Research Article

Effects of Second-Order Slip Flow and Variable Viscosity on Natural Convection Flow of (CNTs – Fe₃O₄)/Water Hybrid Nanofluids due to Stretching Surface

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This study deals with natural convection unsteady flow of CNTs – Fe₃O₄/water hybrid nanofluids due to stretching surface embedded in a porous medium. Both hybrid nanoparticles of SWCNTs – Fe₃O₄ and MWCNTs – Fe₃O₄ are used with water as base fluid. Effects of hybrid nanoparticles volume fraction, second-order velocity slip condition, and temperature-dependent viscosity are investigated. The governing problem of flow is solved numerically employing spectral quasilinearization method (SQLM). The results are presented and discussed via embedded parameters using graphs and tables. The results disclose that the thermal conductivity of (CNTs – Fe₃O₄)/H₂O hybrid nanofluids is higher than that of CNTs – H₂O nanofluids with higher value of hybrid nanoparticle volume fraction. Also, the results show that momentum boundary layer reduces while the thermal boundary layer grows with higher values of temperature-dependent viscosity and second-order velocity slip parameters. The skin friction coefficient improves, and the local heat transfer rate decreases with higher values of nanoparticle volume fraction, temperature-dependent viscosity, and second-order velocity slip parameters. Furthermore, more skin friction coefficients and lower local heat transfer rate are reported in the CNTs – Fe₃O₄/H₂O hybrid nanofluid than in the CNTs – H₂O nanofluid. Thus, the obtained results are promising for the application of hybrid nanofluids in the nanotechnology and biomedicine sectors.

1. Introduction

The broad applications of heat transfer in various sectors of industry and biomedicine have required the accessibility of efficient thermal performance techniques. In the past few decades, several techniques of enhancing the thermal performance of working fluids have been realized by different researchers. Scattering of nanoparticles of metallic structures such as copper, carbides, alumina, nitrides, metal oxides, carbon nanotubes, and graphite in the working fluid is considered to be one of the innovative and efficient methods (Mahanthesh et al. [1]). At present, nanofluids are the noble options for the heat transfer fluids due to their remarkably higher thermal conductivity, and their use is common in heat exchangers, cooling systems, solar energy, biomedicine, and so forth (Aziz et al. [2]). Besides, the porous media have better dissipation area, which results in improved convective

heat transfer. As a result, Reddy and Sreedevi [3] analyzed heat and mass transfer characteristics of nanofluid flow over porous stretching sheet. The non-Newtonian Casson nanofluid flow and heat transfer over stretching cylinder in a porous medium were investigated by Tulu and Ibrahim [4]. Furthermore, the effect of temperature and concentration on the thermal conductivity of ZnO-TiO₂/EG hybrid nanofluid using artificial neural network and curve fitting on experimental data was evaluated by Safaei et al. [5].

At present, heat transfer of carbon nanofluids (CNTs) has received great attention due to their potential applications in the fields of nanotechnology and biomedicine. CNTs are allotropes of carbon prepared in cylindrical tubes of graphite with nanometer in diameter and a few millimeters in length (Hirlekar et al. [6]). CNTs are generally divided into single-wall carbon nanotubes (SWCNTs) and multiwall carbon nanotubes (MWCNTs) depending on their number

of concentric layers of rolled graphene sheets. SWCNTs contain catalyst for their synthesis and encompass a particular graphene sheet encircled around and form a cylinder. MWCNTs are concentrically nested cylinders of graphene sheets and they can be formed without catalyst. The inner and outer diameters of MWCNTs are determined by their number of layers (Khalid et al. [7]). CNTs are the resourceful material of modern technologies because of their special thermal, electrical, and mechanical characteristics, unique morphology, and innovative physicochemical features, and the presence of carbon chains in CNTs does not convey any hazard to the atmosphere (Tulu and Ibrahim [8] and Alsagri et al. [9]).

Modern applications of carbon nanotubes in the biomedical field for drug delivery, cancer therapy, and other applications in medicines have attracted the attention of different researchers in studying the thermophysical properties and nanofluids flow of CNTs. For example, Alnaqi et al. [10] predicted the effect of functionalised MWCNTs on thermal performance factor of water under some Reynolds numbers using artificial neural network. Furthermore, CNTs have encouraging vigorous surface area that permits them to be functionalised and ornamented by numerous kinds of chemical and biological matters. Due to their needle-like shape, CNTs have potential to infiltrate more easily into cell walls and move among various bodies' tissues (Cai et al. [11]). Also, they have the potential to carry drugs in the organism and have the ability to target and destroy specific cancer cells without harming the healthy cells (Srinivasan [12]). Moreover, in the long run, carbon nanotubes with friendly enzymes can be used as enzymatic biosensors that might concurrently sense and measure a multiplicity of biological molecules (Singh et al. [13]).

Carbon nanotubes, however, do not have ample magnetic properties which limit their power for uses in biomedical applications. Furthermore, the weak solubility of CNTs in aqueous solutions and challenging of CNTs in added solvents limit their applications in the biomedical and other technological fields (Yang et al. [14]). Consequently, to overcome the above mentioned limitations of CNTs, researchers have considered different approaches of CNTs nanoparticles preparations (Asadi et al. [15], Asadi et al. [16], Bagherzadeh et al. [17], and Saba et al. [18] to mention few). Among these, magnetic delivery of CNTs is shown to be the best promising approach due to its ability to use a located magnet inside the body or an external magnet in several diagnostic and therapeutic agents at targeted tissues (Samadishadlou et al. [19]). Also, due to the nonhostile behavior of magnetic fields, employing magnetic CNTs in biomedical applications reduces their side effects to adjacent tissues (Mody et al. [20]). In view of these, the hybrid nanoparticles with CNTs that mostly received attention in the biotechnological and biomedical fields are CNTs with magnetite (Fe_3O_4) (Rahmawati et al. [21]). This is due to the fact that Fe_3O_4 exhibits more chemical inertness and stability, but it has lower thermal conductivity than CNTs. Also, CNTs- Fe_3O_4 nanoparticles advance targeting efficiency and enable magnetically engaged delivery. Besides, the magnetite nanocrystals inside the CNTs advance their drug loading

ability, preserve them from agglomeration, and improve their chemical stability (Masotti and Caporali [22]).

Recently, much effort has been given to developing new experimental work to obtain well-defined and highly stable hybrid nanoparticles which enhance the thermal conductivity of base fluid. For example, Tassaddiq et al. [23] studied the transfer of heat and mass over a rotating disk involving carbon nanotubes (CNTs) and magnetic ferrite nanoparticles together with carrier fluids. Also, the heat transfer and friction factor of MWCNT- Fe_3O_4 /water hybrid nanofluids were investigated by Sundar et al. [24]. They reported that these hybrid nanofluids have significantly improved thermal and mechanical properties than regular fluids or mono nanofluids. Hybrid nanofluids have important applications in various engineering and biomedical fields such as in manufacturing, transportation, nuclear safety, military, modern electronic devices and supercomputers, and pharmaceutical and drug delivery (Manjunatha et al. [25]). Due to the mentioned applications, many scholars have been interested in hybrid nanofluids investigation. For example, using the polymer technique, Shi et al. [26] synthesized CNT/ Fe_3O_4 hybrid nanoparticles. The impacts of mixing MgO-MWCNT hybrid nanoparticles in thermal oil were analyzed by Asadi et al. [27]. Recently, Zaresharif et al. [28] have undertaken hydrothermal analysis on natural convection and TiO_2 - SiO_2 /W-EG hybrid nanofluids properties. The free convection heat transfer and entropy generation analysis of water- Fe_3O_4 /CNT hybrid nanofluid were reported by Shahsavar et al. [29]. Also, Sundar et al. [30] experimentally synthesized MWCNT/ Fe_3O_4 water-based hybrid nanofluids. They reported that hybrid nanofluids create higher thermal conductivity and heat transfer compared to single nanoparticles-based nanofluids. Similarly, they observed that the viscosity of (MWCNT- Fe_3O_4)/water hybrid nanofluids significantly improved as compared to base fluid. Further, they suggested that more analyses and experiments are needed to fully realize the means of improving heat transfer of hybrid nanofluids.

Various nanofluids flow studies have considered with constant physical properties of fluid. However, the viscosity of nanofluids may change significantly with temperature and play an indispensable role in nanofluids flow. For example, the heat produced via internal friction increases the temperature, which consecutively affects the stickiness of nanofluids. Thus, to precisely evaluate the flow nature of nanofluids, it is essential to consider this disparity of viscosity with temperature. Kuttan et al. [31] and Manjunatha and Giressha [32] analyzed the flow of fluid over a flat surface with the effects of temperature-dependent viscosity. Also, Udawattha et al. [33] predicted the effective viscosity of nanofluids based on the rheology of suspensions of solid particles. So far, there is a lack of information on CNTs- Fe_3O_4 /H₂O hybrid nanofluids flow considering the effects of temperature-dependent viscosity.

There are situations where no-slip boundary condition is not suitable. For example, for different non-Newtonian fluids, various polymer melts usually express small wall slip; and, normally, they are controlled by a monotonic relation and a nonlinear relationship between the slip velocity and

the adhesive friction (Halim et al. [34]). Fluids offering slip boundary condition have important applications in some technological and biomedical fields, for instance, in expensive lubricating, optical coatings, refrigeration equipment, purifying of artificial heart valve, internal cavities, and other industrial processes. As a result, the effect of slip boundary condition has been considered by some researchers [35]. For example, Oyelakin et al. [36] and Tlili et al. [37] examined the effects of first-order slip boundary condition on the flow and heat transfer of nanofluid over stretching sheet and cylinder, respectively. Recently, the effects of second-order slip condition (Wu's slip features) on nanofluids flow and heat transfer were reported by Khan et al. [38]. They revealed that temperature distribution and thermal boundary layer grow with bigger values of second-order velocity slip conditions. Yet, there is limited information about the effect of second-order slip condition on CNTs – Fe₃O₄/H₂O hybrid nanofluids flow over stretching surface. Thus, one of the aims of this study is to fill the gaps in the above indicated knowledge.

Motivated by the above cited literature survey, the purpose of this study is to analyze CNTs – Fe₃O₄/H₂O hybrid nanofluids flow and heat transfer due to stretching surface. The impacts of temperature-dependent viscosity, free convection, and second-order slip condition are also considered. The governing equations are solved numerically employing the fast convergent and accurate technique, namely, spectral quasilinearization method (SQLM). The effects of governing parameters on hybrid nanofluids flow velocity and temperature distributions are discussed and presented in tables and graphs as well. To the best of our knowledge, no analysis has been published in this direction

yet. The developed model has potential applications in the biomedical fields such as cancer therapy, drug delivery, and enzymatic biosensors. Considering the effects of temperature-dependent viscosity and second-order slip condition on CNTs – Fe₃O₄/H₂O hybrid nanofluids flow and heat transfer and computing it by means of SQLM make this study novel and different from the former studies.

2. Mathematical Description of Problem

We consider an unsteady two-dimensional incompressible viscous flow of CNTs – Fe₃O₄ with H₂O hybrid nanofluids past semi-infinite linearly stretching surface embedded in a porous medium. The flow is situated to $y > 0$, where x and y are, respectively, in the direction of flow and normal to the surface, as shown in Figure 1. Also, we considered the base fluid and the hybrid nanoparticles to be in thermal equilibrium so that no slip occurs between them. It is worth mentioning that, to develop the targeted hybrid nanofluids (CNTs – Fe₃O₄)/H₂O, initially, CNTs are dispersed into base fluid and then Fe₃O₄ is scattered in CNTs/H₂O nanofluids. In the beginning, at time $t = 0$, both the surface and hybrid nanofluids are at rest with uniform temperature $\bar{T}_s$. For time $t > 0$, the surface begins stretching linearly with velocity $\bar{u}_w = (\bar{u}_0/(1 - \zeta t))x$, where $\bar{u}_0$ is constant ($\bar{u}_0 > 0$ for a stretching and $\bar{u}_0 < 0$ for a shrinking surface), $\zeta \geq 0$ is constant, t is time, and $\zeta t < 1$. At the same time, the surface temperature is upturned to $\bar{T}_b$, which is then kept constant.

With foregoing assumptions and Boussinesq approximation, the transport equations of hybrid nanofluid boundary layer flow are established as follows (Manjunatha et al. [25]):

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0, \quad (1)$$

$$\rho_{\text{hnf}} \left(\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = \mu_{\text{hnf}} \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial \bar{u}}{\partial y} \frac{\partial}{\partial y} (\mu_{\text{hnf}}) - \frac{\mu_{\text{hnf}}}{K} \bar{u} + g(\rho\beta_t)_{\text{hnf}} (\bar{T} - \bar{T}_s), \quad (2)$$

$$(\rho c_p)_{\text{hnf}} \left(\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} \right) = k_{\text{hnf}} \frac{\partial^2 \bar{T}}{\partial y^2} + Q_0 (\bar{T} - \bar{T}_s). \quad (3)$$

The boundary conditions with second-order velocity slip and thermal slip are given as follows (Manjunatha et al. [25] and Ibrahim [39]):

$$t \geq 0: \bar{u}(x, t) = \bar{u}_w + \bar{u}_{\text{slip}}, \quad \bar{v} = 0, \bar{T} = \bar{T}_b + N_0 \frac{\partial \bar{T}}{\partial y} \text{ at } y = 0,$$

$$t \geq 0: \bar{u}(x, t) \longrightarrow 0, \quad \bar{v} \longrightarrow 0, \bar{T} \longrightarrow \bar{T}_s \text{ as } y \longrightarrow \infty, \quad (4)$$

and initial condition is

$$t < 0: \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_s, \forall x, y, \quad (5)$$

where $\bar{u}$ and $\bar{v}$ are the hybrid nanofluid velocity components in the x and y orientations, respectively; ρ_{hnf} , μ_{hnf} , k_{hnf} , and $(\rho c_p)_{\text{hnf}}$ are hybrid nanofluid effective density, dynamic viscosity, thermal conductivity, and specific heat capacity, respectively; g , β_{hnf} , Q_0 , and N_0 are gravitational acceleration, hybrid nanofluid volume expansion coefficient, heat generation/absorption rate, and thermal slip factor, respectively.

The temperature-dependent viscosity of the base fluid is given as follows (Manjunatha et al. [25]):

$$\mu_f = \mu_0 e^{-R\theta(\xi)}, \quad (6)$$

FIGURE 1: Physical model of the flow problem.

where μ_0 is reference viscosity of base fluid and R is variable viscosity parameter (note that $R < 0$ for gases and $R > 0$ for liquids).

The slip velocity $\tilde{u}_{\text{slip}}$ at the surface, where Wu's [40] slip velocity equation is usable for random Knudsen number, Kn , and is employed by researcher Ibrahim [39], is specified as

$$\tilde{u}_{\text{slip}} = a \frac{\partial \tilde{u}}{\partial y} + b \frac{\partial^2 \tilde{u}}{\partial y^2}, \quad (7)$$

where a and b are constants given as

$$a = \frac{2}{3} \left[\frac{3 - \alpha r^2}{\alpha} - \frac{3}{2} \frac{1 - r^2}{K_n} \right] \lambda, \quad (8)$$

$$b = -\frac{1}{4} \left[r^4 + \frac{2}{K_n^2} (1 - r^2) \right] \lambda^2,$$

where K_n is Knudsen number, $r = \min[1/K_n, 1]$, λ is the molecular mean free path, and α is the momentum accommodation coefficient with $0 \leq \alpha \leq 1$.

The values of thermophysical properties of nanoparticles CNTs, Fe_3O_4 , and base fluid are given in Table 1 (Tulu and Ibrahim [8], Alsagri et al. [9], Sundar et al. [24], and Hayat et al. [41]).

The hybrid nanofluid effective thermophysical properties models are given in Table 2 (Alsagri et al. [9] and Manjunatha et al. [25]). ϕ_{CNTs} is the CNTs nanoparticle volume fraction, and ϕ_{FO} is the Fe_3O_4 nanoparticle volume fraction. ρ_f , $(\rho c_p)_f$, μ_f , and $(\rho \beta_t)_f$ are, respectively, density, specific heat capacity, dynamic viscosity, and volumetric thermal expansion coefficient of the base fluid. ρ_{CNTs} , $(\rho c_p)_{\text{CNTs}}$, k_{CNTs} , and $(\rho \beta_t)_{\text{CNTs}}$; ρ_{FO} , $(\rho c_p)_{\text{FO}}$, k_{FO} , and $(\rho \beta_t)_{\text{FO}}$ are, respectively, density, specific heat capacity, thermal conductivity, and volumetric thermal expansion coefficient of CNTs and Fe_3O_4 .

The continuity equation (1) is adequately fulfilled introducing the following dimensionless variables based on Oyelakin et al. [36] and Manjunatha et al. [25]:

$$\xi = \sqrt{\frac{\tilde{u}_0}{\nu_f (1 - \zeta t)}} y;$$

$$\tilde{u} = \frac{\tilde{u}_0}{1 - \zeta t} x f'(\xi); \quad (9)$$

$$\tilde{v} = -\sqrt{\frac{\tilde{u}_0 \nu_f}{1 - \zeta t}} f(\xi);$$

$$\theta(\xi) = \frac{\tilde{T} - \tilde{T}_s}{\tilde{T}_b - \tilde{T}_s},$$

where $f'(\xi)$ is a dimensionless velocity and $\theta(\xi)$ is the dimensionless temperature.

Consequently, the transformed equations of momentum and energy are specified as follows.

$$f'''(\xi) + \phi_1 \phi_2 \left[f(\xi) f''(\xi) - A \left(f'(\xi) + \frac{\xi}{2} f''(\xi) \right) - f'^2(\xi) + \phi_3 \tau \theta(\xi) \right] - R \theta'(\xi) f''(\xi) - \kappa f'(\xi) = 0, \quad (10)$$

$$\frac{k_{\text{hnf}}}{k_f} \theta''(\xi) + \phi_4 \text{Pr} \left[f(\xi) \theta'(\xi) - A \left(2\theta(\xi) + \frac{\xi}{2} \theta'(\xi) \right) \right] + \text{Pr} H \theta(\xi) = 0, \quad (11)$$

with transformed boundary conditions:

$$f = 0, f'(0) = 1 + \gamma f''(0) + \delta f'''(0); \theta(0) = 1 + \beta \theta'(0), \quad \xi \geq 0, f'(\infty) \rightarrow 0; \theta(\infty) \rightarrow 0, \xi \geq 0. \quad (12)$$

where

TABLE 1: Thermophysical properties values of base fluid, CNTs, and Fe_3O_4 .

Physical properties	H_2O	SWCNT	MWCNT	Fe_3O_4
ρ (kg/m ³)	997.1	2600	1600	5180
c_p (J/kgK)	4179	425	796	670
k (W/mK)	0.613	6600	3000	9.7
σ (S/m)	0.05	10^6	1.9×10^{-4}	25000
β (10^{-5}K^{-1})	21	27	44	1.05

$$\begin{aligned}
\phi_1 &= (1 - \phi_{\text{FO}}) \left\{ 1 - \phi_{\text{CNTs}} + \phi_{\text{CNTs}} \left(\frac{\rho_{\text{CNTs}}}{\rho_f} \right) \right\} + \phi_{\text{FO}} \left(\frac{\rho_{\text{FO}}}{\rho_f} \right), \\
\phi_2 &= (1 - \phi_{\text{CNTs}})^{2.5} (1 - \phi_{\text{FO}})^{2.5}, \\
\phi_3 &= (1 - \phi_{\text{FO}}) \left\{ 1 - \phi_{\text{CNTs}} + \phi_{\text{CNTs}} \left(\frac{(\beta_t)_{\text{CNTs}}}{(\beta_t)_f} \right) \right\} + \phi_{\text{FO}} \left(\frac{(\beta_t)_{\text{FO}}}{(\beta_t)_f} \right), \\
\phi_4 &= (1 - \phi_{\text{FO}}) \left\{ 1 - \phi_{\text{CNTs}} + \phi_{\text{CNTs}} \left(\frac{(\rho c_p)_{\text{CNTs}}}{(\rho c_p)_f} \right) \right\} + \phi_{\text{FO}} \left(\frac{(\rho c_p)_{\text{FO}}}{(\rho c_p)_f} \right),
\end{aligned} \tag{13}$$

where $A = (\zeta/u_0)$ is unsteady parameter, $\kappa = (\nu_f(1 - \zeta t)/K\tilde{u}_0)$ is permeability parameter, $\text{Pr} = (\nu_f(\rho c_p)_f/k_f)$ is a Prandtl number, $H = ((1 - \zeta t)Q_0/\tilde{u}_0(\rho c_p)_f)$ is heat generation/absorption rate parameter, $\tau = (g\beta_t(\tilde{T}_b - \tilde{T}_s)/x\tilde{u}_0^2)$ is Grashof number, $\gamma = a\sqrt{\tilde{u}_0}/\nu_f$ is the first-order velocity slip parameter, $\delta = b(\tilde{u}_0/\nu_f)$ is the second-order velocity slip parameter, and $\beta = N_0\sqrt{\tilde{u}_0}/\nu_f$ is the thermal slip parameter.

The shearing stress at the wall is defined for hybrid nanofluid as

$$\tau_w = \mu_{\text{hnf}} \left(\frac{\partial \tilde{u}}{\partial y} \right)_{y=0} = \frac{\tilde{u}_0 \mu_f}{(1 - \phi_{\text{CNTs}})^{2.5} (1 - \phi_{\text{FO}})^{2.5}} \sqrt{\frac{\tilde{u}_0}{\nu_f}} x f''(0). \tag{14}$$

The surface heat flux at the wall is given for hybrid nanofluid as

$$q_w = -k_{\text{hnf}} \left(\frac{\partial \tilde{T}}{\partial y} \right)_{y=0} = -k_{\text{hnf}} \sqrt{\frac{\tilde{u}_0}{\nu_f}} (\tilde{T}_b - \tilde{T}_s) \theta'(0). \tag{15}$$

The surface drag and wall heat transfer rates are characterized by the skin friction coefficient C_f and the local Nusselt number Nu , respectively, and they are obtained as

$$\begin{aligned}
C_{fx} &= \frac{\tau_w}{\rho_f \tilde{u}_0^2}, \\
\text{Nu}_x &= \frac{x q_w}{k_f (\tilde{T}_b - \tilde{T}_s)}.
\end{aligned} \tag{16}$$

Thus, the dimensionless skin friction coefficient and local Nusselt number are given by

$$(\text{Re}_x)^{1/2} C_{fx} = \frac{1}{(1 - \phi_{\text{CNTs}})^{2.5} (1 - \phi_{\text{FO}})^{2.5}} f''(0), \tag{17}$$

$$(\text{Re}_x)^{-1/2} \text{Nu}_x = -\frac{k_{\text{hnf}}}{k_f} \theta'(0), \tag{18}$$

where Re_x is the local Reynolds number defined by $\text{Re}_x = (\tilde{u}_0 x / \nu_f)$.

3. Method of Solutions

In this section, the nonlinear differential equations (10) and (11) are solved numerically by SQLM. For complete explanation of the method, the interested person is referred to Motsa [42] and Ibrahim and Tulu [43]. Using this method, the system of nonlinear differential equations in two unknowns f and θ gives the following iterative scheme of linear differential equations:

TABLE 2: Thermophysical models of hybrid nanofluid.

Thermophysical properties	Hybrid nanofluid (CNTs – Fe ₃ O ₄)/H ₂ O
Density (kg/m ³)	$\rho_{\text{hnf}} = (1 - \phi_{\text{FO}})[1 - \phi_{\text{CNTs}}\rho_f + \phi_{\text{CNTs}}\rho_{\text{CNTs}}] + \phi_{\text{FO}}\rho_{\text{FO}}$
Heat capacity (J/kgK)	$(\rho c_p)_{\text{hnf}} = (1 - \phi_{\text{FO}})[(1 - \phi_{\text{CNTs}})(\rho c_p)_f + \phi_{\text{CNTs}}(\rho c_p)_{\text{CNTs}}] + \phi_{\text{FO}}(\rho c_p)_{\text{FO}}$
Viscosity (Ns/m ²)	$\mu_{\text{hnf}} = (\mu_f / ((1 - \phi_{\text{CNTs}})^{2.5} (1 - \phi_{\text{FO}})))$
Thermal conductivity (W/K)	$(k_{\text{hnf}}/k_{nf}) = (1 - \phi_{\text{FO}} + 2\phi_{\text{FO}}(k_{\text{FO}}/(k_{\text{FO}} - k_{\text{nf}}))\ln((k_{\text{FO}} + k_{\text{nf}})/2k_{\text{nf}}))/(1 - \phi_{\text{FO}} + 2\phi_{\text{FO}}(k_{\text{nf}}/(k_{\text{FO}} - k_{\text{nf}}))\ln((k_{\text{FO}} + k_{\text{nf}})/2k_{\text{nf}}))$
Thermal expa. coeff. (K ⁻¹)	where $(k_{\text{nf}}/k_f) = (1 - \phi_{\text{CNTs}} + 2\phi_{\text{CNTs}}(k_{\text{CNTs}}/(k_{\text{CNTs}} - k_f))\ln((k_{\text{CNTs}} + k_f)/2k_f))/(1 - \phi_{\text{CNTs}} + 2\phi_{\text{CNTs}}(k_f/(k_{\text{CNTs}} - k_f))\ln((k_{\text{CNTs}} + k_f)/2k_f))$ $(\beta_t)_{\text{hnf}} = (1 - \phi_{\text{FO}})[(1 - \phi_{\text{CNTs}})(\beta_t)_f + \phi_{\text{CNTs}}(\beta_t)_{\text{CNTs}}] + \phi_{\text{FO}}(\beta_t)_{\text{FO}}$

$$\begin{aligned}
f''_{j+1} + \left(\phi_1 \phi_2 \left(f_j - \frac{A}{2} \xi \right) - R \theta'_j \right) f''_{j+1} - \left(\phi_1 \phi_2 (2f'_j + A) + \kappa \right) f'_{j+1} + \phi_1 \phi_2 f''_j f_{j+1} + \phi_1 \phi_2 \phi_3 \tau \theta_{j+1} - R f''_j \theta'_{j+1} \\
= \phi_1 \phi_2 \left(f_j f''_j - (f'_j)^2 \right) - R \theta'_j f''_j,
\end{aligned} \quad (19)$$

$$\frac{k_{\text{hnf}}}{k_f} \theta''_{j+1} + \phi_4 \Pr \left(f_j - \frac{A}{2} \xi \right) \theta'_{j+1} + \Pr (H - 2A\phi_4) \theta_{j+1} + \phi_4 \Pr \theta'_j f_{j+1} = \phi_4 \Pr f_j \theta'_j, \quad (20)$$

with boundary conditions

$$\begin{aligned}
f_{j+1} = 0, f'_{j+1}(0) = 1 + \gamma f''_{j+1}(0) + \delta f''_{j+1}(0) \theta_{j+1}(0) \\
= 1 + \beta \theta'_{j+1}(0), f'_{j+1}(\infty) \rightarrow 0; \theta_{j+1}(\infty) \rightarrow 0,
\end{aligned} \quad (21)$$

where the terms j and $j+1$ are at the previous and current iteration levels, respectively.

Equations (19) and (20) establish the iterative scheme for the SQLM. A numerical solution is then found employing the Chebyshev spectral collocation method. Beginning with appropriate initial approximations, the iteration schemes are used to obtain f_{j+1} and θ_{j+1} .

First, it is essential to change the semi-infinite domain to a truncated domain $[0, L_\infty]$. Then, the interval $[0, L_\infty]$ is transformed to the interval $[-1, 1]$ using the linear transformation $\xi = (1/2)L_\infty(\chi + 1)$. Further, the Gauss-Lobatto points are selected to define the nodes in $[-1, 1]$ as reported in Trefethen [44]. That is,

$$\chi_i = \cos\left(\frac{\pi i}{N}\right), \quad i = 0, 1, 2, \dots, N; -1 \leq \chi \leq 1, \quad (22)$$

where N is the number of collocation points used.

Using the Chebyshev spectral collocation method, equations (19) and (20) are discretized. The derivatives of the unknown functions f and θ at the collocation points are

defined using the Chebyshev differentiation matrix $\mathbf{D}$ as given in Trefethen's work [44]. That is,

$$\frac{df}{d\xi} = \sum_{p=0}^N \mathbf{D}_{pk} f(\chi_p) = \mathbf{D}_F, \quad p = 1, 2, 3, \dots, N, \quad (23)$$

where $\mathbf{D} = (2D/L_\infty)$ and $\mathbf{F} = [f_0, f_1, \dots, f_N]^T$ is a vector function at the collocation points. Higher-order derivatives are found as powers of $\mathbf{D}$ as

$$\mathbf{F}^n(\xi) = \mathbf{D}^n \mathbf{F}, \quad (24)$$

where n is the order of derivative and $\mathbf{D}$ is matrix of size $(N+1) \times (N+1)$.

Applying the spectral method to the system of equations (19) and (20), it can be solved as a coupled matrix:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{j+1} \\ \boldsymbol{\Theta}_{j+1} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}, \quad (25)$$

with transformed boundary condition

$$\begin{aligned}
\mathbf{F}_{j+1}(\chi_N) = 0, \mathbf{F}_{j+1}(\chi_{N-1}) = 1 + \gamma f''_{j+1}(0) + \delta f''_{j+1}(0), \\
\mathbf{F}_{j+1}(\chi_0) = 0, \\
\boldsymbol{\Theta}_{j+1}(\chi_N) = 1 + \beta \theta'_{j+1}(0), \boldsymbol{\Theta}_{j+1}(\chi_0) = 0,
\end{aligned} \quad (26)$$

where

$$\begin{aligned}
\Psi_{11} &= \mathbf{D}^3 + \left[\phi_1 \phi_2 \left(f_j - \frac{A}{2} \xi \right) - R \theta'_j \right]_d \mathbf{D}^2 - \left[\phi_1 \phi_2 (2f'_j + A) + \kappa \right]_d \mathbf{D} + \phi_1 \phi_2 \left[f''_j \right]_d, \\
\Psi_{12} &= \phi_1 \phi_2 \phi_3 \tau \mathbf{I} - R \left[f''_j \right]_d \mathbf{D}, \\
\Psi_{21} &= \phi_4 \Pr \left[\theta'_j \right]_d, \\
\Psi_{22} &= \frac{k_{\text{hnf}}}{k_f} \mathbf{D}^2 + \phi_4 \Pr \left[f_j - \frac{A}{2} \xi \right]_d \mathbf{D} + \Pr (H - 2A\phi_4) \mathbf{I}, \\
\mathbf{b}_1 &= \phi_1 \phi_2 \left[f_j f''_j - (f'_j)^2 \right]_d - R \theta'_j f''_j, \\
\mathbf{b}_2 &= \phi_4 \Pr f_j \theta'_j, \\
\mathbf{F}_{j+1} &= [f_{j+1,0}, f_{j+1,1}, \dots, f_{j+1,N}]^T, \\
\boldsymbol{\Theta}_{j+1} &= [\theta_{j+1,0}, \theta_{j+1,1}, \dots, \theta_{j+1,N}]^T,
\end{aligned} \quad (27)$$

are vectors of sizes $(N + 1) \times 1$; $\mathbf{I}$ and $[\dots]_d$, respectively, represent an identity and a diagonal matrix of size $(N + 1) \times (N + 1)$.

The stability and convergence of the iteration schemes for velocity and temperature distributions can be evaluated by considering the norm of their differences between two successive iterations. Hence, for each iteration scheme, we can define the maximum error (E_d) for velocity (f') and temperature (θ) at the $(r + 1)^{\text{th}}$ iteration as follows (Motsa [42]):

$$\begin{aligned} E_d &= \text{Max} \|f'_{r+1} - f'_r\|_{\infty}, \\ E_d &= \text{Max} \|\theta_{r+1} - \theta_r\|_{\infty}. \end{aligned} \quad (28)$$

If the iteration scheme converges, the error (E_d) is expected to reduce with an increase in the number of iterations. In this study, the maximum error of the unknown functions f' and θ can be calculated for a given number of collocation points N until the criteria for the convergence tolerance set are fulfilled at iteration r .

When SQLM is employed, the choice of initial guesses is crucial. Thus, the proper initial guesses that fulfill the governing equations of the considered flow problem are

$$\begin{aligned} f_0(\xi) &= \frac{1 - e^{-\xi}}{1 + \gamma - \delta}, \\ \theta_0(\xi) &= \frac{e^{-\xi}}{1 + \beta}. \end{aligned} \quad (29)$$

4. Results and Discussion

Natural convection flow of $(\text{CNTs} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluids due to stretching surface in non-Darcy porous medium with the effects of second-order slip and variable viscosity is considered. For the study, it is assumed that 1% (0.01 volume) of CNTs, 4% (0.04 volume) of Fe_3O_4 , and 95% (0.95 volume) of H_2O are in use for the preparation of $(\text{CNTs} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluid. The thermophysical properties of Fe_3O_4 , SWCNTs, MWCNTs, and water in use are from Table 1. To obtain convergent, stable, and accurate results, the suitable intervals for the parameters used in this study are fixed to

$$\begin{aligned} 0 &< \phi_{\text{CNTs}} < 0.05, \\ 0 &< \phi_{\text{FO}} < 0.1, \\ 0 &< A < 5, \\ 0.0 &< \delta < 0.2, \\ 0 &< \gamma < 3, \\ 0 &< \kappa < 15, \\ 0 &< R < 3, \\ 0 &< \tau < 0.1, \\ 0 &< \beta < 2, \\ -3 &< H < 1.5. \end{aligned} \quad (30)$$

The convergence analysis was done for the skin friction coefficient and local Nusselt number, and almost the

5th-order iteration is enough up to eight digits of approximations for both $f''(0)$ and $-\theta'(0)$, as is shown in Table 3. Also, the accuracy of the employed method is checked using the grid-invariance test choosing mesh with nodes $N = 10, 20, 50$, and 100 for skin friction coefficients and local Nusselt number, as shown in Table 4. Once increasing the number of nodes to more than 50 , the accuracy is not affected up to five decimal-point but only to increase the compilation time. Thus, all the results of this study are obtained with number of nodal points $N = 50$. Besides, the accuracy of the current results is also confirmed comparing the values of $f''(0)$ and $-\theta'(0)$ with formerly available literatures for comparable parameters, as shown in Tables 5 and 6, respectively, and the results are in a very sound agreement.

Variation in fluids thermal conductivity with nanoparticles volume fraction at room temperature is presented in Table 7. The thermal conductivity of both SWCNT – H_2O and MWCNT – H_2O nanofluids improves for bigger values of nanoparticle volume fraction; and it is almost doubled when 5 percent of nanoparticles is added to the base fluid. Also, the thermal conductivity of both $(\text{SWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ and $(\text{MWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluids is better for increasing values of hybrid nanoparticle volume fraction; and it improves nearly by 100 percent when 1 percent of CNTs nanoparticles and 3 percent of Fe_3O_4 nanoparticles are added to the base fluid. These observations are comparable with experimental results of Sundar et al. [30].

The impacts of important physical parameters on velocity $f'(\xi)$ and temperature $\theta(\xi)$ profiles are plotted for both $(\text{SWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ and $(\text{MWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluids when the values of the parameters in use for the entire results to be fixed are $\phi_{\text{CNTs}} = 0.01$, $\phi_{\text{FO}} = 0.04$, $A = 0.8$, $\delta = 0.02$, $\gamma = 0.2$, $\tau = 0.02$, $\kappa = 5$, $\text{Pr} = 6.2$, $\beta = 0.3$, $R = 1$, and $H = -0.5$, unless it is specified. Figures 2 and 3, respectively, represent that the greater value of nanoparticle volume fraction (ϕ_{FO}) leads to increasing both the velocity and temperature profiles of hybrid nanofluids. Due to higher density of SWCNTs compared to MWCNTs, also, it is recognized that there is higher velocity distribution in the $(\text{MWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ than in the $(\text{SWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluids, whereas the opposite trend is perceived for temperature distribution. Thus, physically, $(\text{SWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluid is more efficient than $(\text{MWCNT} - \text{Fe}_3\text{O}_4)/\text{H}_2\text{O}$ hybrid nanofluid for practical applications. The influences of unsteady parameter A on both velocity and temperature profiles of hybrid nanofluids flow are, respectively, reported in Figures 4 and 5. As A increases, near the boundary surface, the velocity profiles rise to its highest value; then it changes downward and finally it reduces to zero. Also, it is revealed that an increase in A leads to dropping the temperature distribution of both hybrid nanofluids, as illustrated in Figure 5.

The effect of variable viscosity parameter R on velocity profiles of hybrid nanofluids flow is shown in Figure 6. Except adjacent to the boundary surface, the velocity profiles show a decreasing tendency as R increases. Also, the thickness of the thermal boundary layer enhances for bigger

TABLE 3: SQLM Convergence for $f''(0)$ and $-\theta'(0)$ of (SWCNTs – Fe₃O₄)/H₂O and (MWCNTs – Fe₃O₄)/H₂O hybrid nanofluids for $N = 50, \xi = 3, \phi_{\text{CNTs}} = 0.01, \phi_{\text{FO}} = 0.04, A = 0.8, \delta = 0.02, \gamma = 0.2, \tau = 0.02, \kappa = 5, \text{Pr} = 6.2, \beta = 0.3, R = 1 \& H = -0.5$.

Order	$f''(0)$		$-\theta'(0)$	
	SWCNTs	MWCNTs	SWCNTs	MWCNTs
1	1.59530902	3.51499284	3.04244662	3.10363049
2	2.07858472	4.43967660	3.05824443	3.13574512
3	2.07869396	4.43950900	3.05824443	3.13575827
4	2.07869395	4.43950871	3.05824443	3.13575823
5	2.07869395	4.43950871	3.05824443	3.13575823
6	2.07869395	4.43950871	3.05824443	3.13575823

TABLE 4: Grid-independence test for (MWCNT – Fe₃O₄)/H₂O hybrid nanofluid when $\xi = 3$ and the values of parameters $\phi_{\text{CNTs}} = 0.01, \phi_{\text{FO}} = 0.04, A = 0.8, \delta = 0.02, \gamma = 0.2, \tau = 0.02, \kappa = 5, \text{Pr} = 6.2, \beta = 0.3, R = 1 \& H = -0.5$ are used.

Order	$f''(0)$				$-\theta'(0)$			
	$N = 10$	$N = 20$	$N = 50$	100	$N = 10$	$N = 20$	$N = 50$	$N = 100$
1	3.25546	3.51499	3.51499	3.51499	3.10307	3.10363	3.10363	3.10363
2	2.95939	4.43126	4.43968	4.43968	3.13865	3.13599	3.13574	3.13574
3	2.96332	4.43229	4.43951	4.43951	2.62121	3.13597	3.13576	3.13576
4	2.96332	4.43229	4.43951	4.43951	2.62121	3.13597	3.13576	3.13576
5	2.96332	4.43229	4.43951	4.43951	2.62121	3.13597	3.13576	3.13576

TABLE 5: Comparison of the present result of $f''(0)$ with γ when $\phi_{\text{CNTs}} = \phi_{\text{FO}} = A = \delta = k = R = \tau = 0$.

γ	Sahoo and Do [45]	Noghrehabadi et al. [46]	Oyelakin et al. [36]	Present result
0.0	1.001154	1.000000	1.000000	1.000000
0.1	0.871447	0.872082	0.872083	0.872083
0.2	0.774933	0.776377	0.776377	0.776377
0.3	0.699738	0.701548	0.701548	0.701548
0.5	0.589195	0.591196	0.591195	0.591196
1.0	0.428450	0.430160	0.430160	0.430160
2.0	0.282893	0.283980	0.283979	0.283981
5.0	0.144430	0.144841	0.144714	0.144842
10.0	0.081091	0.081243	0.080932	0.081245
20.0	0.043748	0.043790	0.043569	0.043792

TABLE 6: Comparison of the present result of $-\theta'(0)$ with Pr when $\phi_{\text{CNTs}} = \phi_{\text{FO}} = A = H = R = \beta = \delta = \gamma = \kappa = \tau = 0$.

Pr	Wang [47]	Khan and Pop [48]	Manjunatha et al. [25]	Present result
0.70	0.4539	0.4539	—	0.453932
2.00	0.9114	0.9113	0.91135	0.91136
7.00	1.8954	1.8954	1.89540	1.89540
20.00	3.3539	3.3539	3.35390	3.35390
70.00	6.4622	6.4621	—	6.46220

TABLE 7: Thermal conductivity (k_{nf}) of CNTs – H₂O nanofluid and k_{hnf} of (CNTs – Fe₃O₄)/H₂O hybrid nanofluid.

k_{nf} (CNTs – H ₂ O)			k_{hnf} (CNTs – Fe ₃ O ₄)/H ₂ O			
ϕ_{CNTs}	SWCNTs	MWCNTs	ϕ_{CNTs}	ϕ_{FO}	SWCNTs	MWCNTs
0.00	0.613000	0.613000	0.01	0.00	0.719390	0.719390
0.01	0.719390	0.709626	0.01	0.01	1.039870	1.040127
0.02	0.827947	0.808217	—	0.02	1.080291	1.080812
0.03	0.938739	0.908834	—	0.03	1.121277	1.122066
0.04	1.051836	1.011541	—	0.04	1.162837	1.163902
0.05	1.167309	1.116402	—	0.05	1.204986	1.206332

values of R , which increases the temperature profile. As well, the same pattern of velocity and temperature distribution in hybrid nanofluids flow was observed by Tulu and Ibrahim [8] and Manjunatha et al. [25]. The impacts of Grashof

number (natural convection parameter) τ on velocity and temperature profiles are presented in Figures 7 and 8, respectively. The increment in τ tends to enhance the fluid velocity profiles and reaches a maximum; then it gradually

FIGURE 2: Velocity profiles for various values of ϕ_{FO} . FIGURE 3: Temperature profiles for some values of ϕ_{FO} .

falls to zero as it is far from the surface. Also, it is observed that the thermal boundary layer thickness gradually reduces with bigger value of τ . Physically, free convection flows are steadily transferred from the stretching surface to the free stream, and increase in τ indicates the progress of free convection currents. Similar result of flow distribution was found by Manjunatha et al. [25]. The effect of permeability parameter κ on velocity field of hybrid nanofluids flow is demonstrated in Figure 9. Here, except near the wall surface, the greater value of κ tends to diminish the velocity profile and the thickness of momentum boundary layer while for the temperature field the opposite behavior is also reported.

Figures 10 and 11 show the variation of second-order velocity slip parameter δ on velocity and temperature profiles of both SWCNTs and MWCNTs hybrid nanofluids. Adjacent to the wall of the surface, the velocity profiles show an increasing tendency. However, at a distance from the wall, it reduces with bigger value of δ . Physically, second-order slip parameter δ increases the resistance of fluid

FIGURE 4: Velocity profiles for some values of A . FIGURE 5: Temperature profiles for some values of A .

motion, which decreases the fluid flow field and momentum boundary layer thickness. Furthermore, improved value of δ leads to improving the temperature profile and thermal boundary layer thickness. Therefore, the occurrence of second-order slip condition in the hybrid nanofluids flow plays an important role in the nanotechnology. Figures 12 and 13, respectively, illustrate the variation of temperature field with respect to thermal slip parameter β and Prandtl number Pr . The temperature distributions of both SWCNTs and MWCNTs hybrid nanofluids fall with added value of β , and more influence is noticed at the surface, as demonstrated in Figure 12. Also, it is illustrated that the temperature profile and thermal boundary layer thickness decrease

FIGURE 6: Velocity profiles for some values of R . FIGURE 8: Temperature profiles for some values of τ . FIGURE 7: Velocity profiles for some values of τ . FIGURE 9: Velocity profiles for some values of κ .

significantly as the value of Prandtl number increases. Physically, the thermal diffusivity of the fluid tends to decline and results in weak diffusion of heat inside the fluid with bigger values of Prandtl number.

The computations of the physical quantities of interest such as the skin friction coefficient $f''(0)$ and local heat transfer rate $-\theta'(0)$ are, respectively, given in Tables 8 and 9 with some embedded parameters for (SWCNTs – Fe_3O_4)/ H_2O and (MWCNTs – Fe_3O_4)/ H_2O hybrid nanofluids. For both hybrid nanofluids, the skin friction coefficient tends to enhance with bigger values of nanoparticle volume fractions ϕ_{CNT} and ϕ_{FO} , Grashof number, and heat absorption parameters. However, the reverse effect occurs for bigger values of Prandtl number, permeability, first-order velocity slip, and thermal slip parameters, as demonstrated in Table 8. Also, for both hybrid nanofluids, the local heat transfer rate shows an increasing

tendency for bigger values of Grashof number, first-order velocity slip parameter, and Prandtl number, as shown in Table 9. Furthermore, the local heat transfer rate tends to reduce for greater values of nanoparticle volume fractions ϕ_{CNT} and ϕ_{FO} , permeability, thermal slip, and heat absorption parameters. Also, a similar result was found in the work of Manjunatha et al. [25]. Physically, the obtained results show that the presence of nanoparticle volume fractions and heat absorption parameters plays an important role in the industrial application of hybrid nanofluids, since the skin friction coefficient tends to enhance, while the local heat transfer rate tends to reduce with bigger values of these parameters.

Furthermore, under unsteady condition, the skin friction coefficient $f''(0)$ and local heat transfer rate $-\theta'(0)$ are illustrated for some parameters in Figures 14–17. The skin

FIGURE 10: Velocity profiles for some values of δ . FIGURE 11: Temperature profiles for some values of δ . FIGURE 12: Temperature profiles for some values of β .

FIGURE 13: Temperature profiles for some of Pr.

TABLE 8: Values of the skin friction coefficient $f''(0)$ of (SWCNTs – Fe₃O₄)/H₂O and (MWCNTs – Fe₃O₄)/H₂O hybrid nanofluids with $N = 50, \xi = 3, A = 0.8, R = 1$ and $\delta = 0.02$ for some embedded physical parameters.

ϕ_{FO}	ϕ_{CNT}	τ	γ	β	κ	H	Pr	SWCNTs	MWCNTs
0.00	0.01	0.02	0.2	0.3	5	-0.5	6.2	1.30265962	3.64339012
0.05								2.26293393	4.61901142
0.10								3.11381860	5.39880158
0.04	0.00							-2.01625523	-2.01625523
	0.01							2.07869395	4.43950871
	0.05							16.41605245	25.84049598
	0.01	0.01						-0.22699785	0.99682266
		0.02						2.07869395	4.43950871
		0.03						4.31748004	7.77084880
		0.02	0.1					2.96255405	6.36885452
			0.2					2.07869395	4.43950871
			0.5					1.08431539	2.27375084
			0.2	0.1				2.40143735	4.95656016
				0.3				2.07869395	4.43950871
				0.5				1.80208058	3.99693631
				0.3	1.0			3.35221262	5.88928374
					5.0			2.07869395	4.43950871
					10.0			1.06304033	3.26839186
					5.0	-5.0		2.07869395	4.43950871
						0.0		2.19475702	4.54461187
						0.5		2.32779645	4.66293515
						-0.5	1.0	3.29783062	5.90702828
							5.0	2.23394557	4.61513735
							10.0	1.73092303	4.06568197

TABLE 9: Values of the local heat transfer $-\theta'(0)$ of (SWCNTs – Fe₃O₄)/H₂O and (MWCNTs – Fe₃O₄)/H₂O hybrid nanofluids with $N = 50, \xi = 3, A = 0.8, R = 1$ and $\delta = 0.02$ for some embedded physical parameters.

ϕ_{FO}	ϕ_{CNT}	τ	γ	β	κ	H	Pr	SWCNTs	MWCNTs
0.00	0.01	0.02	0.2	0.3	5	-0.5	6.2	3.31327594	3.39384115
0.05								3.00074209	3.07725701
0.10								2.74196706	2.81257438
0.04	0.0							3.18247302	3.18247302
	0.01							3.05824943	3.13575823
	0.05							2.71319744	2.89738905
	0.01	0.01						2.99454873	3.04947403
		0.02						3.05824943	3.13575823
		0.03						3.11300552	3.20659355

TABLE 9: Continued.

ϕ_{FO}	ϕ_{CNT}	τ	γ	β	κ	H	Pr	SWCNTs	MWCNTs
		0.02	0.1					3.01799024	3.05269983
			0.2					3.05824943	3.13575823
			0.5					3.10247914	3.22497320
			0.2	0.1				3.26177264	3.34620296
				0.3				3.05824943	3.13575823
				0.5				2.87839652	2.94981629
				0.3	1.0			3.12905932	3.21542468
					5.0			3.05824943	3.13575823
					10.0			3.00984189	3.07764667
					5.0	-5.0		3.05824943	3.13575823
						0.0		2.75330136	2.84337884
						0.5		2.41675689	2.52484894
						-0.5	1.0	1.28303699	1.34185923
							5.0	2.76068035	2.83828676
							10.0	3.84090315	3.91355315

FIGURE 14: Skin friction coefficient C_f for some values of A and R . FIGURE 15: Skin friction coefficient C_f for some values of A and δ .

FIGURE 16: Local heat transfer rate Nu for some values of A and R.

FIGURE 17: Local heat transfer rate Nu for some values of A and δ .

friction coefficient reduces, while the local heat transfer rate increases with bigger values of unsteadiness parameter A . The skin friction coefficient improves with bigger values of both thermal dependent viscosity R and second-order velocity slip parameter δ , as demonstrated in Figures 14 and 15, respectively. Also, the local heat transfer rate reduces gradually with greater value of both thermal dependent viscosity R and second-order velocity slip parameter δ , as reported in Figures 16 and 17, respectively. Besides, there are superior skin friction coefficient and lower local heat transfer rate in the MWCNTs – $\text{Fe}_3\text{O}_4/\text{H}_2\text{O}$ hybrid nanofluid than in the MWCNTs – H_2O mono nanofluid. In general, these findings show us that the presence of both thermal dependent viscosity and second-order velocity slip condition in the nanofluids flow adds a significant role in the nanotechnology and biomedical uses as the skin friction

coefficient tends to improve, while the local heat transfer rate tends to decline with bigger values of these parameters.

5. Conclusion

In this work, unsteady natural convection flow of (CNTs – Fe_3O_4)/ H_2O hybrid nanofluids due to stretching surface embedded in a porous medium with the effects of second-order slip flow and temperature-dependent viscosity is examined. The governing problems of flow are solved numerically using SQLM. The main results of the present study can be summarized as follows:

The thermal conductivity of (CNTs – Fe_3O_4)/ H_2O hybrid nanofluids improves more than CNTs – H_2O nanofluids with increasing values of hybrid nanoparticle volume fraction; and the progress is nearly by 100 percent when 1 percent of CNTs nanoparticles and 3 percent of Fe_3O_4 nanoparticles are added to the base fluid.

Both velocity and temperature profiles of (CNTs – Fe_3O_4)/ H_2O hybrid nanofluids flow enhance with bigger value of nanoparticle volume fraction. Moreover, there is lower velocity distribution in the (SWCNT – Fe_3O_4)/ H_2O than in the (MWCNT – Fe_3O_4)/ H_2O hybrid nanofluids, while the opposite trend is reported in the temperature distribution. Thus, physically, (SWCNT – Fe_3O_4)/ H_2O hybrid nanofluid is more efficient than (MWCNT – Fe_3O_4)/ H_2O hybrid nanofluid in engineering applications.

The momentum boundary layer reduces, while the thermal boundary layer grows with bigger values of temperature-dependent viscosity and second-order velocity slip parameters. Therefore, the occurrence of second-order slip condition and temperature-dependent viscosity in the hybrid nanofluids flow plays an important role in the nanotechnology and biomedicine sectors.

The skin friction coefficient improves with bigger values of nanoparticle volume fraction, Grashof number, temperature-dependent viscosity, and second-order velocity slip parameters. Besides, there are superior skin friction coefficient and lower local heat transfer rate in the CNTs – $\text{Fe}_3\text{O}_4/\text{H}_2\text{O}$ hybrid nanofluids than in the CNTs – H_2O nanofluids. These results show that the hybrid nanofluids have a more important role than working fluids or mono nanofluids in industrial uses.

In general, the obtained results confirm that CNTs – $\text{Fe}_3\text{O}_4/\text{H}_2\text{O}$ hybrid nanofluids have better flow distributions with good stability of thermal properties than CNTs – H_2O nanofluids, and the former can be more recommended for the treatment of cancer therapy and drug delivery in the biomedical sectors.

Nomenclature

- A : Unsteady parameter
- C_{fx} : Local skin friction coefficient
- $(\bar{C}_p)$: Specific heat capacity of the material
- D : Chebyshev differentiation matrix

f :	Dimensionless stream function
g :	Acceleration due to gravity
H :	Heat generation/absorption parameter
I :	Identity matrix
K :	Permeability of porous medium
k :	Thermal conductivity of the material
K_n :	Knudsen number
N :	Number of collocation points
N_0 :	Thermal slip factor
Nu_x :	Local Nusselt number
Pr :	Prandtl number
Q_0 :	Heat generation/absorption rate
q_w :	Wall heat flux
R :	Variable viscosity parameter
Re_x :	Local Reynolds number
$\tilde{T}_b$:	Hybrid nanofluid temp. at the wall
$\tilde{T}_s$:	Ambient/surrounding temperature
$\tilde{u}$:	Hybrid nanofluid velocity in x direction
$\tilde{u}_{slip}$:	Hybrid nanofluid velocity slip
$\tilde{u}_w$:	Stretching/shrinking surface velocity
$\tilde{v}$:	Hybrid nanofluid velocity in y direction.

Greek Symbols

α :	Momentum accommodation coefficient
β :	Thermal slip parameter
β_t :	Volumetric thermal expansion coefficient
γ :	First-order velocity slip parameter
δ :	Second-order velocity slip parameter
ζ :	Dimension reciprocal time
θ :	Dimensionless temperature
κ :	Permeability parameter
λ :	Molecular mean free path
μ :	Dynamic viscosity of the material
ξ :	Dimensionless similarity variable
ρ :	Density of the material
τ :	Grashof number/natural convection parameter
τ_w :	Wall shear stress
ϕ :	Nanoparticle volume fraction
χ_i :	Chebyshev points.

Subscripts

f :	Base fluid
nf :	Nanofluids
hnf :	Hybrid nanofluids
CNT :	Carbon nanotubes
FO :	Fe_3O_4 nanoparticle
s :	Surrounding/ambient condition
w :	Surface condition.

Data Availability

The data used to support the findings of this study are available from the authors upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest concerning the publication of this article.

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