Research Article

A Novel Method for Fault Diagnosis of the Two-Input Two-Output Nonlinear Mass-Spring-Damper System Based on NOFRF and MBPCA

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For fault diagnosis of the two-input two-output mass-spring-damper system, a novel method based on the nonlinear output frequency response function (NOFRF) and multiblock principal component analysis (MBPCA) is proposed. The NOFRF is the extension of the frequency response function of the linear system to the nonlinear system, which can reflect the inherent characteristics of the nonlinear system. Therefore, the NOFRF is used to obtain the original fault feature data. In order to reduce the amount of feature data, a multiblock principal component analysis method is used for fault feature extraction. The least squares support vector machine (LSSVM) is used to construct a multifault classifier. A simplified LSSVM model is adopted to improve the training speed, and the conjugate gradient algorithm is used to reduce the required storage of LSSVM training. A fault diagnosis simulation experiment of a two-input two-output mass-spring-damper system is carried out. The results show that the proposed method has good diagnosis performance, and the training speed of the simplified LSSVM model is significantly higher than the traditional LSSVM.

1. Introduction

At present, fault diagnosis technologies have been widely used in manufacturing equipment, electric machine, wind power system, electronic equipment, and so on. With the increasing requirements of reliability and safety, the studies of fault diagnosis technology become more and more important [1–3]. The mass-spring-damper system is a classical vibration system, which can be used to describe many practical systems [4–7]. During operation, some faults will be occurred due to the influence of aging or external environment. Therefore, it is necessary to study the fault diagnosis of the mass-spring-damper system. In practical engineering, the multivariable nonlinear mass-spring-damper system can be used for modelling [8]. Under normal condition, the nonlinearity of the mass-spring-damper system is weak. When a fault occurred, the parameter or structure will be changed.

Volterra series is an important mathematical model for nonlinear systems. The frequency domain Volterra kernel is called the generalized frequency response function (GFRF), which is a direct generalization of the frequency response function of the linear system in the nonlinear system. The frequency characteristics of the nonlinear system can be described by GFRF [9–12]. The frequency characteristic information is obtained by a generalized frequency response function, which can be used for fault diagnosis of nonlinear systems [13]. The fault diagnosis of a permanent magnet synchronous motor is studied by using the generalized frequency response function and convolutional neural network [14]. For fault diagnosis of nonlinear analog circuit, GFRFs are used to obtain the feature data, and the LSSVM fusion method is used for fault identification [15]. The generalized frequency response function is a multidimensional function, and the computational complexity increases exponentially with the order. In order to reduce the

computational complexity, a nonlinear output frequency response function (NOFRF) is proposed based on GFRF [16]. The NOFRF is a one-dimensional function with less computational complexity. In [17], the stiffness and damper coefficients of the multidegree-of-freedom nonlinear system is obtained according to NOFRF. In [18], the transfer characteristics of NOFRFs of the multidegree-of-freedom system are analyzed. The fault diagnosis of the transmission system of numerical control equipment is studied by using NOFRF [19]. The concept of NOFRF is extended to MIMO nonlinear systems, and the characteristics of NOFRF are analyzed [20].

Principal component analysis (PCA) is a multivariate statistical analysis method, which can be used to extract feature data effectively [21–25]. In [26], the real-time incipient fault diagnosis for the electric drive system in high-speed train is studied based on deep principal component analysis. For degradation of sensor accuracy in the practical system, a hierarchical principal component analysis method based on dynamic fault differential characteristics is used for fault detection [27]. In [28], a distributed fault detection method based on fault-related variable selection and Bayesian reasoning is proposed. Multiblock principal component analysis (MBPCA) methods have been proposed for large-scale data compression and analysis [29–31]. MBPCA divides the data into different blocks according to the characteristics of the data and then conducts principal component analysis. In [32], the measured data of the chemical process are analysed by MBPCA, and the fault is identified by a subblock contribution graph. In [33], the fault detection of semiconductor devices is studied by using multiblock principal component analysis, and the combination index is constructed by using SPE statistics and Hotelling's T^2 statistics.

Support vector machine (SVM) is a typical machine learning method, which is widely used for fault diagnosis [34–37]. The training of the support vector machine is very complex. In [38], the least squares support vector machine is proposed by changing the risk function of the SVM. The

training of LSSVM only needs to solve one linear equation, which is highly efficient. The least square support vector machine model is established to predict the degradation trend of the slewing bearing [39]. In [40], the intelligent location of high-speed train is studied based on LSSVM, and the iterative pruning error minimization and L-0 norm minimization algorithm are used to sparse LSSVM. In [41], S-transform is used for obtaining feature data from induced potential signal and particle swarm optimization LSSVM is used for identifying local demagnetization fault of a permanent magnet linear synchronous motor. An iterative algorithm based on conjugate gradient is used to train LSSVM, and the storage requirement is reduced [42]. In [43], a simplified model of the least squares support vector machine is proposed, which can reduce the computational complexity.

In this study, a fault diagnosis method is proposed for a nonlinear two-input two-output mass-spring-damper system based on nonlinear output frequency response function and multiblock principal component analysis. The nonlinear output frequency response function is used to establish the system model and obtain the original fault feature data. The features are extracted from the amplitudes of NOFRFs by multiblock kernel principal component analysis. A LSSVM multifault classifier is established to identify faults based on a simplified LSSVM model and conjugate gradient algorithm. A simulation experiment for a two-input two-output mass-spring-damper system is used to verify the effectiveness of the proposed fault diagnosis method.

2. NOFRF Estimation of the Two-Input Two-Output Nonlinear Mass-Spring-Damper System

A two-input two-output nonlinear mass-spring-damper system is shown in Figure 1.

The motion equation of the system is represented as

$$\begin{cases} m_1 \ddot{y}_1(t) + c_1 \dot{y}_1(t) + c_2 (\dot{y}_1(t) - \dot{y}_2(t)) + k_1 y_1(t) + \\ k_2 (y_1(t) - y_2(t)) + b_1 y_1^2(t) + b_2 (y_1(t) - y_2(t))^2 = u_1(t), \\ m_2 \ddot{y}_2(t) + c_2 (\dot{y}_2(t) - \dot{y}_1(t)) + c_3 \dot{y}_2(t) + k_2 (y_2(t) - y_1(t)) \\ + k_3 y_2(t) - b_2 (y_1(t) - y_2(t))^2 + b_3 y_2^2(t) = u_2(t), \end{cases} \quad (1)$$

where y_1, y_2 are the outputs, u_1, u_2 are the inputs, and $m_1, m_2, c_1, c_2, c_3, k_1, k_2, k_3, b_1, b_2, b_3$ are the system

parameters: mass, damper, linear stiffness, and nonlinear stiffness, respectively.

The mass-spring-damper system can be expressed as Volterra series:

$$y_i(t) = \sum_{n=1}^N \sum_{p_1+p_2=n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{(i,p_1,p_2)}^{(n)}(\tau_1, \tau_2, \dots, \tau_n) \times u_1(t - \tau_1), \dots, u_1(t - \tau_{p_1}) u_2(t - \tau_{p_1+1}), \dots, u_2(t - \tau_{p_1+p_2}) d\tau_1, \dots, d\tau_n, \quad (2)$$

where $y_i(t)$ is the i^{th} output, $h_{(i,p_1,p_2)}^{(n)}(\tau_1, \tau_2, \dots, \tau_n)$ is the n^{th} Volterra kernel, and N is the order of the nonlinear system, $i = 1, 2$.

The Fourier transform of the n^{th} Volterra kernel is expressed as

$$H_{(i,p_1,p_2)}^{(n)}(j\omega_1, \dots, j\omega_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_{(i,p_1,p_2)}^{(n)}(\tau_1, \tau_2, \dots, \tau_n) e^{-j(\omega_1\tau_1 + \cdots + \omega_n\tau_n)} d\tau_1, \dots, d\tau_n, \quad (3)$$

where $H_{(i,p_1,p_2)}^{(n)}(j\omega_1, \dots, j\omega_n)$ is the n^{th} generalized frequency response function of the nonlinear mass-spring-damper system.

The output spectrum of the nonlinear mass-spring-damper system is described as

$$Y_i(j\omega) = \sum_{n=1}^{\infty} Y_i^{(n)}(j\omega),$$

$$Y_i^{(n)}(j\omega) = \frac{1/\sqrt{n}}{(2\pi)^{n-1}} \sum_{N_1+N_2=n} \int_{\omega_1+\cdots+\omega_n=\omega} H_{(i,p_1,p_2=N_2)}^{(n)}(j\omega_1, \dots, j\omega_n) \times \prod_{j=1}^2 \prod_{k=N_0+\cdots+N_{j-1}+1}^{N_0+\cdots+N_j} U_j(j\omega_k) d\sigma_{n\omega}, \quad (4)$$

where $Y_i(j\omega)$ is the spectrum of the i^{th} output, $Y_i^{(n)}(j\omega)$ is the n^{th} order output spectrum, $U_j(j\omega)$ is the input spectrum, $N_0 = 0$, and $i = 1, 2$.

The generalized frequency response function is a multidimensional function, which requires a lot of calculation.

In order to reduce the computational complexity, the nonlinear output frequency response function of the multivariable system is proposed [20].

The n^{th} NOFRF of the two-input two-output mass-spring-damper system can be expressed as

$$G_{(i,p_1=N_1,p_2=N_2)}^{(n)}(j\omega) = \frac{Y_{(i,p_1=N_1,p_2=N_2)}^{(n)}(j\omega)}{U_{(i,p_1=N_1,p_2=N_2)}^{(n)}(j\omega)}$$

$$= \frac{\int_{\omega_1+\cdots+\omega_n=\omega} H_{(i,p_1=N_1,p_2=N_2)}^{(n)}(j\omega_1, \dots, j\omega_n) \times \prod_{j=1}^2 \prod_{k=N_0+\cdots+N_{j-1}+1}^{N_0+\cdots+N_j} U_j(j\omega_k) d\sigma_{n\omega}}{\int_{\omega_1+\cdots+\omega_n=\omega} \prod_{j=1}^2 \prod_{k=N_0+\cdots+N_{j-1}+1}^{N_0+\cdots+N_j} U_j(j\omega_k) d\sigma_{n\omega}}, \quad (5)$$

where $U_{(i,P_1=N_1,P_2=N_2)}^{(n)}(j\omega) = ((1/\sqrt{n})/(2\pi)^{n-1}) \int_{\omega_1+\dots+\omega_n=\omega} \prod_{j=1}^2 \prod_{k=N_0+\dots+N_{j-1}+1}^{N_0+\dots+N_j} U_j(j\omega_k) d\sigma_{n\omega}$ is the Fourier transformation of $\underbrace{u_1(t), \dots, u_1(t)}_{N_1} \underbrace{u_2(t), \dots, u_2(t)}_{N_2}$.

When the first N order NOFRFs are used to describe the nonlinear mass-spring-damper system, the frequency domain output can be expressed as

$$Y_i(j\omega) = \sum_{n=1}^N \sum_{N_1+\dots+N_m=n} G_{(i,P_1=N_1,P_2=N_2)}^{(n)}(j\omega) U_{(P_1=N_1,P_2=N_2)}^{(n)}(j\omega). \quad (6)$$

The relationship between input and output of the mass-spring-damper system is shown in Figure 2.

Sort $G_{(i,P_1=N_1,P_2=N_2)}^{(n)}(j\omega)$ and $U_{(P_1=N_1,P_2=N_2)}^{(n)}(j\omega)$, respectively:

$$G_{(i,k)}^{(n)}(j\omega), (k = 1, 2, \dots, L_{(n,m)}), \quad (7)$$

$$U_{(k)}^{(n)}(j\omega), (k = 1, 2, \dots, L_{(n,m)}), \quad (8)$$

where $L_{(n,m)} = (m+n-1), \dots, ((m+1)m)/n!$, $m = 2$.

Let $G_{(i,k)}^{(n)}$ represents $G_{(i,k)}^{(n)}(j\omega)$, and $U_{(k)}^{(n)}$ represents $U_{(k)}^{(n)}(j\omega)$. According to equations (7) and (8), equation (6) can be rewritten as

$$Y_i(j\omega) = \mathbf{U}^T \mathbf{G}_i, \quad (9)$$

where $\mathbf{U} = [U_{(1)}^{(1)}, \dots, U_{(L_{(1,m)})}^{(1)}, \dots, U_{(1)}^{(N)}, \dots, U_{(L_{(N,m)})}^{(N)}]^T$, and $\mathbf{G}_i = [G_{(i,1)}^{(1)}, \dots, G_{(i,L_{(1,m)})}^{(1)}, \dots, G_{(i,1)}^{(N)}, \dots, G_{(i,L_{(N,m)})}^{(N)}]^T$.

The NOFRF of the nonlinear system is insensitive to the amplitude of input. Therefore, the NOFRFs of the two-input two-output nonlinear mass-spring-damper system can be estimated based on the least square criterion. Let $u_i(t) = a_j u_i^*(t)$ be the input, where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, \bar{N}$, $\bar{N} \geq L_{(1,m)} + \dots + L_{(n,m)}$, a_j is constant, and $a_{\bar{N}} > \dots > a_1$. The output spectrum of the nonlinear mass-spring-damper system described by NOFRFs can be expressed as

$$Y_i(j\omega) = \mathbf{U}_A \mathbf{G}_i, \quad (10)$$

where $\mathbf{Y}_i(j\omega) = [Y_{i,1}(j\omega), Y_{i,2}(j\omega), \dots, Y_{i,\bar{N}}(j\omega)]^T$, and $\mathbf{U}_A =$

$$\begin{bmatrix} a_1 U_{(1)}^{(1)}, \dots, a_1 U_{(L_{(1,m)})}^{(1)}, \dots, a_1^N U_{(1)}^{(N)}, \dots, a_1^N U_{(L_{(N,m)})}^{(N)} \\ \vdots \\ a_{\bar{N}} U_{(1)}^{(1)}, \dots, a_{\bar{N}} U_{(L_{(1,m)})}^{(1)}, \dots, a_{\bar{N}}^N U_{(1)}^{(N)}, \dots, a_{\bar{N}}^N U_{(L_{(N,m)})}^{(N)} \end{bmatrix}.$$

According to equation (10), the NOFRFs of the mass-spring-damper system can be obtained based on the least square principle:

$$\mathbf{G}_i = [\mathbf{U}_A^{*T} \mathbf{U}_A]^{-1} \mathbf{U}_A^T \mathbf{Y}_i(j\omega). \quad (11)$$

The nonlinear output frequency response function is a one-dimensional function with low computational complexity. When a fault occurred, the nonlinear stiffness coefficient of the mass-spring-damper system will be increased, and the nonlinear output frequency response functions will be changed significantly. Therefore, the original fault feature data obtained by NOFRF can effectively diagnose the

nonlinear mass-spring-damper system. In this study, the amplitudes of NOFRFs are selected for fault diagnosis.

3. Feature Extraction for NOFRF Based on MBPCA

The data amount of NOFRF amplitudes of the two-input two-output nonlinear mass-spring-damper system is large. In order to reduce the amount of feature data, feature extraction is needed. According to the number of system outputs, the system can be divided into two subsystems. In order to make the extracted NOFRF feature data more fully reflect the system characteristics, a multiblock principal component analysis method is used for feature extraction.

Under the normal state of the mass-spring-damper system, several groups of NOFRF amplitude data are obtained as samples. Divide the sample data into two blocks to obtain $\mathbf{S} = [\mathbf{A}_1, \mathbf{A}_2]$, where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the NOFRF amplitude matrices of the two subsystems, respectively. The MBPCA method proposed by Westerhuis et al. [30] is used to extract feature data. In order to establish the MBPCA model of the mass-spring-damper system, the following optimization problems need to be solved.

$$\min_{\mathbf{T}_o, \mathbf{P}_i} \sum_{i=1}^2 \|\mathbf{A}_i - \mathbf{T}_o \mathbf{P}_i^T\|^2, \quad (12)$$

where $\mathbf{T}_o = \sum_{l=1}^2 \mathbf{T}_l \mathbf{W}_l$ is the score matrix of the MBPCA model, $\mathbf{T}_l$ is the score matrix of subblock, $\mathbf{W}_l$ is the weight matrix of subblock, and $\mathbf{P}_1$ and $\mathbf{P}_2$ are the load matrices obtained by the two subblocks.

The nonlinear iterative partial least squares method is used to solve the equation (12). Define $\mathbf{t}_{o1}$ as the first principal component vector of $\mathbf{S}$. Initialize $\mathbf{t}_{o1}$, so that $\|\mathbf{t}_{o1}\| = 1$. Calculate the first load vector of the subblock, respectively.

$$\mathbf{p}_{l1} = (\mathbf{A}_l)^T \mathbf{t}_{o1}, \quad (13)$$

where $l = 1, 2$.

Normalize load vectors $\mathbf{p}_{11}$ and $\mathbf{p}_{21}$ to get $\mathbf{p}'_{11}$ and $\mathbf{p}'_{21}$. Then, calculate the first principal component vector of each block separately:

$$\mathbf{t}'_{l1} = \mathbf{A}_l \mathbf{p}'_{l1}. \quad (14)$$

According to $\mathbf{t}'_{11}$ and $\mathbf{t}'_{21}$, calculate the weight vector:

$$\mathbf{w}_{o1} = \mathbf{T}_1'^T \mathbf{t}_{o1}, \quad (15)$$

where $\mathbf{T}_1' = [\mathbf{t}'_{11}, \mathbf{t}'_{21}]$, $\mathbf{w}_{o1} = [w_{11}, w_{21}]^T$, and w_{11} and w_{21} , respectively, represent the first principal component weight of the two subblock data.

Normalize the weight vector $\mathbf{w}_{o1}$, and calculate the principal component vector $\mathbf{t}_{o1}$.

$$\mathbf{t}_{o1} = \mathbf{T}' \mathbf{w}_{o1}. \quad (16)$$

According to equations (13)–(16), the principal component vector $\mathbf{t}_{o1}$ is iteratively calculated until convergence,

and the weights w_{11}, w_{21} and the load vectors $\mathbf{p}_{11}, \mathbf{p}_{21}$ are obtained.

Calculate the deviation of the estimated value of each subblock matrix from the original matrix separately:

$$\mathbf{E}^{(l)} = \mathbf{A}_l - \hat{\mathbf{A}}_l, \quad (17)$$

where $\hat{\mathbf{A}}_l = \mathbf{t}_{o1} \cdot \mathbf{p}_{l1}^T, l = 1, 2$.

According to the NOFRF amplitude deviation matrix $\mathbf{E}^{(l)}$ of each subblock, the second group weight $\mathbf{w}'_{o2}$ and the second group load vectors $\mathbf{p}'_{12}, \mathbf{p}'_{22}$ can be obtained by equations (13)–(16), and so on, until the γ^{th} weight vector and the γ^{th} group load vector are obtained, where γ represents the number of principal components. According to the weight vectors and the load vector, $\mathbf{P}_1, \mathbf{P}_2, \mathbf{W}_1$, and $\mathbf{W}_2$ can be obtained.

For a group of NOFRF amplitude vectors, the fault feature vectors can be obtained by using the established MBPCA model. The schematic diagram for feature extraction of NOFRF based on multiblock principal component analysis is shown in Figure 3. In Figure 3, $\mathbf{a}_1, \mathbf{a}_2$ are the NOFRF amplitude vectors of the two subsystems, $\mathbf{P}_1, \mathbf{P}_2$ are the load matrices of the subblock, $\mathbf{t}_1, \mathbf{t}_2$ are the principal component vectors of the subsystem, $\mathbf{W}_l = \text{diag}(w_{l1}, w_{l2}, \dots, w_{lA})$ is the weight matrix of the subblock, and $\mathbf{t}_o$ is the

extracted fault feature vector of the mass-spring-damper system.

4. Fault Identification Based on Simplified LSSVM

After the fault features are extracted by MBPCA, they are used to identify the fault of the mass-spring-damper system. The least square support vector machine is used to construct a multifault classifier.

Define the training sample dataset as $S_1: \{(\mathbf{x}_i, y_i)\}_{i=1}^M$, where $\mathbf{x}_i \in \mathbf{R}^n$ is the i^{th} input vector, $y_i \in \{-1, 1\}$ is the i^{th} category label, and M is the sample size. The problem of the binary classification of LSSVM can be described as

$$\min \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{\omega} + \frac{1}{2} C \sum_{i=1}^M \xi_i^2 \quad (18)$$

$$\text{s.t. } y_i [\boldsymbol{\omega}^T \varphi(\mathbf{x}_i) + b] = 1 - \xi_i, \quad i = 1, 2, \dots, M,$$

where $\boldsymbol{\omega}$ is the weight vector of the classification hyperplane, $C > 0$ is the penalty factor, ξ_i is the slack variable, $\varphi(\cdot)$ is the nonlinear mapping, and b is the classification threshold.

Define the Lagrangian function as

$$L(\boldsymbol{\omega}, b, \xi_i, \alpha_i) = \frac{1}{2} \boldsymbol{\omega}^T \boldsymbol{\omega} + \frac{1}{2} C \sum_{i=1}^M \xi_i^2 - \sum_{i=1}^M \alpha_i \{y_i [\boldsymbol{\omega}^T \varphi(\mathbf{x}_i) + b] - 1 + \xi_i\}, \quad (19)$$

where $\alpha_i \geq 0$ is the Lagrange multiplier.

Let the partial derivatives of the Lagrangian function $L(\boldsymbol{\omega}, b, \xi_i, \alpha_i)$ with respect to $\boldsymbol{\omega}$, b , ξ_i , and α_i be zero:

$$\begin{cases} \frac{\partial L}{\partial \boldsymbol{\omega}} = 0 \Rightarrow \boldsymbol{\omega} = \sum_{i=1}^M \alpha_i y_i \varphi(\mathbf{x}_i), \\ \frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^M \alpha_i y_i = 0, \\ \frac{\partial L}{\partial \xi_i} = 0 \Rightarrow \alpha_i = C \xi_i, \\ \frac{\partial L}{\partial \alpha_i} = 0 \Rightarrow y_i [\boldsymbol{\omega}^T \varphi(\mathbf{x}_i) + b] - 1 + \xi_i = 0. \end{cases} \quad (20)$$

By sorting out equation (20), the constrained optimization problem of LSSVM can be transformed into linear equations:

$$\begin{bmatrix} 0 & \mathbf{Y}^T \\ \mathbf{Y} & \mathbf{b} \end{bmatrix} \cdot \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{1} \end{bmatrix}, \quad (21)$$

where $\mathbf{Y} = [y_1, y_2, \dots, y_M]^T$, $\mathbf{H} = \Omega + C^{-1} \cdot \mathbf{I}$, Ω is the M -dimensional symmetric square matrix, $\Omega_{i,j} = y_i y_j K(\mathbf{x}_i, \mathbf{x}_j)$, $K(\cdot, \cdot)$ is the kernel function, $\mathbf{I}$ is the M -dimensional identity matrix, $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_M]^T$ is the Lagrange multiplier vector, and $\mathbf{1} = [1, 1, \dots, 1]^T$.

The decision function is

$$y(\mathbf{x}) = \text{sign} \left[\sum_{i=1}^M \alpha_i y_i K(\mathbf{x}, \mathbf{x}_i) + b \right], \quad (22)$$

where $\mathbf{x}$ is the sample vector to be classified.

It can be seen from equation (21) that the matrix on the left side of the equation is $M + 1$ order square matrix. When M is large, the matrix inversion operation needs a large amount of memory. In order to reduce the required storage, an iterative algorithm based on conjugate gradient can be used to train LSSVM [42].

The conjugate gradient algorithm is used to solve the following two M -variable linear equations:

$$\begin{aligned} \mathbf{H} \boldsymbol{\eta} &= \mathbf{Y}, \\ \mathbf{H} \mathbf{v} &= \mathbf{1}. \end{aligned} \quad (23)$$

According to $\boldsymbol{\eta}$ and $\mathbf{v}$, calculate the classification threshold b and the Lagrangian multiplier vector $\boldsymbol{\alpha}$.

$$b = \frac{(\boldsymbol{\eta}^T \mathbf{1})}{(\boldsymbol{\eta}^T \mathbf{Y})}, \quad (24)$$

$$\boldsymbol{\alpha} = \mathbf{v} - b\boldsymbol{\eta}.$$

There are two M -variable linear equations that need to be solved when using the traditional LSSVM model and conjugate gradient algorithm to train the LSSVM binary classifier. In order to reduce the computational complexity, Li et al. [43] proposed a simplified LSSVM model. In order to improve the training speed and reduce the storage requirement, the LSSVM multifault classifier is trained by the simplified LSSVM model and conjugate gradient algorithm in this study.

The structure of the LSSVM multifault classifier is “one against one.” For the LSSVM multifault classifier, the training sample set of the k^{th} subclassifier is defined as $G_k: \{\mathbf{g}_i, y_i\}, i = 1, 2, \dots, M_k$, where $\mathbf{g}_i \in \mathbf{R}^p$ is the NOFRF feature vector, $y_i \in \{-1, 1\}$ is the category label, and M_k is the sample size.

$$\begin{bmatrix} 0 & \mathbf{Y}_k^T \\ \mathbf{Y}_k & \mathbf{H}_k \end{bmatrix} \cdot \begin{bmatrix} b_k \\ \boldsymbol{\alpha}_k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (25)$$

where $\mathbf{Y}_k = [y_1, y_2, \dots, y_{M_k}]^T$, $\mathbf{H}_k = \Omega_k + c_k^{-1} \cdot \mathbf{I}_k$, Ω_k is the M_k dimensional symmetric square matrix, $\Omega_{i,j} = y_i y_j K(\mathbf{x}_i, \mathbf{x}_j)$, $K(\cdot, \cdot)$ is the kernel function, c_k is the penalty factor, $\mathbf{I}_k$ is the M_k dimensional identity matrix, b_k is the classification threshold, $\boldsymbol{\alpha}_k = [\alpha_1, \alpha_2, \dots, \alpha_{M_k}]$ is the Lagrange multiplier vector, and $\mathbf{1} = [1, 1, \dots, 1]^T$.

The matrix $\mathbf{H}_k$ can be written as

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{H}_k^{M_k-1} & \mathbf{h}_k \\ \mathbf{h}_k^T & H_{M_k M_k} \end{bmatrix}, \quad (26)$$

where $\mathbf{H}_k^{M_k-1}$ is the $(M_k - 1) \times (M_k - 1)$ principal square submatrix of $\mathbf{H}_k$, $\mathbf{h}_k$ is the $M_k - 1$ dimensional vector formed after the last element is removed from the M_k^{th} vector of $\mathbf{H}_k$, $H_{M_k M_k}$ is the element in row M_k , and column M_k of $\mathbf{H}_k$.

Define

$$\begin{aligned} \tilde{H}_k &= \mathbf{H}_k^{M_k-1} - y_{M_k} \mathbf{Y}_k^{(M_k-1)} \mathbf{h}_k^T - y_{M_k} \mathbf{h}_k \left(\mathbf{Y}_k^{(M_k-1)} \right)^T \\ &\quad + H_{M_k M_k} \mathbf{Y}_k^{(M_k-1)} \left(\mathbf{Y}_k^{(M_k-1)} \right)^T, \end{aligned} \quad (27)$$

$$\mathbf{Y}_k^{(M_k-1)} = [y_1, y_2, \dots, y_{M_k-1}]^T, \quad (28)$$

$$\boldsymbol{\alpha}_k^{(M_k-1)} = [\alpha_1, \alpha_2, \dots, \alpha_{M_k-1}]^T. \quad (29)$$

According to equations (25)–(29), the LSSVM simplified model is given by

$$\begin{cases} \tilde{H}_k \boldsymbol{\alpha}_k^{(M_k-1)} = 1 - y_{M_k} \mathbf{Y}_k^{(M_k-1)}, \\ \alpha_{M_k} = -y_{M_k} \left(\mathbf{Y}_k^{(M_k-1)} \right)^T \boldsymbol{\alpha}_k^{(M_k-1)}, \\ b_k = y_{M_k} - y_{M_k} (\mathbf{H}_k \boldsymbol{\alpha}_k)_{M_k}, \end{cases} \quad (30)$$

where $(H_k \boldsymbol{\alpha}_k)_{M_k}$ is the M_k^{th} element of $H_k \boldsymbol{\alpha}_k$.

The subclassifier of the LSSVM multifault classifier is trained according to equation (30). First, the conjugate gradient algorithm is used to solve the linear equation $\tilde{H}_k \boldsymbol{\alpha}_k^{(M_k-1)} = 1 - y_{M_k} \mathbf{Y}_k^{(M_k-1)}$. Then, the Lagrangian multiplier α_{M_k} and classification threshold b_k can be calculated by $\alpha_{M_k} = -y_{M_k} \left(\mathbf{Y}_k^{(M_k-1)} \right)^T \boldsymbol{\alpha}_k^{(M_k-1)}$ and $b_k = y_{M_k} - y_{M_k} (H_k \boldsymbol{\alpha}_k)_{M_k}$.

When the traditional LSSVM model is used to train the k^{th} LSSVM subclassifier of the multifault classifier, there are two M -variable linear equations that need to be solved. The main amount of calculation is $O(3(k_1 + k_2)M_k^2)$, where k_1 and k_2 represent the number of iterations for solving $H_k \boldsymbol{\eta}_k = \mathbf{Y}_k$ and $H_k v_k = 1$, respectively. When the LSSVM simplified model is used to train the k^{th} LSSVM subclassifier of the multifault classifier, there is one $M-1$ -variable linear equation that needs to be solved. The main amount of calculation is $O(3(k_3 + 2)M_k^2)$, where k_3 represents the number of iteration for solving $\tilde{H}_k \boldsymbol{\alpha}_k^{(M_k-1)} = 1 - y_{M_k} \mathbf{Y}_k^{(M_k-1)}$. Generally, $k_1 + k_2 \gg k_3$, so the computational complexity of training the LSSVM multifault classifier is significantly reduced.

The schematic diagram of fault diagnosis for the mass-spring-damper system based on NOFRF and MBPCA is shown in Figure 4. First, the input spectrum and output

FIGURE 1: A two-input two-output nonlinear mass-spring-damper system.

FIGURE 2: The relationship between input and output of the mass-spring-damper system.

FIGURE 3: The schematic diagram for feature extraction of NOFRF based on MBPCA.

FIGURE 4: The schematic diagram of fault diagnosis.

spectrum data are obtained by Fourier transform of time domain data, and then, the NOFRFs are estimated by the least square estimation algorithm. Second, the MBPCA is used to extract fault features. Finally, the LSSVM multifault classifier is used for fault identification.

5. Simulation Experiment

The fault diagnosis simulation experiment of a two-input two-output nonlinear mass-spring-damper system is carried out. The nonlinear equations of the system are given by

FIGURE 5: The outputs of the nonlinear mass-spring-damper system. (a) The first output $y_1(t)$. (b) The second output $y_2(t)$.

FIGURE 6: The amplitudes of the first-order NOFRFs.

FIGURE 7: The amplitudes of the second-order NOFRFs.

FIGURE 8: The amplitudes of the third-order NOFRFs.

FIGURE 9: The amplitudes of the fourth-order NOFRFs.

TABLE 1: The fault description of the mass-spring-damper system.

Fault number	Fault mode	Parameters
F0	Normal	$c_1, c_2, c_3 \in [19.5 \ 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4 \ 1.05 \times 10^4]$ $b_1, b_2, b_3 \in [0.95 \times 10^7 \ 1.05 \times 10^7]$
F1	b_1 increased	$c_1, c_2, c_3 \in [19.5 \ 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4 \ 1.05 \times 10^4]$ $b_1 \in [1.45 \times 10^7 \ 1.55 \times 10^7]$, $b_2, b_3 \in [0.95 \times 10^7 \ 1.05 \times 10^7]$
F2	b_2 decreased	$c_1, c_2, c_3 \in [19.5 \ 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4 \ 1.05 \times 10^4]$ $b_2 \in [1.45 \times 10^7 \ 1.55 \times 10^7]$, $b_1, b_3 \in [0.95 \times 10^7 \ 1.05 \times 10^7]$
F3	b_3 increased	$c_1, c_2, c_3 \in [19.5 \ 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4 \ 1.05 \times 10^4]$ $b_3 \in [1.45 \times 10^7 \ 1.55 \times 10^7]$, $b_1, b_2 \in [0.95 \times 10^7 \ 1.05 \times 10^7]$
F4	b_1 and b_2 increased	$c_1, c_2, c_3 \in [19.5 \ 20.5]$, $k_1, k_2, k_3 \in [0.95 \times 10^4 \ 1.05 \times 10^4]$ $b_1, b_2 \in [1.45 \times 10^7 \ 1.55 \times 10^7]$, $b_3 \in [0.95 \times 10^7 \ 1.05 \times 10^7]$

FIGURE 10: The distributions of the first and second principal components.

FIGURE 11: The distributions of the third and fourth principal components.

TABLE 2: The recognition rate with different kernel functions.

Fault number	Fault mode	Recognition rate				
		Linear kernel (%)	Polynomial kernel (%)	GRB kernel (%)	ERB kernel (%)	MLP kernel (%)
F0	Normal	96	97	100	95	97
F1	b_1 increased	87	87	88	85	82
F2	b_2 decreased	78	94	92	95	82
F3	b_3 increased	85	97	98	93	85
F4	b_1 and b_2 increased	81	85	91	90	86

TABLE 3: The training time.

Kernel function	Training time	
	Traditional LSSVM model (s)	LSSVM simplified model (s)
Linear kernel	0.63	0.56
Polynomial kernel	0.67	0.62
GRB kernel	0.78	0.66
ERB kernel	0.90	0.69
MLP kernel	1.28	0.98

$$\begin{cases}
y_1(t) + 20\dot{y}_1(t) + 20(\dot{y}_1(t) - \dot{y}_2(t)) + 10^4 y_1(t) + \\
10^4 (y_1(t) - y_2(t)) + 10^7 y_1^2(t) + 10^7 (y_1(t) - y_2(t))^2 = u_1(t), \\
\ddot{y}_2(t) + 20(\dot{y}_2(t) - \dot{y}_1(t)) + 20\dot{y}_2(t) + 10^4 (y_2(t) - y_1(t)) \\
+ 10^4 y_2(t) - 10^7 (y_1(t) - y_2(t))^2 + 10^7 y_2^2(t) = u_2(t).
\end{cases} \quad (31)$$

The simulation has been performed using MATLAB R2014a. The CPU clock speed of the computer is 2.3 GHz, and the main memory is 8 GB. Let the input signal be $u_1 = \cos 2\pi t + \cos 6\pi t$ and $u_2 = \cos 4\pi t + \cos 8\pi t$. The sampling frequency is 256 Hz and sampling length is 5 s. The outputs of the system are shown in Figure 5. It can be seen that both outputs of the system are periodic signals.

The first four order NOFRFs are used to describe the mass-spring-damper system. The amplitudes of the NOFRFs are shown in Figures 6–9. It can be seen that the amplitudes of the higher-order NOFRFs of the system are obvious. Therefore, the system's nonlinear characteristic is very significant.

The Monte Carlo method is used for fault diagnosis simulation experiment of the mass-spring-damper system. Assume that under normal conditions, the variation ranges of linear stiffness and nonlinear stiffness are within 5%, and the variation range of damper is within 2.5%. When the system fails, the nonlinear characteristics will increase. Five kinds of faults of the mass-spring-damper system are defined, and the fault description is given in Table 1.

200 sets of input and output samples of the mass-spring-damper system are collected for each fault mode. The NOFRFs of the system are obtained by equation (11). After obtaining the amplitudes of NOFRFs, the features are extracted by MBPCA. The principal component

distributions of the five fault modes are shown in Figures 10 and 11, where PC1 is the first principal component, PC2 is the second principal component, PC3 is the third principal component, and PC4 is the fourth principal component.

For each fault mode, select 100 sets of feature data of NOFRF as training samples and the remaining 100 sets as test samples. The fault diagnosis simulation of the mass-spring-damper system is carried out by the LSSVM simplified model and the traditional LSSVM model based on the conjugate gradient algorithm, respectively. The linear kernel function, polynomial kernel function, Gaussian radial basis (GRB) kernel function, exponential radial basis (ERB) kernel function [44], and multilayered perceptron (MLP) kernel [45] are chosen as kernel functions of the LSSVM multifault classifier, respectively.

The fault recognition rates with different kernel functions are given in Table 2, and the training time is given in Table 3. As can be seen from Table 2, the LSSVM based on the GRB kernel function has the best result for fault identification. As can be seen from Table 3, the training time of the multifault classifier based on the simplified LSSVM model, respectively, is 0.56 s, 0.62 s, 0.66 s, 0.69 s, and 0.98 s, while that based on the traditional LSSVM model, respectively, is 0.63 s, 0.67 s, 0.78 s, 0.90 s, and 1.28 s. The training time of the multifault classifier based on the simplified model is significantly reduced. Therefore, the proposed diagnosis

method for the two-input two-output mass-spring-damper system has good diagnostic performance and fast training speed.

6. Conclusions

In this work, we studied the fault diagnosis of the nonlinear two-input two-output mass-spring-damper system combining NOFRF and MBPCA. In order to obtain the original feature data which can fully reflect the system information, the NOFRFs of the mass-spring-damper system are used to obtain original fault feature data. To reduce the number of feature variables, the multiblock kernel principal component analysis method is used for feature extraction. Based on a simplified LSSVM model and conjugate gradient algorithm, a multifault classifier is constructed for fault identification, which improves the training speed and reduces the storage requirement. A fault diagnosis simulation experiment of a nonlinear two-input two-output mass-spring-damper system is used to verify the effectiveness of the proposed method. The results demonstrate that the performance of the proposed method is good, and the training speed of the multifault classifier is fast.

Due to the serious disturbance in practical engineering, the fault diagnosis accuracy will be affected. Therefore, the identification of NOFRF and the design of the LSSVM multifault classifier will be deeply studied further to improve the estimation accuracy and fault diagnosis rate.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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