Research Article

Production and Financing Strategies of a Distribution Channel under Random Yield and Random Demand

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This study considers a distribution channel consisting of a manufacturer with capital constraint and uncertain yield and a retailer that faces random demand. To maintain production, the manufacturer can either (1) avail bank credit financing from a perfectly competitive market or (2) request advance payment from the retailer. First, we establish two Stackelberg game models under bank credit financing (BCF) and advance payment mechanism (APM). By comparing the equilibrium strategies of the two financing models and the optimal profits of channel members, we conclude that APM is more advantageous than BCF in terms of improving channel performance. Second, we design a revenue sharing (RS) contract based on APM to achieve channel coordination. Finally, numerical analysis is presented to verify the conclusions obtained in this study.

1. Introduction

Small- and medium-sized enterprises (SMEs) play an important role in promoting the economic development in economically developed or developing countries. It is difficult for SMEs to obtain enough financial access to commercial banks due to their low creditworthiness and high bankruptcy risk. The financing difficulty of SMEs hinders their development and further influences the performance of upstream and downstream partners, as well as the entire supply chain. An effective way to solve the financing difficulty of SMEs is to apply supply chain finance (SCF). As a special commercial financing model, SCF can provide comprehensive financial services for upstream and downstream enterprises along the channel. SCF can lower the financing costs, mitigate the risk, and substantially improve the financing efficiency of the entire channel [1, 2]. Bank credit financing (BCF) and trade credit financing (TCF) are the two main financing sources in SCF. BCF is an external financing instrument, and TCF is an internal financing scheme in which one enterprise extends its credit to an upstream/downstream member in the channel through short-term loans [3].

SME suppliers often have capital constraints for their enough production [4]. If the upstream suppliers cannot produce smoothly or deliver products in time, the downstream buyers will face the risk of supply shortage or even supply interruption, and the whole operation of the supply chain will be significantly affected. This motivates downstream participants such as giant retailers and third-party financing institutions to offer various financing programs to those capital-constrained suppliers [5]. In practice, many retailers in various industries, such as auto, aviation, fast-moving consumer goods, and beverage industries, have already provided advance payment to alleviate suppliers' capital constraints. For instance, in the auto industry in Europe, BMW and PSA pay suppliers in advance for parts [6]. In the aviation industry, Boeing paid $590 million to its fuselage supplier Vought Aircraft Industries to ensure normal supply for the Boeing 787 in 2009 [7]. Moreover, in the fast-moving consumer goods industry, the suppliers of Procter & Gamble Company also have opportunities to receive similar early payments to alleviate their financial constraints [8]. Further, in the beverage industry, Coca-Cola Beverages South Africa provided a $20 million fund to boost SME suppliers' development and procurement annuals from 2019 to 2024 [9].

Most studies on SCF have focused on large manufacturers financing SME retailers. Only a few (e.g., Yan et al. [5];

Kouvelis and Zhao [10]) have dealt with SME suppliers' (manufacturers') capital constraints problem. Further, most SCF research has assumed that the yield of the manufacturer is certain. In practice, yield uncertainty of the suppliers (manufacturers) is a common phenomenon. The production process in many industries is affected by yield randomness due to various reasons, such as unplanned machine maintenance, inferior raw materials, and machine failure. This phenomenon results in a mismatch between the planned and actual production [11]. Also in the agriculture industry, the crop output is uncertain due to many uncontrollable factors, such as weather condition, temperature, or natural disasters [12]. This yield randomness influences significantly the operational and financing decisions of a channel.

To address the concerns raised above, the current study considers a distribution channel consisting of a manufacturer with capital constraint and uncertain yield and a retailer that faces random demand. To maintain production, the manufacturer chooses either bank credit financing from a perfectly competitive market or advance payment from the retailer. Our goal in this study is to investigate the joint effects of random yield and financing on the entire channel when the procurement price is endogenously determined by the downstream retailer. Through comparing channel performance under APM and under BCF, this study focuses on assessing the value of APM in managing the manufacturer's financial distress under uncertain yield setting.

The contribution of this research to the field of SCF includes three aspects. Assuming that both the yield and the demand are uncertain, we first derive the optimal operational and financial strategies of channel members under BCF and APM. Through a comparative analysis, we conclude that APM is a better financing mode in improving channel performance compared with BCF. Second, we design an RS contract based on APM to achieve channel coordination. This mechanism enables the retailer and the manufacturer to share the sales revenue and stimulate the manufacturer to increase the production yield to the optimal quantity in the centralized system. Third, through numerical analysis, we find that the retailer is willing to offer APM with high production cost coefficient and with low initial capital. In addition, the retailer has less incentive to provide APM when the yield variability is big.

The remainder of this study is organized as follows. In Section 2, we review the relevant literature. The notations and assumptions used in the models are introduced in Section 3. The equilibrium strategies in the BCF and APM models are analyzed in Sections 4 and 5, respectively. The financing equilibrium is presented in Section 6. The centralized model is solved, and an RS mechanism based on APM is discussed in Section 7. The numerical analysis is presented in Section 8. The conclusion is provided in Section 9.

2. Literature Review

This study is closely related to two areas of the literature: production (inventory) and pricing strategies and supply chain finance. We next review these literatures.

In order to increase competition among companies and supply chains, organizations and enterprises ceaselessly try to extend their activities and also increase their profits or reduce their costs by adopting different inventory, production, and pricing strategies. Assuming that the demand is price-sensitive, Taleizadeh and Noori-daryan [13] investigated the supplier's and producer's optimal pricing policies and the retailers' optimal inventory policies by optimizing the total cost of the supply chain network. Taleizadeh et al. [14] analyzed the optimal pricing and ordering decisions of the manufacturer and the retailers in two competing supply chains by applying different composite coordinating strategies. Noori-daryan et al. [15] studied the optimal pricing and replenishment decisions of the manufacturer and the retailers under three different freight modes and a composite incentive contract. Noori-daryan et al. [16] analyzed the optimal pricing, ordering, promised lead time, and supplier selection decisions under three different game models: decentralization, concentration, and cooperation. In their models, the demand is assumed to be sensitive to selling price and promised delivery lead time. Taleizadeh et al. [17] proposed a pricing inventory model for a two-echelon supply chain in the presence of market segmentation, credit payment, and quantity discount policies. Taleizadeh et al. [18] developed an inventory model which optimizes the price, replenishment, and production policies of a VMI system with deteriorating items. Taleizadeh et al. [19] analyzed the pricing and ordering decisions in a three-level supply chain with imperfect quality item. Yu and Xiao [20] proposed two Stackelberg game models to investigate the pricing and service level decisions of a fresh agri-products supply chain consisting of one supplier, one retailer, and one third-party logistics provider. These above studies explored pricing and inventory decisions in certain yield setting. There are few studies considering pricing, producing, and ordering decisions in random yield situation. For example, Dong et al. [12] examined the effect of random yield on the sourcing decision under two pricing schemes. Gan et al. [21] developed a pricing model that analyzes the random yield effect of product returns on pricing decisions for short life-cycle products in a closed-loop supply chain with random yield and demands. Yu and Fan [22] analyzed the effect of random yield on the optimal decisions under different cost structures in a price-setting newsvendor model composed of a manufacturer with random yield and a retailer with random demand. In these above studies, the wholesale price for the products is determined by the producer. In our study, the retailer, as the buyer of the products, sets the procurement price. We thus extend the current producer-pricing literature by considering buyer-pricing in random yield and random demand setting.

In the supply chain, when upstream or downstream participants have insufficient operating capital, one of the most important issues that should be considered by the decision maker of the companies or supply chain is the interface between operations and financing. In recent years, research in the field of supply chain finance has become one of the most popular fields in operations and management. Xu and Birge [23] first investigated the joint production and

finance decisions in a newsvendor setting by considering different capital structure. Zhou and Groenevelt [24] compared TCF and BCF in a channel with a capital-constrained retailer and discovered that the latter is more advantageous than the former in terms of improving the channel performance. Conversely, Kouvelis and Zhao [25] argued that a retailer prefers TCF over BCF given a suitable trade credit contract. Chen [26] compared the conditions of BCF and TCF under revenue sharing (RS) contracts and found that both mechanisms can achieve channel coordination. Yan et al. [3] designed a credit guarantee scheme for an SCF system with a capital-constrained retailer. This scheme, which is a mixture of BCF and TCF, showed that credit guarantee is an effective way for solving the financing difficulties of SMEs. Yang and Birge [27] designed a portfolio of TCF and BCF to enhance supply chain efficiency by allowing the retailer to partially share the demand risk with the supplier. Wu et al. [28] investigated the operational and financial decisions of a supply chain with asymmetric competing newsvendors. Zhou et al. [29] examined the influence of two different leadership structures on financial policies when the manufacturer or 3PL can act as the guarantor for the retailer's bank loans.

All these above papers mainly assumed the retailer faces capital constraint and focused on supplier financing used to alleviate SME retailers' capital constraints. However, some papers also focused on buyer financing when SME suppliers (manufacturers) face capital constraint. For example, Deng et al. [7] studied the issue of financing multiple heterogeneous capital-constrained suppliers in an assembly system with random demand. Tunca and Zhu [4] modeled a game theoretical model to analyze the role and efficiency of buyer intermediation in supplier finance. They showed that buyer financing can significantly improve the profits of chain members and the whole chain. Given manufacturers' capital constraint, Zhang et al. [30] studied the optimal green investment decisions of the channel under advance payment financing from the retailer. Yan et al. [31] compared two financing schemes provided by the loss-averse retailer to the capital-constrained supplier: loan and investment. They examined the impact of the retailer's loss aversion level on financing decisions. Chen et al. [32] compared three financing schemes, i.e., zero-interest early payment financing and positive-interest in-house factoring financing in a pull supply chain where the manufacturer is capital constrained. They found that early payment financing is more attractive than bank financing for the retailer when the manufacturer's production cost is low.

All these studies on buyer financing assumed that the yield of the suppliers (manufacturers) is certain. However, some studies also explored the issue in the uncertain yield situation. Tang et al. [33] compared buyer financing and bank financing in a channel in which the supplier is unreliable and capital constrained. Yuan et al. [34] investigated the manufacturer's channel preference choice in a channel with a well-capitalized and reliable supplier and a capital-constrained and unreliable supplier with random yield. Guo et al. [35] studied the financing strategies for a coal-electricity supply chain under yield uncertainty. They showed

that the profits of the channel members under APM are higher than those under BCF. Ding and Wan [36] investigated the financing and coordinating issues of a supply chain with random yield. They found that the manufacturer is willing to pay in advance to the capital-constrained supplier.

Although a few studies have examined the joint effect of random yield and financing in a channel, they assumed that the wholesale price is exogenous and seldom theoretically explored the impact of wholesale pricing on the production and financing strategies. In the current study, we investigate the production, pricing, and financing decisions of a distribution channel consisting of a manufacturer with capital constraint and a retailer. The key difference of the present study from the existing ones is that the operational and financing strategies of the channel are analyzed under random yield and random demand. In our models, the manufacturer is assumed to be capital constrained and the procurement price of the product is endogenously determined by the retailer.

3. Model Description

3.1. Problem Description. In this study, we consider a single-period distribution channel where the manufacturer faces capital constraint and the retailer possesses abundant capital. The retailer purchases all products produced by the manufacturer before the sales season. The market demand is assumed to be uncertain. The manufacturer faces yield randomness and capital pressure during production. To maintain production, the manufacturer must rely on the support of external funds. The manufacturer borrows from a commercial bank or applies request advance payment from the retailer. If the yield at the end of production is not sufficient to cover the bank loan or the advance payment, the manufacturer becomes bankrupt. In this case, the bank or the retailer faces the loss risk. In other words, the manufacturer has limited liability [26].

3.2. Notations and Assumptions. The notations and assumptions used in this study are enumerated as follows:

(i) Parameters

  (i) c: manufacturer's production cost coefficient

  (ii) p: retailer's unit retail price

  (iii) r_f: risk-free interest rate

  (iv) X: random demand, $X \geq 0$

  (v) Y: random yield rate, $Y \in [0, n]$

  (vi) r_b: bank's interest rate

  (vii) ϕ: manufacturer's sharing ratio of sales revenue

(ii) Decision variables

  (i) q: manufacturer's production quantity

  (ii) w: unit procurement price set by the retailer

We use the notation $\pi_i^j(\cdot)$, where $i = m, r$ and $j = b, t, \text{RS}, 0$ to express the expected profit of each decision

maker, where subscripts m and r denote the manufacturer and retailer, respectively, and superscripts b, t, RS and 0 represent the BCF model, APM model, RS mechanism based on advance payment, and centralized model, respectively. In addition, q_j and $w_j (j = b, t, \phi, 0)$ represent the manufacturer's production quantity and the retailer's procurement price, respectively, where subscripts b, t, ϕ, and 0 denote the BCF model, APM model, RS mechanism based on advance payment, and centralized model, respectively. The symbol "*" indicates the optimal solution.

In this study, we assume that the bank market is perfectly competitive and the manufacturer can receive unlimited financial support at a risk-free interest rate r_f. Without loss of generality, r_f is normalized to zero. We also assume that initial capital of the manufacturer is zero.

Given the increasingly expensive production capacity (or input), we assume that the manufacturer's production technology exhibits diseconomies of scale [37, 38]. By adopting the ideas of Anand and Mendelson [39] and Ha et al. [40], we set $C(q) = cq^2/2$ as the production cost function. A large value of this function signifies a great production diseconomy.

We assume that the manufacturer, as the Stackelberg game leader, determines the procurement price (similar assumptions have been used by Niu et al. [41] and Chen et al. [32]). In practice, Dongfanghong, one of the largest green onion packers in China, sets the wholesale price to the farmers. Those farmers are provided technical assistance in the production process.

The notations $f(\cdot)$ and $F(\cdot)$ denote the probability density function (PDF) and cumulative density function (CDF) of demand X, respectively. We assume that $F(\cdot)$ is a differential and strictly increasing function and $f(\cdot)$ is absolutely continuous within $[0, \infty)$. The PDF and CDF of the yield rate Y are represented by $g(\cdot)$ and $G(\cdot)$, respectively, and the complementary CDF is denoted by $\overline{G}(\cdot) = 1 - G(\cdot)$. We assume that Y is within the range $[0, n]$, where $n > 0$ is the upper bound of the random yield. Moreover, we presume that $G(\cdot)$ is a differential and strictly increasing function and $g(\cdot)$ is absolutely continuous over $[0, n]$. The hazard rate of the random yield and the generalized failure rate are represented by $h(y) = g(y)/\overline{G}(y)$ and $H(y) = yh(y)$, respectively. We assume that $h(y)$ is increasing in y and $H(y)$ monotonically increases with y. These properties are satisfied by common distributions, such as the uniform, normal, exponential, and Weibull families [3, 26]. Moreover, X and Y are assumed to be independent random variables with expected values μ_X and μ_Y, respectively.

The residual value and return policy for unsold products are not considered in this study. Furthermore, the manufacturer and retailer are considered risk neutral, and all information are common knowledge to all channel members.

4. Bank Financing Model

In this section, we analyze the channel equilibrium adopted by the manufacturer under BCF. In this model, the retailer is assumed to be the Stackelberg game leader and the manufacturer is the follower. First, the retailer selects the procurement price w_b of the product to maximize its own expected profit. Second, the manufacturer plans the amount of production quantity q_b after observing w_b. Third, the bank sets the interest rate r_b in accordance with the zero-profit condition.

We use the backward inductive method to solve the channel equilibrium. To support production, the manufacturer's $L(q_b)$ from the bank is equal to the production cost $C(q_b)$. When lending loan to the manufacturer, the expected profit of the bank is zero because the bank market is assumed to be completely competitive.

At the end of production, the manufacturer's revenue is Yq_bw_b, and the expected repayment to the lending bank is $\min\{Yq_bw_b, (1 + r_b^*)L(q_b)\}$. The lending bank's cost is $(1 + r_f)L(q_b)$, where $r_f = 0$ based on the assumption in Section 3.2.

The expected profit of the manufacturer under BCF is expressed as

$$\pi_m^b(q_b) = E\left[\left(Yq_bw_b - (1 + r_b^*)L(q_b)\right)^+\right], \qquad (1)$$

where $z^+ = \max(z, 0)$, Yq_bw_b is the revenue of the manufacturer, and $(1 + r_b^*)L(q_b)$ is the cost of the loan.

Lemma 1. *The decision-making problem of the manufacturer under BCF is equivalent to her decision-making problem without capital constraint.*

Lemma 1 demonstrates that under BCF and limited liability, the manufacturer's capital constraint has no influence on the production decision because the complete competitiveness of the bank market makes the profit of the lending bank zero. The break-even rate r_b^* serves as a compensation for the losses incurred by the bank due to the default of the manufacturer. In this case, the manufacturer uses bank loans as her own funds to produce products.

According to Lemma 1, the expected profit of the retailer under BCF is expressed as

$$\pi_r^b(w_b) = E[p\min(Yq_b, X) - w_bq_bY], \qquad (2)$$

where $p\min(Yq_b, X)$ and w_bq_bY are the sales revenue and purchasing cost of the retailer, respectively.

Proposition 1. *Under BCF, the optimal production quantity q_b^* and procurement price w_b^* satisfy*

$$w_b^*\mu_Y = cq_b^*, \qquad (3)$$

$$p\left[\mu_Y - \int_0^n yg(y)F(q_b^*y)dy\right] = 2cq_b^*, \qquad (4)$$

respectively.

The proof for this proposition is provided in Appendix A.

According to Proposition 1, the optimal production quantity increases with the optimal procurement price. This conclusion is consistent with the actual scenario. The higher the procurement price set by the retailer, the more product

produced by the manufacturer. In addition, the optimal production quantity depends on the demand and yield distributions, production cost coefficient, and retail price.

5. Advance Payment Financing Model

This section presents the analysis of the equilibrium strategies of the channel members under APM. As the leader, the retailer sets the procurement price w_t of the product to maximize the expected profit. The manufacturer, as the follower, then selects the production quantity q_t after observing w_b. The backward inductive method is used to solve the channel equilibrium.

5.1. Manufacturer's Production Quantity Decision.
According to the assumption in Section 3.2, the initial capital of the manufacturer is zero. At the beginning of production, the amount of advance payment made by the retailer is $L(q_t)$, which is equal to the manufacturer's production cost $C(q_t)$. If the sales revenue Yq_tw_t at the end of production is not sufficient to pay the advance payment, the manufacturer becomes bankrupt, and the expected profit is zero. Otherwise, the manufacturer can obtain the remaining balance after deducting the advance payment amount. It is worth noting that we assume here that the retailer will pay the remaining payment to the manufacturer at the end of production regardless of the retailer's sales revenue (or whether the realized demand is high or low). Therefore, the expected profit of the manufacturer under APM is

$$\pi_m^t(q_t) = E\left[\left(Yq_tw_t - L(q_t)\right)^+\right].\tag{5}$$

Let

$$\widehat{y}(q) = \frac{cq}{2w},\tag{6}$$

which represents a critical value. If the yield is lower than the critical value, that is $Y < \widehat{y}(q)$, the amount of advance payment cannot be deducted from the revenue, and the manufacturer becomes bankrupt.

Proposition 2. *Under APM, for any given w_t, the manufacturer's optimal production quantity q_t^* satisfies*

$$w_t \int_{\widehat{y}(q_t^*)}^n yg(y)dy - cq_t^*\overline{G}(\widehat{y}(q_t^*)) = 0.\tag{7}$$

The proof of this proposition is discussed in Appendix A.

Proposition 2 provides the manufacturer's optimal response function to describe the relationship between the optimal production quantity q_t^* and the procurement price w_t set by the retailer.

Corollary 1. *Under APM, q_t^* is increasing in w_t (i.e., $dq_t^*/dw_t > 0$).*

The proof is provided in Appendix A.

Corollary 1 shows that under APM, the manufacturer will increase the production quantity when the retailer increases the procurement price. This phenomenon is consistent with the conclusion under BCF.

5.2. Retailer's Optimal Pricing Decision.
Under APM, the expected profit function of the retailer given the manufacturer's best response function $q_t^* = q_t^*(w_t)$ is

$$\pi_r^t(w_t) = pE[\min(Yq_t^*, X)] - L(q_t^*) - E\left[\left(Yq_t^*w_t - L(q_t^*)\right)^+\right],\tag{8}$$

where the first, second, and third items represent the sales revenue, advance payment, and subsequent payment to the manufacturer of the retailer, respectively.

Proposition 3. *Under APM, the retailer's optimal procurement price w_t^* satisfies*

$$w_t^* = \begin{cases} w_t^\Delta, & \mu_Y \leq \overline{\mu}_Y, \\ w_t^{\Delta\Delta}, & \mu_Y > \overline{\mu}_Y, \end{cases}\tag{9}$$

where w_t^Δ and $w_t^{\Delta\Delta}$ satisfy

$$p\left[\mu_Y - \int_0^n yg(y)F(q_t^*y)dy\right] = cq_t^* + w_t^\Delta \int_{\widehat{y}(q_t^*)}^n yg(y)dy,\tag{10}$$

$$w_t^{\Delta\Delta} = \frac{\mu_Y w_b^*}{2\widehat{y}(q_t^*)\sqrt{\overline{G}(\widehat{y}(q_t^*))}},\tag{11}$$

respectively, and $\overline{\mu}_Y$ is defined as $\overline{\mu}_Y = (2\widehat{y}(q_t^*)w_t^\Delta \sqrt{\overline{G}(\widehat{y}(q_t^*))}/w_b^*$.

The proof of this proposition is presented in Appendix A.

Proposition 3 indicates that there exists an random yield expected value threshold point $\overline{\mu}_Y$. The bankruptcy risk of the manufacturer is high when $\mu_Y \leq \overline{\mu}_Y$. In this case, the bank will set a high break-even rate to balance the manufacturer's default risk. Thus, the manufacturer is willing to participate in APM. Then, the retailer will choose the optimal procurement price w_t^Δ (equation (10)) to maximize her expected profit. By contrast, the bankruptcy risk of the manufacturer is low when $\mu_Y > \overline{\mu}_Y$. In the situation, the bank will set a low break-even rate. To attract the manufacturer to participate in APM, the procurement price w_t is adjusted to $w_t^{\Delta\Delta}$ by using equation (11) which makes the manufacturer's profit under APM not lower than that under BCF. That is, the retailer compromises some profits to the manufacturer. This proposition also suggests that the retailer plays a dual role under APM as the purchaser and the financier. Therefore, the retailer's procurement price w_t^* includes a discount due to the provision of advance payment to the manufacturer.

6. The Financing Equilibrium

In this section, we compare the optimal profits of all channel members and the whole channel under APM and BCF and present the financing equilibrium of the channel.

Definition 1. One value of the procurement price $\underline{w}$ under APM is defined as follows. When $w_t = \underline{w}$ and $q_t = q_b^*$, the manufacturer's profits under APM are equal to her optimal profits under BCF; that is, $\pi_m^t(q_b^*, \underline{w}) = \pi_m^b(q_b^*, w_b^*)$.

Lemma 2

  (i) $\underline{w}$ represents the lower bound of the procurement price under APM (i.e., $w_t \geq \underline{w}$)

  (ii) When $w_t = \underline{w}$ and $q_t = q_b^*$, the retailer's profits under APM are equal to her optimal profits under BCF (i.e., $\pi_r^t(q_b^*, \underline{w}) = \pi_r^b(q_b^*, w_b^*)$)

The proofs are provided in Appendix A.

Lemma 2 (i) indicates that if the production quantity under APM is equal to the optimal production quantity under BCF, w_t is not lower than $\underline{w}$. This implies that the manufacturer would opt for bank financing if w_t is lower than $\underline{w}$. Lemma 2 (ii) shows that similar to the manufacturer, the retailer gains the same profits under APM as that under BCF when $w_t = \underline{w}$ and $q_t = q_b^*$.

Proposition 4. *Under APM, $q_t^* > q_b^*$ for $w_t \geq \underline{w}$.*

The proof for this proposition is provided in Appendix A.

Proposition 4 states that for any given procurement price $w_t \geq \underline{w}$, the manufacturer produces more products under APM than under BCF. Hence, the retailer gains high sales revenue, which is beneficial for promoting the efficiency of the entire channel.

Proposition 5. *The manufacturer's optimal profit under APM is not lower than that under BCF, and the retailer's optimal profits under APM are larger than those under BCF.*

The proof for this proposition is provided in Appendix A.

Proposition 5 suggests that channel performance is better under APM than under BCF, and APM is the financing equilibrium of the channel. Under BCF, the retailer only plays a role as the distributor. In contrast, under APM the retailer plays a dual role as the distributor and the financier. Therefore, APM leads to a higher degree of channel integration over BCF which mitigates inefficiency caused by double marginalization.

7. RS Incentive Mechanism Based on APM

The previous analysis focuses on the equilibrium strategies in a decentralized system, wherein each channel member tries to maximize its own profit without considering the other members. In this case, the total channel profit is low due to the "double marginalization effect." To eliminate this effect, common contracts, such as RS [41], buyback [42], and quantity discount [43, 44], are designed to coordinate the channel. Channel coordination aims to make the profit of a decentralized system under a contract equal to that of a centralized system.

We solve the centralized model and perform a comparison analysis of the optimal production quantities in centralized and decentralized systems. Then, we design an RS contract based on APM to achieve channel coordination. RS contract is a popular mechanism in channel coordination.

7.1. Centralized Model. In a centralized system, the channel, as a whole, can be regarded as a noncapital constraint because the retailer is well-funded. The sole decision maker of the channel decides the production quantity q_0 and maximizes the expected profit of the entire channel.

The expected profit of the channel in the centralized model is expressed as

$$\pi^0(q_0) = pE\min(Yq_0, X) - C(q_0), \qquad (12)$$

where $pE\min(Yq_0, X)$ and $C(q_0)$ represent the sales revenue and production cost of the channel, respectively.

Similar to the analysis in Section 5, the optimal production quantity q_0^* of the channel in the centralized model is obtained as

$$p\left[\mu_Y - \int_0^n yg(y)F(q_0^* y)dy\right] = cq_0^*. \qquad (13)$$

Similar to the proof of Proposition 4, we can easily get the following result.

Proposition 6. $q_t^* < q_0^*$.

Proposition 6 implies that the optimal production quantity under APM is smaller than that in the centralized model. Therefore, APM does not achieve channel coordination. This mechanism will achieve channel coordination if the optimal production quantity decision in the decentralized model is consistent with that in the centralized one (i.e., $q_0^* = q_t^*$). We next explore such a mechanism which realizes channel coordination under APM.

7.2. RS Contract Based on APM. We design an RS contract (ϕ, w_ϕ) based on APM to achieve channel coordination, where w_ϕ denotes the procurement price paid by the retailer per unit product and $\phi (0 < \phi < 1)$ is the ratio of sales revenue shared by the manufacturer. The retailer obtains the remaining ratio $(1 - \phi)$ of the sales revenue.

The expected profits of the manufacturer and the retailer under the RS contract based on APM are expressed as

$$\pi_m^{TR}(q_\phi, w_\phi) = E\left[\phi p\min(Yq_\phi, X) + (Yq_\phi w_\phi - L(q_\phi))^+\right] \qquad (14)$$

$$\pi_r^{TR}(q_\phi, w_\phi) = (1 - \phi)pE\min(Yq_\phi, X) - C(q_\phi)$$
$$- E\left[(Yq_\phi w_\phi - L(q_\phi))^+\right], \qquad (15)$$

respectively.

Proposition 7. *An APM-based RS contract (ϕ, w_ϕ) with parameters satisfying*

$$\int_{\widehat{y}(q_\phi)}^n yg(y)dy = 2\widehat{y}(q_\phi)\left[\overline{G}(\widehat{y}(q_\phi)) - \phi\right], \qquad (16)$$

where $\widehat{y}(q_\phi) = cq_\phi/2w_\phi$, can coordinate the channel.

TABLE 1: Relative payoffs of the manufacturer and the retailer under APM and BCF.

c	N					
	0.8	1	1.2	1.4	1.6	1.8
0.01	(1, 1.06)	(1, 1.05)	(1, 1.04)	(1, 1.03)	(1, 1.03)	(1, 1.03)
0.02	(1, 1.09)	(1, 1.07)	(1, 1.06)	(1, 1.05)	(1, 1.04)	(1, 1.04)
0.03	(1, 1.11)	(1, 1.09)	(1, 1.07)	(1, 1.06)	(1, 1.06)	(1, 1.05)
0.04	(1, 1.12)	(1, 1.10)	(1, 1.09)	(1, 1.07)	(1, 1.06)	(1, 1.06)
0.05	(1, 1.12)	(1, 1.11)	(1, 1.09)	(1, 1.08)	(1, 1.07)	(1, 1.06)
0.06	(1, 1.13)	(1, 1.12)	(1, 1.10)	(1, 1.09)	(1, 1.08)	(1, 1.07)
0.07	(1, 1.14)	(1, 1.12)	(1, 1.11)	(1, 1.10)	(1, 1.08)	(1, 1.08)
0.08	(1, 1.14)	(1, 1.13)	(1, 1.11)	(1, 1.10)	(1, 1.09)	(1, 1.08)
0.09	(1, 1.14)	(1, 1.13)	(1, 1.12)	(1, 1.10)	(1, 1.09)	(1, 1.09)
0.1	(1, 1.15)	(1, 1.13)	(1, 1.12)	(1, 1.11)	(1, 1.10)	(1, 1.09)

The proof for this proposition is provided in Appendix A.

Proposition 7 suggests that under APM, the RS contract enables the retailer and the manufacturer to share the yield and sales risks, as well as the sales revenue. This mechanism encourages the manufacturer to increase the production quantity to the optimal amount in the centralized model. As a result, channel coordination is realized. This proposition also indicates that yield randomness greatly influences the structure of the contract parameters.

If the manufacturer's revenue sharing ratio ϕ and the procurement price w_ϕ satisfy equation (16), the expected profit of the channel can reach the expected profit of the centralized system. However, this mechanism is advantageous only when the profits of all channel members under it are not lower than their profits under the APM decentralized system. Therefore, the parameters of this mechanism must ensure that $\pi_r^{TR} \geq \pi_r^t, \pi_m^{TR} \geq \pi_m^t$ to determine the mutually beneficial range of ϕ, which is referred to as the Pareto zone. In addition, the specific value of ϕ depends on the bargaining power of the channel members.

8. Numerical Analysis

In this section, a numerical experiment is conducted to illustrate the conclusions obtained from the previous analysis. In addition, the coordination effect of the RS contract based on APM in the channel is examined. We assume that the random variable Y of the yield fluctuation is uniformly distributed within $[0, n]$, where $n \in [0.8, 1.8]$, and has a mean of $\mu_Y = (n/2), n \in [0.8, 1.8]$. The demand X is assumed to be exponentially distributed with mean $\mu_X = 100$. Moreover, the retail price is assumed to be $p = 10$, and the production cost coefficient is $c \in [0.01, 0.1]$.

Table 1 presents the effect of the production cost coefficient and the upper bound of the random yield rate on the payoffs of the channel members under APM with respect to their payoffs under BCF $(\pi_m^t/\pi_m^b, \pi_r^t/\pi_r^b)$. The retailer's profit under APM is higher than that under BCF for any given n and c. Meanwhile, the manufacturer's profit under APM is not lower than that under BCF. Therefore, the channel performance can be improved under APM. In Table 1, the

procurement prices are adjusted to ensure that the manufacturer could participate in APM. In addition, the retailer is willing to provide APM with high production cost when the upper bound of the random yield rate remains unchanged. When the yield variability increases, the incentive for the retailer to provide APM decreases.

Next, we examine the impact of the manufacturer's initial working capital on the financing strategies of channel members. Assumed that $n = 1$, $c = 0.05$, and $p = 10$, Figures 1–3 show the impact of the manufacturer's initial capital B on the profits of the manufacturer, retailer, and whole channel, respectively. From Figures 1 and 2, we see that the manufacturer prefers BCF over APM while the retailer prefers APM over BCF when the manufacturer's initial capital B is low. When the manufacturer's initial capital B is high, the opposite situation appears. Moreover, manufacturer's profit is monotonically increasing in B under APM, while the retailer's situation is opposite. Figure 3 shows the decentralized channel profit is smaller than the centralized channel profit, and channel performance is better off under APM when the manufacturer's initial capital B is low. This is mainly because the manufacturer with less initial capital would produce more products due to her limited liability, which could realize a higher channel profit.

Subsequently, we examine the coordination effect of the RS contract based on APM on the channel. The assumed parameter values are $n = 1$, $c = 0.05$, and $p = 10$. Figure 4 depicts the effect of the manufacturer's revenue sharing ratio on the coordinated procurement price.

According to equation (16), we obtain the feasible region of ϕ is (0, 0.134). The coordinated procurement price increases with ϕ (Figure 4), which might be because the manufacturer's marginal revenue is lower than the marginal cost when ϕ is within the feasible region. This phenomenon results in a negative profit of the manufacturer before sharing the sales revenue. Therefore, ϕ must be increased to compensate the loss of the manufacturer.

Figure 5 illustrates the influence of ϕ on the coordinated profits of the channel members. The coordinated profit of the manufacturer and retailer increases and decreases, respectively, as ϕ increases. The figure also shows that a Pareto zone $[a, b]$
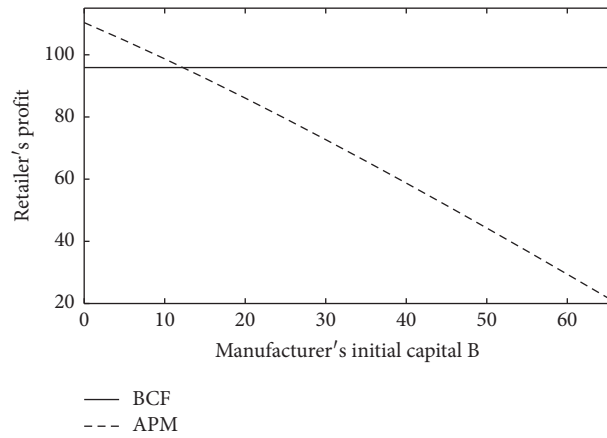
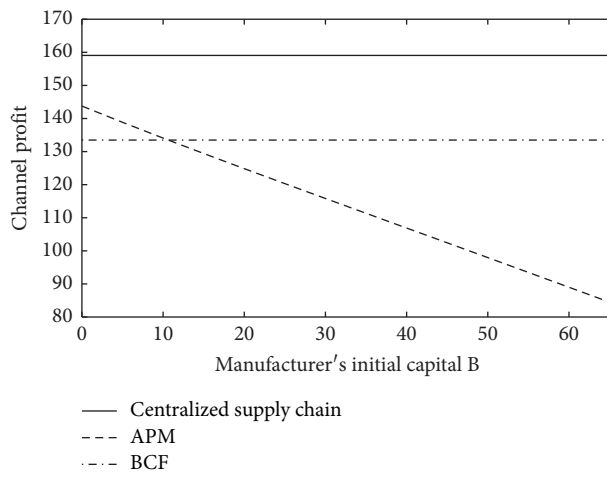FIGURE 1: Impact of the manufacturer's initial capital on the retailer's profit.

FIGURE 2: Impact of the manufacturer's initial capital on the manufacturer's profit.
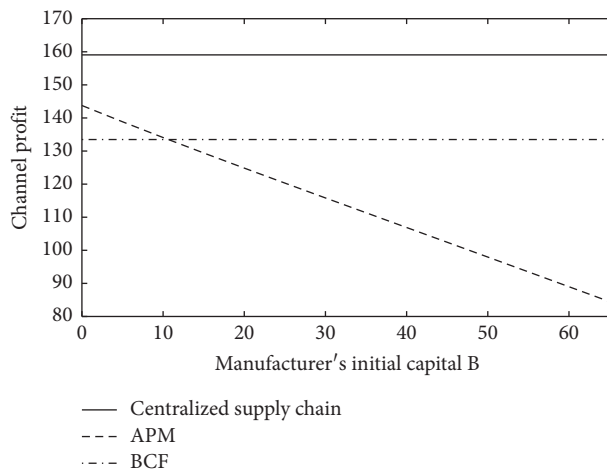

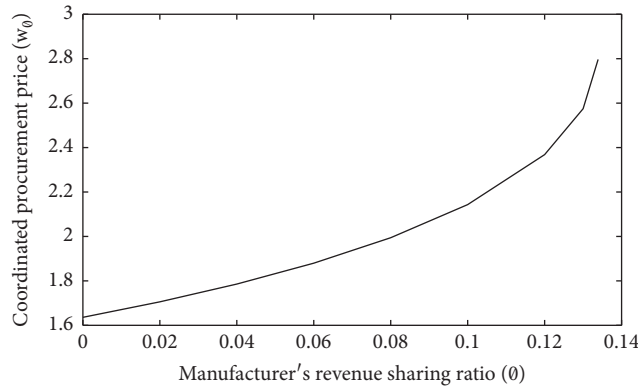FIGURE 3: Impact of the manufacturer's initial capital on the channel profit.

FIGURE 4: Impact of the manufacturer's revenue sharing ratio on the coordinated procurement price.
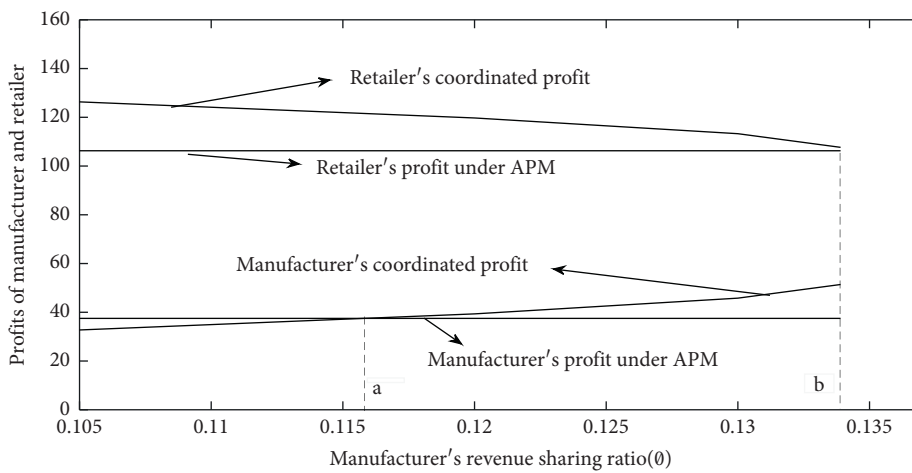

FIGURE 5: Impact of the manufacturer's revenue sharing ratio on the coordinated profits of channel members.

exists under the RS contract based on APM. This result implies that as long as ϕ is within the Pareto zone, the RS mechanism can enable all channel members to realize Pareto improvement. In other words, this mechanism renders a "win-win" result.

9. Conclusions

This study investigates a distribution channel where the retailer with abundant capital can finance the manufacturer with capital constraint and uncertain yield. The manufacturer's production scale is assumed to be uneconomical due to the expensive production capacity or input. Assuming that the retailer is the Stackelberg game leader, we derive the optimal operational and financial strategies of channel members under BCF and APM. Through a comparative analysis, we conclude that the manufacturer's optimal profit under APM is not lower than that under BCF, and the retailer's optimal profits under APM are larger than those under BCF. Therefore, APM can improve channel performance compared with BCF. Furthermore, an RS mechanism based on APM is introduced to coordinate the channel.

Through theoretical and numerical analysis, some significant conclusions can be draw. First, the retailer has less incentive to provide APM when the yield variability is big. This is because the retailer faces a higher financial risk when

the yield variability is big. Second, considering a quadratic production cost function in our model, we find that the retailer is willing to offer APM with high production cost coefficient which is different from the situation where the production yield of the manufacturer is certain. In the retailer financing supply chain where the yield is certain and linear production cost is considered, the retailer has no incentive to offer finance the manufacturer when the production cost is substantially high [32]. Third, the retailer is willing to offer APM when the manufacturer's initial capital is low. In addition, the retailer has no incentive to provide APM when the initial capital is high. This result is different from that presented in the manufacturer financing supply chain in which the manufacturer provides trade credit to the capital-constrained retailer. In the manufacturer financing supply chain, the manufacturer is always better off under trade credit independent of the retailer's initial capital [26]. Finally, a Pareto zone exists in the RS contract based on APM. When the revenue sharing ratio is within the Pareto zone, this mechanism can achieve a "win-win" result.

Some managerial insights are conveyed in this study. First, when SME manufacturers are subject to capital constraints for their production, downstream powerful retailers can provide manufacturers financing schemes, such as advance payment, to ensure better operations and achieve

higher performance for the entire channel. Second, as the leader of the supply chain, the core enterprise should design an effective coordination mechanism to promote cooperation between the two parties and facilitate the negotiations about splitting of additional profits. Third, some other factors, such as yield randomness and initial working capital, impact the implementation of APM. Therefore, the managers should evaluate all these factors before implementing a specific financing scheme.

Several issues will be investigated in the future. This study can be extended to a three-level supply chain or used to explore the equilibrium decisions when all channel members possess asymmetric information. Another possible extension is the analysis of the case where all channel members face financial problems.

Appendix

A. Proof of Lemma 1

Proof. In accordance with the zero-profit condition, the prevailing interest of the bank r_b^* satisfies the following equation:

$$L(q_b) = E[\min\{Yq_b w_b, (1 + r_b^*)L(q_b)\}]. \quad (A.1)$$

Equation (A.1) can be rewritten as

$$E[\min\{Yq_b w_b - L(q_b), L(q_b)r_b^*\}] = 0. \quad (A.2)$$

Note that $(x - y)^+ = x - \min\{x, y\}$, where $x = Yq_b w_b - L(q_b)$ and $y = L(q_b)r_b^*$. Therefore, equation (1) can be written as

$$\pi_m^b(q_b) = E[Yq_b w_b - L(q_b) - \min\{Yq_b w_b - L(q_b), L(q_b)r_b^*\}]. \quad (A.3)$$

Substituting equation (A.2) to (A.3) yields

$$\pi_m^b(q_b) = E[(Yq_b w_b - L(q_b))]. \quad (A.4)$$

□

B. Proof of Proposition 1

Proof. From equation (A.4), $\pi_m^b(q_b)$ can be rewritten as

$$\pi_m^b(q_b) = q_b w_b \mu_Y - \frac{cq_b^2}{2}. \quad (A.5)$$

Taking the first-order and second-order derivatives of $\pi_m^b(q_b)$ with respect to q_b, we get

$$\frac{d\pi_m^b(q_b)}{dq_b} = w_b \mu_Y - cq_b, \quad (A.6)$$

$$\frac{d^2\pi_m^b(q_b)}{dq_b^2} = -c. \quad (A.7)$$

Since $d^2\pi_m^b(q_b)/dq_b^2 < 0$, $\pi_m^b(q_b)$ is a strictly concave function in q_b. By the first-order condition of $d\pi_m^b(q_b)/dq_b = 0$, we get

$$q_b^* = \frac{w_b^* \mu_Y}{c}. \quad (A.8)$$

From equation (A.8), it follows that $dq_b^*/dw_b = \mu_Y/c$.
From equation (2), $\pi_r^b(w_b)$ can be further rewritten as

$$\pi_r^b(w_b) = pq_b^* \mu_Y - p\int_0^n g(y)dy \int_0^{yq_b^*}(yq_b^* - x)f(x)dx - w_b q_b^* \mu_Y. \quad (A.9)$$

Taking the first-order and second-order derivatives of $\pi_r^b(w_b)$ with respect to w_b, we get

$$\frac{d\pi_r^b(w_b)}{dw_b} = \frac{\mu_Y}{c}\left[p\mu_Y - w_b\mu_Y - p\int_0^n yg(y)F(q_b^* y)dy\right] - q_b^* \mu_Y,$$

$$\frac{d^2\pi_r^b(w_b)}{dw_b^2} = -\frac{\mu_Y}{c}\left[2\mu_Y + \frac{p\mu_Y}{c}\int_0^n y^2 g(y)f(q_b^* y)dy\right]. \quad (A.10)$$

Similarly, we can see that $\pi_r^b(w_b)$ is a strictly concave function in w_b since $d^2\pi_r^b(w_b)/dw_b^2 < 0$. Then, we have the first-order condition of $d\pi_r^b(w_b)/dw_b = 0$ as follows:

$$\left[p\mu_Y - w_b^* \mu_Y - p\int_0^n yg(y)F(q_b^* y)dy\right] = cq_b^*. \quad (A.11)$$

Combining equations (A.8) and (A.11), we conclude that equations (3) and (4) hold. □

C. Proof of Proposition 2

Proof. According to equations (5) and (6), $\pi_m^t(q_t)$ can be rewritten as

$$\pi_m^t(q_t) = \int_{\widehat{y}(q_t)}^n \left[yq_t w_t - \frac{cq_t^2}{2}\right]g(y)dy. \quad (A.12)$$

Similar to equations (A.6) and (A.7), we get

$$\frac{d\pi_m^t(q_t)}{dq_t} = w_t \int_{\widehat{y}(q_t)}^n yg(y)dy - cq_t\overline{G}(\widehat{y}(q_t)),$$

$$\frac{d^2\pi_m^t(q_t)}{dq_t^2} = -\frac{c}{2}\left[2\overline{G}(\widehat{y}(q_t)) - \widehat{y}(q_t)g(\widehat{y}(q_t))\right]. \quad (A.13)$$

According to the assumption in Section 2.2, we further get that

$$\frac{d^2\pi_m^t(q_t)}{dq_t^2} = -\frac{c}{2}\overline{G}(\widehat{y}(q_t))[2 - H(\widehat{y}(q_t))]. \quad (A.14)$$

Define $\overline{q}$ as the least upper bound on the set of points such that $H(\widehat{y}(q_t)) < 2$. Therefore, when $q_t \in [0, \overline{q}]$, we have $d^2\pi_m^t(q_t)/dq_t^2 < 0$ according to IGFR distribution of yield. Thus, $\pi_m^t(q_t)$ is a strictly concave function when $q_t \in [0, \overline{q}]$. Denote $\widetilde{q} = 2nw_t/c$ and $S(q_t) = d\pi_m^t(q_t)/dq_t$. Hence, when $q_t \in [\overline{q}, \widetilde{q}]$, we have $H(\widehat{y}(q_t)) \geq 2$. That is, $d^2\pi_m^t(q_t)/dq_t^2 > 0$. This means that $S(q_t)$ is an increasing function when $q_t \in [\overline{q}, \widetilde{q}]$. Note that $S(\widetilde{q}) = 0$ when $q_t = \widetilde{q}$. Thus, $S(q_t) \leq 0$

when $q_t \in [\overline{q}, \widetilde{q}]$. Therefore, $\pi_m^t(q_t)$ is an decreasing function when $q_t \in [\overline{q}, \widetilde{q}]$. In addition, note that $\pi_m^t(q_t) = 0$ when $q_t \in [\widetilde{q}, \infty]$.

Synthesizing the above analysis, we conclude that $\pi_m^t(q_t)$ is a unimodal function of q_t over $[0, \infty)$. Thus, q_t^* is unique. Equation (13) can be proved by first-order condition of $d\pi_m^t(q_t)/dq_t$. □

D. Proof of Corollary 1

Proof. Applying the implicit function theorem and taking the derivative with respect to w_t on both sides of equation (7), we have

$$\int_{\widehat{y}(q_t^*)}^n yg(y)dy + [cq_t^* - w_t\widehat{y}(q_t^*)]g(\widehat{y}(q_t^*))\frac{d\widehat{y}(q_t^*)}{dw_t}$$
$$- c\overline{G}(\widehat{y}(q_t^*))\frac{dq_t^*}{dw_t} = 0. \tag{A.15}$$

By equation (6), we have

$$\frac{d\widehat{y}(q_t^*)}{dw_t} = \frac{c}{2w_t}\left(\frac{dq_t^*}{dw_t} - \frac{q_t^*}{w_t}\right). \tag{A.16}$$

Combining equations (A.15) and (A.16), we get

$$\frac{dq_t^*}{dw_t} = \frac{c^2 q_t^{*2} g(\widehat{y}(q_t^*)) - 4w_t^2 \int_{\widehat{y}(q_t^*)}^n yg(y)dy}{c^2 q_t^* w_t g(\widehat{y}(q_t^*)) - 4w_t^2 c\overline{G}(\widehat{y}(q_t^*))}. \tag{A.17}$$

Moreover, from equations (7) and (A.17), we have

$$\frac{dq_t^*}{dw_t} = \frac{c^2 q_t^{*2} g(\widehat{y}(q_t^*)) - 4w_t cq_t^* \overline{G}(\widehat{y}(q_t^*))}{c^2 q_t^* w_t g(\widehat{y}(q_t^*)) - 4w_t^2 c\overline{G}(\widehat{y}(q_t^*))}. \tag{A.18}$$

Note that $h(y) = g(y)/\overline{G}(y)$ and $H(y) = yh(y)$. Then, by the assumption in Section 2.2, we can get $dq_t^*/dw_t = q_t^*/w_t > 0$. □

E. Proof of Proposition 3

Proof. According to equations (5)–(7), we get

$$\pi_m^t(w_t) = \frac{2(w_t\widehat{y}(q_t^*))^2 \overline{G}(\widehat{y}(q_t^*))}{c}. \tag{A.19}$$

By equations (2)–(4), we have

$$\pi_m^b(w_b^*) = \frac{(\mu_Y w_b^*)^2}{2c}. \tag{A.20}$$

We divide our proof into the following cases:

(i) *Case A.* $\pi_m^b(w_b^*) \le \pi_m^t(w_t)$: In this case, we have $\mu_Y \le \overline{\mu}_Y$, where $\overline{\mu}_Y = 2\widehat{y}(q_t^*)w_t\sqrt{\overline{G}(\widehat{y}(q_t^*))}/w_b^*$. At this time, the retailer chooses the optimal procurement price, denoted by w_t^Δ, to maximize its profit. Next, we deduce the equation satisfied by w_t^Δ.

From equation (8), $\pi_r^t(w_t)$ can be rewritten as

$$\pi_r^t(w_t) = p\left[q_t^*\mu_Y - \int_0^n g(y)dy \int_0^{yq_t^*}(yq_t^* - x)f(x)dx\right]$$
$$- \int_{\widehat{y}(q_t^*)}^n \left[yq_t^* w_t - \frac{cq_t^{*2}}{2}\right]g(y)dy - \frac{cq_t^{*2}}{2}. \tag{A.21}$$

Taking the first-order and second-order derivatives of $\pi_r^t(w_t)$ with respect to w_t, we get

$$\frac{d\pi_r^t(w_t)}{dw_t} = \frac{q_t^*}{w_t}\left[p\mu_Y - cq_t^* - p\int_0^n yg(y)F(yq_t^*)dy\right]$$
$$- q_t^* \int_{\widehat{y}(q_t^*)}^n yg(y)dy,$$

$$\frac{d^2\pi_r^t(w_t)}{dw_t^2} = -\frac{q_t^*}{w_t}\left[\frac{cq_t^*}{w_t} + \frac{pq_t^*}{w_t}\int_0^n y^2 g(y)f(q_t^* y)dy\right.$$
$$\left. + \int_{\widehat{y}(q_t^*)}^n yg(y)dy\right]. \tag{A.22}$$

Since $d^2\pi_r^t(w_t)/dw_t^2 < 0$, $\pi_r^t(w_t)$ is a strictly concave function about w_t. According to the first-order condition $d\pi_r^t(w_t)/dw_t = 0$, the retailer's unique optimal procurement price w_t^Δ satisfies

$$p\left[\mu_Y - \int_0^n yg(y)F(q_t^* y)dy\right] = cq_t^* + w_t^\Delta \int_{\widehat{y}(q_t^*)}^n yg(y)dy. \tag{A.23}$$

(ii) *Case B.* $\pi_m^b(w_b^*) > \pi_m^t(w_t)$: From equations (A.19) and (A.20), we have $\mu_Y > \overline{\mu}_Y$. In this time, the retailer chooses the procurement price $w_t^{\Delta\Delta}$ to satisfy $\pi_m^b(w_b^*) = \pi_m^t(w_t^{\Delta\Delta})$, which yields

$$w_t^{\Delta\Delta} = \frac{\mu_Y w_b^*}{2\widehat{y}(q_t^*)\sqrt{\overline{G}(\widehat{y}(q_t^*))}}. \tag{A.24}$$

Thus, Proposition 3 has been proved by the above two cases. □

F. Proof of Lemma 2

Proof

(i) From equation (A.12), we can verify for a given q_t that

$$\frac{\partial\pi_m^t(q_t, w_t)}{\partial w_t} = q_t \int_{\widehat{y}(q_t)}^n yg(y)dy > 0. \tag{A.25}$$

The above inequality implies that $\pi_m^t(q_t, w_t)$ is increasing in w_t. Assuming that $w_t < \underline{w}$, we then have $\pi_m^t(q_b^*, w_t) < \pi_m^t(q_b^*, \underline{w}) = \pi_m^b(q_b^*, w_b^*)$. Therefore, the manufacturer would opt for BCF. Hence, to attract the manufacturer to participate in APM, the feasible value of w_t will be set to satisfy $w_t \ge \underline{w}$.

(ii) According to the Definition 1, we know that $\pi_m^t(q_b^*, \underline{w}) = \pi_m^b(q_b^*, w_b^*)$ when $w_t = \underline{w}$ and $q_t = q_b^*$. From equations (1) and (5), we get

$$E\left[\left(Yq_b^* w_b^* - (1 + r_b^* L(q_b^*))\right)^+\right] = E\left[\left(Yq_b^* \underline{w} - L(q_b^*)\right)^+\right]. \quad \text{(A.26)}$$

Combining equations (8) and (A.26), we have

$$\pi_r^t(q_b^*, \underline{w}) == pE[\min(Yq_b^*, X)] - L(q_b^*) \\ - E\left[\left(Yq_b^* w_b^* - (1 + r_b^*)L(q_b^*)\right)^+\right]. \quad \text{(A.27)}$$

According to Lemma 1, $\pi_r^t(q_b^*, \underline{w})$ can be further re-written as

$$\begin{aligned} \pi_r^t(q_b^*, \underline{w}) &= pE[\min(Yq_b^*, X)] - L(q_b^*) \\ &\quad - E[(Yq_b^* w_b^* - L(q_b^*))] \\ &= pE[\min(Yq_b^*, X)] - E[Yq_b^* w_b^*] \\ &= \pi_r^b(q_b^*, w_b^*). \end{aligned} \quad \text{(A.28)}$$

Therefore, the retailer makes the same profits under APM and BCF when $w_t = \underline{w}$ and $q_t = q_b^*$. □

G. Proof of Proposition 4

Proof. We claim that $q_t^* > q_b^*$. To finish it, we by contradiction assume that $q_t^* \le q_b^*$. Then, we have $cq_t^* \le cq_b^*$. Since $F(x)$ is strictly increasing in x, $F(q_t^* y) \le F(q_b^* y)$. Therefore, we have

$$\int_0^n yg(y)F(q_t^* y)dy \le \int_0^n yg(y)F(q_b^* y)dy. \quad \text{(A.29)}$$

From equations (7) and (10), we have

$$p\left[\mu_Y - \int_0^n yg(y)F(q_t^* y)dy\right] = cq_t^*\left[1 + \overline{G}(\hat{y}(q_t^*))\right]. \quad \text{(A.30)}$$

By equations (A.30) and (4), we get

$$(cq_b^* - cq_t^*) + \left[\int_0^n yg(y)F(q_b^* y)dy - \int_0^n yg(y)F(q_t^* y)dy\right] \\ + cq_b^* - cq_t^*\overline{G}(\hat{y}(q_t^*)) = 0. \quad \text{(A.31)}$$

Since $(cq_b^* - cq_t^*) \ge 0$ and also

$$\int_0^n yg(y)F(q_b^* y)dy - \int_0^n yg(y)F(q_t^* y)dy \ge 0, \quad \text{(A.32)}$$

then $cq_b^* \le cq_t^*\overline{G}(\hat{y}(q_t^*)) \le cq_t^*$. It contradicts the assumption of $cq_t^* \le cq_b^*$. Therefore, the assumption is invalid, and we have $q_t^* > q_b^*$. □

H. Proof of Proposition 5

Proof. According to the proof of Proposition 3, we know that the manufacturer's optimal profit under APM is not

lower than that under BCF. Next, we prove that the retailer makes greater profits under APM than under BCF.

From equation (A.21), we can verify that

$$\frac{\partial \pi_r^t(q_t^*, w_t)}{\partial q_t^*} = p\mu_Y - p\int_0^n yg(y)F(yq_t)dy - cq_t^*. \quad \text{(A.33)}$$

By substituting equation (A.26) into equation (A.33), we get

$$\frac{\partial \pi_r^t(q_t^*, w_t)}{\partial q_t^*} = cq_t^*\overline{G}(\hat{y}(q_t^*)) > 0. \quad \text{(A.34)}$$

The above inequality implies that $\pi_r^t(q_t^*, w_t)$ is increasing in q_t^*. Therefore, when $w_t^* = \underline{w}$ and $q_t > q_b^*$, we get $\pi_r^t(q_t^*, \underline{w}) > \pi_r^t(q_b^*, \underline{w})$.

Under APM, the retailer's optimal profit is

$$\pi_r^t(q_t^*, w_t^*) \ge \pi_r^t(q_t^*, \underline{w}) > \pi_r^t(q_b^*, \underline{w}) = \pi_r^b(q_b^*, w_b^*), \quad \text{(A.35)}$$

where the last equality holds according to Lemma 2 (ii). □

I. Proof of Proposition 7

Proof. From equation (14), taking the derivative of $\pi_m^{\text{TR}}(q_\phi, w_\phi)$ with respect to q_ϕ, we get

$$\begin{aligned} \frac{d\pi_m^{\text{TR}}}{dq_\phi} &= \phi p\mu_Y - \phi p\int_0^n yg(y)F(yq_\phi)dy \\ &\quad + w_\phi \int_{\hat{y}(q_\phi)}^n yg(y)dy - cq_\phi\overline{G}(\hat{y}(q_\phi)). \end{aligned} \quad \text{(A.36)}$$

From first-order condition of $d\pi_m^{\text{TR}}(q_\phi, w_\phi)/dq_\phi = 0$, we have

$$\phi p\mu_Y - \phi p\int_0^n yg(y)F(yq_\phi)dy + w_\phi \int_{\hat{y}(q_\phi)}^n yg(y)dy \\ - cq_\phi\overline{G}(\hat{y}(q_\phi)) = 0. \quad \text{(A.37)}$$

To achieve channel coordination, we can easily see that $q_\phi = q_0^*$. Combining equations (13) and (A.37), we can obtain equation (16). □

Data Availability

The data used to support the findings of this study are included within the paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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