Research Article

Treating Measurement Errors in the Run Rule Schemes Integrated with Shewhart $\bar{X}$ Chart

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In modern quality control applications, there often exist significant measurement errors because observations are measured quickly in time order. As a result, the errors influence the power of a control chart to detect a given change in the process parameter(s) of a quality characteristic. In this paper, by using a covariate error model, the properties of the Shewhart $\bar{X}$ chart integrated with run rules are investigated when errors exist in the measurement of quality characteristic. Two metrics, the average run length and 95% quantile of the run length, are adopted to evaluate the chart's performance for different mean shifts and sample sizes. Numerous simulations are conducted, and the results indicate that the errors in the measurement significantly affect the performance of the run rule $\bar{X}$ chart, especially when the errors are large. To reduce this negative effect on the run rule $\bar{X}$ chart, measuring more times of each item in each subgroup and increasing the coefficient in the covariate error model are shown to be good choices for practitioners.

1. Introduction

As it is known in statistical process control (SPC), the Shewhart chart is effective in detecting large shifts in the process but is insensitive to small or moderate shifts. Even though the Shewhart-type charts still received much attention for the simple representation and implementation, as an intermediate solution, Western Electric [1] suggested the run rule scheme integrated with Shewhart-type charts to improve the chart's performance. Then, run rules combined with control charts were studied by many researchers, such as Divoky and Taylor [2]; Champ and Woodall [3]; Klein [4]; Fu et al. [5]; and so on. More recent works on control charts integrated with run rule schemes can be found in Hu and Castagliola [6]; Chew et al. [7]; Shongwe and Graham [8]; Shongwe and Graham [8]; Shongwe et al. [9]; Khaw and Chew [10]; Chew et al. [11]; Shongwe [12]; Malela-Majika and Rapoo [13]; Malela-Majika et al. [14]; Hu et al. [15]; and Ali et al. [16]. All these existing research studies focused on how to improve the chart's performance by incorporating run rules. They all

assumed that observations are measured with no errors in the quality characteristic.

However, in the actual measuring equipment, an exact measurement of a quality characteristic is a rare phenomenon, causing measurement errors in practice. The properties of a control chart for monitoring observations with measurement errors differ from the chart for monitoring observations with precise measurements. Most existing works have only studied the measurement errors' effect on a specific monitoring scheme. For example, by using a linear covariate model, Linna and Woodall [17] investigated the measurement errors' effect on the Shewhart mean ($\bar{X}$) and variance (S^2) charts. By extending the covariate model to the multivariate case, Linna et al. [18] studied the performance of multivariate control charts when measurement errors exist. Using the same linear model introduced in Linna and Woodall [17], the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts' performance for monitoring observations with measurement errors was studied in Maravelakis et al. [19] and Maravelakis [20]. More recently, considering the autocorrelation and

measurement errors, Costa and Castagliola [21] studied the Shewhart $\bar{X}$ chart's performance and suggested a new skipping sampling technique to reduce the negative effect of autocorrelation. Hu et al. [22, 23] investigated the overall performance of the synthetic and VSS $\bar{X}$ chart when measurement errors exist. For a full account of publications on measurement errors prior to 2017, readers are referred to Maleki et al. [24]. For the most recent works on control charts with measurement errors, we suggest the research in Cheng and Wang [25]; Shongwe et al. [26]; Nguyen et al. [27]; Nguyen et al. [28]; Thanwane et al. [29]; Thanwane et al. [30]; and so on. Even the above research works are on the control charts with measurement errors. To the best of our knowledge, no research on the run rule schemes integrated with Shewhart $\bar{X}$ chart with measurement errors can be found.

In this paper, we focus on the properties of 2-of-3 run rules integrated with Shewhart-type charts in the presence of measurement errors. The run rule schemes are set by using warning limits, and it gives a signal when the selected run rule pattern occurs. This research can be extended to other run rule schemes, which are not presented here. It is noted that this paper is mainly to investigate the run rule $\bar{X}$ chart's performance when measurement errors exist, not to increase the chart's performance.

This paper is organized as follows. In Section 2, the linearly covariate error model used to represent the true value and the measurement value of observations is introduced. Then, the 2-of-3 run rule $\bar{X}$ chart under this measurement error model is presented in Section 3. The detailed effect of measurement errors on the run rule $\bar{X}$ chart is presented in Section 4. Finally, some conclusions and future research directions are given in Section 6.

2. The Linearly Covariate Measurement Error Model

In a process, $n \geq 1$ consecutive items $\{Y_{i,1}, Y_{i,2}, \dots, Y_{i,n}\}$ of the quality characteristic Y are collected at each sampling point $i = 1, 2, \dots$. These items are assumed to be *independent* normal random variables with mean μ_0 and standard deviation σ_0 , i.e., $Y_{i,j} \sim N(\mu = \mu_0 + \delta\sigma_0, \sigma_0)$. δ is the mean shift parameter in the process. Due to the measurement errors, the true value of quality characteristic $Y_{i,j}$ is observed from $m \geq 1$ measurements $\{X_{i,j,1}, X_{i,j,2}, \dots, X_{i,j,m}\}$, with each $X_{i,j,k}$ being equal to the following equation:

$$X_{i,j,k} = A + BY_{i,j} + \varepsilon_{i,j,k}, \quad (1)$$

where A and B are two constants from the analysis of measurement device and $\varepsilon_{i,j,k} \sim N(0, \sigma_M)$ is an independent random error term.

In practice, if there are errors in the measurements of observations, it is suggested to measure each item several times and average these measurements for each item. By doing this, a smaller variance in the measurement error component than for a single measurement can be obtained. This can be simplified to the following equation:

$$\bar{X}_i = \frac{1}{mn} \sum_{j=1}^n \sum_{k=1}^m X_{i,j,k} = A + \frac{1}{n} \left(B \sum_{j=1}^n Y_{i,j} + \frac{1}{m} \sum_{j=1}^n \sum_{k=1}^m \varepsilon_{i,j,k} \right), \quad (2)$$

where m is the number of measurements per item. It can be derived that the expectation $E(\bar{X}_i)$ and the variance $V(\bar{X}_i)$ of $\bar{X}_i$,

$$E(\bar{X}_i) = A + B(\mu_0 + \delta\sigma_0), \quad (3)$$

$$V(\bar{X}_i) = \frac{1}{n} \left(B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right). \quad (4)$$

Moreover, considering that the measurement error variance σ_M^2 sometimes linearly depends on the process mean μ , ε is normally distributed with mean 0 and variance $\sigma_M^2 = C + D\mu$, with two known constants C and D . $E(\bar{X}_i)$ and $V(\bar{X}_i)$ are derived to be

$$E(\bar{X}_i) = A + B(\mu_0 + \delta\sigma_0), \quad (5)$$

$$V(\bar{X}_i) = \frac{1}{n} \left(B^2 \sigma_0^2 + \frac{C + D\mu}{m} \right). \quad (6)$$

3. The Shewhart $\bar{X}$ Chart Integrated with Run Rules under Linearly Covariate Measurement Errors

Considering the constant measurement error variance in equations (3) and (4), by setting $\delta = 0$ in the in-control state, two warning limits, LWL (the lower warning limit) and UWL (the upper warning limit), are given as follows:

$$\text{LWL} = A + B\mu_0 - W \sqrt{\frac{1}{n} \left(B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right)}, \quad (7)$$

$$\text{UWL} = A + B\mu_0 + W \sqrt{\frac{1}{n} \left(B^2 \sigma_0^2 + \frac{\sigma_M^2}{m} \right)}.$$

Second, considering the linearly increasing measurement error variance in equations (5) and (6), the warning limits of the chart are given as

$$\text{LWL} = A + B\mu_0 - W \sqrt{\frac{1}{n} \left(B^2 \sigma_0^2 + \frac{C + D\mu}{m} \right)}, \quad (8)$$

$$\text{UWL} = A + B\mu_0 + W \sqrt{\frac{1}{n} \left(B^2 \sigma_0^2 + \frac{C + D\mu}{m} \right)}.$$

In the implementation of the Shewhart $\bar{X}$ chart integrated with 2-of-3 run rules, if two out of the three successive points fall above UWL or fall below LWL, an out-of-control signal is triggered. By modeling the 2-of-3 run rule chart with a Markov chain, Figure 1 shows all the transient states corresponding to the 2-of-3 run rules.

When a new sample is collected and the corresponding sample mean (point) falls into one of the above seven transient states or into the 8th state, i.e., the absorbing state,

FIGURE 1: The transient states of 2-of-3 run rules. (a) (1). (b) (2). (c) (3). (d) (4). (e) (5). (f) (6). (g) (7).

where two out of the three successive points fall outside [LWL, UWL], then the run length properties of the Shewhart $\bar{X}$ chart integrated with 2-of-3 run rules can be obtained by using the Markov chain matrix $\mathbf{P}$:

$$\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{r} \\ \mathbf{0}^T & \mathbf{1} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & p_C & p_L & 0 & 0 & p_U \\ 0 & 0 & 0 & 0 & 0 & 0 & p_C & p_L + p_U \\ p_C & p_L & 0 & 0 & 0 & 0 & 0 & p_U \\ 0 & 0 & p_U & p_C & p_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_U & p_C & p_L \\ p_C & 0 & 0 & 0 & 0 & 0 & 0 & p_L + p_U \\ 0 & 0 & p_U & p_C & 0 & 0 & 0 & p_L \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

with the probabilities $p_L = P(\bar{X}_i < \text{LWL})$, $p_C = P(\text{LWL} \leq \bar{X}_i \leq \text{UWL})$, and $p_U = P(\bar{X}_i > \text{UWL})$. $\mathbf{Q}$ refers to the transient probabilities between 7 states in the figure and the vector $\mathbf{r} = \mathbf{1} - \mathbf{Q}\mathbf{1}$, with $\mathbf{1} = (1, 1, 1, 1, 1, 1, 1)^T$. For more details of the run rule charts, readers can refer to the related research works introduced in Section 1.

For the case in equations (3) and (4), the transient probabilities are equal to

$$p_L = \Phi\left(-W - \frac{\delta\sqrt{n}}{\sqrt{1 + (\gamma^2/B^2m)}}\right), \quad (10)$$

$$p_C = \Phi\left(W - \frac{\delta\sqrt{n}}{\sqrt{1 + (\gamma^2/B^2m)}}\right) - \Phi\left(-W - \frac{\delta\sqrt{n}}{\sqrt{1 + (\gamma^2/B^2m)}}\right), \quad (11)$$

$$p_U = \Phi\left(-W + \frac{\delta\sqrt{n}}{\sqrt{1 + (\gamma^2/B^2m)}}\right), \quad (12)$$

where $\gamma^2 = \sigma_M^2/\sigma_0^2$ is the measurement error ratio and $\Phi(\cdot)$ is the cumulative distribution function (c.d.f.) of the standard normal distribution.

For the case in equations (5) and (6), the transient probabilities are equal to

$$p_L = \Phi\left(\frac{-W\sqrt{B^2\sigma_0^2 + (C + D\mu_0/m)} - B\delta\sigma_0\sqrt{n}}{\sqrt{B^2\sigma_0^2 + (C + D\mu/m)}}\right), \quad (13)$$

$$p_C = \Phi\left(\frac{W\sqrt{B^2\sigma_0^2 + (C + D\mu_0/m)} - B\delta\sigma_0\sqrt{n}}{\sqrt{B^2\sigma_0^2 + (C + D\mu/m)}}\right) - \Phi\left(\frac{-W\sqrt{B^2\sigma_0^2 + (C + D\mu_0/m)} - B\delta\sigma_0\sqrt{n}}{\sqrt{B^2\sigma_0^2 + (C + D\mu/m)}}\right), \quad (14)$$

$$p_U = \Phi\left(\frac{-W\sqrt{B^2\sigma_0^2 + (C + D\mu_0/m)} + B\delta\sigma_0\sqrt{n}}{\sqrt{B^2\sigma_0^2 + (C + D\mu/m)}}\right). \quad (15)$$

From equations (10)–(15), it is shown that the parameter A has no influence on the chart, while in practice, as it has been explained in Linna and Woodall [17], this parameter should be taken into account for the process location or the process capability.

Using the transition matrix $\mathbf{Q}$ in the Markov chain, the probability mass function (p.m.f.) $f_L(\ell|\mathbf{Q}, \mathbf{q})$ and the c.d.f. $F_L(\ell|\mathbf{Q}, \mathbf{q})$ of the run length distribution of the chart with measurement errors are given as

$$f_L(\ell|\mathbf{Q}, \mathbf{q}) = \mathbf{q}^T \mathbf{Q}^{\ell-1} \mathbf{r}, \quad (16)$$

$$F_L(\ell|\mathbf{Q}, \mathbf{q}) = 1 - \mathbf{q}^T \mathbf{Q}^\ell \mathbf{1},$$

with $\mathbf{q} = (0, 0, 0, 1, 0, 0, 0)^T$ being the initial probabilities of transient states.

Furthermore, the ARL of the chart can be derived as

$$\text{ARL} = \mathbf{q}^T (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{1}, \quad (17)$$

and the $\alpha \in (0, 1)$ quantile RL_α of the Shewhart $\bar{X}$ chart with 2-of-3 run rules under linearly covariate measurement error model can be obtained as (see Gan [31])

$$F_L(\text{RL}_\alpha - 1|\mathbf{Q}, \mathbf{q}) \leq \alpha, \quad (18)$$

$$F_L(\text{RL}_\alpha|\mathbf{Q}, \mathbf{q}) > \alpha. \quad (19)$$

With equations (18) and (19), any quantile of the run length distribution can be computed. By setting $\alpha = 0.95$, the 95% quantile $\text{RL}_{0.95}$ of the run length distribution can be obtained.

4. Influence of Linearly Covariate Measurement Errors on the Run Rule Shewhart $\bar{X}$ Chart

Some characteristics of run length are usually used to show the control chart's performance. Two metrics, ARL and $RL_{0.95}$, are selected to evaluate the performance of the run rule $\bar{X}$ chart. As it has been stated in Khoo et al. [32], by computing a higher percentile of the run length distribution, say $RL_{0.95}$ in this paper, a practitioner can state with 95% confidence that when a process shift $\delta = 0.5$ occurs (see the case $\gamma^2 = 0$, $n = 5$ in Table 1), a signal will be triggered by the 50th sample. The information from the percentiles of the run length acts as practical guidance to study the behavior of control charts. The subscripts "0" and "1" of ARL represent the corresponding in-control and out-of-control performance of the chart, respectively. Without loss of generality, it is assumed that $ARL_0 = 370.4$. Consequently, the parameter W of the run rule Shewhart $\bar{X}$ chart with linearly covariate measurement errors satisfies the following constraint:

$$ARL(m, n, B, \gamma^2, W, \delta = 0) = ARL_0. \quad (20)$$

In this case, assuming $\delta = 0$ in equations (10)–(15), we can compute the only value of W using the "vert solvevert" function in MATLAB, irrespective of m , n , B , and γ^2 .

4.1. A Constant Measurement Error Variance. It can be seen from equations (17)–(19) that ARL_1 and $RL_{0.95}$ are functions of n , m , B , γ^2 , W , and δ . Table 1 presents ARL and $RL_{0.95}$ of the run rule $\bar{X}$ chart with linearly covariate error model for $\gamma^2 \in \{0, 0.1, 0.2, 0.3, 0.5, 0.7, 1\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $n \in \{1, 3, 5, 7\}$ when $m = 1$ and $B = 1$. The case $\gamma^2 = 0$ corresponds to the "no measurement error" case. For example, when $n = 1$, $\gamma^2 = 0$, and $\delta = 0$, the chart parameter $W = 1.9293$. With this value, we have $ARL = 198.43$ and $RL_{0.95} = 591$ at $\delta = 0.3$. As it can be seen from Table 1, for a specified δ , ARL and $RL_{0.95}$ increase when the measurement error ratio γ^2 increases. This fact demonstrates the negative effect of the measurement errors on the run rule $\bar{X}$ chart. For instance, for the example above but with $\gamma^2 = 1$, we have $ARL = 260.93$ and $RL_{0.95} = 778$. We can see that with the measurement error ratio increasing, larger ARL and $RL_{0.95}$ are needed to detect a shift.

The conclusions drawn above can also be seen from the four graphics in Figure 2 where ARL and $RL_{0.95}$ are plotted for $\delta \in \{0.5, 1.5\}$, $n \in \{1, 3, 5, 7\}$ and γ^2 when $m = 1$ and $B = 1$. This figure confirms the conclusion that larger ARL and $RL_{0.95}$ are observed with increasing measurement errors.

Table 2 presents ARL and $RL_{0.95}$ of the chart for $B \in \{1, 2, 3, 4, 5\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $n \in \{1, 3, 5, 7\}$ when $m = 1$ and $\gamma^2 = 1$. It can be seen that for specified δ and n , the negative effect of measurement errors on the run rule $\bar{X}$ chart decreases as B increases. Moreover, through the comparison between Tables 1 and 2, it can be seen that when $B = 5$, the difference between ARL and SDRL corresponding to $\gamma^2 = 0$ (no measurement error) and $\gamma^2 = 1$ is negligible. For instance, when $\delta = 0.3$, $m = 1$, $\gamma^2 = 1$, $n = 1$, and $B = 5$

(Table 2), $ARL = 202.26$ and $RL_{0.95} = 602$, while in Table 1, for the same values of m and δ , $ARL = 198.43$ and $RL_{0.95} = 591$ when $\gamma^2 = 0$.

In Figure 3, ARL and $RL_{0.95}$ of the run rule $\bar{X}$ chart in the presence of measurement errors are plotted for $\delta \in \{0.5, 1.5\}$, $n \in \{1, 3, 5, 7\}$, and B when $m = 1$ and $\gamma^2 = 1$. This figure also confirms the conclusion that with the value of B increasing, both ARL and $RL_{0.95}$ decrease.

Table 3 presents ARL and $RL_{0.95}$ for $m \in \{1, 2, 3, 4, 5\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $n \in \{1, 3, 5, 7\}$ when $B = 1$ and $\gamma^2 = 1$. It can be seen from Table 3 that as m increases, the negative effect of the measurement errors on the run rule $\bar{X}$ chart decreases. For instance, for the case where $\delta = 0.3$, $B = 1$, $n = 1$, and $\gamma^2 = 1$, $ARL = 260.93$ and $RL_{0.95} = 778$ decrease down to $ARL = 216.03$ and $RL_{0.95} = 643$, respectively, when $m = 1$ increases up to $m = 5$. So, it is concluded that if there is a relative high variability of the measurement error compared with the process variability, a large m is needed to reduce the negative effect of measurement errors on the run rule $\bar{X}$ chart.

In Figure 4, ARL and $RL_{0.95}$ of the run rule $\bar{X}$ chart in the presence of measurement errors are plotted for $\delta \in \{0.5, 1.5\}$, $n \in \{1, 3, 5, 7\}$, and m when $\gamma^2 = 1$ and $B = 1$. This figure confirms the conclusion that by measuring the sample item several times, the negative effect of measurement errors on the run rule $\bar{X}$ chart can be reduced to some extent.

Since measuring the item in each sample several times can reduce the measurement errors' negative effect on control charts, the run length performance of run rule $\bar{X}$ chart with linearly covariate error is presented in Figure 5, for the case of $m = 5$ measurements per item, using γ^2 , $\delta \in \{0.5, 1.5\}$ and $n \in \{1, 3, 5, 7\}$ when $B = 1$. It can be noted that for fixed n , with $m = 5$ measurements per item, ARL and $RL_{0.95}$ only have a slightly increasing trend with γ^2 increasing. Moreover, through the comparisons between the four graphics in Figure 2, the negative effect of measurement errors on the run rule $\bar{X}$ chart is reduced by taking multiple measurements of $m = 5$, especially for the cases when $\gamma^2 \geq 0.5$.

Finally, Figure 6 presents the run length performance of the run rule $\bar{X}$ chart for B , $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$ when $\gamma^2 = 1$. From Figure 3, it is known that increasing B reduces the negative effect of measurement errors on the run rule $\bar{X}$ chart. Meanwhile, through the comparison of Figures 3 and 6, it is noted that taking $m = 5$ measurements and increasing B enable the negative effect of measurement errors on the run rule $\bar{X}$ chart to be diminished faster compared to the cases in Figure 3.

4.2. Linearly Increasing Measurement Error Variance. Considering the linearly increasing variance case, it can be seen that parameter C in equations (5) and (6) behaves like σ_M^2 in equation (4). If there is a shift in the process mean, then it is amplified by parameter D , which also changes the variance of the measurement error. So, the influence of D on the run rule $\bar{X}$ chart is investigated in Figure 7. In this figure, for specified values of $n \in \{1, 3, 5, 7\}$, $B = 1$, $C = 5$, $m = 1$, $\mu_0 = 10$, and $\sigma_0^2 = 10$ are set for illustration. Through this

TABLE 1: Out-of-control $(ARL, RL_{0.95})$ when $ARL_0 = 370.4$, $m = 1$, and $B = 1$ for $\gamma^2 \in \{0, 0.1, 0.2, 0.3, 0.5, 0.7, 1\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $n \in \{1, 3, 5, 7\}$.

γ^2	0	0.1	0.2	0.3	0.5	0.7	1
δ	$n = 1$						
0.1	(339.52, 1013)	(342.12, 1021)	(344.32, 1028)	(346.20, 1033)	(349.25, 1042)	(351.62, 1050)	(354.31, 1058)
0.3	(198.43, 591)	(207.70, 618)	(216.03, 643)	(223.55, 666)	(236.60, 705)	(247.52, 738)	(260.93, 778)
0.5	(100.86, 299)	(109.07, 323)	(116.82, 346)	(124.15, 368)	(137.66, 409)	(149.80, 445)	(165.86, 493)
0.7	(53.16, 156)	(58.74, 172)	(64.17, 189)	(69.47, 205)	(79.62, 235)	(89.21, 264)	(102.57, 304)
1	(23.30, 66)	(26.19, 75)	(29.10, 84)	(32.01, 93)	(37.82, 110)	(43.58, 127)	(52.03, 152)
1.5	(8.38, 22)	(9.44, 25)	(10.53, 28)	(11.65, 32)	(13.98, 39)	(16.39, 46)	(20.12, 57)
2	(4.33, 10)	(4.78, 11)	(5.26, 13)	(5.76, 14)	(6.81, 17)	(7.93, 21)	(9.71, 26)
δ	$n = 3$						
0.1	(290.13, 865)	(296.08, 883)	(301.21, 899)	(305.67, 912)	(313.07, 934)	(318.95, 952)	(325.81, 972)
0.3	(94.46, 279)	(102.41, 303)	(109.95, 326)	(117.10, 347)	(130.34, 387)	(153.17, 455)	(158.23, 470)
0.5	(32.98, 95)	(36.86, 107)	(40.71, 119)	(44.53, 130)	(52.03, 152)	(59.32, 174)	(69.82, 206)
0.7	(14.34, 40)	(16.19, 45)	(18.08, 51)	(19.99, 57)	(23.88, 68)	(27.83, 80)	(33.77, 98)
1	(5.93, 15)	(6.63, 17)	(7.36, 19)	(8.12, 21)	(9.71, 26)	(11.37, 31)	(13.98, 39)
1.5	(2.76, 5)	(2.96, 6)	(3.16, 6)	(3.38, 7)	(3.85, 9)	(4.36, 10)	(5.19, 13)
2	(2.14, 3)	(2.19, 3)	(2.26, 3)	(2.33, 3)	(2.50, 4)	(2.68, 5)	(3.01, 6)
δ	$n = 5$						
0.1	(252.34, 752)	(260.12, 775)	(266.95, 796)	(272.98, 814)	(283.15, 844)	(291.40, 869)	(301.21, 899)
0.3	(58.13, 171)	(64.06, 188)	(69.82, 206)	(75.41, 222)	(86.08, 254)	(96.09, 284)	(109.95, 326)
0.5	(17.62, 50)	(19.87, 56)	(22.15, 63)	(24.50, 70)	(29.10, 84)	(33.76, 98)	(40.71, 119)
0.7	(7.55, 20)	(8.49, 22)	(9.46, 25)	(10.46, 28)	(12.54, 34)	(14.71, 41)	(18.08, 51)
1	(3.49, 7)	(3.81, 8)	(4.15, 9)	(4.51, 11)	(5.26, 13)	(6.07, 15)	(7.36, 19)
1.5	(2.17, 3)	(2.24, 3)	(2.31, 3)	(2.40, 4)	(2.59, 5)	(2.80, 5)	(3.16, 1)
2	(2.01, 2)	(2.02, 2)	(2.03, 2)	(2.05, 2)	(2.09, 3)	(2.15, 3)	(2.26, 3)
δ	$n = 7$						
0.1	(222.52, 663)	(231.30, 689)	(239.10, 713)	(246.07, 733)	(258.01, 769)	(267.85, 799)	(279.77, 834)
0.3	(40.35, 117)	(44.89, 131)	(49.38, 144)	(53.79, 158)	(62.38, 183)	(70.63, 208)	(82.34, 243)
0.5	(11.49, 31)	(12.97, 36)	(14.49, 40)	(16.04, 45)	(19.22, 54)	(22.47, 64)	(27.43, 79)
0.7	(5.10, 12)	(5.67, 14)	(6.28, 16)	(6.90, 18)	(8.21, 22)	(9.60, 26)	(11.79, 32)
1	(2.70, 5)	(2.88, 6)	(3.07, 6)	(3.28, 7)	(3.72, 8)	(4.20, 10)	(4.99, 12)
1.5	(2.04, 2)	(2.07, 3)	(2.10, 3)	(2.13, 3)	(2.22, 3)	(2.33, 3)	(2.52, 4)
2	(2.00, 2)	(2.00, 2)	(2.00, 2)	(2.01, 2)	(2.02, 2)	(2.03, 2)	(2.07, 3)

FIGURE 2: Continued.

FIGURE 2: The influence of γ^2 on the run rule Shewhart $\bar{X}$ chart when $m = 1$, $B = 1$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.

TABLE 2: Out-of-control (ARL, $RL_{0.95}$) when $ARL_0 = 370.4$, $m = 1$, and $\gamma^2 = 1$ for $n \in \{1, 3, 5, 7\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $B \in \{1, 2, 3, 4, 5\}$.

B	1	2	3	4	5
δ	$n = 1$				
0.1	(354.31, 1058)	(345.30, 1031)	(342.39, 1022)	(341.20, 1018)	(340.61, 1017)
0.3	(260.93, 778)	(219.88, 655)	(208.67, 621)	(204.34, 608)	(202.26, 602)
0.5	(165.86, 493)	(120.54, 357)	(109.95, 326)	(106.04, 314)	(104.20, 309)
0.7	(102.57, 304)	(66.84, 197)	(59.35, 174)	(56.66, 166)	(55.41, 162)
1.0	(52.03, 152)	(30.55, 88)	(26.51, 76)	(25.10, 72)	(24.45, 70)
1.5	(20.12, 57)	(11.09, 30)	(9.56, 26)	(9.04, 24)	(8.80, 23)
2.0	(9.71, 26)	(5.51, 14)	(4.84, 12)	(4.61, 11)	(4.51, 11)
δ	$n = 3$				
0.1	(325.81, 972)	(303.51, 905)	(296.69, 885)	(293.95, 877)	(292.62, 873)
0.3	(158.23, 470)	(113.57, 337)	(103.27, 306)	(99.48, 294)	(99.69, 289)
0.5	(69.82, 206)	(42.63, 124)	(37.29, 108)	(35.41, 103)	(34.53, 100)
0.7	(33.77, 98)	(19.03, 54)	(16.40, 46)	(15.49, 43)	(15.08, 42)
1.0	(13.98, 39)	(7.74, 20)	(6.71, 17)	(6.37, 16)	(6.21, 16)
1.5	(5.19, 13)	(3.27, 7)	(2.98, 6)	(2.88, 6)	(2.84, 6)
2.0	(3.01, 6)	(2.29, 3)	(2.20, 3)	(2.17, 3)	(2.16, 3)
δ	$n = 5$				
0.1	(301.21, 899)	(270.05, 805)	(260.93, 778)	(257.33, 767)	(255.58, 762)
0.3	(109.95, 326)	(72.63, 214)	(64.70, 190)	(61.85, 182)	(60.52, 178)
0.5	(40.71, 119)	(23.30, 66)	(20.12, 57)	(19.02, 54)	(18.51, 52)
0.7	(18.08, 51)	(9.96, 27)	(8.59, 23)	(8.13, 21)	(7.92, 21)
1.0	(7.36, 19)	(4.33, 10)	(3.85, 9)	(3.69, 8)	(3.62, 8)
1.5	(3.16, 6)	(2.36, 4)	(2.25, 3)	(2.21, 3)	(2.20, 3)
2.0	(2.26, 3)	(2.04, 2)	(2.02, 2)	(2.02, 2)	(2.01, 2)
δ	$n = 7$				
0.1	(279.77, 834)	(242.68, 723)	(232.21, 692)	(228.13, 680)	(226.16, 674)
0.3	(82.34, 243)	(51.59, 151)	(45.39, 133)	(43.19, 126)	(42.17, 123)
0.5	(27.43, 79)	(15.26, 43)	(13.14, 36)	(12.41, 34)	(12.08, 33)
0.7	(11.79, 32)	(6.59, 17)	(5.74, 14)	(5.46, 13)	(5.33, 13)
1.0	(4.99, 12)	(3.17, 6)	(2.90, 6)	(2.81, 5)	(2.77, 5)
1.5	(2.52, 4)	(2.11, 3)	(2.07, 3)	(2.06, 3)	(2.05, 2)
2.0	(2.07, 3)	(2.01, 2)	(2.00, 2)	(2.00, 2)	(2.00, 2)

FIGURE 3: The influence of B on the run rule Shewhart $\bar{X}$ chart when $m = 1$, $\gamma^2 = 1$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.TABLE 3: Out-of-control $(ARL, RL_{0.95})$ when $ARL_0 = 370.4$, $B = 1$, and $\gamma^2 = 1$ for $n \in \{1, 3, 5, 7\}$, $\delta \in \{0.1, 0.3, 0.5, 0.7, 1, 1.5, 2\}$, and $m \in \{1, 2, 3, 4, 5\}$.

m	1	2	3	4	5
δ	$n = 1$				
0.1	(354.31, 1058)	(349.25, 1042)	(346.77, 1035)	(345.30, 1031)	(344.32, 1028)
0.3	(260.93, 778)	(236.60, 705)	(225.90, 673)	(219.88, 655)	(216.03, 643)
0.5	(165.86, 493)	(137.66, 409)	(126.51, 375)	(120.54, 357)	(116.82, 346)
0.7	(102.57, 304)	(79.62, 235)	(71.20, 210)	(66.84, 197)	(64.17, 189)
1.0	(52.03, 152)	(37.82, 110)	(32.98, 95)	(30.55, 88)	(29.10, 84)
1.5	(20.12, 57)	(13.98, 39)	(12.03, 33)	(11.09, 30)	(10.53, 28)
2.0	(9.71, 26)	(6.81, 17)	(5.93, 15)	(5.51, 14)	(5.26, 13)
δ	$n = 3$				
0.1	(325.81, 972)	(313.07, 934)	(307.03, 916)	(303.51, 905)	(301.21, 899)
0.3	(158.23, 470)	(130.34, 387)	(119.40, 354)	(113.57, 337)	(109.95, 326)
0.5	(69.82, 206)	(52.03, 152)	(45.79, 134)	(42.63, 124)	(40.71, 119)
0.7	(33.77, 98)	(23.88, 68)	(20.63, 59)	(19.03, 54)	(18.08, 51)
1.0	(13.98, 39)	(9.71, 26)	(8.38, 22)	(7.74, 20)	(7.36, 19)
1.5	(5.19, 13)	(3.85, 9)	(3.46, 7)	(3.27, 7)	(3.16, 6)
2.0	(3.01, 6)	(2.50, 4)	(2.36, 4)	(2.29, 3)	(2.26, 3)
δ	$n = 5$				
0.1	(301.21, 899)	(283.15, 844)	(274.83, 820)	(270.05, 805)	(266.95, 796)
0.3	(109.95, 326)	(86.08, 254)	(77.23, 228)	(72.63, 214)	(69.82, 206)

TABLE 3: Continued.

m	1	2	3	4	5
0.5	(40.71, 119)	(29.10, 84)	(25.22, 72)	(23.30, 66)	(22.15, 63)
0.7	(18.08, 51)	(12.54, 34)	(10.80, 29)	(9.96, 27)	(9.46, 25)
1.0	(7.36, 19)	(5.26, 13)	(4.63, 11)	(4.33, 10)	(4.15, 9)
1.5	(3.16, 6)	(2.59, 5)	(2.43, 4)	(2.36, 4)	(2.31, 3)
2.0	(2.26, 3)	(2.09, 7)	(2.05, 3)	(2.04, 2)	(2.03, 2)
δ	$n = 7$				
0.1	(279.77, 834)	(258.01, 769)	(248.23, 740)	(242.68, 723)	(239.10, 713)
0.3	(82.34, 243)	(62.38, 183)	(55.24, 162)	(51.59, 151)	(49.38, 144)
0.5	(27.43, 79)	(19.22, 54)	(16.57, 46)	(15.26, 43)	(14.49, 40)
0.7	(11.79, 32)	(8.21, 22)	(7.11, 18)	(6.59, 17)	(6.28, 16)
1.0	(4.99, 12)	(3.72, 8)	(3.35, 7)	(3.17, 6)	(3.07, 6)
1.5	(2.52, 4)	(2.22, 3)	(2.15, 3)	(2.11, 3)	(2.10, 3)
2.0	(2.07, 3)	(2.02, 2)	(2.01, 2)	(2.01, 2)	(2.00, 2)

FIGURE 4: The effect of m on the run rule Shewhart $\bar{X}$ chart when $B = 1$, $\gamma^2 = 1$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.

figure, it is seen that increasing D can significantly increase the values of ARL and $RL_{0.95}$. This fact indicates the negative effect of linearly increasing variance on the run rule $\bar{X}$ chart.

For instance, if $n = 3$ and $\delta = 0.5$, when $D = 1$ increases up to $D = 3$, $ARL = 74.80$ and $RL_{0.95} = 221$ increase up to $ARL = 104.61$ and $RL_{0.95} = 310$, respectively.

FIGURE 5: The effect of γ^2 on the run rule Shewhart $\bar{X}$ chart when $m = 5$, $B = 1$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.

FIGURE 6: Continued.

FIGURE 6: The effect of B on the run rule Shewhart $\bar{X}$ chart when $m = 5$, $\gamma^2 = 1$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.

FIGURE 7: The effect of D on the run rule Shewhart $\bar{X}$ chart when $C = 5$, $m = 1$, $\mu_0 = 10$, $\sigma_0^2 = 10$, $\delta \in \{0.5, 1.5\}$, and $n \in \{1, 3, 5, 7\}$.

TABLE 4: 20 samples corresponding to a sequence of production.

i	$X_{i,1,k}$		$X_{i,2,k}$		$X_{i,3,k}$		$X_{i,4,k}$		$X_{i,5,k}$		$\bar{X}_i$
1	125.24	125.35	125.12	125.15	123.38	123.13	124.75	125.38	126.18	126.50	125.02
2	124.79	124.93	124.42	124.61	123.90	124.21	123.46	123.68	123.89	124.63	124.25
3	124.96	124.95	124.40	124.54	124.93	124.94	125.54	125.59	124.96	124.80	124.96
4	125.55	124.96	125.76	125.44	124.88	124.53	125.85	126.35	123.76	124.20	125.13
5	124.92	125.01	123.86	124.27	124.16	124.29	124.77	125.29	125.93	125.52	124.80
6	124.95	125.27	125.92	125.95	124.23	124.15	124.90	125.61	124.50	124.28	124.98
7	124.16	124.41	122.80	123.15	125.76	124.96	126.34	126.21	125.04	125.06	124.79
8	125.73	126.04	125.08	124.87	123.98	124.28	124.29	124.18	124.28	124.55	124.73
9	125.75	125.63	124.58	124.41	125.38	125.07	123.71	123.78	124.86	124.73	124.79
10	123.33	123.48	124.22	124.49	125.13	125.06	126.51	126.10	124.99	125.16	124.85
11	124.29	124.86	124.67	125.03	124.78	125.32	122.70	122.39	123.22	122.95	124.02
12	123.86	123.63	123.34	123.84	122.87	123.04	123.73	123.93	123.87	123.83	123.59
13	125.32	125.12	123.68	123.48	123.41	122.97	124.77	124.00	123.75	123.73	124.02
14	124.02	124.43	124.05	124.43	122.86	122.94	124.65	124.55	124.16	124.09	124.02
15	123.93	123.85	123.93	123.47	124.47	124.96	124.45	124.61	122.82	122.56	123.90
16	125.83	125.87	123.89	124.09	124.40	124.15	124.55	124.35	123.44	123.14	124.37
17	125.33	124.97	124.57	124.37	123.97	124.29	124.90	125.24	124.38	124.16	124.62
18	123.02	123.10	124.50	124.45	124.96	125.15	125.66	125.23	124.88	124.34	124.53
19	122.93	123.35	125.18	124.79	124.12	124.15	124.18	124.03	125.04	125.08	124.28
20	124.59	124.64	121.48	121.21	122.63	123.08	123.68	123.42	123.79	123.21	123.17

FIGURE 8: $\bar{X}$ chart with 2-of-3 run rules corresponding to the data in Table 4.

5. An Illustrative Example

Similar to the research work in Costa and Castagliola [21], a yogurt cup filling process is used here to illustrate the implementation of the $\bar{X}$ chart integrated with run rules when measurement error exists. The critical quality characteristic in the filling process is the weight Y of each yogurt cup. From the historical database of yogurt cup weights (Phase I), the in-control mean $\mu_0 = 124.9$ and the standard deviation $\sigma_0 = 0.76$ are estimated accurately. Moreover, an independent R&R (repeatability and reproducibility) study has showed an estimated measurement standard deviation $\sigma_M = 0.24$, which leads to $\gamma = 0.24/0.76 = 0.316$. By fixing the parameter settings $A = 0$ and $B = 1$ in the linearly covariate error model, the quality practitioner decides to use Shewhart $\bar{X}$ chart integrated with 2-of-3 run rules to

monitor the process. Taking $n = 5$ yogurt cups and measuring each item $m = 2$ times, in a sample, the parameter W of the run rule $\bar{X}$ chart can be calculated as $W = 1.9293$ using equation (20). Then, the warning limits in equations (7) and (8) are given as follows:

$$\begin{aligned} \text{LWL} &= 124.9 - 1.9293 \times \sqrt{\frac{1}{5} \left(0.76^2 + \frac{0.24^2}{2} \right)} = 124.23, \\ \text{UWL} &= 124.9 + 1.9293 \times \sqrt{\frac{1}{5} \left(0.76^2 + \frac{0.24^2}{2} \right)} = 125.57. \end{aligned} \quad (21)$$

In addition, 20 samples corresponding to a sequence of production (Phase II) are recorded in Table 4. We have 10 values for each sample, i.e., the weight of $n = 5$ yogurt cups weighed $m = 2$ times. The mean $\bar{X}_i$ of these samples is presented in the right side of Table 4. In Figure 8, these mean values $\bar{X}_i$ with the warning limits $\text{LWL} = 124.23$ and $\text{UWL} = 125.57$ are also plotted. As it can be seen in the figure, the Shewhart $\bar{X}$ chart with 2-of-3 run rules actually triggers an out-of-control signal from samples 11 and 12 (shown in the frame), suggesting a downward shift in the process mean. One possible reason is that the pipe used for filling the cups is clogged.

6. Conclusion

The properties of Shewhart $\bar{X}$ chart integrated with run rules are investigated when errors exist in the measurement of observations in a quality characteristic. By using a linearly covariate error model, $(\text{ARL}, \text{RL}_{0.95})$ of the chart is derived using a Markov chain approach. The simulations results show that $(\text{ARL}, \text{RL}_{0.95})$ of the run rule Shewhart $\bar{X}$ chart is seriously affected by the measurement errors. With the measurement error increasing, the run rule Shewhart $\bar{X}$

chart's performance deteriorates. To reduce the negative effect of measurement errors, it is beneficial to measure each item in each subgroup more times and increase the coefficient B in the linearly covariate error model whenever possible. Both are good alternatives to implement the run rule Shewhart $\bar{X}$ chart. It should be careful that measuring each item more times practically causes more cost.

All the results are based on the covariate error model, and it would be interesting to extend our research to different control charts with some other measurement error models.

Data Availability

All relevant data are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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