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Research Article

Group Consensus of Heterogeneous Multiagent Systems with Time Delay

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In this paper, a neighbour-based control algorithm of group consensus is designed for a class of hybrid-based heterogeneous multiagent systems with communication time delay. We consider the statics leaders and active leaders, respectively. The original systems are transformed into new error systems by transformation. On the basis of the systems, applying Lyapunov stability theory and adopting the linear matrix inequality method, sufficient conditions which guarantee the heterogeneous multiagent systems stability are obtained. To illustrate the validity of theoretical results, some numerical simulations are given at the end of the paper.

1. Introduction

Consensus problem of multiagent systems (MAS) has always been a hot topic in the control field. In recent years, due to the huge leap in electronic technology, the application of MAS is extremely extensive, mainly involving mechanical engineering, unmanned aerial vehicles, robot formations, neural networks and so on [1–4]. In previous studies, each agent had the same dynamic system, and their own attributes and traits were the same. But in today's ever-changing era, any electronic device is constantly improving, and the cycle of updating is becoming shorter and shorter. Therefore, in order to adapt to the development of the times, many scholars have focused on heterogeneity. On the agent system, the dynamic model of the agent in the system is not exactly the same, and consensus issues for heterogeneous MAS have been investigated in [5–7].

Due to the complexity of practical network international circumstances and the diversity of control systems, it is impossible for all agents to only tend to be in a stable state. In view of this, the group consensus issue for heterogeneous MAS has attracted the favour of many academics at home and abroad, among which the most of it is common for heterogeneous systems composed of second-order MAS in continuous time [8]. In Hu et al.'s study [8], the authors considered the heterogeneous MAS with uncertain parameters. There are also heterogeneous systems which are consisted of a first-order system and a second-order MAS [9]. In Yu et al.'s study [9], investigated group consensus matters for heterogeneous multiagent networks in a rival system. In Wen et al.'s study [10], the authors discussed group consensus issues for heterogeneous MAS in the situation of input saturation. It was composed of two first-order MAS with or without nonlinear and second-order multiagent systems. In addition to continuous time, there are a large number of works dealing with the matters of consensus of cluster and group for discrete-time heterogeneous multiagent systems [11]. For example, in Shi et al.'s study [11], based on discrete time, scholars investigated dynamic interactions nonsynchronous group consistency for heterogeneous MAS in the topology of message interaction. In Jiang et al.'s study [12], the authors studied couple-group consensus for heterogeneous MAS in discrete time though adopting synergetic-competitive interaction and time-lags. In the study by Feng and Zheng [13], the issues of group consensus control for the first-order and second-order heterogeneous MAS in the situation of discrete time was discussed minutely. At present, most of the research focuses on the interactive messages of agent's neighbours to obtain relative status information of each agent, but in the process of the information transmission often has some physical factors such as transmission channels, maybe lead to the inability of messages between agents to arrive in time. This requires scholars to refer to the important factor of communication delay in research. In Wen et al.'s study [14], researched dynamical group consensus matters for heterogeneous MAS with communication lags. Under the circumstances directional structure of message interactions, Li et al. considered group consensus issues for the MAS with sampled and quantized data in detail in [15]. However, in practical applications, many engineering problems are not a single linear system, in order to make works that are closer to the actual application, in the study of group consistency issue for heterogeneous MAS, not only time lag but also the consideration of the influence of nonlinear term. Under the situation of parametric uncertainties, Hu et al. researched group consensus for heterogeneous multiagent systems in [8]. In Liu et al.'s study [16], they investigated consensus for heterogeneous MAS under fixed and switching topologies. It enriches the content of heterogeneous multiagent systems.

Inspired by the above-mentioned results, this article studies the group consensus problem for mixed-order heterogeneous MAS with time delay, which is grouped into two and three portions. Part one, in the circumstance of the presence of time lags, group consensus issues are investigated for the second-order MAS with or without nonlinear; in the second part, the group consensuses for the systems which are consisted of the second-order nonlinear multiagent system and the second-order and first-order MAS in a linear environment are mainly studied. The analysis and conclusion of three parts can be proceeded primarily via utilizing Lyapunov stability theory. The approach of linear matrix inequality is used to prove it. Finally, the results are numerically simulated by Matlab, and the validity of the conclusion is further demonstrated.

The construction of the remainder of the article is described below. Section 2 mainly illustrated textual preparatory work, which contains the two portions of graph theory and dynamical systems description. There are couple-part contents in Section 3, part one under the premise of active leaders in multiagent systems, a sufficient criterion is provided to implement group consensus; the other part in the presence of static leaders, we also acquire a sufficient condition in regard to group consistency of heterogeneous MAS with time-lags. In the sequel, numerical modelling and conclusion are expounded in Sections 4 and 5, separately.

Notations: for simplicity of the proof process of the paper, some mathematical notations are adopted throughout this article. Suppose $\mathbb{R}^{n\times n}$ denotes the real matrix with $n \times n$. Let diag$\{\cdots\}$ denote the matrix with the elements are all zero except the main diagonal elements. Let R^T signify the transpose of matrix R. Suppose 1_n denotes a column vector with all elements are 1.

2. Preliminaries

2.1. Graph Theory. Suppose $G = (I, T, \text{and}\, C)$ is a weighted digraph which has a set of node $I = \{1, \ldots, n\}$, a set of trajectory $T \subseteq I \times I$, and a weighted adjacency matrix $C = [c_{ij}] \in \mathbb{R}^{n\times n}$. A trajectory of $G$is denoted by $(i\,\text{and}\, j)$, it means to from j to i. If element c_{ij} fulfilled the trajectory in the digraph, then it is positive, denotes $c_{ij} \neq 0 \Longleftrightarrow (i, j) \in T$. Let $c_{ii} = 0$ for all agents $i \in I$. The set of neighbours of node i is expressed by $N_i = \{j \in I\colon\ (i, j) \in T\}$. Then, Laplacianof the weighted digraph G is defined as

$$L = D - C \in \mathbb{R}^{n\times n}, \qquad L = [l_{ij}] \in \mathbb{R}^{n\times n},$$

$$l_{ij} = \begin{cases} \sum_{j=1, j\neq i}^{n} c_{ij}, i = j \\ -c_{ij}, i \neq j \end{cases}.$$

On the basis of the peculiarity of Laplacian can be aware of the all row-sums of L are zero. Therefore, L has a zero eigenvalue corresponding to a right eigenvector $1 = (1, \ldots, 1)^T \in \mathbb{R}^n$ 1 = (1,. . .1) T ∈ Rn + m. Let $D = \text{diag}\,(d_1, d_2, \ldots, d_n)$ be an in-degree matrix and $d_i = \text{deg}_{in}(i) = \sum_{i=1}^{n} c_{ij}$.

2.2. Dynamic Systems Description. In this subsection, a generalized graph $\widetilde{G}$ is defined which contains followers and two active leaders. There is no absence of generality, graph $\widetilde{G}$ is divided into two portions, the first subgroup $\widetilde{G}_1$ contains the former m followers which follow their leader l_1, and the rest of the followers belong to the second group $\widetilde{G}_2$ which follows l_2. Let $G_j^* = (I_j^*, T_j^*, C_j^*)\,(j) = (1, 2)$ be a subgroup of $G^* = (I^*, T^*, C^*)$ and its leader l_j for $j = (1, 2)$.

The dynamical dynamics of follower$i$can be indicated as

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = u_i(t), \end{cases} \quad i \in I_1 := \{1, 2, \ldots, m\}, \tag{1}$$

where $p_i(t), q_i(t)$, and $u_i(t) \in \mathbb{R}$ f (pit, qi(t)) are position status, velocity status, and control input of agent i.

The dynamics of the leader of heterogeneous MAS can be constructed as

$$\dot{p}_1^*(t) = q_1^*(t), \tag{2}$$

where $p_1^*(t) \in \mathbb{R}$ and $q_1^*(t) \in \mathbb{R}$ are the location state and speed status of the leader l_1.

The dynamical modelling of the other follower i can be depicted as

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = u_i(t) + f_i(p_i(t), t), \end{cases} \quad i \in I_2 := \{m+1, \ldots, n\}, \tag{3}$$

where $p_i(t), q_i(t)$, and $u_i(t) \in \mathbb{R}$ and $f_i(p_i(t), t)$ are location status, velocity status, control input, and the inherent nonlinear term of agent i.

The dynamic state of leader can be depicted as follows:

$$\begin{cases} \dot{p}_2^*(t) = q_2^*(t), \\ \dot{q}_2^*(t) = f(p_2^*(t), t), \end{cases} \tag{4}$$

where $p_2^*(t)$ and $q_2^*(t) \in \mathbb{R}$ and $f(p_2^*(t), t)$ are position state, speed state, and the consecutive nonlinear function of the leader l_2.

On account of the presence of time lags, all agents may not receive the messages from others and their leaders.

Hence, for agent i, a coupled control protocol is put forward in the following form:

$$u_i(t) = \begin{cases} \sum_{j=1}^{m} c_{ij}\big(p_j(t-r(t)) - p_i(t-r(t))\big) + \sum_{j=m+1}^{n} c_{ij}\big(p_j(t-r(t)) + q_j(t)\big) + kb_i\left(q_1^*(t) - q_i(t)\right) \\ +b_i\left(p_1^*(t-r(t)) - p_i(t-r(t))\right), \quad i = 1,\ldots,m, \\ \sum_{j=m+1}^{n} c_{ij}\big(p_j(t-r(t)) - p_i(t-r(t))\big) + b_i\left(p_2^*(t-r(t)) - p_i(t-r(t))\right) + kb_i\left(q_2^*(t) - q_i(t)\right) \\ +\sum_{j=1}^{m} c_{ij}\big(p_j(t-r(t)) + q_j(t)\big) \quad i = m+1,\ldots,n. \end{cases} \tag{5}$$

where the lag $r(t)$ is ever-changing and differentiable and $0 < r(t) < \tau_M, \dot{r}(t) < u$. b_i is a diagonal element of an adjacency matrix of the leader, matrix denoted as $B = \text{diag}\{b_i\} \in \mathbb{R}^{n\times n}$. And if agent i can receive a message from leader l_j, then $b_i > 0$; otherwise, $b_i = 0$. $k > 0$ is a gain constant.

In this subsection, the followers can be divided into three groups as follows. And we consider static leaders. Then, the dynamical model of the second-order nonlinear agent i can be written as

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = u_i(t) + f_i(p_i(t), t), \end{cases} \quad i \in I_1 := \{1, 2, \ldots, l\}, \tag{6}$$

where $p_i(t), q_i(t)$, and $u_i(t) \in \mathbb{R}$ and $f_i(p_i(t), t)$ are location status, velocity status, control input, and nonlinear portion of agent i, respectively.

The dynamical modality of the second-order agent i can be described as

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = u_i(t), \end{cases} \quad i \in I_2 := \{l+1, 2, \ldots, m\}, \tag{7}$$

where $p_i(t), q_i(t)$, and $u_i(t) \in \mathbb{R}$ are position state, velocity state, and control input of agent i, respectively.

The dynamical model of the first-order agent i can be described as

$$\dot{p}_i(t) = u_i(t) \quad i \in I_3 := \{m+1, \ldots, n\}, \tag{8}$$

where $p_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are position state and control input of agent i.

The control protocol of systems (6)–(8) are as follows:

$$u_i(t) = \begin{cases} k_1 \sum_{j=1}^{l} c_{ij}\big(p_j(t-r(t)) - p_i(t-r(t))\big) + k_1 \sum_{j=1}^{l} l_{ij} p_{\sigma_j}(t-r(t)) + \sum_{j\notin I_1} c_{ij}\big(p_j(t-r(t)) + q_j(t)\big) + \\ k_2 \sum_{j=1}^{l} c_{ij}\big(q_j(t) - q_i(t)\big) - k_3 q_i(t), \quad i \in I_1, \\ k_1 \sum_{j=l+1}^{m} c_{ij}\big(p_j(t-r(t)) - p_i(t-r(t))\big) + k_1 \sum_{j=l+1}^{m} l_{ij} p_{\sigma_j}(t-r(t)) + \sum_{j\notin I_2} c_{ij}\big(p_j(t-r(t)) + q_j(t)\big) + \\ k_2 \sum_{j=l+1}^{m} c_{ij}\big(q_j(t) - q_i(t)\big) - k_3 q_i(t), \quad i \in I_2, \\ \sum_{j=m+1}^{n} c_{ij}\big(p_j(t-r(t)) - p_i(t-r(t))\big) + \sum_{j=m+1}^{n} l_{ij} p_{\sigma_j}(t-r(t)) + \sum_{j\notin I_3} c_{ij}\big(p_j(t-r(t)) + q_j(t)\big) \quad i \in I_3, \end{cases} \tag{9}$$

where k_1, k_2, and k_3 are all positive constants, which standing for the positive coupling strengths, severally. And p_{σ_j} represents the consistent position equilibrium of the group in which the $j-$thagent resides, and $\sigma_i = 1, 2$, and 3.

In order to the simplicity of presentation, we lay emphasis on one-dimensional space. However, for any high-dimensional space, we can generalize the results through applying the characteristics of the Kronecker Kronecker product, denoted as $\otimes$.

Remark 1. The portion of agents which are second-order agents and second-order nonlinear agents, the adjacency matrix C is partitioned as

$$C = \begin{pmatrix} C_{S_1} & C_{S_1S_2} \\ C_{S_2S_1} & C_{S_2} \end{pmatrix}, \tag{10}$$

with $C_{S_1} \in \mathbb{R}^{m\times m}$ and $C_{S_2} \in \mathbb{R}^{(n-m)\times(n-m)}$. Suppose L_{S_1} and L_{S_2} signify Laplacian matrix of the second-order agents in the two situations of linearity and nonlinearity, respectively. Then, the matrix can be attained as follows:

$$L = \begin{pmatrix} L_{S_1} + D_{S_1S_2} & -C_{S_1S_2} \\ -C_{S_2S_1} & L_{S_2} + D_{S_2S_1} \end{pmatrix}, \tag{11}$$

with

$$D_{S_1S_2} = \text{diag}\left(\sum_{j\in N_{1,S_2}} c_{1j}, \dots, \sum_{j\in N_{m,S_2}} c_{mj}\right);$$
$$D_{S_2S_1} = \text{diag}\left(\sum_{j\in N_{m+1,S_1}} c_{m+1j}, \dots, \sum_{j\in N_{n,S_1}} c_{nj}\right). \tag{12}$$

Remark 2. The portion of agents are consisted of the second-order agents with or without nonlinearity and first-order agents, then the segmentation of C as follows:

$$C = \begin{pmatrix} C_{s_1} & C_{s_1s_2} & C_{s_1f} \\ C_{s_2s_1} & C_{s_2} & C_{s_2f} \\ C_{fs_1} & C_{fs_2} & C_f \end{pmatrix}, \tag{13}$$

with $C_{s_1} \in \mathbb{R}^{l\times l}, C_{s_2} \in \mathbb{R}^{(m-l)\times(m-l)}$, and $C_f \in \mathbb{R}^{(n-m)\times(n-m)}$.

Presume L_{s_1}, L_{s_2}, and L_f are Laplacian matrix of the second-order agents with nonlinearity and linearity, and first-order agents, severally. Then, L can be depicted as

$$L = \begin{pmatrix} L_{s_1} + D_{s_1s_2} + D_{s_1f} & -A_{s_1s_2} & -A_{s_1f} \\ -A_{s_2s_1} & L_{s_2} + D_{s_2s_1} + D_{s_2f} & -A_{s_2f} \\ -A_{fs_1} & -A_{fs_2} & L_f + D_{fs_1} + D_{fs_2} \end{pmatrix}, \tag{14}$$

with $L_{s_1} = D_{s_1} - C_{s_1}; L_{s_2} = D_{s_2} - C_{s_2}; L_f = D_f - C_f$

$$D_{s_1s_2} = \text{diag}\left(\sum_{j\in N_{1,s_2}} c_{1j}, \dots, \sum_{j\in N_{l,s_2}} c_{lj}\right);$$
$$D_{s_1f} = \text{diag}\left(\sum_{j\in N_{1,f}} c_{1j}, \dots, \sum_{j\in N_{l,f}} c_{lj}\right);$$
$$D_{s_2s_1} = \text{diag}\left(\sum_{j\in N_{1,s_1}} c_{l+1j}, \dots, \sum_{j\in N_{l,s_1}} c_{mj}\right);$$
$$D_{s_2f} = \text{diag}\left(\sum_{j\in N_{1,f}} c_{l+1j}, \dots, \sum_{j\in N_{l,f}} c_{mj}\right);$$
$$D_{fs_1} = \text{diag}\left(\sum_{j\in N_{m+1,s_1}} c_{m+1j}, \dots, \sum_{j\in N_{n,s_1}} c_{nj}\right);$$
$$D_{fs_2} = \text{diag}\left(\sum_{j\in N_{m+1,s_2}} c_{m+1j}, \dots, \sum_{j\in N_{n,s_2}} c_{nj}\right). \tag{15}$$

Next, we will give the definition of the two-group consensus and three-group for heterogeneous multiagent systems. And the similar definition can be given for multigroup consensus problem.

Definition 1. The group consensus of heterogeneous multiagent systems can be achieved if the states of followers tend to the states of leaders in the sense of

$$\lim_{t\longrightarrow\infty}\left|p_i(t) - p_1^*(t)\right| = 0, \lim_{t\longrightarrow\infty}\left|q_i(t) - q_1^*(t)\right| = 0, \quad \forall i \in I_1;$$
$$\lim_{t\longrightarrow\infty}\left|p_i(t) - p_2^*(t)\right| = 0, \lim_{t\longrightarrow\infty}\left|q_i(t) - q_2^*(t)\right| = 0, \quad \forall i \in I_2. \tag{16}$$

Definition 2. Group consensus of heterogeneous multiagent systems can be achieved if the states of followers tend to the states of leaders in the sense of

$$\lim_{t\longrightarrow\infty}\left|p_i(t) - p_{\sigma_i}(t)\right| = 0, \lim_{t\longrightarrow\infty}\left|q_i(t)\right| = 0, \quad \forall i \in I_1;$$
$$\lim_{t\longrightarrow\infty}\left|p_i(t) - p_{\sigma_i}(t)\right| = 0, \lim_{t\longrightarrow\infty}\left|q_i(t)\right| = 0, \quad \forall i \in I_2;$$
$$\lim_{t\longrightarrow\infty}\left|p_i(t) - p_{\sigma_i}(t)\right| = 0, \quad \forall i \in I_3. \tag{17}$$

Assumption 1 (see in [13]).

$$\sum_{j=m+1}^{n} c_{ij} = 0, \quad i = 1, \ldots, m;$$
$$\sum_{j=1}^{m} c_{ij} = 0, \quad i = m+1, \ldots, n. \tag{18}$$

Assumption 2 (see in [15]).

$$\sum_{j=m+1}^{n} c_{ij} = 0, \quad \forall i \in I_1;$$
$$\sum_{j=1, j\notin I_2}^{n} c_{ij} = 0, \quad \forall i \in I_2; \tag{19}$$
$$\sum_{j=1}^{m} c_{ij} = 0, \quad \forall i \in I_3.$$

For the sake of demonstrating group consensus of heterogeneous MAS, we need to give several lemmas which are useful in the proof of the main results as follows:

Lemma 1 (see in [14]). *$f(p_i(t), t)$ fpit, qit, and $f(p_j^*(t), t)$ are continuous differentiable function vectors. In order to achieve group consensus of the heterogeneous MAS containing the nonlinear term, the following assumption is given as follows:*

$$\left\| f(p(t), t) - f(p_2^*(t), t) \cdot 1_n \right\| \le \rho \left\| p(t) - p_2^*(t) \cdot 1_n \right\|, \tag{20}$$

$\forall p_i(t), p_2^*(t) \in \mathbb{R}^n, i = m+1, \ldots, n$; $p(t) = (p_{m+1}(t), \ldots, p_n(t))^T$; $1_n = (1, \ldots, 1)^T$, *and a nonnegative constant ρ.*

Lemma 2 (see in [15]). *For arbitrary constant vectors $a, b \in \mathbb{R}^n$ and a positive definite matrix$R \in \mathbb{R}^{n\times n}$, then the following inequality holds*

$$\pm 2a^T b \le a^T R a + b^T R^{-1} b. \tag{21}$$

Lemma 3 (see in [14]) (SchurSchur complement). *Given a matrix $\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12}^T & \Delta_{22} \end{bmatrix}$. Thus the following conditions are equivalent as follows:*

$$\begin{aligned} &\Delta < 0; \\ &\Delta_{11} < 0, \\ &\Delta_{22} - \Delta_{12}^T \Delta_{11}^{-1} \Delta_{12} < 0; \\ &\Delta_{22} < 0, \\ &\Delta_{11} - \Delta_{12} \Delta_{22}^{-1} \Delta_{12}^T < 0. \end{aligned} \tag{22}$$

3. Results

3.1. Active Leaders. In systems with active leaders, the models change as follows:

Let

$$\begin{aligned} &\overline{p}_i^1(t) = p_i(t) - p_1^*(t); \\ &\overline{q}_i^1(t) = q_i(t) - q_1^*(t); \quad i = 1, \ldots, m, \\ &\overline{p}_i^2(t) = p_i(t) - p_2^*(t); \\ &\overline{q}_i^2(t) = q_i(t) - q_2^*(t); \quad i = m+1, \ldots, n, \\ &\overline{p}^1(t) = \left(\overline{p}_1^1(t), \ldots, \overline{p}_m^1(t)\right)^T; \\ &\overline{p}^2(t) = \left(\overline{p}_{m+1}^2(t), \ldots, \overline{p}_n^2(t)\right)^T; \\ &\overline{q}^1(t) = \left(\overline{q}_1^1(t), \ldots, \overline{q}_m^1(t)\right)^T; \\ &\overline{q}^1(t) = \left(\overline{q}_{m+1}^2(t), \ldots, \overline{q}_n^2(t)\right)^T; \\ &\overline{p}(t) = \left(\overline{p}^{1T}(t), \overline{p}^{2T}(t)\right)^T; \\ &\overline{q}(t) = \left(\overline{q}^1(t), \overline{q}^2(t)\right). p^*(t) = \left(p_1^*(t), p_2^*(t)\right)^T; \\ &q^*(t) = \left(q_1^*(t), q_2^*(t)\right). \end{aligned} \tag{23}$$

Then, according to the characteristic of L, (1) and (2) can be substituted as

$$\left\{ \dot{\overline{p}}^1(t) = \overline{q}^1(t), \dot{\overline{q}}^1(t) = -(\overline{L}_{11} + B_1)\overline{p}^1(t - r(t)) - \overline{L}_{12}\overline{p}^2(t - r(t)) - kB_1\overline{q}^1(t), \right. \tag{24}$$

$$\left\{ \dot{\overline{p}}^2(t) = \overline{q}^2(t), \dot{\overline{q}}^2(t) = -\overline{L}_{21}\overline{p}^1(t - r(t)) - (\overline{L}_{22} + B_2)\overline{p}^2(t - r(t)) - kB_2\overline{q}^2(t) + \overline{f}\left(\overline{p}^2(t), t\right), \right. \tag{25}$$

where $\overline{f}(\overline{p}^2(t), t) = f(p(t), t) - 1f(p_2^*(t), t)$, $f(p(t), t) = (f_{m+1}(p_{m+1}(t), t), \ldots, f_n(p_n(t), t))$; $\overline{L}_{11} = L_{S_1} + D_{S_1S_2}$; $\overline{L}_{21} = -C_{S_2S_1}$; $\overline{L}_{22} = L_{S_2} + D_{S_2S_1}$; $B = \mathrm{diag}(B_1, B_2)$, $B_1 = \mathrm{diag}(b_1, \ldots, b_m)$, and $B_2 = \mathrm{diag}(b_{m+1}, \ldots, b_n)$.

Then, let $\chi_1(t) = (\overline{p}^{1T}(t), \overline{q}^{1T}(t))^T$; $\chi_2(t) = (\overline{p}^{2T}(t), \overline{q}^{2T}(t))^T$

$$\begin{aligned} \dot{\chi}_1(t) &= \overline{M}_1\chi_1(t - r(t)) + \tilde{M}_1\chi_2(t - r(t)) + N_1\chi_1(t), \\ \dot{\chi}_2(t) &= \overline{M}_2\chi_1(t - r(t)) + \tilde{M}_2\chi_2(t - r(t)) + N_2\chi_2(t) + F, \end{aligned} \tag{26}$$

where

$$\overline{M}_1 = \begin{pmatrix} 0 & 0 \\ -M_1 & 0 \end{pmatrix},$$
$$\widetilde{M}_1 = \begin{pmatrix} 0 & 0 \\ -\overline{L}_{12} & 0 \end{pmatrix},$$
$$N_1 = \begin{pmatrix} 0 & I_m \\ 0 & -kB_1 I_m \end{pmatrix},$$
$$\overline{M}_2 = \begin{pmatrix} 0 & 0 \\ -\overline{L}_{21} & 0 \end{pmatrix},$$
$$\widetilde{M}_2 = \begin{pmatrix} 0 & 0 \\ -M_2 & 0 \end{pmatrix}, \tag{27}$$
$$N_2 = \begin{pmatrix} 0 & I_{n-m} \\ 0 & -kB_2 I_{n-m} \end{pmatrix},$$
$$F = \begin{pmatrix} 0 \\ \overline{f}\left(\overline{p}^2(t), t\right) \end{pmatrix},$$
$$M_1 = \overline{L}_{11} + B_1,$$
$$M_2 = \overline{L}_{22} + B_2.$$

Theorem 1. *Group consensus of systems (24) and (25) can be achieved under control protocol (5) if the following condition holds:*

$$\begin{pmatrix} \Theta_1 & \tau_M R_1 & \tau_M R_2 \\ * & -\tau_M R & 0 \\ * & * & -\tau_M R \end{pmatrix} < 0, \tag{28}$$

where R_1 and R_2 both are arbitrary matrices; τ_M is a constant; $*$ denotes symmetry elements of the matrix.

$$\Theta_1 = \begin{pmatrix} N_1^T P_1 + P_1 N_1 + \tau_M N_1^T R N_1 & \overline{M}_1^T P_1 + \tau_M \overline{M}_1^T R N_1 & 0 & \widetilde{M}_1^T P_1 + \tau_M \widetilde{M}_1^T R N_1 \\ P_1 \overline{M}_1 + \tau_M N_1^T R \overline{M}_1 & \tau_M \overline{M}_1^T R \overline{M}_1 & P_2 \overline{M}_2 & \tau_M \widetilde{M}_1^T R \overline{M}_1 \\ 0 & \overline{M}_2^T P_2 & N_2^T P_2 + P_2 N_2 + P_2 R^{-1} P_2 + \rho^2 R + Q & \widetilde{M}_2^T P_2 \\ P_1 \widetilde{M}_1 + \tau_M N_1^T R \widetilde{M}_1 & \tau_M \overline{M}_1^T R \widetilde{M}_1 & P_2 \widetilde{M}_2 & -(1-u)Q + \tau_M \widetilde{M}_1^T R \widetilde{M}_1 \end{pmatrix}. \tag{29}$$

Proof. According to Lyapunov functional theory, we construct Lyapunov functions as follows:

$$V = V_1 + V_2 + V_3 + V_4. \tag{30}$$

where

$$V_1(t) = \chi_1^T(t) P_1 \chi_1(t);$$
$$V_2(t) = \chi_2^T(t) P_2 \chi_2(t);$$
$$V_3(t) = \int_{t-r(t)}^{t} \chi_2^T(s) Q \chi_2(s) \mathrm{d}s;$$
$$V_4(t) = \int_{-\tau_M}^{0} \int_{t+s}^{t} \dot{\chi}_1^T(v) R \dot{\chi}_1(v) \mathrm{d}v \mathrm{d}s.$$

And P_1, P_2, Q, and $R$are all positive definite matrices. Take the derivation of $V(t)$ along (26), we have

$$\begin{aligned}
\dot{V}_1(t) &= \dot{\chi}_1^T(t)P_1\chi_1(t) + \chi_1^T(t)P_1\dot{\chi}_1(t),\\
&= \left(\overline{M}_1\chi_1(t-r(t)) + \widetilde{M}_1\chi_2(t-r(t)) + N_1\chi_1(t)\right)^T P_1\chi_1(t) + \chi_1^T(t)P_1\left(\overline{M}_1\chi_1(t-r(t)) + \widetilde{M}_1\chi_2(t-r(t)) + N_1\chi_1(t)\right),\\
&= \chi_1^T(t-r(t))\overline{M}_1^T P_1\chi_1(t) + \chi_2^T(t-r(t))\widetilde{M}_1^T P_1\chi_1(t) + \chi_1^T(t)N_1^T P_1\chi_1(t) + \chi_1^T(t)P_1\widetilde{M}_1\chi_2(t-r(t)) + \chi_1^T(t)P_1N_1\chi_1(t)+\\
&\quad \chi_1^T(t)P_1\overline{M}_1\chi_1(t-r(t)),\\
\dot{V}_2(t) &= \dot{\chi}_2^T(t)P_2\chi_2(t) + \chi_2^T(t)P_2\dot{\chi}_2(t),\\
&= \left(\overline{M}_2\chi_1(t-r(t)) + \widetilde{M}_2\chi_2(t-r(t)) + N_2\chi_2(t) + F\right)^T P_2\chi_2(t) + \chi_2^T(t)P_2\left(\overline{M}_2\chi_1(t-r(t))\right.\\
&\quad \left.+ \widetilde{M}_2\chi_2(t-r(t)) + N_2\chi_2(t) + F\right),\\
&= \chi_1^T(t-r(t))\overline{M}_2^T P_2\chi_2(t) + \chi_2^T(t-r(t))\widetilde{M}_2^T P_2\chi_2(t) + \chi_2^T(t)N_2^T P_2\chi_2(t) + \chi_2^T(t)P_2\widetilde{M}_2\chi_2(t-r(t)) + \chi_2^T(t)P_2N_2\chi_2(t)+\\
&\quad 2\chi_2^T(t)P_2F + \chi_2^T(t)P_2\overline{M}_2\chi_1(t-r(t)),\\
\dot{V}_3 &= \left(\int_{t-r(t)}^{t} \chi_2^T(s)Q\chi_2(s)\mathrm{d}s\right)'\\
&= \chi_2^T(t)Q\chi_2(t) - (1-\dot{r}(t))\chi_2^T(t-r(t))Q\chi_2(t-r(t)) \le \chi_2^T(t)Q\chi_2(t) - (1-u)\chi_2^T(t-r(t))Q\chi_2(t-r(t)),\\
\dot{V}_4 &= \left(\int_{-\tau_M}^{0}\int_{t+s}^{t} \dot{\chi}_1^T(v)R\dot{\chi}_1(v)\mathrm{d}v\mathrm{d}s\right)'\\
&= \tau_M\dot{\chi}_1^T(t)R\dot{\chi}_1(t) - \tau_M\dot{\chi}_1^T(t+s)R\dot{\chi}_1(t+s),\\
&= \tau_M\dot{\chi}_1^T(t)R\dot{\chi}_1(t) + \int_{t}^{t-\tau_M} \dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s,\\
&= \tau_M\left(\overline{M}_1\chi_1(t-r(t)) + \widetilde{M}_1\chi_2(t-r(t)) + N_1\chi_1(t)\right)^T \cdot R\left(\overline{M}_1\chi_1(t-r(t)) + \widetilde{M}_1\chi_2(t-r(t)) + N_1\chi_1(t)\right)\\
&\quad + \int_{t}^{t-\tau_M} \dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s,\\
&= \tau_M\chi_1^T(t-r(t))\overline{M}_1^T R\overline{M}_1\chi_1(t-r(t)) + \tau_M\chi_1^T(t-r(t))\overline{M}_1^T R\widetilde{M}_1\chi_2(t-r(t)) + \tau_M\chi_1^T(t-r(t))\overline{M}_1^T RN_1\chi_1(t)+\\
&\quad \tau_M\chi_2^T(t-r(t))\widetilde{M}_1^T R\overline{M}_1\chi_1(t-r(t)) + \tau_M\chi_2^T(t-r(t))\widetilde{M}_1^T R\widetilde{M}_1\chi_2(t-r(t)) + \tau_M\chi_2^T(t-r(t))\widetilde{M}_1^T RN_1\chi_1(t)+\\
&\quad \tau_M\chi_1^T(t)N_1^T R\overline{M}_1\chi_1(t-r(t)) + \tau_M\chi_1^T(t)N_1^T R\widetilde{M}_1\chi_2(t-r(t)) + \tau_M\chi_1^T(t)N_1^T RN_1\chi_1(t) + \int_{t}^{t-\tau_M} \dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s.
\end{aligned} \tag{31}$$

By Lemma 2, we get

$$2\chi_2^T(t)P_2F \le \chi_2^T(t)P_2R^{-1}P_2\chi_2(t) + F^T RF. \tag{32}$$

According to Lemma 1, (32) become

$$\begin{aligned}
2\chi_2^T(t)P_2F &\le \chi_2^T(t)P_2R^{-1}P_2\chi_2(t) + \rho^2\chi_2^T(t)R\chi_2(t)\\
&= \chi_2^T(t)\left(P_2R^{-1}P_2 + \rho^2R\right)\chi_2(t).
\end{aligned} \tag{33}$$

Thus, the derivative of $V_2(t)$ can be written as

$$\begin{aligned}
\dot{V}_2(t) &= \dot{\chi}_2^T(t)P_2\chi_2(t) + \chi_2^T(t)P_2\dot{\chi}_2(t),\\
&= \left(\overline{M}_2\chi_1(t-r(t)) + \widetilde{M}_2\chi_2(t-r(t)) + N_2\chi_2(t) + F\right)^T P_2\chi_2(t) + \chi_2^T(t)P_2\left(\overline{M}_2\chi_1(t-r(t))\right.\\
&\quad \left.+ \widetilde{M}_2\chi_2(t-r(t)) + N_2\chi_2(t) + F\right),\\
&= \chi_1^T(t-r(t))\overline{M}_2^T P_2\chi_2(t) + \chi_2^T(t-r(t))\widetilde{M}_2^T P_2\chi_2(t) + \chi_2^T(t)N_2^T P_2\chi_2(t) + \chi_2^T(t)P_2\overline{M}_2\chi_1(t-r(t))+\\
&\quad \chi_2^T(t)\left(P_2R^{-1}P_2 + \rho^2R\right)\chi_2(t) + \chi_2^T(t)P_2\widetilde{M}_2\chi_2(t-r(t)) + \chi_2^T(t)P_2N_2\chi_2(t).
\end{aligned} \tag{34}$$

Let $Z_1(t) = (\chi_1^T(t), \chi_1^T(t-r(t)), \chi_2^T(t), \chi_2^T(t-r(t)))^T$, then one can get

$$\dot{V}(t) \le Z_1^T(t)\Theta_1 Z_1(t) + \int_t^{t-\tau_M} \dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s + 2Z_1^T(t)R_1 \cdot \left[\chi_1(t) - \chi_1(t-r(t)) - \int_{t-r(t)}^{t} \dot{\chi}_1(s)\mathrm{d}s\right] + 2Z_1^T(t)R_2\left[\chi_1(t-r(t)) - \chi_1(t-\tau_M) - \int_{t-\tau_M}^{t-r(t)} \dot{\chi}_1(s)\mathrm{d}s\right]. \tag{35}$$

As known by Lemma 2,

$$\begin{aligned} &-2Z_1^T(t)R_1\int_{t-r(t)}^{t}\dot{\chi}_1(s)\mathrm{d}s \le r(t)Z_1^T(t)R_1R^{-1}R_1^TZ_1(t) + \int_{t-r(t)}^{t}\dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s, \\ &-2Z_1^T(t)R_2\int_{t-\tau_M}^{t-r(t)}\dot{\chi}_1(s)\mathrm{d}s \le (\tau_M - r(t))Z_1^T \cdot (t)R_2R^{-1}R_2^TZ_1(t) + \int_{t-\tau_M}^{t-r(t)}\dot{\chi}_1^T(s)R\dot{\chi}_1(s)\mathrm{d}s. \end{aligned} \tag{36}$$

Thus, the derivative of $V(t)$ can become

$$\begin{aligned} \dot{V}(t) &\le Z_1^T(t)\Theta_1Z_1(t) + r(t)Z_1^T(t)R_1R^{-1}R_1^TZ_1(t) + (\tau_M - r(t))Z_1^T(t)R_2R^{-1}R_2^TZ_1(t) \le \\ &Z_1^T(t)\left(\Theta_1 + \tau_MR_1R^{-1}R_1^T + \tau_MR_2R^{-1}R_2^T\right)Z_1(t). \end{aligned} \tag{37}$$

Therefore, we can able to see the second-order heterogeneous MAS can attain group consensus if the above-mentioned systems satisfy the condition which has been given in Theorem 1. □

3.2. Static Leaders. In systems with static leaders, the models change as follows.

Let

$$\begin{aligned} &\widehat{p}_i^1(t) = p_i(t) - p_{\sigma_j}(t); \\ &\widehat{q}_i^1(t) = q_i(t), \quad i = 1, \dots, l, \\ &\widehat{p}_i^2(t) = p_i(t) - p_{\sigma_j}(t); \\ &\widehat{q}_i^2(t) = q_i(t), \quad i = l+1, \dots, m, \\ &\widehat{p}_i^3(t) = p_i(t) - p_{\sigma_j}(t); \\ &\widehat{q}_i^3(t) = q_i(t), \quad i = m+1, \dots, n, \\ &\widehat{p}^1(t) = \left(\widehat{p}_1^1(t), \dots, \widehat{p}_l^1(t)\right)^T; \\ &\widehat{p}^2(t) = \left(\widehat{p}_{l+1}^2(t), \dots, \widehat{p}_m^2(t)\right)^T; \\ &\widehat{p}^3(t) = \left(\widehat{p}_{m+1}^3(t), \dots, \widehat{p}_n^3(t)\right)^T; \\ &\widehat{p}(t) = \left(\widehat{p}^{1T}(t), \widehat{p}^{2T}(t), \widehat{p}^{3T}(t)\right)^T \cdot \widehat{q}^1(t) \\ &\quad = \left(\widehat{q}_1^1(t), \dots, \widehat{q}_l^1(t)\right)^T; \\ &\widehat{q}^2(t) = \left(\widehat{q}_{l+1}^2(t), \dots, \widehat{q}_m^2(t)\right)^T; \\ &\widehat{q}^3(t) = \left(\widehat{q}_{m+1}^3(t), \dots, \widehat{q}_n^3(t)\right)^T; \\ &\widehat{q}(t) = \left(\widehat{q}^{1T}(t), \widehat{q}^{2T}(t), \widehat{q}^{3T}(t)\right). \end{aligned} \tag{38}$$

Then, according to the characteristic of L, (6)–(8) can be substituted as

$$\begin{cases} \dot{\widehat{p}}^1(t) = \widehat{q}^1(t), \\ \dot{\widehat{q}}^1(t) = -k_1\widehat{L}_{11}\widehat{p}^1(t-r(t)) - k_1\widehat{L}_{12}\widehat{p}^2(t-r(t)) - k_1\widehat{L}_{13}\widehat{p}^3(t-r(t)) + \left(-k_2\widehat{L}_1 - k_3I_l\right)\widehat{q}^1(t) - k_2\widehat{L}_2\widehat{q}^2(t) + f(p(t),t), \end{cases} \tag{39}$$

$$\begin{cases} \dot{\widehat{p}}^2(t) = \widehat{q}^2(t), \\ \dot{\widehat{q}}^2(t) = -k_1\widehat{L}_{21}\widehat{p}^1(t-r(t)) - k_1\widehat{L}_{22}\widehat{p}^2(t-r(t)) - k_1\widehat{L}_{23}\widehat{p}^3(t-r(t)) - k_2\widehat{L}_3\widehat{q}^1(t) + \left(-k_2\widehat{L}_4 - k_3I_{m-l}\right)\widehat{q}^2(t), \end{cases} \tag{40}$$

$$\dot{\widehat{p}}^3(t) = -\widehat{L}_{31}\widehat{p}^1(t-r(t)) - \widehat{L}_{32}\widehat{p}^2(t-r(t)) - \widehat{L}_{33}\widehat{p}^3(t-r(t)), \tag{41}$$

where

$$\begin{aligned}
\widehat{L}_{11} &= L_{s_1} + D_{s_1 s_2} + D_{s_1 f};\\
\widehat{L}_{12} &= -C_{s_1 s_2};\\
\widehat{L}_{13} &= -C_{s_1 f};\\
L_{s_1} &= D_{s_1} - C_{s_1};\\
\widehat{L}_{21} &= -C_{s_2 s_1};\\
\widehat{L}_{22} &= L_{s_2} + D_{s_2 s_1} + D_{s_2 f};\\
\widehat{L}_{23} &= -C_{s_2 f};\\
L_{s_2} &= D_{s_2} - C_{s_2};\\
\widehat{L}_{31} &= -C_{f s_1};\\
\widehat{L}_{32} &= -C_{f s_2};\\
\widehat{L}_{33} &= L_f + D_{f s_1} + D_{f s_2};\\
L_f &= D_f - C_f;\\
\widehat{L}_1 &= L_{s_1} + D_{s_1 s_2};\\
\widehat{L}_2 &= -C_{s_1 s_2};\\
\widehat{L}_3 &= -C_{s_2 s_1};\\
\widehat{L}_4 &= L_{s_2} + D_{s_2 s_1}.
\end{aligned} \tag{42}$$

Let
$\phi_1(t) = (\widehat{p}^{1T}(t), \widehat{q}^{1T}(t))^T$
$\phi_2(t) = (\widehat{p}^{2T}(t), \widehat{q}^{2T}(t))^T$
$\phi_3(t) = (\widehat{p}^{3T}(t), \widehat{q}^{3T}(t))$

Then, one have

$$\begin{aligned}
\dot{\phi}_1(t) &= M_5\phi_1(t-r(t)) + \overline{M}_5\phi_2(t-r(t)) + \widetilde{M}_5\phi_3(t-r(t))\\
&\quad + \overline{N}_5\phi_1(t) + \widetilde{N}_5\phi_2(t) + F,\\
\dot{\phi}_2(t) &= M_6\phi_1(t-r(t)) + \overline{M}_6\phi_2(t-r(t)) + \widetilde{M}_6\phi_3(t-r(t))\\
&\quad + \overline{N}_6\phi_1(t) + \widetilde{N}_6\phi_2(t),\\
\dot{\phi}_3(t) &= M_7\phi_1(t-r(t)) + \overline{M}_7\phi_2(t-r(t)) + \widetilde{M}_7\phi_3(t-r(t)),
\end{aligned} \tag{43}$$

where

$$\begin{aligned}
M_5 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{11} & 0\end{pmatrix},\\
\overline{M}_5 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{12} & 0\end{pmatrix},\\
\widetilde{M}_5 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{13} & 0\end{pmatrix},\\
\overline{N}_5 &= \begin{pmatrix} 0 & I_l\\ 0 & N_5\end{pmatrix},\\
\widetilde{N}_5 &= \begin{pmatrix} 0 & 0\\ 0 & -k_2\widehat{L}_2\end{pmatrix},\\
N_5 &= -k_2\widehat{L}_1 - k_3 I_l;\\
M_6 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{21} & 0\end{pmatrix},\\
\overline{M}_6 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{22} & 0\end{pmatrix},\\
\widetilde{M}_6 &= \begin{pmatrix} 0 & 0\\ -k_1\widehat{L}_{23} & 0\end{pmatrix},\\
\overline{N}_6 &= \begin{pmatrix} 0 & 0\\ 0 & -k_2\widehat{L}_3\end{pmatrix},\\
\widetilde{N}_6 &= \begin{pmatrix} 0 & I_{m-l}\\ 0 & N_6\end{pmatrix},\\
N_6 &= -k_2\widehat{L}_4 - k_3 I_{m-l};\\
M_7 &= \begin{pmatrix} 0 & 0\\ -\widehat{L}_{31} & 0\end{pmatrix},\\
\overline{M}_7 &= \begin{pmatrix} 0 & 0\\ -\widehat{L}_{32} & 0\end{pmatrix},\\
\widetilde{M}_7 &= \begin{pmatrix} 0 & 0\\ -\widehat{L}_{33} & 0\end{pmatrix},\\
F &= \begin{pmatrix} 0\\ f(p(t),t)\end{pmatrix}.
\end{aligned} \tag{44}$$

Theorem 2. *Group consensus of systems (39)–(41) can be attained under control protocol (9) if the following criterion holds:*

$$\begin{pmatrix} \Theta_3 & P_1 & \tau_M R_5 & \tau_M R_6\\ * & -R & 0 & 0\\ * & * & -\tau_M R & 0\\ * & * & * & -\tau_M R \end{pmatrix} < 0, \tag{45}$$

where R_5 *and* R_6 *both are arbitrary matrices;* $P_1, P_2, P_3, Q_1, Q_2, Q_3,$ *and* R *are positive definite matrices;* τ_M *is a constant;* $*$ *denotes symmetry.*

$$\Theta_3 = \begin{pmatrix} \overline{N}_5^T P_1 + P_1\overline{N}_5 + Q_1 + \tau_M \overline{N}_6^T R\overline{N}_6 + \rho^2 R & M_5^T P_1 + \tau_M M_6^T R\overline{N}_6 & \widetilde{N}_5^T P_1 + P_2\overline{N}_6 + \tau_M \widetilde{N}_6^T R\overline{N}_6 & \overline{M}_5^T P_1 + \tau_M \overline{M}_6^T R\overline{N}_6 & 0 & \widetilde{M}_5^T P_1 + \tau_M \widetilde{M}_6^T R\overline{N}_6 \\ P_1 M_5 + \tau_M \overline{N}_6^T R M_6 & -(1-u)Q_1 + \tau_M M_6^T R M_6 & P_2 M_6 + \tau_M \widetilde{N}_6^T R M_6 & \tau_M \overline{M}_6^T R M_6 & P_3 M_7 & \tau_M \widetilde{M}_6^T R M_6 \\ P_1\widetilde{N}_5 + \overline{N}_6^T P_2 + \tau_M \overline{N}_6^T R\widetilde{N}_6 & M_6^T P_2 + \tau_M M_6^T R\widetilde{N}_6 & P_2 M_6 + \tau_M \widetilde{N}_6^T R M_6 & \overline{M}_6^T P_2 + \tau_M \overline{M}_6^T R\widetilde{N}_6 & 0 & \widetilde{M}_6^T P_2 + \tau_M \widetilde{M}_6^T R\widetilde{N}_6 \\ P_1\overline{M}_5 + \tau_M \overline{N}_6^T R\overline{M}_6 & \tau_M M_6^T R\overline{M}_6 & \widetilde{N}_6^T P_2 + P_2\widetilde{N}_6 + Q_2 + \tau_M \widetilde{N}_6^T R\widetilde{N}_6 & -(1-u)Q_2 + \tau_M \overline{M}_6^T R\overline{M}_6 & P_3\overline{M}_7 & \tau_M M_6^T R\overline{M}_6 \\ 0 & M_7^T P_3 & 0 & \overline{M}_7^T P_3 & Q_3 & M_7^T P_3 \\ P_1\widetilde{M}_5 + \tau_M \overline{N}_6^T R\widetilde{M}_6 & \tau_M M_6^T R\widetilde{M}_6 & \tau_M \widetilde{N}_6^T R\widetilde{M}_6 + P_2\widetilde{M}_6 & \tau_M \overline{M}_6^T R\widetilde{M}_6 & P_3\widetilde{M}_7 & \tau_M M_6^T R\widetilde{M}_6 \end{pmatrix}. \tag{46}$$

Proof:. through Lyapunov functional theory, one set Lyapunov functions $V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7$ as follows:

$$\begin{aligned} V_1(t) &= \phi_1^T(t)P_1\phi_1(t); \\ V_2(t) &= \phi_2^T(t)P_2\phi_2(t); \\ V_3(t) &= \phi_3^T(t)P_3\phi_3(t); \\ V_4(t) &= \int_{t-r(t)}^{t} \phi_1^T(s)Q_1\phi_1(s)\mathrm{d}s; \\ V_5(t) &= \int_{t-r(t)}^{t} \phi_2^T(s)Q_2\phi_2(s)\mathrm{d}s; \\ V_6(t) &= \int_{t-r(t)}^{t} \phi_3^T(s)Q_3\phi_3(s)\mathrm{d}s; \\ V_7(t) &= \int_{-\tau_M}^{0}\int_{t+s}^{t} \dot{\phi}_2^T(v)R\dot{\phi}_2(v)\mathrm{d}v\mathrm{d}s, \end{aligned} \tag{47}$$

where $P_1, P_2, P_3, Q_1, Q_2, Q_3$, and R are all positive definite matrices. Then, calculating the derivation of the Lyapunov functions along the system (43), we have

$$\begin{aligned}
\dot{V}_3(t) &= \dot{\phi}_3^T(t)P_3\phi_3(t) + \phi_3^T(t)P_3\dot{\phi}_3(t),\\
&= \left(M_7\phi_1(t-r(t)) + \overline{M}_7\phi_2(t-r(t)) + \widetilde{M}_7\phi_3(t-r(t))\right)^T P_3\phi_3(t) + \phi_3^T(t)P_3\left(M_7\phi_1(t-r(t))\right.\\
&\quad \left.+ \overline{M}_7\phi_2(t-r(t)) + \widetilde{M}_7\phi_3(t-r(t))\right),\\
&= \phi_1^T(t-r(t))M_7^TP_3\phi_3(t) + \phi_2^T(t-r(t))\overline{M}_7^TP_3\phi_3(t) + \phi_3^T(t-r(t))\widetilde{M}_7^TP_3\phi_3(t) + \phi_3^T(t)P_3M_7\phi_1(t-r(t)) +\\
&\quad \phi_3^T(t)P_3\overline{M}_7\phi_2(t-r(t)) + \phi_3^T(t)P_3\widetilde{M}_7\phi_3(t-r(t)),\\
\dot{V}_4 &= \left(\int_{t-r(t)}^{t}\phi_1^T(s)Q_1\phi_1(s)\mathrm{d}s\right)'\\
&= \phi_1^T(t)Q_1\phi_1(t) - (1-\dot{r}(t))\phi_1^T(t-r(t))Q_1\phi_1(t-r(t)) \le \phi_1^T(t)Q_1\phi_1(t) - (1-u)\phi_1^T(t-r(t))Q_1\phi_1(t-r(t)),\\
\dot{V}_5 &= \left(\int_{t-r(t)}^{t}\phi_2^T(s)Q_2\phi_2(s)\mathrm{d}s\right)'\\
&= \phi_2^T(t)Q_2\phi_2(t) - (1-\dot{r}(t))\phi_2^T(t-r(t))Q_2\phi_2(t-r(t)) \le \phi_2^T(t)Q_2\phi_2(t) - (1-u)\phi_2^T(t-r(t))Q_2\phi_2(t-r(t)),\\
\dot{V}_6 &= \left(\int_{t-r(t)}^{t}\phi_3^T(s)Q_3\phi_3(s)\mathrm{d}s\right)'\\
&= \phi_3^T(t)Q_3\phi_3(t) - (1-\dot{r}(t))\phi_3^T(t-r(t))Q_3\phi_3(t-r(t)) \le \phi_3^T(t)Q_3\phi_3(t) - (1-u)\phi_3^T(t-r(t))Q_3\phi_3(t-r(t)),\\
\dot{V}_7 &= \left(\int_{-\tau_M}^{0}\int_{t+s}^{t}\dot{\phi}_2^T(v)R\dot{\phi}_2(v)\mathrm{d}v\mathrm{d}s\right)\prime,\\
&= \tau_M\dot{\phi}_2^T(t)R\dot{\phi}_2(t) - \tau_M\dot{\phi}_2^T(t+s)R\phi_2(t+s) = \tau_M\dot{\phi}_2^T(t)R\dot{\phi}_2(t) + \int_{t}^{t-\tau_M}\dot{\phi}_2^T(s)R\dot{\phi}_2(s)\mathrm{d}s,\\
&= \tau_M\left(M_6\phi_1(t-r(t)) + \overline{M}_6\phi_2(t-r(t)) + \widetilde{M}_6\phi_3(t-r(t)) + \overline{N}_6\phi_1(t) + \widetilde{N}_6\phi_2(t)\right)^T R\left(M_6\phi_1(t-r(t)) + \overline{M}_6\phi_2(t-r(t)) +\right)\\
&\quad \widetilde{M}_6\phi_3(t-r(t)) + \overline{N}_6\phi_1(t) + \widetilde{N}_6\phi_2(t)) + \int_{t}^{t-\tau_M}\dot{\phi}_2^T(s)R\dot{\phi}_2(s)\mathrm{d}s,\\
&= \tau_M\phi_1^T(t-r(t))M_6^TRM_6\phi_1(t-r(t)) + \tau_M\phi_1^T(t-r(t))M_6^TR\overline{N}_6\phi_1(t) + \tau_M\phi_1^T(t-r(t))M_6^TR\widetilde{N}_6\phi_2(t)\\
&\quad + \tau_M\phi_1^T(t-r(t))M_6^TR\overline{M}_6\phi_2(t-r(t)) + \tau_M\phi_1^T(t-r(t))M_6^TR\widetilde{M}_6\phi_3(t-r(t)) + \tau_M\phi_2^T(t-r(t))\overline{M}_6^TRM_6\phi_1(t-r(t))\\
&\quad + \tau_M\phi_2^T(t-r(t))\overline{M}_6^TR\overline{M}_6\phi_2(t-r(t)) + \tau_M\phi_2^T(t-r(t))\overline{M}_6^TR\widetilde{M}_6\phi_3(t-r(t)) + \tau_M\phi_2^T(t-r(t))\overline{M}_6^TR\overline{N}_6\phi_1(t)\\
&\quad + \tau_M\phi_3^T(t-r(t))\widetilde{M}_6^TRM_6\phi_1(t-r(t)) + \tau_M\phi_3^T(t-r(t))\widetilde{M}_6^TR\overline{M}_6\phi_2(t-r(t)) + \tau_M\phi_3^T(t-r(t))\widetilde{M}_6^TR\widetilde{M}_6\phi_3(t-r(t))\\
&\quad + \int_{t}^{t-\tau_M}\dot{\phi}_2^T(s)R\dot{\phi}_2(s)\mathrm{d}s + \tau_M\phi_2^T(t-r(t))\overline{M}_6^TR\widetilde{N}_6\phi_2(t) + \tau_M\phi_3^T(t-r(t))\widetilde{M}_6^TR\overline{N}_6\phi_1(t) + \tau_M\phi_3^T(t-r(t))\widetilde{M}_6^TR\widetilde{N}_6\phi_2(t)\\
&\quad + \tau_M\phi_1^T(t)\overline{N}_6^TRM_6\phi_1(t-r(t)) + \tau_M\phi_1^T(t)\overline{N}_6^TR\overline{M}_6\phi_2(t-r(t)) + \tau_M\phi_1^T(t)\overline{N}_6^TR\widetilde{M}_6\phi_3(t-r(t)) + \tau_M\phi_1^T(t)\overline{N}_6^TR\overline{N}_6\phi_1(t)\\
&\quad + \tau_M\phi_1^T(t)\overline{N}_6^TR\widetilde{N}_6\phi_2(t) + \tau_M\phi_2^T(t)\widetilde{N}_6^TRM_6\phi_1(t-r(t)) + \tau_M\phi_2^T(t)\widetilde{N}_6^TR\overline{M}_6\phi_2(t-r(t)) + \tau_M\phi_2^T(t)\widetilde{N}_6^TR\widetilde{M}_6\phi_3(t-r(t))\\
&\quad + \tau_M\phi_2^T(t)\widetilde{N}_6^TR\overline{N}_6\phi_1(t) + \tau_M\phi_2^T(t)\widetilde{N}_6^TR\widetilde{N}_6\phi_2(t).
\end{aligned} \tag{48}$$

As known by Lemma 2, we can obtain

$$2\phi_1^T(t)P_1F \le \phi_1^T(t)P_1R^{-1}P_1\phi_1(t) + F^TRF. \tag{49}$$

Through Lemma 1, (49) can become

$$2\phi_1^T(t)P_1F \le \phi_1^T(t)P_1R^{-1}P_1\phi_1(t) + \rho^2\phi_1^T(t)R\phi_1(t) = \phi_1^T(t)\left(P_1R^{-1}P_1 + \rho^2R\right)\phi_1(t). \tag{50}$$

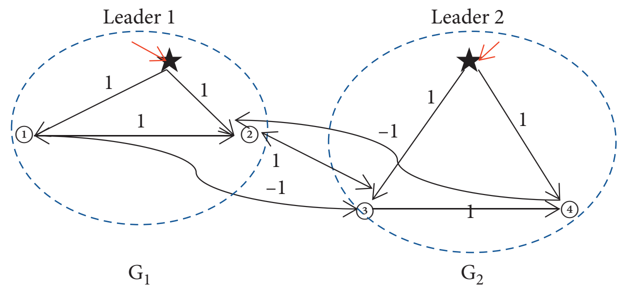

Figure 1: The topology construction of followers and leaders.

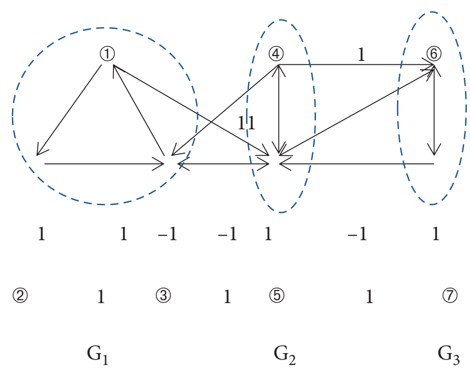

Figure 2: The topology of the followers and leaders.

Let $Z_3(t) = (\phi_1^T(t), \phi_1^T(t - r(t)), \phi_2^T(t), \phi_2^T(t - r(t)))^T$, then, we get

$$\dot{V}(t) \leq Z_3^T(t)\left(\Theta_3 + P_1 R^{-1} P_1\right) Z_3(t) + \int_t^{t-\tau_M} \dot{\phi}_2^T(s) R \dot{\phi}_2(s) ds + 2Z_3^T(t) R_5 \left[\phi_2(t) - \phi_2(t - r(t)) - \int_{t-r(t)}^t \dot{\phi}_2(s) ds\right] + 2Z_3^T(t) R_6 \left[\phi_2(t - r(t)) - \phi_2(t - \tau_M) - \int_{t-\tau_M}^{t-r(t)} \dot{\phi}_2(s) ds\right]. \tag{51}$$

From Lemma 2, one can obtain

$$\begin{aligned} -2Z_3^T(t) R_5 \int_{t-r(t)}^t \dot{\phi}_2(s) ds &\leq r(t) Z_3^T(t) R_5 R^{-1} R_5^T Z_3(t) + \int_{t-r(t)}^t \dot{\phi}_2^T(s) R \dot{\phi}_2(s) ds - 2Z_3^T(t) R_6 \int_{t-\tau_M}^{t-r(t)} \dot{\phi}_2(s) ds \\ &\leq \left(\tau_M - r(t)\right) Z_3^T(t) R_6 R^{-1} R_6^T Z_3(t) + \int_{t-\tau_M}^{t-r(t)} \dot{\phi}_1^T(s) R \dot{\phi}_1(s) ds. \end{aligned} \tag{52}$$

Thus, $\dot{V}(t)$ becomes

$$\begin{aligned} \dot{V}(t) &\leq Z_3^T(t)\left(\Theta_3 + P_1 R^{-1} P_1\right) Z_3(t) + r(t) Z_3^T(t) R_5 R^{-1} R_5^T Z_3(t) + \left(\tau_M - r(t)\right) Z_3^T(t) R_6 R^{-1} R_6^T Z_3(t) \\ &\leq Z_3^T(t)\left(\Theta_3 + P_1 R^{-1} P_1 + \tau_M R_5 R^{-1} R_5^T + \tau_M R_6 R^{-1} R_6^T\right) Z_3(t). \end{aligned} \tag{53}$$

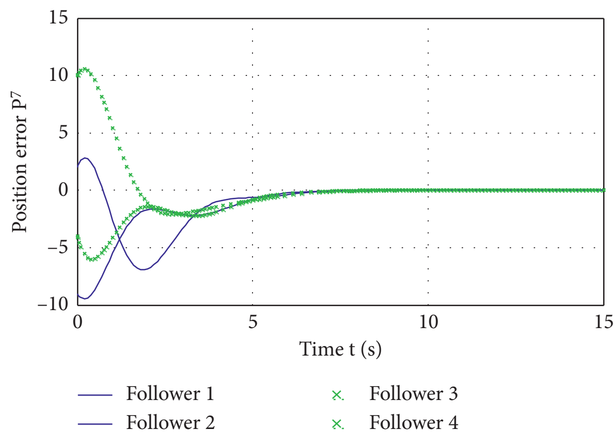

FIGURE 3: The position error trajectory.

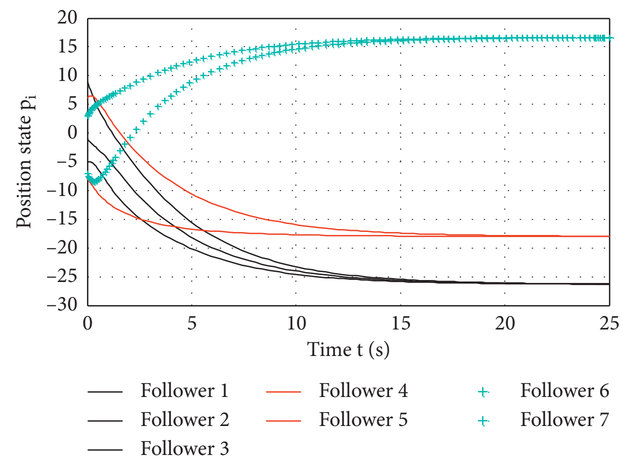

FIGURE 5: The position state trajectory.

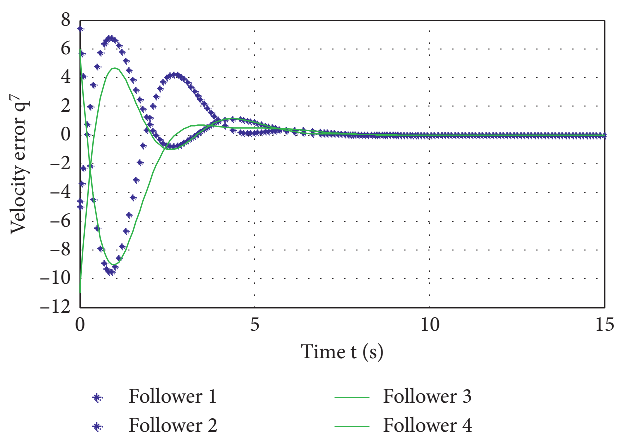

FIGURE 4: The velocity error trajectory.

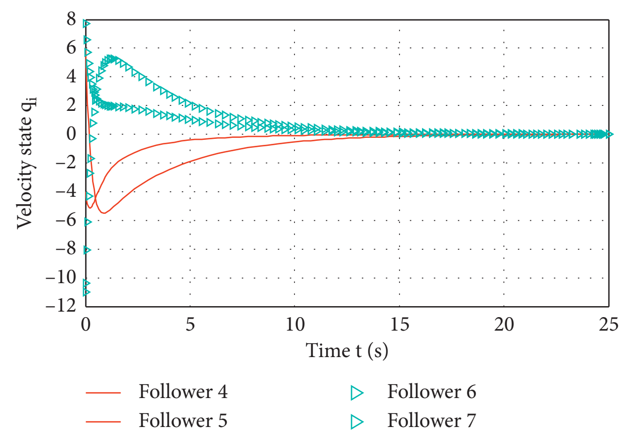

FIGURE 6: The velocity state trajectory.

Therefore, if the error system (43) contents the above condition which has been given in Theorem 2, the heterogeneous MAS can reach group consensus. □

4. Numerical Simulations

4.1. Active Leaders. In the condition of $f_i(p_i(t), t) = 0.15\sin(p_i(t))$ and $k = 2, \tau_M = 0.1$, when the agents belong of the second-order linear and nonlinear heterogeneous MAS, and the topology of the multiagent systems are given by Figures 1 and 2, then the error figures of the position and velocity graphs are shown as follows, respectively:

4.2. Static Leaders. When $k_1 = 1, k_2 = 2, k_3 = 1, \tau_M = 0.1$, and $f_i(p_i(t), t) = 0.15\sin(p_i(t))$, the agents of the first-order, second-order linear and nonlinear heterogeneous MAS, and relative state figures are as follows, respectively:

From Figures 3 and 4, it can be seen that all the tracks of the agents trend to balance position; in other words, each error curves all go to zero. From Figure 5, it can be observed that all trajectories of the agents which belong to the same group trend to the identical line. As you can see from Figure 6, all curves verge to a straight line; then, these graphs can illustrate that the systems achieve group consensus.

5. Conclusions

Based on neighbour's status information, the group consensus problem of heterogeneous MAS in the case of communication time lag was considered, and the corresponding control protocol was designed. For the active and static leaders, we considered the two-group and three-group problems for the multiagent systems. Via the methods of Lyapunov stability theory and linear matrix inequality, the sufficient condition for the heterogeneous MAS with a time delay to achieve group consensus. Finally, the given simulation examples are given to show the effectiveness of the obtained results.

Data Availability

The data used to support the findings of this study are available from the corresponding authors upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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