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Research Article

Robust Stabilization of Controlled Positive Switched Linear Systems by Output Feedback

Jinjin Liu and Ting Zhang

Department of Mathematics and Information Science, Henan University of Economics and Law, Zhengzhou 450046, China

Correspondence should be addressed to Jinjin Liu; jjliu2020@126.com

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This study focuses on the controller synthesis issues for constrained switched linear systems with uncertainties under mode-dependent average dwell time (MDADT) switching strategy. First, output feedback controllers ensure that the closed-loop systems are positive and asymptotically stable. Second, the bounded controllers are acquired based on system states with interval and polytopic uncertainties. Also, the proposed approach can be applied to the systems with the constrained output. Then, the presented conditions can be formulated in terms of linear programming. Finally, illustrative example is provided to show the effectiveness of the theoretical results.

1. Introduction

A switched system consists of a family of subsystems and a law determining which subsystem is active during a certain time interval [1, 2]. In real world, the associated states are always nonnegative, such as the population size of species, level of liquid, and concentration of substances. Such models are described by positive systems [3, 4]. As a special switched system, the positive switched system has not only the properties of the switched system but also the nature of the positive system. The positive switched linear system receives much attention in recent years due to the widely practical applications, such as congestion control in communication networks [5], medical treatment [6], general anesthesia [7, 8], multi-agent system [9], and so on.

Many researchers have been concerned with the controller design problem, especially the constrained controller synthesis for the general system and positive system [10–12]. In fact, it is inevitable that the control system encounters limited situations, for example, maximum current or highest voltage in the circuits, the maximum temperature in thermal systems, and so on. Therefore, it is of great practical significance to design a controller which makes the control signal and state of the closed-loop system be bounded by the specified boundary. The study of bounded control for the positive system starts from [13, 14], where stability and stabilization conditions for the positive system by means of linear composite Lyapunov function are proposed under the constrained state feedback controller. Then, the results are extended to the positive system with delay in [15, 16], Takagi–Sugeno fuzzy system [17], fractional order system [18], and event-triggered system [19]. Meanwhile, stability and stabilization issues of positive switched systems are developing rapidly. Reference [20] considered the stability analysis problem for a class of positive switched systems with average dwell time switching; then, [21] continued this idea and further studied this issue under MDADT switching, that is, each subsystem has its own average dwell time. Reference [22] tried to adapt the constrained control problem to both continuous and discrete-time positive switched systems with delays, focusing on the nonnegative controller design under a prescribed upper bound with the arbitrary switching signal. [23, 24] investigated the controller synthesis for a class of discrete-time switched linear systems with bounds on the controls and the states, which guarantees positivity and stability of the closed-loop system. However, most of these results are based on the state feedback.

On the other hand, output variables are easy to directly measure and technically implement and also have a clear physical meaning in most cases, so the output feedback is a common form of the feedback mode. Based on linear programming, [25] solved the stabilization issue by the output feedback controller having one rank gains. On the basis of the singular value decomposition approach, [26] revisited the output feedback stabilization problem with and without interval uncertainties. By using the system augmentation approach, an iterative linear matrix inequality algorithm is presented to compute the feedback gain matrix [27]. Turning to positive switched systems, [28] continued the linear programming method and addressed feedback control under system input matrix. Reference [29] involved controller synthesis via a constrained output feedback, but the rank of the controller gain is one. Furthermore, it is well known that because of modeling errors and variations of plant parameters, there always exists uncertainty due to some unpredictable factors in many real systems, and some effective results have been achieved in this area [30–32].

In this context, we consider robust stabilization of enforced positive switched linear systems by the output feedback with and without boundary under the MDADT switching scheme. The so-called controlled positive systems mean that the resulting closed-loop systems we received is positive, even if the open-loop system is not positive at all. In order to overcome the incomplete controller gain, we start from the elements view of the system matrix. By introducing quantity matrices, sufficient conditions for the existence of the output feedback controller are formulated, such that the closed-loop system can not only guarantee robust asymptotic stability but also satisfy positive. Simultaneously, inputs and outputs are limited by the prescribed boundary provided that the initial condition is within the same boundary.

The rest of study is structured as follows. Section 2 deals with problem statement and presents some preliminaries. Robust stabilization of controlled positive switched linear systems interval uncertainties and polytopic uncertainties are solved in in Sections 3 and 4, respectively. Section 5 gives a numerical example to reveal our design. Finally, Section 6 concludes this study.

1.1. Notations. $\mathbb{R}^n$ denotes the vector of n-tuples of real numbers, and $\mathbb{R}^{m\times n}$ means the space of $m \times n$ matrices with real entries. For a vector $v \in \mathbb{R}^n$ and a matrix $A \in \mathbb{R}^{m\times n}$, $v \succ 0$ ($v \succeq 0$) and $A \succ 0$ ($A \succeq 0$) imply that all its elements are positive (nonnegative), respectively. Furthermore, $v \preceq 0$ ($v \preceq 0$) and $A \preceq 0$ ($A \preceq 0$) stand for all its elements are negative (not positive), respectively. For two matrices A, $B \in \mathbb{R}^{m\times n}$, a_{ij} and b_{ij} are the elements in the ith row and jth column of A and B for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. So, $A \succ B$ ($A \succeq B$) means that $a_{ij} > b_{ij}$ ($a_{ij} \geq b_{ij}$), and $A \preceq B$ ($A \preceq B$) denotes $a_{ij} < b_{ij}$ ($a_{ij} \leq b_{ij}$) for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, respectively.

2. Problem Statement and Preliminaries

In this section, we present precise problem description for switched systems under consideration. Consider the following switched linear systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) = C_{\sigma(t)}x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^s$ denote the state vector, control input, and output, respectively. The switching signal $\sigma(t)$: $[0, \infty) \longrightarrow \mathbb{P} = \{1, 2, \ldots, N\}$ is a piecewise constant and right continuous function of time t, with N being the number of modes of the overall switched system. $\sigma(t) = p \in \mathbb{P}$ implies that the pth subsystem is active, and $A_p = [a_{pij}] \in \mathbb{R}^{n\times n}$, $B_p = [b_{pij}] \in \mathbb{R}^{n\times m}$, and $C_p = [c_{pij}] \in \mathbb{R}^{s\times n}$ are the system matrices.

The main problem we studied in this article is the following: under the premise of system matrices are not precisely known, the output feedback must be set up in such a way that the resulting closed-loop system is positive and asymptotically stable. In addition, the control law we designed should be not restricted in sign and limited within certain boundary. To be specific, consider the output feedback controller $u(t) = G_p y(t)$, leading to the corresponding closed-loop system.

$$\dot{x}(t) = A_{cp}x(t) \tag{2}$$

is positive and asymptotically stable under MDADT switching signal, while the open-loop system may not be positive. Here, $A_{cp} = A_p + B_p G_p C_p$, $\forall p \in \mathbb{P}$.

Before presenting the main results of this study, the following definitions and lemmas need to list.

Definition 1 (see [22]). The continuous-time linear system

$$\dot{x}(t) = A_p x(t) \tag{3}$$

is said to be positive if for any switching signal and any initial condition $x_0 \succeq 0$, and the corresponding trajectory of system $x(t) \succeq 0$ holds for all $t \geq 0$.

Definition 2 (see [4]). Matrix A is called a Metzler matrix if the off-diagonal entries of A are nonnegative.

Definition 3 (see [21]). For a switching signal $\sigma(t)$ and any $t_2 > t_1 \geq 0$, let $N_{\sigma p}(t_1, t_2)$ be the switching number that the p^{th} subsystem is activated in time interval $[t_1, t_2]$ and $T_p(t_1, t_2)$ denotes the total running of the p^{th} subsystem in time interval $[t_1, t_2]$. If there exists a nonnegative constant N_{0p}, such that

$$N_{\sigma p}(t_1, t_2) \leq N_{0p} + \frac{T_p(t_1, t_2)}{\tau_{\alpha p}}, \tag{4}$$

then $\tau_{\alpha p}$ is a MDADT of $\sigma(t)$, and N_{0p} is the mode-dependent chattering bound.

Lemma 1 (see [22]). *System (3) is positive if and only if A_p is a Metzler matrix for each $p \in \mathbb{P}$.*

Here, it is worth mentioning that the closed-loop system (2) is positive only if system matrix A_{cp} is Metzler for each $p \in \mathbb{P}$.

Lemma 2 (see [21]). *If there exist vector $\lambda_p \succ 0$ and constant $\gamma_p > 0$, such that $(A_p + \gamma_p I)\lambda_p \preceq 0$, a SPLS $\dot{x}(t) = A_p x(t)$ is asymptotically stable under the MDADT switching signal $\tau_{\alpha p} > \tau^*_{\alpha p} = (\ln \mu_p/\gamma_p)$, where $\mu_p = \max(\lambda_p/\lambda_q)$ for $(p, q) \in \mathbb{P} \times \mathbb{P}$.*

Lemma 3 (see [22]). *Consider a SPLS $\dot{x}(t) = A_p x(t)$; if there exist vector $\lambda_p \succ 0$ and constant $\gamma > 0$, such that $(A_p + \gamma_p I)\lambda_p \preceq 0$ sets up for $0 \preceq x_0 \preceq \lambda_p$, then the state trajectory of SPLS satisfies that $0 \preceq x(t) \preceq \lambda_p$ for $p \in \mathbb{P}$.*

3. Robust Stabilization with Interval Uncertainties

In this section, we consider the controller synthesis issue if switched systems with interval uncertainties. Here, the subsystem matrices A_p and B_p are not precisely known but belong to the following interval uncertainty domain: $\underline{A}_p \preceq A_p \preceq \overline{A}_p$ and $\underline{B}_p \preceq B_p \preceq \overline{B}_p$. In addition, we divide this section into three cases.

3.1. Sign-Restricted Controls. First of all, a fundamental result for robust stabilization of switched linear systems under MDADT switching strategy with nonnegative or negative controls is now put forward. Let us begin by the condition $u(t) \succcurlyeq 0$.

Theorem 1. *Consider the switched linear system (1) with interval uncertainties. For prescribed constants $\gamma_p > 0$, if there exist diagonal matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ with positive diagonal entries and matrices $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ subjected to*

$$\underline{a}_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} \underline{b}_{pik} z_{pkl} c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \tag{5}$$

$$\sum_{j=1}^{n} \overline{a}_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{s} \sum_{k=1}^{m} \overline{b}_{pik} z_{pkl} c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \tag{6}$$

$$\sum_{l=1}^{s} z_{pil} c_{plj} \geq 0, \quad \forall 1 \leq i, \leq m, 1 \leq j, \leq n, \tag{7}$$

then the closed-loop system (2) with the output feedback gain $G_p = Z_p D_p^{-1}$ is positive and asymptotically stable under MDADT switching strategy $\tau_{\alpha p} > \tau^*_{\alpha p} = (\ln \mu_p/\gamma_p)$, where $\mu_p = \max(d_p/d_q)$, $\forall (p, q) \in \mathbb{P} \times \mathbb{P}$. Furthermore, $u(t) \succcurlyeq 0$ holds for initial condition $0 \preceq x_0 \preceq d_{vp}$, where $d_{vp} = [d_p, d_p, \ldots, d_p]^T \in \mathbb{R}^n$.

Proof. Using the output feedback gain matrix, it follows that

$$a_{cpij} d_p = a_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} b_{pik} g_{pkl} c_{plj} d_p, \quad \forall 1 \leq i, j \leq n. \tag{8}$$

Note the fact that $z_{pkj} = g_{pkj} d_p$, and it is easy to give

$$\begin{aligned} a_{cpij} d_p &= a_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} b_{pik} z_{pkl} c_{plj} \\ &\geq \underline{a}_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} \underline{b}_{pik} z_{pkl} c_{plj}. \end{aligned} \tag{9}$$

Using the fact (5) and recalling the condition $d_p > 0$, one can derive $a_{cpij} \geq 0$ for $1 \leq i \neq j \leq n$. In consequence, A_{cp} is the Metzler matrix and the closed-loop (2) system is positive by Lemma 1.

With the vector d_{vp} and matrix operation rule in mind, we have

$$\begin{aligned} \left[A_{cp} d_{vp}\right]_i &= \sum_{j=1}^{n} \left[\left(a_{pij} + \sum_{l=1}^{s} \sum_{k=1}^{m} b_{pik} g_{pkl} c_{plj}\right) d_p\right] \\ &= \sum_{j=1}^{n} a_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{n} \sum_{k=1}^{m} b_{pik} z_{pkl} c_{plj}, \\ &\leq \sum_{j=1}^{n} \overline{a}_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{n} \sum_{k=1}^{m} \overline{b}_{pik} z_{pkl} c_{plj}. \end{aligned} \tag{10}$$

By (6), we can get $(A_{cp} + \gamma_p I) d_{vp} = (A_p + B_p G_p C_p + \gamma_p I) d_{vp} \preceq 0$, that is, the closed-loop system (2) is asymptotically stable by Lemma 2.

Finally, it derives that $Z_p C_p = G_p C_p d_p \succcurlyeq 0$ from (7). Since $d_p > 0$, it yields $G_p C_p \succcurlyeq 0$. Thus, under the output feedback controller and (7), $u(t) = G_p y(t) = G_p C_p x(t) \succcurlyeq 0$ sets up. □

Remark 1. We do not restrict the state of the original system because our goal is to design the output feedback controller, so that the closed-loop system is positive. This is so-called the controlled positive system.

Next, we turn our attention on negative control, that is, $u(t) \preceq 0$.

Theorem 2. *Consider the switched linear system (1) with interval uncertainties. For predetermined constants $\gamma_p > 0$, if there exist diagonal matrices $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ with positive diagonal entries and matrices $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, such that*

$$\underline{a}_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} \overline{b}_{pik} z_{pkl} c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \tag{11}$$

$$\sum_{j=1}^{n} \overline{a}_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{s} \sum_{k=1}^{m} \underline{b}_{pik} z_{pkl} c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \tag{12}$$

$$\sum_{l=1}^{s} z_{pil} c_{plj} \leq 0, \quad \forall 1 \leq i \leq m, 1 \leq j \leq n, \tag{13}$$

then the closed-loop system (2) is positive and asymptotically stable under MDADT switching strategy $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$, $\forall (p,q) \in \mathbb{P} \times \mathbb{P}$. *Moreover, the output feedback gain is given by* $G_p = Z_p D_p^{-1}$. *Also,* $u(t) \preceq 0$ *sets up, and assume that the initial condition is* $0 \preceq x_0 \preceq d_{vp}$ *for* $d_{vp} \in \mathbb{R}^n$.

Proof. By (11) and (12), it verifies that the closed-loop system (2) is positive and asymptotically stable using a similar proof to Theorem 1. Taking $d_p > 0$ and (13) into account, we can obtain $G_p C_p \preceq 0$. So, it follows that $u(t) = G_p y(t) = G_p C_p x(t) \preceq 0$. □

Remark 2. For the model of a positive system, system control is also required to be nonnegative. On the contrary, negative controllers are also very common in real life. So, it is important to talk about sign-constrained controllers.

3.2. Bounded Controls. In this subsection, the bounded control problems are investigated for the switched system with interval uncertainties under MDADT switching. That is, the designed output feedback controllers allow positivity and asymptotic stability of the closed-loop system with the initial condition $0 \preceq x_0 \preceq d_{vp}$, and the controllers should be limited to boundaries decided in advance. In this sequel, we consider the condition of $0 \preceq u(t) \preceq \overline{u}$ for fixed $\overline{u} \succcurlyeq 0$.

Theorem 3. *Consider the switched linear system (1) with interval uncertainties. For fixed constants* $\gamma_p > 0$, *if there exist diagonal matrices* $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, *such that*

$$\underline{a}_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} \underline{b}_{pik} z_{pkl} c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \tag{14}$$

$$\sum_{j=1}^{n} \overline{a}_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{s} \sum_{k=1}^{m} \overline{b}_{pik} z_{pkl} c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \tag{15}$$

$$\sum_{l=1}^{s} z_{pil} c_{plj} \geq 0, \quad \forall 1 \leq i \leq m, 1 \leq j \leq n, \tag{16}$$

$$\sum_{j=1}^{n} \sum_{l=1}^{s} z_{pil} c_{plj} \leq \overline{u}_i, \quad \forall 1 \leq i \leq m, \tag{17}$$

then the closed-loop system (2) with the output feedback gain $G_p = Z_p D_p^{-1}$ *is positive and asymptotically stable under MDADT switching strategy* $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$ *and* $\forall (p,q) \in \mathbb{P} \times \mathbb{P}$. *Furthermore, the restricted condition* $0 \preceq u(t) \preceq \overline{u}$ *meets if initial condition is* $0 \preceq x_0 \preceq d_{vp}$, *where* $d_{vp} \in \mathbb{R}^n$.

Proof. By Theorem 1, the closed-loop system is positive and asymptotically stable, and $u(t) \succcurlyeq 0$ also sets up. Furthermore, it is clear that $0 \preceq x(t) \preceq d_{vp}$ holds for the initial condition meets $0 \preceq x_0 \preceq d_{vp}$ by Lemma 3. Then, we have

$$u(t) = G_p y(t) = G_p C_p x(t) \preceq G_p C_p d_{vp}. \tag{18}$$

Let $e = [1, 1 \ldots, 1]^T \in \mathbb{R}^s$. We have $d_{vp} = d_p e$. Recalling the output feedback gain matrix, we can obtain

$$G_p C_p d_{vp} = G_p d_p C_p e = Z_p C_p e. \tag{19}$$

Then, the inequality $u(t) \preceq \overline{u}$ holds by (17).

In the following, we aim to the condition $-\widetilde{u} \preceq u(t) \preceq 0$ of controlled positive switched systems for prescribed $\widetilde{u} \succ 0$. □

Theorem 4. *Consider the switched linear system (1) with interval uncertainties. For given constants* $\gamma_p > 0$, *if there exist diagonal matrices* $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, *such that*

$$\begin{aligned}
&\underline{a}_{pij} d_p + \sum_{l=1}^{s} \sum_{k=1}^{m} \overline{b}_{pik} z_{pkl} c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \\
&\sum_{j=1}^{n} \overline{a}_{pij} d_p + \sum_{j=1}^{n} \sum_{l=1}^{s} \sum_{k=1}^{m} \underline{b}_{pik} z_{pkl} c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \\
&\sum_{l=1}^{s} z_{pil} c_{plj} \leq 0, \quad \forall 1 \leq i \leq m, 1 \leq j \leq n, \\
&-\widetilde{u}_i \leq \sum_{j=1}^{n} \sum_{l=1}^{s} z_{pil} c_{plj}, \quad \forall 1 \leq i \leq m,
\end{aligned} \tag{20}$$

then the closed-loop system (2) with the output feedback gain $G_p = Z_p D_p^{-1}$ *is positive and asymptotically stable under MDADT switching strategy* $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$, $\forall (p,q) \in \mathbb{P} \times \mathbb{P}$. *Moreover,* $-\widetilde{u} \preceq u(t) \preceq 0$ *fulfills suppose that initial condition satisfies* $0 \preceq x_0 \preceq d_{vp}$, *where* $d_{vp} \in \mathbb{R}^n$.

Proof. The proof is similar to Theorem 3 and therefore omitted here.

At the end of this subsection, asymmetrically bounded control $-\widetilde{u} \preceq u(t) \preceq \overline{u}$ is addressed to stabilize switched systems with interval uncertainties for given $\widetilde{u}$ and $\overline{u}$. □

Theorem 5. *Consider the switched linear system (1) with interval uncertainties. For given constants* $\gamma_p > 0$, *if there exist diagonal matrices* $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, $W_p = [w_{pij}] \in \mathbb{R}^{m\times s}$, *such that*

$$\underline{a}_{pij}d_p+\sum_{l=1}^{s}\sum_{k=1}^{m}\min\left\{\underline{b}_{pik}\left(z_{pkl}-w_{pkl}\right)c_{plj},\overline{b}_{pik}\left(z_{pkl}-w_{pkl}\right)c_{plj}\right\}\geq 0,\quad \forall 1\leq i\neq j\leq n, \tag{21}$$

$$\sum_{j=1}^{n}\overline{a}_{pij}d_p+\sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m}\max\left\{\underline{b}_{pik}\left(z_{pkl}-w_{pkl}\right)c_{plj},\overline{b}_{pik}\left(z_{pkl}-w_{pkl}\right)c_{plj}\right\}+\gamma_p d_p<0,\quad \forall 1\leq i\leq n, \tag{22}$$

$$\sum_{l=1}^{s}z_{pil}c_{plj}\geq 0,\quad \forall 1\leq i\leq m, 1\leq j\leq n, \tag{23}$$

$$\sum_{k=1}^{s}w_{pil}c_{plj}\geq 0,\quad \forall 1\leq i\leq m, 1\leq j\leq n, \tag{24}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{s}z_{pil}c_{plj}\leq \overline{u}_i,\quad \forall 1\leq i\leq m, \tag{25}$$

$$\sum_{j=1}^{n}\sum_{l=1}^{s}w_{pil}c_{plj}\leq \tilde{u}_i,\quad \forall 1\leq i\leq m, \tag{26}$$

then the closed-loop system (2) with the output feedback gain $G_p=(Z_p-W_p)D_p^{-1}$ *is positive and asymptotically stable under MDADT switching strategy* $\tau_{\alpha p}>\tau_{\alpha p}^{*}=(\ln\mu_p/\gamma_p)$, *where* $\mu_p=\max(d_p/d_q)$, $\forall(p,q)\in\mathbb{P}\times\mathbb{P}$. *Moreover,* $-\tilde{u}\preceq u(t)\preceq\overline{u}$ *holds if the initial condition is* $0\preceq x_0\preceq d_{vp}$, *where* $d_{vp}\in\mathbb{R}^n$.

Proof. According to the proof of Theorem 1, the closed-loop system is positive and asymptotically stable for the output feedback gain matrix $G_p=Z_pD_p^{-1}-W_pD_p^{-1}$. Since, inequalities (21) and (22) are together with Lemma 3, $0\preceq x(t)\preceq d_{vp}$, sets up for the initial condition $0\preceq x_0\preceq d_{vp}$. Note that vector e and combining with (23)–(26), we can get

$$0\preceq Z_pD_p^{-1}y(t)=Z_pD_p^{-1}C_px(t)\preceq Z_pD_p^{-1}C_pd_{vp}=Z_pC_pe, \tag{27}$$

$$0\preceq W_pD_p^{-1}y(t)=W_pD_p^{-1}C_px(t)\preceq W_pD_p^{-1}C_pd_{vp}=W_pC_pe. \tag{28}$$

Using the properties of the inequality, (27) can be rewritten as

$$-W_pC_pe\preceq -W_pD_p^{-1}y(t)\preceq 0. \tag{29}$$

So, one has

$$-\tilde{u}\preceq u(t)G_py(k)=\left(Z_p-W_p\right)D_p^{-1}y(t)\preceq\overline{u}. \tag{30}$$

This completes the proof. □

3.3. Constrained Output. The matrix C_p is nonnegative. Under this general premise, this subsection handles the constrained output issue for controlled positive switched systems. The objective is to design the output feedback controller, such that the corresponding closed-loop system is positive and asymptotically stable via MDADT switching signals. Here, the output satisfies the bound $0\preceq y(t)\preceq\overline{y}$ for fixed $\overline{y}\succ 0$.

Theorem 6. *Consider the switched linear system (1) with interval uncertainties. Assume that* $C_p\succcurlyeq 0$. *For given constants* $\gamma_p>0$, *if there exist diagonal matrices* $D_p=diag\left\{d_p,d_p,\ldots,d_p\right\}\in\mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p=[z_{pij}]\in\mathbb{R}^{m\times s}$ *satisfying*

$$\underline{a}_{pij}d_p+\sum_{l=1}^{s}\sum_{k=1}^{m}\min\left\{\underline{b}_{pik}z_{pkl}c_{plj},\overline{b}_{pik}z_{pkl}c_{plj}\right\}\geq 0,\quad \forall 1\leq i\neq j\leq n, \tag{31}$$

$$\sum_{j=1}^{n}\overline{a}_{pij}d_p+\sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m}\max\left\{\underline{b}_{pik}z_{pkl}c_{plj},\overline{b}_{pik}z_{pkl}c_{plj}\right\}+\gamma_pd_p<0,\quad \forall 1\leq i\leq n, \tag{32}$$

$$\sum_{j=1}^{n}c_{pij}d_p\leq\overline{y}_i,\quad \forall 1\leq i\leq s, \tag{33}$$

then the closed-loop system (2) with the output feedback gain $G_p = Z_p D_p^{-1}$ *is positive and asymptotically stable under MDADT switching strategy* $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$, $\forall (p,q) \in \mathbb{P}\times\mathbb{P}$. *In addition,* $0 \preceq y(t) \preceq \overline{y}$ *satisfies if initial condition is* $0 \preceq x_0 \preceq d_{vp}$, *where* $d_{vp} \in \mathbb{R}^n$.

Proof. It is similar to the previous proof of Theorem 1, and we know that the closed-loop system (2) is positive and asymptotically stable from (31) and (32). By using $C_p \succcurlyeq 0$ and Lemma 3, it follows $y(t) = C_p x(t) \preceq C_p d_{vp}$. Together with (33), we immediately arrive at constraints $0 \preceq y(t) \preceq \overline{y}$. □

4. Robust Stabilization with Polytopic Uncertainties

This section is focused on robust stabilization for controlled positive switched systems with polytopic uncertainties. Suppose that the subsystem matrices A_p and B_p are not precisely known but belong to the following polytopic uncertainty set:

$$\Theta_p = \left\{ \sum_{s=1}^{M} \gamma_s \left[A_p^{(s)}, B_p^{(s)} \right] \Big| \sum_{s=1}^{M} \gamma_s = 1, \gamma_s \geq 0 \right\}, \quad (34)$$

where $A_p^{(s)}$ and $B_p^{(s)}$ are the certain matrices and stand for extreme points of the p^{th} subsystem, and M denotes the total number of extreme points. Except, we are talking about asymmetrically constrained control and constrained output cases. Others can be discussed similarly.

Theorem 7. *Consider the switched linear system (1) with polytopic uncertainties. For given constants* $\gamma_p > 0$, *if there exist diagonal matrices* $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$, $W_p = [w_{pij}] \in \mathbb{R}^{m\times s}$, *such that*

$$a_{pij}^{(s)} d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} \left(z_{pkl} - w_{pkl}\right) c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \quad (35)$$

$$\sum_{j=1}^{n} a_{pij}^{(s)} d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} \left(z_{pkl} - w_{pkl}\right) c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \quad (36)$$

$$\sum_{l=1}^{s} z_{pil} c_{plj} \geq 0, \quad \forall 1 \leq i \leq m, 1 \leq j \leq n, \quad (37)$$

$$\sum_{k=1}^{s} w_{pil} c_{plj} \geq 0, \quad \forall 1 \leq i \leq m, 1 \leq j \leq n, \quad (38)$$

$$\sum_{j=1}^{n}\sum_{l=1}^{s} z_{pil} c_{plj} \leq \overline{u}_i, \quad \forall 1 \leq i \leq m, \quad (39)$$

$$\sum_{j=1}^{n}\sum_{l=1}^{s} w_{pil} c_{plj} \leq \widetilde{u}_i, \quad \forall 1 \leq i \leq m, \quad (40)$$

then the closed-loop system (2) with the output feedback gain $G_p = (Z_p - W_p) D_p^{-1}$ *is positive and asymptotically stable under MDADT switching strategy* $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$, $\forall (p,q) \in \mathbb{P}\times\mathbb{P}$. *Moreover,* $-\widetilde{u} \preceq u(t) \preceq \overline{u}$ *can be achieved if the initial condition* $0 \preceq x_0 \preceq d_{vp}$, *where* $d_{vp} \in \mathbb{R}^n$.

Proof. First, from the closed-loop system matrix, one has

$$a_{cpij}^{(s)} = a_{pij}^{s} + \sum_{l=1}^{(s)}\sum_{k=1}^{m} b_{pik}^{(s)} g_{pkl} c_{plj}, \quad \forall 1 \leq i, j \leq n. \quad (41)$$

Then, it is not hard to see that

$$a_{cpij}^{(s)} d_p = a_{pij}^{(s)} d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} g_{pkl} c_{plj} d_p, \quad (42)$$

Notice the matrix element of the output feedback gain, and it follows that $g_{pkj} d_p = z_{pkj} - w_{pkj}$. Hence,

$$a_{cpij}^{(s)} d_p = a_{pij}^{(s)} d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} \left(z_{pkl} - w_{pkj}\right) c_{plj}. \quad (43)$$

By virtue of (35), $a_{cpij}^{(s)} d_p \geq 0$ holds for $1 \leq i \neq j \leq n$. Thus, $A_{cp}^{(s)}$ is the Metzler matrix due to $d_p > 0$. This means that A_{cp} is also the Metzler matrix by a convexity of argument, so the closed-loop (2) system is positive by Lemma 1.

Next, consider vector d_{vp}, and we have

$$\begin{aligned}\left[A_{cp}^{(s)} d_{vp}\right]_i &= \sum_{j=1}^{n} \left[\left(a_{pij}^{(s)} + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} g_{pkl} c_{plj} \right) d_p \right] \\ &= \sum_{j=1}^{n} a_{pij}^{(s)} d_p + \sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{m} b_{pik}^{(s)} \left(z_{pkl} - w_{pkj}\right) c_{plj}.\end{aligned} \quad (44)$$

The above together with (36) means that $(A_{cp}^{(s)} + \gamma_p I) d_{vp} \preceq 0$, and it follows $(A_{cp} + \gamma_p I) d_{vp} = (A_p + B_p G_p C_p + \gamma_p I) d_{vp} \preceq 0$. Thus, the closed-loop system is asymptotically stable by Lemma 2.

The rest of the proof is similar to Theorem 5. □

Remark 3. In fact, the processes of interval uncertainties and polytopic uncertainties can convert each other. If $\underline{A_p} \preceq A_p^{(s)} \preceq \overline{A_p}$ and $\underline{B_p} \preceq B_p^{(s)} \preceq \overline{B_p}$, Theorem 7 can be treated in the same way as Theorem 5.

Theorem 8. *Consider the switched linear system (1) with polytopic uncertainties. Assume that* $C_p \succcurlyeq 0$. *For given constants* $\gamma_p > 0$, *if there exist diagonal matrices* $D_p = diag\{d_p, d_p, \ldots, d_p\} \in \mathbb{R}^{s\times s}$ *with positive diagonal entries and matrices* $Z_p = [z_{pij}] \in \mathbb{R}^{m\times s}$ *subjected to*

$$a_{pij}^{(s)} d_p + \sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} z_{pkl} c_{plj} \geq 0, \quad \forall 1 \leq i \neq j \leq n, \quad (45)$$

$$\sum_{j=1}^{n} a_{pij}^{(s)} d_p + \sum_{j=1}^{n}\sum_{l=1}^{s}\sum_{k=1}^{m} b_{pik}^{(s)} z_{pkl} c_{plj} + \gamma_p d_p < 0, \quad \forall 1 \leq i \leq n, \quad (46)$$

$$\sum_{j=1}^{n} c_{pij} d_p \le \overline{y}_i, \quad \forall 1 \le i \le s, \tag{47}$$

then the closed-loop system (2) with the output feedback gain $G_p = Z_p D_p^{-1}$ *is positive and asymptotically stable under the MDADT switching strategy* $\tau_{\alpha p} > \tau_{\alpha p}^* = (\ln \mu_p/\gamma_p)$, *where* $\mu_p = \max(d_p/d_q)$, $\forall (p,q) \in \mathbb{P} \times \mathbb{P}$. *And then,* $0 \preceq y(t) \preceq \overline{y}$ *holds if* $0 \preceq x_0 \preceq d_{vp}$, *where* $d_{vp} \in \mathbb{R}^n$.

Proof. It is similar to the previous proof of Theorem 7, and we know that the closed-loop system (2) is positive and asymptotically stable from (45) and (46). The rest section can be deduced by Theorem 6. □

Remark 4. It should be pointed out that we discuss the general case of switching signals in this study. When we choose $\mu_p = \mu$ and $\gamma_p = \gamma$ for $\forall p \in \mathbb{P}$, MDADT switching strategy degenerates into the simple case of average dwell time. Furthermore, if we make $\mu_p = \mu$, $\gamma_p = \gamma$, and $d_p = d$, the presentation of our conclusion becomes that under the arbitrary switching signal.

5. Numerical Examples

Consider system (1) with the interval uncertainties, where the bounds of the system matrices are given by

$$\begin{aligned}
\underline{A}_1 &= \begin{pmatrix} -2.1 & -0.9 \\ -0.6 & -1.3 \end{pmatrix}, \\
\overline{A}_1 &= \begin{pmatrix} -1.9 & -0.7 \\ -0.4 & -1.1 \end{pmatrix}, \\
\underline{B}_1 &= \begin{pmatrix} 1.7 & -1.3 \\ -0.7 & 0.9 \end{pmatrix}, \\
\overline{B}_1 &= \begin{pmatrix} 1.9 & -1.1 \\ -0.5 & 1.1 \end{pmatrix}, \\
\underline{A}_2 &= \begin{pmatrix} -1.3 & -0.6 \\ 0.4 & -1.1 \end{pmatrix}, \\
\overline{A}_2 &= \begin{pmatrix} -1.1 & -0.4 \\ 0.6 & -0.9 \end{pmatrix}, \\
\underline{B}_2 &= \begin{pmatrix} -2.3 & 0.7 \\ -0.9 & -1.9 \end{pmatrix}, \\
\overline{B}_2 &= \begin{pmatrix} -2.1 & 0.9 \\ -0.7 & -1.7 \end{pmatrix}, \\
C_1 &= \begin{pmatrix} 1 & 0.8 \\ 0.5 & 1 \end{pmatrix}, \\
C_2 &= \begin{pmatrix} 0.6 & 1 \\ 1 & 0.5 \end{pmatrix}.
\end{aligned} \tag{48}$$

In this part, we take the constrained output feedback controller $0 \preceq u(t) \preceq \overline{u}$, for example, take $\overline{u} = [6,8]^T$. Address inequalities (14)–(17) in Theorem 3 by utilizing the linear programming algorithm in MATLAB, and it can readily obtain that $D_1 = \mathrm{diag}\{6.5164, 6.5164\}$, $D_2 = \mathrm{diag}\{6.7756, 6.7756\}$, and matrices

$$\begin{aligned}
Z_1 &= \begin{pmatrix} -2.6754 & 6.6434 \\ 9.5755 & -7.2464 \end{pmatrix}, \\
Z_2 &= \begin{pmatrix} -1.2436 & 3.0280 \\ 9.5813 & -5.5690 \end{pmatrix}.
\end{aligned} \tag{49}$$

Then, the output feedback gain matrix can be calculated by

$$\begin{aligned}
G_1 &= \begin{pmatrix} -0.4106 & 1.0195 \\ 1.4694 & -1.1120 \end{pmatrix}, \\
G_2 &= \begin{pmatrix} -0.1835 & 0.4469 \\ 1.4141 & -0.8219 \end{pmatrix}.
\end{aligned} \tag{50}$$

It is easy to see that the gain matrices G_1 and G_2 are nonsingular, and the rank of the matrix is full. Furthermore, the extreme closed-loop system matrices are

$$\begin{aligned}
\underline{A}_{c1} &= \underline{A}_1 + \underline{B}_1 G_1 C_1 = \begin{pmatrix} -3.1189 & 0.1922 \\ 0.1527 & -1.7265 \end{pmatrix}, \\
\underline{A}_{c2} &= \underline{A}_2 + \underline{B}_2 G_2 C_2 = \begin{pmatrix} -2.0560 & 0.0104 \\ 0.0465 & -3.0419 \end{pmatrix}, \\
\overline{A}_{c1} &= \overline{A}_1 + \overline{B}_1 G_1 C_1 = \begin{pmatrix} -2.7163 & 0.5431 \\ 0.5552 & -1.3756 \end{pmatrix}, \\
\overline{A}_{c2} &= \overline{A}_2 + \overline{B}_2 G_2 C_2 = \begin{pmatrix} -1.7833 & 0.4190 \\ 0.3192 & -2.6333 \end{pmatrix}.
\end{aligned} \tag{51}$$

We can observe that $\underline{A}_{c1}$ and $\underline{A}_{c2}$ are the Metzler matrices. Select $A_i = (1/2)(\underline{A}_i + \overline{A}_i)$ and $B_i = (1/2)(\underline{B}_i + \overline{B}_i)$ for $i = 1, 2$; hence, the close-loop system is positive. Let $\gamma_1 = 0.4$, $\gamma_2 = 0.6$ and $\mu_1 = 1.2$, $\mu_2 = 1.25$; we get the minimum MDADT $\tau_1^* = 0.1990$ and $\tau_2^* = 0.4368$. The state trajectories of the closed-loop system together with the extreme plants under initial $x_0 = (6,6)^T$ are given in Figures 1 and 2. Figures 3 and 4 plot the control input, which illustrates that the

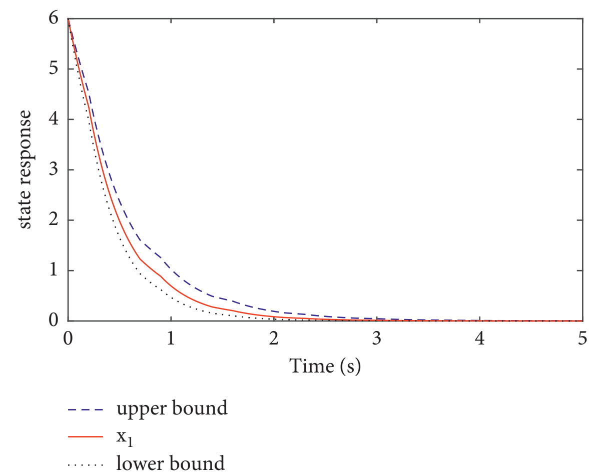

Figure 1: Simulation of closed-loop system state trajectories.

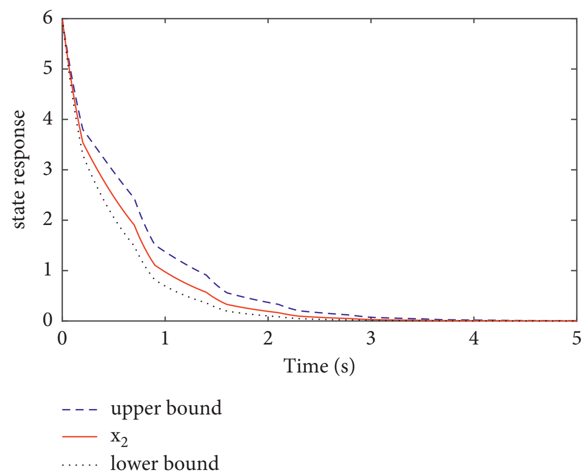

FIGURE 2: Simulation of control input.

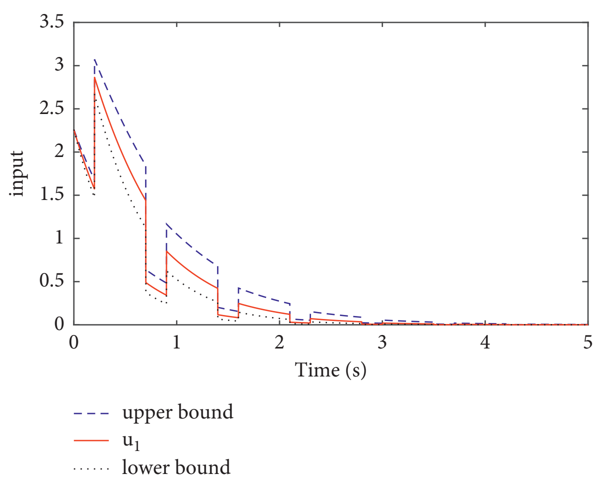

FIGURE 3: Simulation of closed-loop system state trajectories.

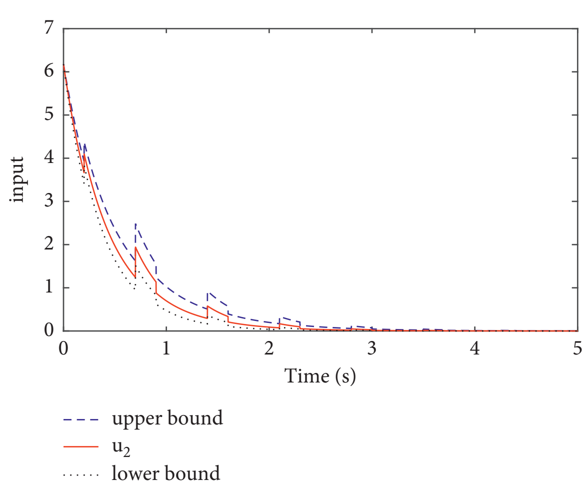

FIGURE 4: Simulation of control input.

controls meet the fixed bound $0 \preceq u(k) \preceq [6,8]^T$ when the initial value is limited to $0 \preceq x_0 \preceq d_{v1} = (6.5164, 6.5164)^T$. The switching signal $\sigma(t)$ with MDADT is shown in Figure 5.

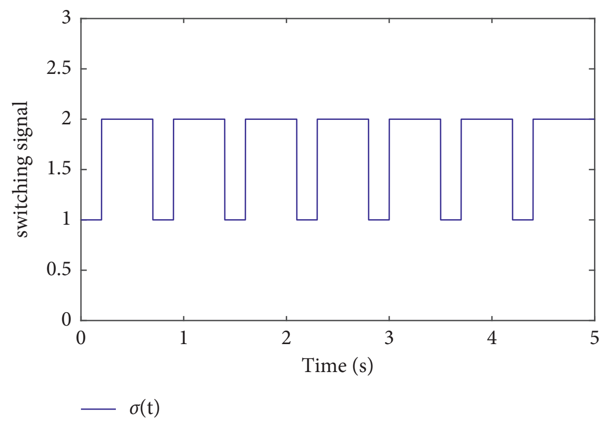

FIGURE 5: Simulation of switching signal.

6. Conclusion

The problem of robust stabilization of controlled positive switched linear systems by the output feedback is addressed in this article. First, we pay attention to find constrained output feedback controllers rendering the positiveness and asymptotic stability of the closed-loop systems, eventhough the open-loop system is not positive. Second, by utilizing the MDADT switching scheme, the robust stabilization of the controlled positive switched system is solved under interval and polytopic uncertainties. It is worth pointing out that all the controllers we designed satisfy certain limiting conditions, including bounded control inputs and constrained outputs. All the conditions can be solvable in terms of linear programming, instead of LMI. Along the approach of this study, further works may refer to delay problems and nonfragile reliable control.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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