as

$$(\beta; q)_n = \begin{cases} 1, & \text{if } n = 0, \\ (1-\beta)(1-\beta q) \cdots (1-\beta q^{n-1}), & \text{if } n \in \mathbb{N}, \end{cases} \quad (16)$$

or

$$(\beta; q)_n = \frac{(\beta; q)_{\infty}}{(\beta q^n; q)_{\infty}}. \quad (17)$$

The definition for q -analogue of the I -function in expressions of the Mellin-Barnes type basic contour integral given by Saxena and Kumar [18] is as follows:

$$I_{P_i, Q_i}^{m, n} \left[y; q \middle| \begin{matrix} (a_j, \alpha_j)_{1, n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1, m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right] \quad (18)$$

$$= \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi y^s}{\sum_{i=1}^r \{ \prod_{j=m+1}^{Q_i} G(q^{1-b_{ji} + \beta_{ji} s}) \prod_{j=m+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \} G(q^{1-s}) \sin \pi s} ds,$$

where $0 \leq m \leq Q_i; 0 \leq n \leq P_i; i = 1, 2, \dots, r; r$ is finite; $\omega = \sqrt{-1}$; and

$$G(q^a) = \left\{ \prod_{n=0}^{\infty} (1 - q^{a+n}) \right\}^{-1} = \frac{1}{(q^a; q)_{\infty}}, \quad (19)$$

where $\alpha_j, \alpha_{ji}, \beta_j, \beta_{ji}$ are real with positive and a_j, a_{ji}, b_j, b_{ji} are complex numbers.

As of the contour C all of the poles of $G(q^{b_j - \beta_j s})$, $1 \leq j \leq m$, are to its right side, and those of $G(q^{1-a_j + \alpha_j s})$,

$1 \leq j \leq n$, are to its left side and at least some $\varepsilon > 0$ distance away. The basic integral converges if $\Re[s \log(y) - \log \sin \pi s] < 0$, for huge values of $|s|$ on the contour, that is, if $|\arg y| < \pi$. It reduces to the contour of integration C that can be replaced by other properly concave contours parallel to the imaginary axis.

It is interesting to note that for $r = 1, P_1 = P; Q_1 = Q$; definition (18) yields the q -analogue of Fox's H -function due to Saxena et al. [19], namely,

$$H_{P,Q}^{m,n} \left[y; q \left| \begin{matrix} (a, \alpha) \\ (b, \beta) \end{matrix} \right. \right] = \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1 - a_j + \alpha_j s}) \pi y^s}{\prod_{j=m+1}^Q G(q^{1 - b_j + \beta_j s}) \prod_{j=n+1}^P G(q^{a_j - \alpha_j s}) G(q^{1-s}) \sin \pi s} ds. \quad (20)$$

If we put $\alpha_i = \beta_j = 1$, $\forall i$ and j in definition (20), it reduces to a basic analogue of Meijer's G -function defined by Saxena et al. [19], that is,

$$H_{P,Q}^{m,n} \left[y; q \left| \begin{matrix} (a, 1) \\ (b, 1) \end{matrix} \right. \right] \equiv G_{P,Q}^{m,n} \left[y; q \left| \begin{matrix} a_1, \dots, a_P \\ b_1, \dots, b_Q \end{matrix} \right. \right] = \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - s}) \prod_{j=1}^n (q^{1 - a_j + s}) \pi y^s}{\prod_{j=m+1}^Q G(q^{1 - b_j + s}) \prod_{j=n+1}^P G(q^{a_j - s}) G(q^{1-s}) \sin \pi s} ds, \quad (21)$$

where $0 \leq m_1 \leq Q, 0 \leq n_1 \leq P$, and $\Re[s \log(y) - \log \sin \pi s] < 0$.

Additionally, if we take $n_1 = 0, m_1 = Q$ in definition (21), we attain the q -analogue of MacRobert's E -function:

$$G_{P,Q}^{Q,0} \left[y; q \left| \begin{matrix} a_1, \dots, a_P \\ b_1, \dots, b_Q \end{matrix} \right. \right] = E_q \left[Q; b_j; P; a_j; y \right]. \quad (22)$$

A comprehensive description of many functions expressible in terms of Fox's H -function can be found in research editions published by Mathai and Saxena [20, 21] and Mathai et al. [22].

2. Main Results

In this section, we will prove theorems involving fractional order q -integrals of Erdélyi-Kober type and generalized Weyl type for the product of q -analogue of general class of polynomial and q -analogue of I -function.

Theorem 1. Let $\Re(\mu) > 0, \eta, \lambda \in \Re, |q| < 1$, and $I_q^{\eta, \mu} \{\cdot\}$ be the Kober fractional q -integral operator for multiplication of q -general class of polynomial and q -analogue of I -function as

$$\begin{aligned} I_q^{\eta, \mu} \left\{ y^{\lambda-1} f_{n,N}(y, q) I_{P_i, Q_i}^{m,n} \left[\rho y^k; q \left| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right. \right] \right\} \\ = y^{\lambda-1} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \\ \times I_{P_i, Q_i+1}^{m+1,n} \left[\rho y^k; q \left| \begin{matrix} (1-j-\lambda-\eta, k)(a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} (1-j-\mu-\lambda-\eta, k) \end{matrix} \right. \right], \end{aligned} \quad (23)$$

provided $\Re[s \log(y) - \log \sin \pi s] < 0$.

Proof. By using definitions (13) and (18) on the left-hand side of equation (23), we have

$$\begin{aligned}
 L &= I_q^{\eta, \mu} \left\{ y^{\lambda-1} \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \right. \\
 &\quad \times \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi (\rho y^k)^s}{\sum_{i=1}^r \left\{ \prod_{j=m+1}^{Q_i} G(q^{1-b_{ji} + \beta_{ji} s}) \prod_{j=n+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \right\} G(q^{1-s}) \sin \pi s} ds \Big\} \\
 &= \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\
 &\quad \times \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi \rho^s}{\sum_{i=1}^r \left\{ \prod_{j=m+1}^{Q_i} G(q^{1-b_{ji} + \beta_{ji} s}) \prod_{j=n+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \right\} G(q^{1-s}) \sin \pi s} \times I_q^{\eta, \mu} (y^{j+\lambda+ks-1}) ds.
 \end{aligned} \tag{24}$$

Using known result (10), we get

$$\begin{aligned}
 L &= \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\
 &\quad \times \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi \rho^s}{\sum_{i=1}^r \left\{ \prod_{j=m+1}^{Q_i} G(q^{1-b_{ji} + \beta_{ji} s}) \prod_{j=n+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \right\} G(q^{1-s}) \sin \pi s} ds \\
 &\quad \times (1-q)^\mu \frac{(q^{j+\lambda+ks+\mu+\eta}; q)_\infty}{(q^{j+\lambda+ks+\eta}; q)_\infty} y^{j+\lambda+ks-1},
 \end{aligned} \tag{25}$$

We arrived at the right-hand side of result (23) by further simplification of above term. $\square$

Theorem 2. Let $\Re(\mu) > 0, |q| < 1$ and the generalized Weyl fractional q -integral of the multiplication of basic analogue of general class of polynomials and q -analogue of I -function exist, and we have

$$\begin{aligned}
 &K_q^{\eta, \mu} \left\{ y^\lambda f_{n,N}(y, q) I_{P_i, Q_i}^{m, n} \left[\rho y^k; q \middle| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right] \right\} \\
 &= y^\lambda q^{-\mu\lambda} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \\
 &\quad \times I_{P_i+1, Q_i+1}^{m+1, n} \left[\rho (y q^{-\mu})^k; q \middle| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i}, (\eta - j - \lambda, k) \\ (\eta - \lambda - j + \mu, k) (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right],
 \end{aligned} \tag{26}$$

where $0 \leq m \leq Q_i$; $0 \leq n \leq P_i$; $i = 1, 2, \dots, r$; r is finite, $|q| < 1$, and λ, ρ are arbitrary.

Proof. By using definition (13) and (18) on the left-hand side of equation (26), we have

$$L = \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \times \frac{1}{2\pi\omega} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1 - a_j + \alpha_j s}) \pi \rho^s}{\prod_{i=1}^r \left\{ \prod_{j=m+1}^{Q_i} G(q^{1 - b_{ji} + \beta_{ji} s}) \prod_{j=n+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \right\} G(q^{1-s}) \sin \pi s} \times K_q^{\eta, \mu} (y^{j+\lambda+ks}) ds. \quad (27)$$

If we use known result (12), the expression reduces to

$$L = \frac{y^\lambda q^{-\mu(\lambda+ks)}}{2\pi\omega} \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \times \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1 - a_j + \alpha_j s}) \pi (\rho y^k)^s}{\prod_{i=1}^r \left\{ \prod_{j=m+1}^{Q_i} G(q^{1 - b_{ji} + \beta_{ji} s}) \prod_{j=n+1}^{P_i} G(q^{a_{ji} - \alpha_{ji} s}) \right\} G(q^{1-s}) \sin \pi s} \times \frac{\Gamma_q(\eta - j - \lambda - ks)}{\Gamma_q(\eta - j - \lambda - ks + \mu)}. \quad (28)$$

By simplifying further, we got to the right-hand side of result (26). $\square$

Theorem 3. Let $\Re(\mu) > 0$, $\eta \in \Re$, $|q| < 1$, and $I_q^{\eta, \mu} \{\cdot\}$ be the Kober fractional q -integral operator; then, the following result holds:

$$I_q^{\eta, \mu} \left\{ y^{\lambda-1} f_{n,N}(y, q) H_{P,Q}^{m_1, n_1} \left[y^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right] \right\} \\ = y^{\lambda-1} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \times H_{P+1,Q}^{m_1, n_1+1} \left[y^k; q \middle| \begin{matrix} (1-j-\lambda-\eta, k), (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q), (1-j-\mu-\lambda-\eta, k) \end{matrix} \right], \quad (29)$$

provided $\Re[s \log(y) - \log \sin \pi s] < 0, k > 0$.

Proof. By using definitions (13) and (20) on the left-hand side of equation (29), we have

$$\begin{aligned}
 L &= I_q^{\eta, \mu} \left\{ y^{\lambda-1} \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \right. \\
 &\quad \times \frac{1}{2\pi i} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi (y^k)^s}{\prod_{j=m+1}^Q G(q^{1-b_j + \beta_j s}) \prod_{j=n+1}^P G(q^{a_j - \alpha_j s}) G(q^{1-s}) \sin \pi s} ds \Big\} \\
 &= \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\
 &\quad \times \frac{1}{2\pi i} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi}{\prod_{j=m+1}^Q G(q^{1-b_j + \beta_j s}) \prod_{j=n+1}^P G(q^{a_j - \alpha_j s}) G(q^{1-s}) \sin \pi s} \times I_q^{\eta, \mu} (y^{j+\lambda+ks-1}) ds.
 \end{aligned} \tag{30}$$

On using known result (10),

and the above expression reduces to

$$I_q^{\eta, \mu} (y^{v-1}) = \frac{\Gamma_q(v + \eta)}{\Gamma_q(v + \eta + \mu)} y^{v-1}, \tag{31}$$

$$\begin{aligned}
 L &= \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\
 &\quad \times \frac{1}{2\pi i} \int_C \frac{\prod_{j=1}^m G(q^{b_j - \beta_j s}) \prod_{j=1}^n G(q^{1-a_j + \alpha_j s}) \pi (1-q)^{j+\lambda+ks}}{\prod_{j=m+1}^Q G(q^{1-b_j + \beta_j s}) \prod_{j=n+1}^P G(q^{a_j - \alpha_j s}) G(q^{1-s}) \sin \pi s} \\
 &\quad \times (1-q)^\mu \frac{(q^{j+\lambda+ks+\mu+\eta}; q)_\infty}{(q^{j+\lambda+ks+\eta}; q)_\infty} y^{j+\lambda+ks-1}.
 \end{aligned} \tag{32}$$

The desired right-hand side of (29) may be obtained by further simplification. $\square$

Theorem 4. If $\Re(\mu) > 0, \eta \in \Re, |q| < 1$, then the generalized Weyl fractional q -integral operator for the basic analogue of product of general class of polynomial and basic analogue of H -function is given by

$$\begin{aligned}
 &K_q^{\eta, \mu} \left\{ y^\lambda f_{n,N}(x, q) H_{P,Q}^{m_1, n_1} \left[\rho y^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right]; \right\} \\
 &= y^\lambda q^{-\mu\lambda} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} y^j \\
 &\quad \times H_{P+1,Q}^{m_1, n_1+1} \left[\rho (y q^{-\mu})^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P), (\eta - j - \lambda, k) \\ (\eta - \lambda - j + \mu, k), (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right].
 \end{aligned} \tag{33}$$

Proof. We can prove above theorem similar to the proof of Theorems 2 and 3. $\square$

3. Applications of the Main Results

In this section, we shall discuss some special cases of the main result, by assigning appropriate values to the parameters involved in the main results.

If we use the relation between basic analogue of Weyl fractional q -integral operator and generalized Weyl fractional q -integral operator for particular $f(x) = x^\lambda$ as

$$K_q^\mu \{f(y)\} = y^\mu q^{-\mu(\mu+1)/2} K_q^{-\mu, \mu} \{f(y)\}, \quad (34)$$

then for the basic analogue of the product of the general class of polynomials and the q -analogue of the I -function, we include Weyl fractional q -integral

$$\begin{aligned} & K_q^\mu \left\{ y^\lambda f_{n,N}(x, q) I_{P_i, Q_i}^{m,n} \left[\rho y^k; q \middle| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right] \right\} \\ &= y^{\lambda+\mu} q^{-\mu\lambda - (\mu(\mu+1)/2)} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\ & \times I_{P_i+1, Q_i+1}^{m+1,n} \left[\rho (yq^{-\mu})^k; q \middle| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i}, (-\mu-j-\lambda, k) \\ (-\lambda-j, k), (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right], \end{aligned} \quad (35)$$

and also Weyl fractional q -integral image of the q -analogue for product of general class of polynomial and q -analogue of H -function

$$\begin{aligned} & K_q^\mu \left\{ y^\lambda f_{n,N}(x, q) H_{P,Q}^{m_1, n_1} \left[\rho y^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right] \right\} \\ &= y^{\lambda+\mu} q^{-\mu\lambda + (\mu(\mu+1)/2)} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\ & \times H_{P+1, Q}^{m_1+1, n_1+1} \left[\rho (yq^{-\mu})^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P), (-\mu-j-\lambda, k) \\ (-\lambda-j, k), (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right]. \end{aligned} \quad (36)$$

Similarly, if we use the relation between Kober fractional q -integral and Riemann–Liouville fractional q -integral operator (cf. Yadav et al. [6]) as

$$I_q^\mu \{f(y)\} = y^\mu I_q^{0, \mu} \{f(y)\}, \quad (37)$$

then we have Riemann–Liouville fractional q -integral for the basic analogue of product of general class of polynomial and basic analogue of I -function

$$\begin{aligned} & I_q^\mu \left\{ y^{\lambda-1} f_{n,N}(y, q) I_{P_i, Q_i}^{m,n} \left[\rho y^k; q \middle| \begin{matrix} (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i} \end{matrix} \right] \right\} \\ &= y^{\lambda+\mu-1} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\ & \times I_{P_i+1, Q_i+1}^{m+1,n} \left[\rho y^k; q \middle| \begin{matrix} (1-j-\lambda, k), (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1, P_i} \\ (b_j, \beta_j)_{1,m}, (b_j, \beta_j)_{m+1, Q_i}, (1-j-\mu-\lambda, k) \end{matrix} \right]. \end{aligned} \quad (38)$$

Furthermore, Riemann–Liouville fractional q -integral for the basic analogue of product of general class of polynomial and basic analogue of H -function is given by

$$\begin{aligned} & I_q^\mu \left\{ y^{\lambda-1} f_{n,N}(y, q) H_{P,Q}^{m_1, n_1} \left[y^k; q \middle| \begin{matrix} (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q) \end{matrix} \right] \right\} \\ &= y^{\lambda+\mu-1} (1-q)^\mu \sum_{j=0}^{[n/N]} \begin{bmatrix} n \\ N_j \end{bmatrix} S_{j,q} \\ & \times H_{P+1,Q}^{m_1, n_1+1} \left[y^k; q \middle| \begin{matrix} (1-j-\lambda, k), (a_1, \alpha_1), \dots, (a_P, \alpha_P) \\ (b_1, \beta_1), \dots, (b_Q, \beta_Q) (1-j-\mu-\lambda, k) \end{matrix} \right]. \end{aligned} \quad (39)$$

4. Conclusion

We arrive at the conclusion that the results presented in this article are general and contribute to the q -calculus theory. It is also important to note that the results presented in this paper can be applied to the arrangements of specific q -differentials and q -integrals related to the q -analogues of Meijer's G -function, Fox's H -function, and I -function. Furthermore, by using the appropriate coefficient $S_{j,q}$ the q -polynomial family $f_{n,N}(y, q)$ yields a number of known q -polynomials as special cases. These include, among others, the q -Hermite polynomials, the q -Laguerre polynomials, the q -Jacobi polynomials, the Wall polynomials, the q -Konhauser polynomials, and several others.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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