

Research Article

Sharing Logistics Service Supply Chain with Revenue-Sharing vs. Cost-Sharing Contracts

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The purpose of this study is to explore the design of equity cooperation mechanism in the sharing logistics service supply chain. This study designs a two-echelon logistics service supply chain composed of an urban joint distribution company and *N* logistics companies. The urban joint distribution company is jointly established by *N* logistics companies based on specific shares of equity investment. We establish sharing logistics service supply chain models under revenue-sharing or cost-sharing contracts. Revenue-sharing factor or cost-sharing factor is the equity cooperation parameter. When the members of the supply chain choose to cooperate in revenue-sharing or cost-sharing mechanism, not all cooperation scenarios considered in the study can achieve Pareto improvement of the total profit of the supply chain, but at least one situation can achieve Pareto improvement. This study provides feasible solutions for logistics companies to join the sharing logistics service platform and provides a reference for the operation of a joint distribution platform established by logistics companies. New results and managerial insights are derived by the sharing logistics service supply chain with revenue-sharing vs cost-sharing contracts, which enriches the interfaces of the operation of the sharing logistics service supply chain.

1. Introduction

The development of the sharing economy has provided great convenience to our lives, such as sharing cars and sharing bicycles to ease travel problems. At present, the impact of COVID-19 has made the shared medical and shared education industries grow against the trend. It is predicted that China's sharing economy will account for 10% of GDP in 2020, and it will reach 20% in 2025 [1]. The sharing economy was written into China's "Government Work Report" from 2017 to 2020, which shows that China attaches great importance to the development of the sharing economy.

In recent years, typical business models of shared logistics such as shared distribution, shared warehousing, and shared logistics dedicated lines have been favored by the capital market. Sichuan Tongyue Supply Chain Management Co., Ltd. (Tongyue), Dazhou Dayun Logistics Co., Ltd. (Dayun), Sichuan Wangyong Logistics Co., Ltd. (Wangyong), and Sichuan Yusheng Logistics Development Co., Ltd. (Yusheng) made a joint investment in Sichuan Yuanyao Supply Chain Management Co., Ltd. (Yuanyao) in 2018 in Sichuan, China. The shareholder units of Yuanyao are mainly engaged in the logistics transportation business from Dazhou, Yibin, and Xichang to Chengdu. Yuanyao's shareholder units have effectively cooperated to develop the feasibility of urban joint distribution from 2018 to 2019.

Some questions about sharing logistics service supply chain deserve attention, such as why do companies with different logistics service capabilities choose to cooperate? Why do many logistics companies choose to cooperate to build a distribution platform in urban distribution? How do logistics companies with different logistics services and logistics capabilities solve the "free-riding" behavior of small enterprises in cooperation?

Motivated by these questions, we propose a new operating mechanism and design a two-echelon service supply chain for a common distribution logistics system composed of urban joint distribution companies and N logistics companies. Meanwhile, the two-echelon service supply chain is based on equity cooperation. What we need to decide is whether to choose revenue-sharing or cost-sharing contracts.

The contributions of our study are as follows. First, we study the 1 - N supply chain structure, which is more realistic. Second, we introduce the concept of sharing economy into the field of urban distribution and design a path for cooperation and sharing in the study. Third, we summarize and refine the business-based equity cooperation phenomenon that often occurs in the modern logistics industry after field research on many logistics companies and e-commerce logistics companies. Based on this, we propose a joint distribution cooperation mechanism and combine the core parameters of supply chain contract with equity design.

The study is organized as follows. Section 2 presents the literature review. Section 3 introduces the research problem hypothesis and model setting. Section 4 shows the benchmark model. The analysis of the optimal strategy of revenue-sharing and cost-sharing contract is evaluated in Sections 5 and 6. Section 7 presents the case analysis. Conclusions are summarized in Section 8.

2. Literature Review

The study focuses on logistics service supply chain and sharing mechanism. The research on the sharing mechanism in the field of operation management mainly has two directions: one is the research on the business model of sharing economy, such as shared bikes and cars, and the other is to study the sharing and cooperation mechanism in operation management, such as information-sharing, capacitysharing, revenue-sharing, and cost-sharing.

2.1. Logistics Service Supply Chain. Logistics service supply chain is composed of a logistics service integrator and some functional logistics service providers [2-5]. Liu et al. [6] study the fairest revenue-sharing coefficient when logistics service integrators and functional logistics service providers implement revenue-sharing contracts under random demand. Liu et al. [7] discuss the order distribution of the logistics service supply chain. Liu et al. [8] consider how a two-echelon logistics service supply chain can minimize the average unit operating cost of logistics service integrators and maximize the average satisfaction of logistics service providers due to their capacities and time constraints. The influence of loss aversion preference on logistics service supply chain members' decision-making is discussed by Liu et al. [9]. Shen et al. [10] focus on the trade-off between transportation time and cost in the global supply chain and provide insights on how to manage logistics services. Wang et al. [11] investigate the selection of service providers and order allocation in different processes for mass customized logistics service models and design an improved genetic algorithm to solve.

2.2. Business Model of Sharing Economy. The sharing economy, in which ordinary consumers also act as sellers, is attracting great interest from scholars and practitioners. The rapid growth of this emerging economic form and the emergence of several big brands such as Uber and Airbnb raise many interesting questions to be studied [12]. Transaction costs in the shared market have a non-monotonic effect on company profits, consumer surplus, and social welfare. When a company strategically chooses retail prices, it is a win-win situation for consumers to share products with high marginal costs, but it may be a lose-lose situation for them if they share products with low marginal costs [13].

More than 400 cities around the world have deployed or plan to deploy bicycle sharing systems. However, the factors driving their use and the amount of rebalancing they require are unclear [14]. In view of the increasing importance of bike sharing systems, Caggiani et al. [15] propose a new integrated dynamic bike redistribution method. With the rapid development of bicycle sharing, more and more issues related to it have been studied: such as bicycle sharing stations based on maximum coverage location, factors affecting bicycle sharing members, and its economic contribution to the sustainability and efficient operation of the city [16].

Revenue-Sharing and Cost-Sharing 2.3. Contracts. Revenue-sharing contracts are popular in the cassette tape rental industry [17]. Liu et al. [6] show that related concepts of sharing mechanism can be introduced into the study of logistics service supply chain management. Li et al. [18] propose that consignment contract under revenue-sharing has been widely used in many industries, especially in the online retail market. Pan et al. [19] show that the revenuesharing contract can improve supply chain performance. Kimms and Cetiner [20] propose a nucleolus-based revenuesharing cooperative game strategy based on operational optimization to solve the problem of equitable income distribution in airline alliances. Van der Rhee et al. [21] point out that although there have been many studies on revenuesharing contracts, the application rate of their research results in the industry is not high. Zhang et al. [22] design the revenue-sharing and cooperative investment contract to coordinate the supply chain. Wei et al. [23] reveal a joint optimal decision-making strategy that involves revenuesharing and cooperative investment contracts based on an order flow proportion (OFP) and a revenue-sharing factor (RSF) and find that an OFP system offers the best solution in designing revenue-sharing contracts based on RSFs. Costsharing contract can improve profits of green food supply chain members [24, 25]. Revenue-sharing and cost-sharing contracts are widely used together in the researches [26–35].

The works that are of particular relevance to our study are by Zhang et al. [22], Yang and Chen [31], Wei et al. [23], and Yu et al. [33]. As only the implicit solution of the optimal decision can be obtained, the optimal decision is finally presented through numerical examples. Similar to Zhang et al. [22], our optimal decision can only be obtained by an implicit solution, and finally, a numerical example is used to choose a cooperative contract. Wei et al. [23] discuss an omnichannel supply chain structure in the joint distribution logistics cooperation model, but we design a mechanism of revenue-sharing and cost-sharing based on equity cooperation. Yang and Chen [31] and Yu et al. [33] both discuss whether supply chain members choose sharing of revenue, cost, or both in emission reduction cooperation. However, we consider whether urban joint distribution company and many logistics companies should choose revenue-sharing contract or cost-sharing contract to cooperate to improve the performance of logistics service supply chain.

3. Model Description and Assumptions

We design a two-echelon omnichannel service supply chain of a joint distribution logistics system composed of an urban joint distribution company and N logistics companies, in which logistics companies only operate the intercity logistics transportation, and the urban distribution service is outsourced to the urban joint distribution company. Unlike the traditional urban logistics operation model, the urban joint distribution company is jointly established by N logistics companies according to certain proportions of equity investment. In this study, the proportion of revenue-sharing or cost-sharing. We summarize all notations as follows.

Similar to the assumption about the demand function by Ma et al. [36] and Wei et al. [23], we assume that the demand function of logistics company *i*, as shown in the following equation:

$$x_{i} = \lambda_{i}d - \alpha p_{i} + \beta \sum_{\substack{j=1\\j\neq i}}^{n} p_{j} + \varepsilon,$$

$$(1)$$

$$i = 1, 2, 3, \dots, n, \text{ where } i = 1, 2, 3, \dots, n.$$

In equation (1), λ_i represents the market share of logistics company i, $\lambda_i \in [0, 1]$, and $\sum_{i=1}^n \lambda_i = 1$. The demand x_i of logistics company i is negatively correlated with the unit service price p_i provided by logistics company i and positively correlated with the unit service price p_j of all other logistics company j. \hat{x}_i is the average value of the market demand of logistics company i. ε is the disturbance term of the random market demand, which obeys uniform distribution and represents other influencing factors of demand uncertainty and satisfies $\varepsilon \sim U[-h,h]$, where h is an interval value of this uniform distribution $E(\varepsilon) = 0$.

As an independent service entity, the urban joint distribution company provides all logistics companies with urban joint distribution services. The profit function of the urban joint distribution company is as follows:

$$\pi_{s} = w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \right].$$
(2)

In equation (2), the operating cost of the urban joint distribution company not only includes unit service cost but also includes nonscale economy costs $e(\sum_{i=1}^{n} q_i)^2$, where $e(\sum_{i=1}^{n} q_i)^2$ is the non-scale economic cost, which is mainly incurred due to the increase in costs caused by management capabilities and user service access [37, 38].

The profit equals the revenue minus the costs, so the profit function of logistics company i is as follows:

$$\pi_{i} = \int_{\widehat{x_{i}-h}}^{q_{i}} (p_{i} - w - c_{i}) x_{i} f(x_{i}) dx_{i} + \int_{q_{i}}^{\widehat{x_{i}+h}} (p_{i} - w - c_{i}) q_{i} f(x_{i}) dx_{i}.$$
(3)

There will be an oversupply or undersupply of logistics services due to the asymmetry of market information and the inaccuracy of market demand forecasts in practice. Therefore, overcapacity cost and undercapacity cost caused by random fluctuations in market demand will be considered in this study.

Overcapacity cost is as follows:

$$O_{i}(\overrightarrow{p},\overrightarrow{q}) = \int_{\widehat{x_{i}}} -h^{q_{i}}(w-v)(q_{i}-x_{i})f(x_{i})dx_{i}, \qquad (4)$$

where $O_i(\vec{p}, \vec{q})$ is the cost of providing logistics service at a level lower than the market price due to excess logistics service capacity and v is the unit residual value of logistics service in equation (4). The specific connotation of services and general physical products residual value is different. The unit residual value of general physical products may be the value that can be generated after the physical product is used for other purposes or processed; the residual value of logistics service capabilities represents the profit obtained by temporarily providing the logistics service capacity to other social customers at a level lower than the market price.

Undercapacity cost is as follows:

$$U_{i}(\overrightarrow{p},\overrightarrow{q}) = \int_{q_{i}}^{\widehat{x}_{i}+h} k(x_{i}-q_{i})f(x_{i})\mathrm{d}x_{i}, \qquad (5)$$

where $U_i(\vec{p}, \vec{q})$ is the cost incurred by missing market opportunities due to insufficient supply of logistics service capabilities.

The assumption is as follows:

A1: there is positive market demand, that is, d > 0.

A2: the unit service cost of urban joint distribution company is significantly higher than the diseconomies of scale, and it is greater than zero, that is, $c \gg e > 0$. A3: $k > v \ge 0$.

4. Centralized and Decentralized Service Supply Chain: The Benchmark

We study the optimal decision-making problem of centralized service supply chain and decentralized service supply chain of the urban joint distribution cooperation model. 4.1. Centralized Service Supply Chain. The centralized service supply chain takes the total profit of the supply chain as the objective function and the members of the supply chain jointly bear the risk loss incurred due to the oversupply or undersupply of logistics services. Therefore, the total profit of the centralized service supply chain can be expressed as follows, and the detail of the following equation is shown in Appendix (A.1):

$$\pi_{sc}^{c} = \sum_{i=1}^{n} \left\{ \frac{(p_{i} - c_{i} - v)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{vq_{i}}{2h} (q_{i} - \hat{x}_{i} + h) + \frac{q_{i}(p_{i} - c_{i} + k)}{2h} (\hat{x}_{i} + h - q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i} + h)^{2} - q_{i}^{2} \right] \right\}$$

$$- c \sum_{i=1}^{n} q_{i} - e \left(\sum_{i=1}^{n} q_{i} \right)^{2}.$$
(6)

By analyzing equation (6), we obtain Proposition 1.

Proposition 1. The optimal solution for the centralized service supply chain satisfies the following equation: $(\overrightarrow{p}^*, \overrightarrow{q}^*) = \arg \max \left\{ \pi_{sc}^c(\Lambda_0^c), \pi_{sc}^c(\Lambda_{\overrightarrow{p}}^c, \Lambda_{\overrightarrow{q}}^c) \right\}, \text{ where } \Lambda_0^c \text{ is }$ the boundary of the feasible region of the decision variable and $(\Lambda_{\overrightarrow{P}}^c, \Lambda_{\overrightarrow{q}}^c)$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} -\frac{q_{i}^{*2}}{4h} + \frac{q_{i}^{*}}{2h} (\hat{x}_{i}^{*} + h + \alpha v - \alpha p_{i}^{*} + \alpha c_{i} - \alpha k) - \frac{(\hat{x}_{i}^{*} - h^{2})}{4h} + \frac{\alpha (p_{i}^{*} - c_{i} - v)(\hat{x}_{i}^{*} - h)}{2h} + \frac{\alpha k(\hat{x}_{i}^{*} + h)}{2h} \\ \sum_{\substack{j=1\\j\neq 1}}^{n} \left[-\frac{\beta (p_{j}^{*} - c_{j} - v)(\hat{x}_{j}^{*} - h)}{2h} + \frac{\beta q_{j}^{*}(p_{j}^{*} - c_{j} + k - v)}{2h} - \frac{\beta k(\hat{x}_{j}^{*} + h)}{2h} \right] = 0, \end{cases}$$

$$(7)$$

$$\frac{(\hat{x}_{i}^{*} - q_{i}^{*})(p_{j}^{*} = c_{i} - v + k)}{2h} + \frac{p_{i}^{*} - c_{i} + k + v}{2} - c - 2e \sum_{j=1}^{n} q_{j}^{*} = 0,$$

$$\Lambda_{\frac{p}{p}}^{c} = \{p_{1}^{*}, p_{2}^{*}, \dots, p_{n}^{*}\}, p_{i}^{*} \in \Lambda_{\frac{p}{p}}^{c}; \Lambda_{\frac{p}{q}}^{c} = \{q_{1}^{*}, q_{2}^{*}, \dots, q_{n}^{*}\}, q_{i}^{*} \in \Lambda_{\frac{q}{q}}^{c}.$$

Proof. Refer to the Appendix.

We can derive from equation (7) that the decision variable is the order quantity (or logistics service price) of each logistics company, and as it is a multivariate cubic equation, it is difficult to solve for the explicit expression of the optimal decision, but the implicit expression can be derived through Proposition 1. After giving other corresponding parameters, we could simulate and present the optimal decision through numerical examples.

4.2. Decentralized Service Supply Chain. In the analysis of the decentralized supply chain, two scenarios are taken into consideration: urban joint distribution company bearing

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cost risk and logistics companies independently bearing cost risk. In the decentralized supply chain, the urban joint distribution company and logistics companies will pursue profit maximization.

4.2.1. Logistics Companies Bear Cost Risks. The optimal decision of the urban joint distribution company is as follows:

$$\frac{d\pi_s^{d1}}{dq_i} = w - \left(c + 2e\sum_{j=1}^n q_j\right) = 0, \text{ that is, } w = c + 2e\sum_{j=1}^n q_j.$$
(8)

The profit of the logistics company i is as follows, and the detail of the following equation is shown in (A.2):

$$\pi_{i}^{d1} = \frac{\left(p_{i} - c_{i} - \nu\right)}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h\right)^{2}\right] + \frac{\nu q_{i}}{2h} \left(q_{i} - \hat{x}_{i} + h\right) + \frac{q_{i}\left(p_{i} - c_{i} + k\right)}{2h} \left(\hat{x}_{i} + h - q_{i}\right) - \frac{k}{4h} \left[\left(\hat{x}_{i} + h\right)^{2} - q_{i}^{2}\right] - cq_{i} - 2eq_{i} \sum_{j=1}^{j=n} q_{j}.$$
(9)

Proposition 2. In a decentralized service supply chain where logistics companies independently bear operational cost risks, the optimal solution satisfies the following system of equations $(p_i^*, q_i^*) = \arg \max \{\pi_i^{d1}(\Lambda_0^{d1}), \pi_i^{d1}(\Lambda_{p_i}^{d1}, \Lambda_{q_i}^{d1})\}$, where Λ_0^{d1} is

the boundary of the feasible region of the decision variable and $(\Lambda_{p_i}^{d_1}, \Lambda_{q_i}^{d_1})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\frac{-q_i^{*2}}{4h} + \frac{q_i^{*}(\hat{x}_i^{*} + h)}{2h} - \frac{\alpha q_i^{*}(p_i^{*} - c_i - v + k)}{2h} - \frac{(\hat{x}_i^{*} - h)^2}{4h} + \frac{\alpha (p_i^{*} - c_i - v)(\hat{x}_i^{*} - h)}{2h} + \frac{\alpha k}{2h}(\hat{x}_i^{*} + h) = 0,$$

$$\frac{(p_i^{*} - c_i + k - v)(\hat{x}_i^{*} - q_i^{*})}{2h} + \frac{p_i^{*} - c_i + k + v}{2} - c - 2e\sum_{j=1}^{n} q_j^{*} - 2eq_i^{*} = 0,$$

$$\Lambda_{a}^{d1} = \{p_i^{*}\}, p_i^{*} \in \Lambda_{a}^{d1}; \Lambda_{a}^{d1} = \{q_i^{*}\}, q_i^{*} \in \Lambda_{a}^{d1}.$$
(10)

Proof. Refer to the Appendix.

As shown in equation (10), similar to the centralized optimal solution, the decision variable of the decentralized supply chain is the order quantity (or logistics service price) of each logistics company and it is a binary cubic equation system. It is difficult to solve for its explicit expression, but the implicit expression can be derived through Proposition 2. 4.2.2. Urban Joint Distribution Company Bears Cost Risks. The profit function of the urban joint distribution company is derived as follows, and the detail of the following equation is shown in Appendix (A.3):

$$\pi_{s}^{d2} = w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \right]$$

$$- \sum_{i=1}^{n} \left\{ \frac{(w-v)q_{i}}{2h} \left(q_{i} - \hat{x}_{i} + h \right) - \frac{(w-v)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{k}{4h} \left[\left(\hat{x}_{i} + h \right)^{2} - q_{i}^{2} \right] - \frac{kq_{i}}{2h} \left(\hat{x}_{i} + h - q_{i} \right) \right\}.$$

$$(11)$$

We solve first derivatives of the function π_s^{d2} with respect to the variable q_i and equate it to 0 to obtain equation (12), and the detail of the first derivatives of the function π_s^{d2} with respect to the variable q_i is shown in (A.4):

$$w = \frac{2h(c+2e\sum_{j=1}^{n}q_{j}) + (k-v)(q_{i}-\hat{x}_{i}) - h(k+v)}{\hat{x}_{i}+h-q_{i}}.$$
(12)

As $\forall i, j \in N, i \neq j$, we can obtain $q_i - \hat{x}_i = q_j - \hat{x}_j$. Thus, assuming $q_1 - \hat{x}_1 = q_2 - \hat{x}_2 = q_3 - \hat{x}_3 = \cdots = q_n - \hat{x}_n = Q \in [-h, h]$, then we have $\hat{x}_i = q_i - Q, i = 1, 2, 3, \dots, n\hat{x}_i = \lambda_i d - \lambda_i d -$

 $\alpha p_i + \beta \sum_{\substack{j=1 \ j \neq i}}^n p_j$ can be obtained from equation (1). If

$$\sum_{i=1}^{n} \hat{x}_{i} = \sum_{i=1}^{n} \left(\lambda_{i} d - \alpha p_{i} + \beta \sum_{\substack{j=1\\j \neq i}}^{n} p_{j} \right) = d - \alpha \sum_{i=1}^{n} p_{i} + (n - 1)\beta \sum_{i=1}^{n} p_{i} = \sum_{i=1}^{n} q_{i} - nQ$$
, then we obtain the expression of p_{i} , as the following equation:

$$p_i = \frac{\lambda_i d + Q - q_i}{\alpha + \beta} + \frac{\sum_{i=1}^n q_i - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}.$$
 (13)

Equations (12) and (13) are the optimal decisions for the urban joint distribution company. Combining equations (12) and (13), we can obtain the profit of logistics company i as follows and derive Proposition 3, and the detail of the following equation is shown in Appendix (A.5):

$$\pi_{i}^{d2} = \frac{1}{4h} \left[\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{i=1}^{n} q_{i} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}) + (k - \nu)Q - h(k + \nu)}{h - Q} - c_{i} \right]$$
(14)
$$\left[-q_{i}^{2} - (q_{i} - Q - h)^{2} + 2q_{i}(q_{i} - Q + h) \right].$$

Proposition 3. The optimal solution in the decentralized supply chain when the urban joint distribution company bearing cost risks satisfies the following equation: $(q_i^*, Q^*) = \arg \max \left\{ \pi_i^{d2} (\Lambda_0^{d2}), \pi_i^{d2} (\Lambda_{q_i}^{d2}, \Lambda_Q^{d2}) \right\}$, where Λ_0^{d2} is

the boundary of the feasible region of the decision variable and $(\Lambda_{q_i}^{d^2}, \Lambda_Q^{d^2})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} \frac{\lambda_{i}d + Q^{*} - q_{i}^{*}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j}^{*} - nQ^{*} - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}^{*}) + (k - \nu)Q^{*} - h(k + \nu)}{h - Q^{*}} - c_{i} \\ + \frac{-q_{i}^{*2} - (q_{i}^{*} - Q^{*} - h)^{2} + 2q_{i}^{*}(q_{i}^{*} - Q^{*} + h)}{4h} \times \left[\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{h - Q^{*}} \right] = 0,$$

$$(15)$$

$$\Lambda_{q_{i}}^{d2} = \{q_{i}^{*}\}, q_{i}^{*} \in \Lambda_{q_{i}}^{d2}; \Lambda_{Q}^{d2} = \{Q^{*}\}, Q^{*} \in [-h, h].$$

Proof. Refer to the Appendix.

Similar to the centralized optimal solution, the implicit expression can be derived through Proposition 3. However, we could see that the dimensions of the decision-making variables in Proposition 3 are lower than that in Propositions 1 and 2, which means the decision complexity has been reduced. After giving other corresponding parameters, we could simulate and present the optimal decision through numerical examples.

5. Revenue-Sharing Cooperative Decision

According to the analysis of the decentralized service supply chain, it is feasible for the urban joint distribution company and each logistics company to independently bear the cost risk. Therefore, in revenue-sharing cooperation, these two situations are also considered. At the same time, the revenue-sharing strategies of an order flow proportion type and a factor combination type are analyzed, respectively, in each case of cost-sharing.

5.1. Revenue-Sharing When Logistics Company Bears Cost Risk. We can obtain the optimal decision of the urban joint distribution company as follows:

$$w = c + 2e \sum_{j=1}^{n} q_j.$$
 (16)

5.1.1. Revenue-Sharing under Order Flow Proportion When Logistics Companies Bear Cost Risk. Each logistics company

independently bears its own operating cost risk and determines its revenue-sharing strategy according to the proportion of its order flow contribution. The proportion of order flow contribution for logistics company *i* is as follows:

$$\left(\frac{q_1}{\sum_{j=1}^n q_j}, \frac{q_2}{\sum_{j=1}^n q_j}, \frac{q_3}{\sum_{j=1}^n q_j}, \dots, \frac{q_n}{\sum_{j=1}^n q_j}\right).$$
(17)

The profit of logistics company i in an order flow proportion revenue-sharing supply chain when logistics companies bearing cost risks is as follows, and the detail of the following equation is shown in Appendix (A.6):

$$\pi_{i}^{rf_{1}} = \frac{(p_{i} - c_{i} - \nu)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{\nu q_{i}}{2h} (q_{i} - \hat{x}_{i} + h) + \frac{q_{i}(p_{i} - c_{i} + k)}{2h} (\hat{x}_{i} + h - q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i} + h)^{2} - q_{i}^{2} \right] - cq_{i} - eq_{i} \sum_{j=1}^{n} q_{j}.$$
(18)

By analyzing equation (18), we can obtain Proposition 4.

Proposition 4. The optimal solution in an order flow proportion revenue-sharing supply chain when logistics companies bearing cost risks satisfies the following equation: $(p_i^*, q_i^*) = \arg \max \{ \pi_i^{rf1}(\Lambda_0^{rf1}), \pi_i^{rf1}(\Lambda_{p_i}^{rf1}, \Lambda_{q_i}^{rf1}) \},$ where Λ_0^{rf1} is the boundary of the feasible region of the decision variable and $(\Lambda_{p_i}^{rf1}, \Lambda_{q_i}^{rf1})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} -\frac{q_{i}^{*2}}{4h} + \frac{q_{i}^{*}(\hat{x}_{i}^{*} + h)}{2h} - \frac{\alpha q_{i}^{*}(p_{i}^{*} - c_{i} - \nu + k)}{2h} - \frac{(\hat{x}_{i}^{*} - h)^{2}}{4h} + \frac{\alpha (p_{i}^{*} - c_{i} - \nu)(\hat{x}_{i}^{*} - h)}{2h} + \frac{\alpha k}{2h}(\hat{x}_{i}^{*} + h) = 0, \\ \frac{(p_{i}^{*} - c_{i} + k - \nu)(\hat{x}_{i}^{*} - q_{i}^{*})}{2h} + \frac{p_{i}^{*} - c_{i} + k + \nu}{2} - c - e\sum_{j=1}^{n} q_{j}^{*} - eq_{i}^{*} = 0, \\ \Lambda_{p_{i}}^{rf1} = \{p_{i}^{*}\}, p_{i}^{*} \in \Lambda_{p_{i}}^{rf1}; \Lambda_{q_{i}}^{rf1} = \{q_{i}^{*}\}, q_{i}^{*} \in \Lambda_{q_{i}}^{rf1}. \end{cases}$$

$$(19)$$

Proof. Refer to the Appendix.

Similar to the centralized and the decentralized optimal solution, the implicit expression of the optimal decision variable can be derived through Proposition 4. The implicit expression in Proposition 4 is similar to that in Proposition 2 because the logistics company bears the cost risks in both the situations. After presenting the other corresponding parameters, we could present the optimal decision through numerical examples.

5.1.2. Revenue-Sharing under Factor Combination When Logistics Companies Bear Cost Risk. The shared factor combination coefficient for logistics company *i* is as follows:

$$(\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_n), \quad \forall \gamma_i \in [0, 1],$$

$$\sum_{i=1}^n \gamma_i = 1.$$
(20)

Then, we can obtain π_i^{rr1} as follows, and the detail of the following equation is shown in Appendix (A.7):

$$\pi_{i}^{rr1} = \frac{(p_{i} - c_{i} - v)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{vq_{i}}{2h} (q_{i} - \hat{x}_{i} + h) + \frac{q_{i}(p_{i} - c_{i} + k)}{2h} (\hat{x}_{i} + h - q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i} + h)^{2} - q_{i}^{2} \right] - \left(c + 2e \sum_{j=1}^{n} q_{j} \right) q_{i} + \gamma_{i} \left\{ \left(c + 2e \sum_{j=1}^{n} q_{j} \right) \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \right] \right\}.$$
(21)

By analyzing equation (21), we can obtain Proposition 5.

Proposition 5. The optimal solution in a factor combination revenue-sharing supply chain when logistics companies bearing cost risks satisfies the following equation: $(p_i^*, q_i^*) = \arg \max \{\pi_i^{rr1}(\Lambda_0^{rr1}), \pi_i^{rr1}(\Lambda_{p_i}^{rr1}, \Lambda_{q_i}^{rr1})\},$ where Λ_0^{rr1} is the boundary of the feasible region of the decision variable and $(\Lambda_{p_i}^{rr1}, \Lambda_{q_i}^{rr1})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} -\frac{q_{i}^{*2}}{4h} + \frac{q_{i}^{*}\left(\hat{x}_{i}^{*}+h\right)}{2h} - \frac{\alpha q_{i}^{*}\left(p_{i}^{*}-c_{i}-\nu+k\right)}{2h} - \frac{\left(\hat{x}_{i}^{*}-h\right)^{2}}{4h} + \frac{\alpha \left(p_{i}^{*}-c_{i}-\nu\right)\left(\hat{x}_{i}^{*}-h\right)}{2h} + \frac{\alpha k}{2h}\left(\hat{x}_{i}^{*}+h\right) = 0, \\ \frac{\left(p_{i}^{*}-c_{i}+k-\nu\right)\left(\hat{x}_{i}^{*}-q_{i}^{*}\right)}{2h} + \frac{p_{i}^{*}-c_{i}+k+\nu}{2} - c - 2e\sum_{j=1}^{n} q_{j}^{*} - 2eq_{i}^{*} + 2\gamma_{i}e\sum_{j=1}^{n} q_{j}^{*} = 0, \\ \Lambda_{p_{i}}^{rf1} = \{p_{i}^{*}\}, p_{i}^{*} \in \Lambda_{p_{i}}^{rf1}; \Lambda_{q_{i}}^{rf1} = \{q_{i}^{*}\}, q_{i}^{*} \in \Lambda_{q_{i}}^{rf1}. \end{cases}$$

$$(22)$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression of the optimal decision variable can be derived through Proposition 5. We can see that the first subexpression both in the implicit expression of the Propositions 4 and 5 is the same one and there is no any difference, while the second subexpression is different at the eq_i^* and eq_j^* terms. We could present the optimal decision through numerical examples after giving other corresponding parameters.

5.2. Revenue-Sharing When the Urban Joint Distribution Company Bears Cost Risk. The profit of the urban joint distribution company is as follows:

$$\pi_{s}^{r^{2}} = \underbrace{w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]}_{\pi_{s}} \qquad (23)$$
$$- \sum_{i=1}^{n} \left[O_{i}(\overrightarrow{p}, t\overrightarrow{q}) + U_{i}(\overrightarrow{p}, \overrightarrow{q})\right].$$

Similar to the analysis of the profit and optimal decisionmaking of the urban joint distribution company, we can obtain the optimal decisions for the urban joint distribution company as follows:

$$w = \frac{2h(c+2e\sum_{j=1}^{n}q_{j}) + (k-\nu)(q_{i}-\hat{x}_{i}) - h(k+\nu)}{\hat{x}_{i} + h - q_{i}},$$

$$\forall i \in N,$$

(24)

$$p_{i} = \frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{i=1}^{n} q_{i} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}, \quad \forall i \in N.$$
(25)

Combined equations (24) and (25), we can obtain the profit of the logistics company *i* as follows and Proposition 6 can be obtained.

5.2.1. Revenue-Sharing under Order Flow Proportion When the Urban Joint Distribution Company Bears Cost Risk. Logistics companies make decisions on their revenuesharing ratio according to their respective order flow contribution proportion, which is shown in the following equation:

$$\left(\frac{q_1}{\sum_{j=1}^n q_j}, \frac{q_2}{\sum_{j=1}^n q_j}, \frac{q_3}{\sum_{j=1}^n q_j}, \dots, \frac{q_n}{\sum_{j=1}^n q_j}\right),$$
(26)

where π_i^{rf2} can be obtained as follows, and the detail of the following equation is shown in Appendix (A.8):

$$\pi_{i}^{rf2} = \left(\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}) + (k - \nu)Q - h(k + \nu)}{h - Q} - c_{i}\right)$$

$$\frac{\left[-q_{i}^{2} - (q_{i} - Q - h)^{2} + 2q_{i}(q_{i} - Q + h)\right]}{4h} + wq_{i} - \left(cq_{i} + eq_{i}\sum_{j=1}^{n} q_{j}\right).$$
(27)

By analyzing equation (27), we can obtain Proposition 6.

Proposition 6. The optimal solution in an order flow proportion revenue-sharing supply chain when the urban joint distribution company bearing cost risks satisfies the following equation: $(q_i^*, Q^*) = \arg \max \{\pi_i^{rf2}(\Lambda_0^{rf2}), \pi_i^{rf2}\}$

 $(\Lambda_{q_i}^{rf2}, \Lambda_Q^{rf2})$, where Λ_0^{rf2} is the boundary of the feasible region of the decision variable and $(\Lambda_{q_i}^{rf2}, \Lambda_Q^{rf2})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations.

$$\begin{cases} \left(-\frac{1}{\alpha+\beta} + \frac{1}{(\alpha+\beta)(n-1(\alpha/\beta))} - \frac{4he}{h-Q^*}\right) \frac{\left[-q_i^{*\,2} - (q_i^* - Q^* - h)^2 + 2q_i^* (q_i^* - Q^* + h)\right]}{4h} \\ + \left(\frac{\lambda_i d + Q^* - q_i^*}{\alpha+\beta} + \frac{\sum_{j=1}^n q_j^* - nQ^* - d}{(\alpha+\beta)(n-1(\alpha/\beta))} - \frac{2h\left(c + 2e\sum_{j=1}^n q_j^*\right) + (k-\nu)Q^* - h(k+\nu)}{h-Q^*} - c_i\right) \\ + \frac{2h\left(c + 2e\sum_{j=1}^n q_j^*\right) + (k-\nu)Q^* - h(k+\nu)}{h-Q^*} + \frac{4he}{h-Q^*}q_i^* - \left(c + eq_i^* + e\sum_{j=1}^n q_j^*\right) = 0, \\ \Lambda_{q_i}^{rf2} = \{q_i^*\}, q_i^* \in \Lambda_{q_i}^{rf2}; \Lambda_Q^{rf2} = \{Q^*\}, Q^* \in [-h, h]. \end{cases}$$

$$(28)$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression can be derived through Proposition 6. We could see that the dimensions of the decision-making variables in Proposition 6 are similar to that in Proposition 3 because the urban joint distribution company bears cost risk in both the situations. After giving other corresponding parameters, we could simulate and present the optimal decision through numerical examples. 5.2.2. Revenue-Sharing under Factor Combination When the Urban Joint Distribution Company Bears Cost Risk. The shared factor combination coefficient for logistics company *i* is as follows:

$$(\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_n), \quad \forall \gamma_i \in [0, 1],$$

 $\sum_{i=1}^n \gamma_i = 1,$ (29)

where π_i^{rr2} can be obtained as follows, and the detail of the following equation is shown in Appendix (A.9):

$$\pi_{i}^{rr2} = \left(\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}) + (k - \nu)Q - h(k + \nu)}{h - Q} - c_{i}\right)$$

$$\frac{\left[-q_{i}^{2} - (q_{i} - Q - h)^{2} + 2q_{i}(q_{i} - Q + h)\right]}{4h} + \gamma_{i}\left\{w\sum_{i=1}^{n} q_{j} - \left[c\sum_{i=1}^{n} q_{j} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]\right\}.$$
(30)

By analyzing equation (30), we can obtain Proposition 7.

Proposition 7. The optimal solution in a factor combination revenue-sharing supply chain when the urban joint distribution company bearing cost risks satisfies the following equation: $(q_i^*, Q^*) = \arg \max \{\pi_i^{rr2}(\Lambda_0^{rr2}), \pi_i^{rr2}(\Lambda_{q_i}^{rr2}, \Lambda_Q^{rr2})\},\$

where Λ_0^{rr2} is the boundary of the feasible region of the decision variable and $(\Lambda_{q_i}^{rr2}, \Lambda_Q^{rr2})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} \left(-\frac{1}{\alpha+\beta} + \frac{1}{(\alpha+\beta)(n-1-(\alpha/\beta))} - \frac{4he}{h-Q^*}\right) \frac{\left[-q_i^{*\,2} - \left(q_i^* - Q^* - h\right)^2 + 2q_i^*\left(q_i^* - Q^* + h\right)\right]}{4h} \\ + \left(\frac{\lambda_i d + Q^* - q_i^*}{\alpha+\beta} + \frac{\sum_{j=1}^n q_j^* - nQ^* - d}{(\alpha+\beta)(n-1-(\alpha/\beta))} - \frac{2h\left(c+2e\sum_{j=1}^n q_j^*\right) + (k-\nu)Q^* - h(k+\nu)}{h-Q^*} - c_i\right) \\ + \gamma_i \frac{2h\left(c+2e\sum_{j=1}^n q_j^*\right) + (k-\nu)Q^* - h(k+\nu)}{h-Q^*} + \gamma_i \frac{4he}{h-Q^*} \sum_{i=1}^n q_i^* - \gamma_i \left(c+2e\sum_{j=1}^n q_j^*\right) = 0, \\ \Lambda_{q_i}^{rr2} = \{q_i^*\}, q_i^* \in \Lambda_{q_i}^{rr2}; \Lambda_Q^{rr2} = \{Q^*\}, Q^* \in [-h,h]. \end{cases}$$

$$(31)$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression can be derived through Proposition 7. We could will simulate and present the optimal decision through numerical examples after giving other corresponding parameters.

6. Cost-Sharing Cooperative Decision

In this section, we will further study the cooperation mechanism problem of the sharing logistics distribution service mode with the risk cost-sharing contracts, which mainly focuses on the analysis of the following three different risk cost-sharing strategies:

- (1) Each logistics enterprise shares the risk cost caused by insufficient logistics service supply and excess logistics service supply according to the proportion of order flow (see Section 6.1).
- (2) The urban joint distribution company and each logistics company bear the risk cost of insufficient and excess logistics service supply according to the single factor combination (φ₀, φ₁, φ₂, φ₃,..., φ_n), where φ₀ = 1 − ∑ⁱ⁼ⁿ_{i=1}φ_i, ∀φ_i ∈ [0, 1], i = 0, 1, 2, 3, ..., n (see Section 6.2).

(3) The urban joint distribution company and each logistics company shall bear the costs caused by the insufficient supply of logistics services and the oversupply according to the multifactor combination (θ₀, θ₁, θ₂, θ₃, ..., θ_n) (γ₀, γ₁, γ₂, γ₃, ..., γ_n), where θ₀ = 1 - ∑ⁱ⁼ⁿ_{i=1}θ_i, γ₀ = 1 - ∑ⁱ⁼ⁿ_{i=1}γ_i, ∀φ_i, γ_i ∈ [0, 1], i = 0, 1, 2, 3, ..., n (see Section 6.3).

6.1. Order Flow Proportion Cost-Sharing Strategy. Each logistics company shares the risk cost due to insufficient and excessive supply of logistics services according to the proportion of order flow contribution, which is as follows:

$$\left(\frac{q_1}{\sum_{j=1}^n q_j}, \frac{q_2}{\sum_{j=1}^n q_j}, \frac{q_3}{\sum_{j=1}^n q_j}, \dots, \frac{q_n}{\sum_{j=1}^n q_j}\right).$$
 (32)

The optimal decision of the urban joint distribution company is given by the following equation:

$$w = c + 2e \sum_{j=1}^{n} q_j,$$
 (33)

where π_i^{rs1} can be obtained as follows, and the detail of the following equation is shown in Appendix (A.10):

$$\pi_{i}^{rs1} = \frac{\left(p_{i} - c - 2e\sum_{j=1}^{n} q_{j} - c_{i}\right)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2}\right] + \frac{q_{i}\left(p_{i} - c - 2e\sum_{j=1}^{n} q_{j} - c_{i}\right)}{2h} \left(\hat{x}_{i} + h - q_{i}\right) \\ - \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \sum_{j=1}^{n} \left\{\frac{\left(c + 2e\sum_{j=1}^{n} q_{j} - \nu + k\right)}{4h}q_{j}^{2} - q_{j}\left[\frac{\left(c + 2e\sum_{j=1}^{n} q_{j} - \nu\right)\left(\hat{x}_{j} - h\right)}{2h} + \frac{k\left(\hat{x}_{j} + h\right)}{2h}\right] \\ + \frac{\left(c + 2e\sum_{j=1}^{n} q_{j} - \nu\right)}{4h} \left(\hat{x}_{j} - h\right)^{2} + \frac{k}{4h}\left(\hat{x}_{j} + h\right)^{2}\right\}.$$

$$(34)$$

By analyzing equation (34), we can obtain Proposition 8.

Proposition 8. The optimal solution under order flow proportion cost-sharing strategy satisfies the following equation $(p_i^*, q_i^*) = \arg \max \{\pi_i^{rs1}(\Lambda_0^{rs1}), \pi_i^{rs1}(\Lambda_{p_i}^{rs1}, \Lambda_{q_i}^{rs1})\},\$

where Λ_0^{rs1} is the boundary of the feasible region of the decision variable and $(\Lambda_{p_i}^{rs1}, \Lambda_{q_i}^{rs1})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} -\frac{e}{2h} \Big[q_i^{*2} - (\hat{x}_i^* - h)^2 \Big] + \frac{(p_i^* - c - 2e\sum_{j=1}^n q_j^* - c_j)}{2h} (\hat{x}_i^* - h - q_i^*) - \frac{eq_i^*}{h} (\hat{x}_i^* - h - q_i^*) - \frac{\sum_{j=1}^n q_j^* - q_i^*}{(\sum_{j=1}^n q_j^*)^2} \times \\ \sum_{j=1}^n \left\{ \frac{(c + 2e\sum_{j=1}^n q_j^* - v + k)}{4h} q_j^{*2} - q_j^* \Big[\frac{(c + 2e\sum_{j=1}^n q_j^* - v)(\hat{x}_i^* - h)}{2h} + \frac{k(\hat{x}_i^* + h)}{2h} \Big] + \frac{(c + 2e\sum_{j=1}^n q_j^* - v)}{4h} (\hat{x}_j^* - h)^2 + \frac{k}{4h} (\hat{x}_j^* + h)^2 \Big\} \\ - \frac{q_i^*}{\sum_{j=1}^n q_j^*} \left\{ \sum_{j=1}^n \Big[\frac{e}{2h} q_j^{*2} - \frac{q_j^* e(\hat{x}_j^* + h)}{h} + \frac{e}{2h} (\hat{x}_j^* - h)^2 \Big] + \frac{c + 2e\sum_{j=1}^n q_j^* - v + k}{2h} q_i^* - \frac{(c + 2e\sum_{j=1}^n q_j^* - v)(\hat{x}_i^* - h)}{2h} - \frac{k(\hat{x}_i^* + h)}{2h} \right\} = 0, \\ \frac{1}{4h} \Big[q_1^{*2} - (\hat{x}_i^* - h)^2 \Big] + \frac{\alpha(p_i^* - c - 2e\sum_{j=1}^n q_j^* - c_i)}{2h} (\hat{x}_i^* - h - q_i^*) + \frac{q_i^*}{2h} (\hat{x}_i^* + h - q_i^*) - \frac{q_i^*}{2n} q_j^*}{2h} \times \\ \\ \int \frac{1}{4h} \Big[q_1^{*2} - (\hat{x}_i^* - h)^2 \Big] + \frac{\alpha(p_i^* - c - 2e\sum_{j=1}^n q_j^* - c_i)}{2h} (\hat{x}_i^* - h - q_i^*) + \frac{q_i^*}{2h} (\hat{x}_i^* + h) - \frac{q_i^*}{2n} q_j^* - 2h + \frac{2e\sum_{j=1}^n q_j^* - v}{2h} (\hat{x}_i^* - h - q_i^*) + \frac{q_i^*}{2h} (\hat{x}_i^* + h) - \frac{q_i^*}{2n} q_j^* \times 2h + \frac{2e\sum_{j=1}^n q_j^* - v}{2h} (\hat{x}_j^* - h) - \frac{e}{2h} (\hat{x}_j^* - h) + \frac{e}{2h} (\hat{x}_j^* + h) \Big] \\ \\ \int \sum_{j=1}^{n_{j=1}^n} \frac{\left[\frac{\beta q_i^* (c + 2e\sum_{j=1}^n q_j^* - v)}{2h} - \frac{\beta k q_j^*}{2h} + \frac{\beta(c + 2e\sum_{j=1}^n q_j^* - v)}{2h} (\hat{x}_i^* - h) + \frac{\beta k}{2h} (\hat{x}_j^* + h) \Big] \\ \\ \int \sum_{j=1}^{n_{j=1}^n} \frac{\alpha q_i^* (c + 2e\sum_{j=1}^n q_j^* - v)}{2h} + \frac{\alpha k q_i^*}{2h} - \frac{\alpha(c + 2e\sum_{j=1}^n q_j^* - v)}{2h} (\hat{x}_i^* - h) + \frac{\alpha k}{2h} (\hat{x}_i^* + h) \Big] \\ \\ \int \sum_{j=1}^{n_{j=1}^n} \frac{\alpha q_j^* (c + 2e\sum_{j=1}^n q_j^* - v)}{2h} + \frac{\alpha k q_i^*}{2h} - \frac{\alpha(c + 2e\sum_{j=1}^n q_j^* - v)}{2h} (\hat{x}_i^* - h) + \frac{\alpha k k}{2h} (\hat{x}_i^* + h) \Big]$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression can be derived through Proposition 8. After giving other corresponding parameters, we could simulate and present the optimal decision through numerical examples.

6.2. Single-Factor Combined Cost-Sharing Strategy. The urban joint distribution company and logistics companies share the risk cost due to insufficient and excessive supply of logistics services according to the unified factor combination coefficient, which is as follows:

$$(\phi_0, \phi_1, \phi_2, \phi_3, \dots, \phi_n)$$
, where $\phi_0 = 1 - \sum_{i=1}^n \phi_i$,
 $\forall \phi_i \in [0, 1], i = 0, 1, 2, 3, \dots, n$, (36)

where π_s^{rs2} can be obtained as follows, and the detail of the following equation is shown in Appendix (A.11):

$$\pi_{s}^{rs2} = w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \right] - \phi_{0} \sum_{i=1}^{n} \left\{ \frac{(w - v)q_{i}}{2h} \left(q_{i} - \hat{x}_{i} + h \right) - \frac{(w - v)}{4h} \right. \left. \left. \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{k}{4h} \left[\left(\hat{x}_{i} + h \right)^{2} - q_{i}^{2} \right] \right.$$
(37)
$$\left. - \frac{kq_{i}}{2h} \left(\hat{x}_{i} + h - q_{i} \right) \right\}.$$

We solve first derivatives of π_s^{rs2} with respect to the variable q_i and equate it to 0, thus obtaining the following equation:

$$\frac{\partial \pi_s^{rs2}}{\partial q_i} = w - \left(c + 2e\sum_{j=1}^n q_j\right) - \phi_0 \left[\frac{(w-v)}{2h} \left(q_i - \hat{x}_i + h\right) - \frac{k}{2h} \left(\hat{x}_i + h - q_i\right)\right] \\
= w - \left(c + 2e\sum_{j=1}^n q_j\right) - \phi_0 \left[\frac{(w-v+k)}{2h} \left(q_i - \hat{x}_i\right) + \frac{w-v-k}{2}\right] = 0,$$
(38)
$$w = \frac{2h \left(c + 2e\sum_{j=1}^n q_j\right) + \phi_0 \left(k - v\right) \left(q_i - \hat{x}_i\right) - \phi_0 h \left(k + v\right)}{\left(2 - \phi_0\right)h + \phi_0 \left(\hat{x}_i - q_i\right)},$$

where $\forall i, j \in N, i \neq j$, and we can obtain $q_i - \hat{x_i} = q_j - \hat{x_j}$. Thus, assuming $q_1 - \hat{x_1} = q_2 - \hat{x_2} = q_3 - \hat{x_3} = \cdots = q_n - \hat{x_n}$ $= Q \in [-h, h]$, then there is $\hat{x_i} = q_i - Q$, $i = 1, 2, 3, \ldots, n$. Thus, we obtain the expression of p_i as follows:

$$p_{i} = \frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{i=1}^{n} q_{i} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}.$$
 (39)

Equations (38) and (39) are the optimal decisions for the urban joint distribution company. Combining equations (38) and (39), we can obtain the profit of the logistics company i as follows and Proposition 9 can be obtained, and the detail of the following equation is shown in Appendix (A.12):

$$\pi_{i}^{rs2} = \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - (q_{i} - Q - h)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} (h - Q) - \phi_{i} \sum_{j=1}^{n} \left\{ \frac{(w - v + k)}{4h} q_{j}^{2} - q_{j} \left[\frac{(w - v)(q_{j} - Q - h)}{2h} + \frac{k(q_{j} - Q + h)}{2h} \right] + \frac{(w - v)}{4h} (q_{j} - Q - h)^{2} + \frac{k}{4h} (q_{j} - Q + h)^{2} \right\}.$$

$$(40)$$

Proposition 9. The optimal solution under single-factor combined cost-sharing strategy satisfies the following equation: $(q_i^*, Q^*) = \arg \max \{\pi_i^{rs2}(\Lambda_0^{rs2}), \pi_i^{rs2}(\Lambda_{q_i}^{rs2}, \Lambda_Q^{rs2})\}$, where Λ_0^{rs2} is the boundary of the feasible region of the decision

variable and $(\Lambda_{q_i}^{rs2}, \Lambda_Q^{rs2})$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} \frac{\lambda_{i}d + Q^{*} - q_{i}^{*}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j}^{*} - nQ^{*} - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}^{*}) + \phi_{0}(k - \nu)Q^{*} - \phi_{0}h(k + \nu)}{(2 - \phi_{0})h - \phi_{0}Q^{*}} \\ -c_{i} + \frac{q_{i}^{*2} - (q_{i}^{*} - Q^{*} - h)^{2} + 2q_{i}^{*}(h - Q^{*})}{4h} \\ \times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{(2 - \phi_{0})h - \phi_{0}Q^{*}} \right] - \phi_{i} \left[\frac{(h + Q^{*})^{2}}{4h} \times \frac{4nhe}{(2 - \phi_{0})h - \phi_{0}Q^{*}} + \frac{k}{2h} + \frac{k(2q_{i}^{*} - Q^{*} + h)}{2h} \right] = 0, \\ \Lambda_{q_{i}}^{rs2} = \{q_{i}^{*}\}, q_{i}^{*} \in \Lambda_{q_{i}}^{rs2}; \Lambda_{Q}^{rs2} = \{Q^{*}\}, Q^{*} \in [-h, h]. \end{cases}$$

$$(41)$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression can be derived through

Proposition 9. We could simulate and present the optimal decision through numerical examples after giving other corresponding parameters.

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6.3. Multifactor Combined Cost-Sharing Strategy. They bear the cost incurred due to the insufficient supply of logistics services according to the coefficient θ_i and bear the cost incurred due to the excess supply of logistics services according to the coefficient γ_i , which is as follows:

$$(\theta_0, \theta_1, \theta_2, \theta_3, \dots, \theta_n), (\omega_0, \omega_1, \omega_2, \omega, \dots, \omega_n),$$

where $\theta_0 = 1 - \sum_{i=1}^n \theta_i, \gamma_0 = 1 - \sum_{i=1}^n \omega_i,$
 $\forall \phi_i, \omega_i \in [0, 1], i = 0, 1, 2, 3, \dots, n,$ (42)

where π_s^{rs3} can be obtained as follows, and the detail of the following equation is shown in Appendix (A.13):

$$\pi_{s}^{rs3} = w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{i=n} q_{i} \right)^{2} \right] - \theta_{0} \sum_{j=1}^{n} \left\{ \frac{(w-v)q_{j}}{2h} \left(q_{j} - \hat{x}_{j} + h \right) - \frac{(w-v)}{4h} \left[q_{j}^{2} - \left(\hat{x}_{j} - h \right)^{2} \right] \right\}$$

$$- \omega_{0} \sum_{j=1}^{n} \left\{ \frac{k}{4h} \left[\left(\hat{x}_{j} + h \right)^{2} - q_{j}^{2} \right] - \frac{kq_{j}}{2h} \left(\hat{x}_{j} + h - q_{j} \right) \right\}.$$

$$(43)$$

We solve first derivatives of the function π_s^{rs3} with respect to the variable q_i and equate it to 0, thus obtaining equation (38):

$$\frac{\partial \pi_s^{rs3}}{\partial q_i} = w - \left(c + 2e\sum_{j=1}^n q_j\right) - \theta_0 \frac{(w - v)(q_i - \hat{x}_i + h)}{2h} - \omega_0 \frac{k(q_i - \hat{x}_i - h)}{2h} = 0,$$

$$w = \frac{2h(c + 2e\sum_{j=1}^n q_j) - \theta_0 v(q_i - \hat{x}_i + h) + \omega_0 k(q_i - \hat{x}_i - h)}{(2 - \theta_0)h - \theta_0(q_i - \hat{x}_i)},$$
(44)

Where $\forall i, j \in N, i \neq j$, and we can obtain $q_i - \hat{x}_i = q_j - \hat{x}_j$. Thus, assuming $q_1 - \hat{x}_1 = q_2 - \hat{x}_2 = q_3 - \hat{x}_3 = \cdots = q_n - \hat{x}_n$ $= Q \in [-h, h]$, then there is $\hat{x}_i = q_i - Q, i = 1, 2, 3, \dots, n$, obtaining the expression of p_i as equation (39):

$$p_i = \frac{\lambda_i d + Q - q_i}{\alpha + \beta} + \frac{\sum_{i=1}^n q_i - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}.$$
(45)

Equations (44) and (45) are the optimal decisions for the urban joint distribution company. Combining equations (44) and (45), we can obtain the profit of the logistics company i as follows and derive Proposition 10, and the detail of equation (46) is shown in Appendix (A.14):

$$\pi_{i}^{rs3} = \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - (q_{i} - Q - h)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} (h - Q) - \theta_{i} \sum_{j=1}^{n} \left\{ \frac{(w - v)q_{j}}{2h} (Q + h) - \frac{(w - v)}{4h} \left[q_{j}^{2} - (q_{j} - Q - h)^{2} \right] \right\} - \omega_{i} \sum_{j=1}^{n} \left\{ \frac{k}{4h} \left[\left(q_{j} - Q + h \right)^{2} - q_{j}^{2} \right] - \frac{kq_{j}}{2h} (h - Q) \right\}.$$
(46)

Proposition 10. The optimal solution under multifactor combined cost-sharing strategy satisfies the following equation: $qi*,Q* = argmax\pi irs3\Lambda 0rs3,\pi irs3\Lambda qirs3,\Lambda Qrs3$, where $\Lambda 0rs3$ is the boundary of the feasible region of the decision

variable and $\Lambda qirs3$, $\Lambda Qrs3$ denotes all the maximum points of the decision function in the feasible region of the decision variable. It also satisfies the following system of equations:

$$\begin{cases} \frac{\lambda_{i}d + Q^{*} - q_{i}^{*}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j}^{*} - nQ^{*} - d}{(\alpha + \beta)(n - 1(\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}^{*}) - \theta_{0}v(Q^{*} + h) - w_{0}k(h = Q^{*})}{(2 - \theta_{0})h - \theta_{0}Q^{*}} \\ -c_{i} + \frac{q_{i}^{*} - (q_{i}^{*} - Q^{*} - d) + 2(h - Q^{*})q_{i}^{*}}{4h} \times \\ \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1(\alpha/\beta))} - \frac{4he}{(2 - \theta_{0})h - \theta_{0}Q^{*}} \right] - \theta_{i} \times \frac{(Q^{*} + h)^{2}}{4h} \times \frac{4\eta he}{(2 - \theta_{0})h - \theta_{0}Q^{*}} = 0, \end{cases}$$

$$(47)$$

$$\Lambda_{q_{i}}^{rs3} = \{q_{i}^{*}\}, q_{i}^{*} \in \Lambda_{q_{i}}^{rs3}; \Lambda_{Q}^{rs3} = \{Q^{*}\}, Q^{*} \in [-h, h].$$

Proof. Refer to the Appendix.

Similar to the centralized and decentralized optimal solution, the implicit expression can be derived through Proposition 10. After giving other corresponding parameters, we could simulate and present the optimal decision through numerical examples.

7. Case Analysis

7.1. Case 1: Provincial Joint Distribution. We obtained relevant data for the two-echelon logistics service supply chain composed of Tongyue, Yusheng, Xinye, Wangyong, and Yuanyao in Sichuan Province for October, 2019, through interviews and surveys. Tongyue, Yusheng, and Xinye are selected for the analysis, and the model parameters are shown in Table 1.

The change trend of the total profit of the provincial supply chain under different cooperation model decisions is shown in Table 2 and Figure 1. We obtain some conclusions from Table 2. The total profit of the decentralized supply chain when the urban joint distribution company bears the cost risk is higher than that of the decentralized supply chain when the logistics company bears the cost risk. When the logistics companies bear the cost risks, the order flow proportion and the factor combination revenue-sharing strategy cannot achieve Pareto improvement of the total profit of the supply chain, where their values are equal to the total profit of the decentralized supply chain when the logistics company bears the cost risk. When the urban joint distribution company bears cost risks, the order flow proportion and the factor combination revenue-sharing strategy can achieve Pareto improvement of the total profit of the supply chain and the order flow proportion revenue-sharing strategy is better than factor combination revenue-sharing strategy. Although the Pareto improvement of the total profit of the supply chain failed to achieve supply chain coordination, compared to the total profit of the

decentralized supply chain, the profit of the supply chain in the order flow proportion revenue-sharing supply chain when urban joint distribution company bears cost risks increased by 55.48%.

As shown in Figure 1, under most factor combinations, the total profit of the supply chain under the single-factor combination cost-sharing strategy is significantly higher than that under the two-factor combination cost-sharing strategy. At the same time, the total profit of the supply chain under the cost-sharing strategy of logistics company order flow proportion is higher than that under the single factor combination-sharing strategy in the vast majority of factor combinations. However, there are certain factor combinations that make the total supply chain profit under the single-factor combined cost-sharing strategy higher than that under the logistics company's order flow proportion cost-sharing strategy. The total profit of the supply chain under the cost-sharing strategy of logistics company order flow proportion is higher than that under the two-factor combined cost-sharing strategy. Moreover, the total profit of the supply chain under the cost-sharing strategy is higher than that under the revenue-sharing strategy in most factor combinations. In particular, the order flow proportion costsharing strategy is better than the revenue-sharing strategy.

7.2. *Case 2: Interprovincial Joint Distribution*. The interprovincial joint distribution case uses data from the annual financial reports of the listed express companies A, B, C, and D for October, 2018, as shown in Tables 3 and 4.

The market share and market scale of various express delivery companies are shown in Table 3. The market shares are 19.63%, 25.43%, 23.67%, and 31.26% for the listed express companies A, B, C, and D, respectively.

Referring to Wei et al. [23]; we estimate α and β by the least square method. As shown in Table 4, the sensitivity coefficients of the market demand of each express company

	IABLE 1: Model parameters.					
α	β	λ_1	λ_2	λ_3	d	С
12	2	0.26	0.63	0.11	120	2
<i>c</i> ₁	<i>c</i> ₂	<i>c</i> ₃	ν	k	h	е
1	0.8	1.5	5	6	6	0.01

TABLE 2: Total profit of the provincial supply chain under different cooperative decision-making situations (×103).

$\pi_{\rm sc}^c$	$\pi^{d1}_{ m sc}$	$\pi^{d2}_{ m sc}$	$\pi^{rf1}_{ m sc}$	$\pi^{rf2}_{ m sc}$	$\pi^{rr1}_{ m sc}$	$\pi^{rr2}_{ m sc}$	$\pi^{rs1}_{ m sc}$	$\pi^{rs2}_{ m sc}$	$\pi^{rs3}_{ m sc}$
21.1	1.94	5.84	1.94	9.08	1.94	5.76	1.58	5.84	5.84
21.1	1.94	5.84	1.94	9.08	1.94	7.23	1.58	1.00	6.26
21.1	1.94	5.84	1.94	9.08	1.94	8.29	1.58	1.18	7.39
21.1	1.94	5.84	1.94	9.08	1.94	7.87	1.58	1.18	9.99
21.1	1.94	5.84	1.94	9.08	1.94	6.83	1.58	1.76	1.14
21.1	1.94	5.84	1.94	9.08	1.94	7.49	1.58	1.84	1.25
21.1	1.94	5.84	1.94	9.08	1.94	8.06	1.58	1.89	1.23

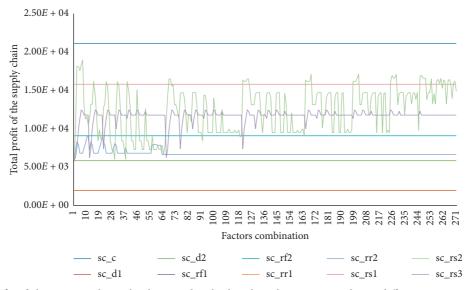


FIGURE 1: Total profit of the provincial supply chain under the benchmark situation and two different cooperative decision-making situations in case 1. Note: the abscissa in Figure 1 is the factor combination for revenue-sharing or cost-sharing. To facilitate an intuitive comparison, we have summarized the profit results in all relevant situations into a single graph. However, due to the different factor combinations of revenue-sharing or cost-sharing in different situations, some are single factors, while others are double factors, and the factor combinations are multidimensional. Hence, the abscissa only represents the optimal case of all factor combinations, not a specific parameter value. The abscissa in Figure 2 is the same as that in Figure 1.

TABLE 3: Market size and market share of listed express companies.

d	λ_1	λ_2	λ_3	λ_4
19954817621	0.1971	0.2633	0.2406	0.3209
	$\lambda_1 d$	$\lambda_2 d$	$\lambda_3 d$	$\lambda_4 d$
	3933094553	5254103480	4801129120	6403500975

TABLE 4: Estimated parameters of α and β .

$\hat{\alpha}$	\widehat{eta}
1342985031.67	7601243.14

to its own express service price and the service prices of other express companies are 1342985031.67 and 7601243.14, respectively.

As shown in Table 5, the unit distribution cost of the shared logistics distribution company industry is 1.55 yuan/

TABLE 5: Cost structure information.

С	c_1	<i>c</i> ₂	<i>c</i> ₃	c_4
1.55	1.03	1.24	1.08	0.95

piece and the unit transportation cost of each express delivery company, namely, the intercity logistics transportation cost, is 1.03, 1.24, 1.08, and 0.95, respectively.

As shown in Table 6, the value range of the nonscale economic cost coefficient e for shared logistics distribution companies to provide shared logistics distribution services is set between 0.01 and 0.11 yuan, the residual value of the excess supply of express services v is 1.20 yuan/piece, and the supply of express services is insufficient. The unit opportunity $\cos k$ is 2 yuan/piece. At the same time, the characteristic variable of uncertain itemsh is 200,000,000, which is about 4.44% of the average market size of several listed express companies.

TABLE 6: Risk costs.

е	ν	k	h
$0.01 \longrightarrow 0.11$	1.25	2.03	20000000

TABLE 7: Total profit of the interprovincial supply chain under different cooperative decision-making situations (×1016).

$\pi_{\rm sc}^c$	$\pi^{d1}_{ m sc}$	$\pi^{d2}_{ m sc}$	$\pi^{rf1}_{ m sc}$	$\pi^{rf2}_{ m sc}$	$\pi^{rr1}_{ m sc}$	$\pi^{rr2}_{ m sc}$	$\pi^{rs1}_{ m sc}$	$\pi^{rs2}_{ m sc}$	$\pi^{rs3}_{ m sc}$
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199
975	2.74	93.6	2.74	140	2.74	296	86.6	93.6	199

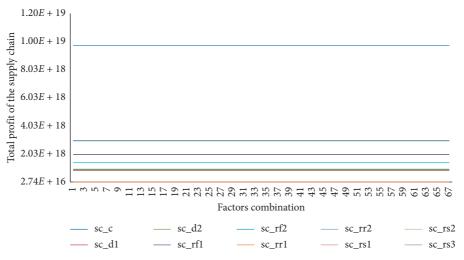


FIGURE 2: Total profit of the interprovincial supply chain under the benchmark situation and two different cooperative decision-making situations in case 2.

Next, we analyze the change in trend of the total profit of the supply chain under several supply chain decision situations when the combination sharing factor changes, taking centralized and decentralized supply chains as benchmarks. We randomly selected three from the four listed express companies in China for sharing logistics and distribution cooperation to graphically display the effect of the shared cooperation mechanism.

The change in trend of the total profit of the interprovincial supply chain under different cooperation model decisions is shown in Table 7 and Figure 2. We obtain the following conclusions. Under revenue-sharing or cost-sharing cooperation mechanism, not all cooperation scenarios under consideration can achieve Pareto improvement of the total profit of the supply chain, but at least one situation can achieve Pareto improvement. The total profit of the supply chain in an order flow proportion revenue-sharing supply chain and a factor combination revenuesharing supply chain when urban codistribution company bears cost risks and under two-factor combined cost-sharing strategy are higher than that under the decentralized supply chain. The optimal decision of the logistics service supply chain is to choose a factor-combined revenue-sharing cooperation strategy when the city codistribution company bears the risk cost. The total profit of the supply chain has increased by 216.24% compared to the total profit of the decentralized supply chain.

8. Conclusions

We design a two-echelon logistics service supply chain composed of an urban joint distribution company and N logistics companies. The urban joint distribution company is jointly established by N logistics companies that cooperate through equity investment. The equity investment ratio is uniquely determined by the revenue-sharing or cost-sharing factor that achieves Pareto improvement of the total profit of the supply chain. Then, based on the centralized and decentralized service supply chain, four different types of cooperation strategies are comparatively analyzed under the revenue-sharing mechanism, and three different types of cooperation strategies are analyzed under the cost-sharing mechanism. As the solution of optimal decision-making involves binary cubic equations, only implicit expressions could be obtained. Similar to Zhang et al. [22], we use some algorithms to solve the optimal decision in the case analysis. Both case 1 and case 2 show that whether to choose revenue-sharing or cost-sharing mechanism, not all cooperation scenarios considered in the study can achieve Pareto improvement of the total profit of the supply chain, but at least one situation can achieve Pareto improvement; thus, cooperation is the better choice.

The optimal decision of the logistics service supply chain in case 1 is to choose the logistics company's order flow proportion cost-sharing cooperation strategy. The equity investment proportion of the logistics company *i* is $(q_i / \sum_{j=1}^n q_j)$. This is contrary to the conclusion of Yang and Chen [31] and Yu et al. [33] who find that revenue-sharing contracts are preferred for the supply chain. A possible reason is that the two sides of the supply chain are independent in their studies, which is not the case in our study. The joint distribution company is jointly established by the logistics companies through equity cooperation. As it is related to revenue-sharing, cost-sharing is much more important.

The optimal decision of the logistics service supply chain in case 2 is to choose a factor combination revenue-sharing cooperation strategy when the urban joint distribution company bears the risk cost. The equity investment proportion of the logistics company *i* is γ_i . This is consistent with the research conclusions of Yang and Chen [31] and Yu et al. [33]; that is, revenue-sharing contracts are preferred for the supply chain. Even when revenue-sharing and costsharing are both optional, cost-sharing is dispensable.

The findings of the study have some implications for managements. First, the sharing economy model is promising in the field of urban distribution and cost reduction and efficiency increase could be achieved through building a common distribution platform. Second, different logistics companies can learn from the equity investment cooperation methods mentioned in this study when deciding which way to cooperate to build a joint distribution platform. The joint distribution company is jointly established by the logistics companies through equity cooperation, which is related to both the revenue-sharing and cost-sharing contracts. We propose an actual strategy that guides the logistics company scientifically steps into the cooperation based on both the supply chain coordination theory and the corporate governance (equity investment) theory. Third, our research conclusions have pointed out a stable vertical integration path for the scientific development of small and medium logistics companies, and this path can avoid the detours that many enterprises have taken on the road of integration to a certain extent.

There are several limitations worth further exploration. First, this study does not consider the issue of logistics service supply chain coordination. Second, it does not consider the risk preference of logistics service supply chain members.

Some future research directions are as follows. First, the issue of equity cooperation mechanism in the sharing logistics service supply chain can be considered when the service price changes with the distribution product, region, distance, and time. Second, the sharing logistics service supply chain coordination can be considered in future research. Third, the different risk preferences of logistics service supply chain members can be discussed in the future study on the design of equity cooperation mechanism in the sharing logistics service supply chain.

Notations

x_i :	Demand function of logistics company <i>i</i>
\widehat{x}_i :	Mean of x_i
$f'(x_i)$:	Probability density function of x_i
$F(x_i)$:	Probability distribution function of x_i
ε:	Error of x_i , $\varepsilon \sim U[-h, h]$
<i>h</i> :	The interval value of the uniform distribution
	$\varepsilon \sim U[-h,h]$
<i>d</i> :	Market size of the entire subdivision
	distribution field
<i>e</i> :	Unit nonscale economic cost
p_i :	Unit service price of logistics company <i>i</i>
q_i :	Service order quantity of logistics company <i>i</i>
<i>c</i> :	Unit service cost of the urban joint distribution
	company
c_i :	Unit service cost of logistics company <i>i</i>
α:	The sensitivity coefficient of the demand of
	logistics company <i>i</i> to its unit service price
β:	The sensitivity coefficient of the demand of
-	logistics company <i>i</i> to the unit service price of
	other logistics companies
λ_i :	Market share of the logistics company <i>i</i>
<i>w</i> :	Unit price of the urban joint distribution service
<i>v</i> :	Unit residual value of the urban joint
	distribution company
<i>k</i> :	Unit opportunity cost of the urban joint
	distribution company
γ_i :	Shared factor combination coefficient for
,	logistics company <i>i</i>
ϕ_i :	Unified factor combination coefficient of the urban
	joint distribution company and logistics companies that share the risk cost due to insufficient and
θ_i :	excessive supply of logistics services Coefficient of the urban joint distribution
o_i .	company and logistics companies that bear the
	cost incurred due to the insufficient supply of
	logistics services
<i>ω</i> _i :	Coefficient of the urban joint distribution
ω_{i} .	company and logistics companies that bear the
	cost incurred due to the excess supply of
	logistics services
π_s :	Profit function of the urban joint distribution
5	company
π_i :	Profit function of logistics company i
$O_i(\overrightarrow{p}, \overrightarrow{q})$:	Overcapacity cost Undercapacity cost
$U_i(\vec{p},\vec{q})$:	Undercapacity cost
$\pi_{\rm sc}^c$:	Profit of the centralized supply chain under joint
	distribution cooperation
π_s^{d1} :	Profit of the urban joint distribution company
	in a decentralized supply chain when logistics
<i>d</i> 1	companies bear cost risks
π_i^{d1} :	Profit of logistics company <i>i</i> in a decentralized
	supply chain when logistics companies bear cost
π^{d1} :	risks $-d1 - \sqrt{1 + \Sigma} - d1$
$\pi^{"}$:	$\pi^{d_1} = \pi^{d_1} + \Sigma \pi^{d_1}$

$$\pi_{\rm sc}^{d1} = \pi_s^{d1} + \sum \pi_i^{d1}$$

 $\pi_{\rm sc}^{d2}$

Profit of the urban joint distribution company in a decentralized supply chain when the urban joint distribution company bears cost risks $\pi_i^{d_2}$: Profit of logistics company *i* in a decentralized supply chain when the urban joint distribution company bears cost risks

$$\pi_{sc}^{d2}: \qquad \pi_{sc}^{d2} = \pi_{s}^{d2} + \sum \pi_{i}^{d2}$$

$$\pi_{s}^{r1}: \qquad \text{Profit of the urban joint distribution company}$$

in a revenue-sharing supply chain when logistics
companies bear cost risks

 π_i^{rf1} : Profit of logistics company *i* in an order flow proportion revenue-sharing supply chain when logistics companies bear cost risks π^{rf1} : $\pi^{rf1} = \pi^{r1} + \sum \pi^{rf1}$

$$\pi_{\rm sc}^{r_{\rm J}1}$$
: $\pi_{\rm sc}^{r_{\rm J}1} = \pi_{\rm s}^{r_{\rm J}1} + \sum_{r_{\rm J}1} \pi_{\rm sc}^{r_{\rm J}1}$

- $\pi_i^{rr1}: \qquad \text{Profit of logistics company } i \text{ in a factor} \\ \text{combination revenue-sharing supply chain} \\ \text{when logistics companies bear cost risks} \\ \pi_{sc}^{rr1}: \qquad \pi_{sc}^{rr1} = \pi_s^{r1} + \sum \pi_i^{rr1}$
- $\pi_{sc}^{rr1}: \qquad \pi_{sc}^{rr1} = \pi_s^{r1} + \sum \pi_i^{rr1}$ $\pi_s^{r2}: \qquad \text{Profit of the urban joint distribution company}$ in a revenue-sharing supply chain when the urban joint distribution company bears cost risks
- π_i^{rf2} : Profit of logistics company *i* in an order flow proportion revenue-sharing supply chain when the urban joint distribution company bears cost risks

$$\pi_{sc}^{rf2}: \qquad \pi_{sc}^{rf2} = \pi_{s}^{r2} + \sum \pi_{i}^{rf2}$$
$$\pi_{i}^{rr2}: \qquad \text{Profit of logistics c}$$

Profit of logistics company *i* in a factor combination revenue-sharing supply chain

when the urban joint distribution company bears cost risks

Appendix

The detail of equation (6) is as follows:

$$\begin{aligned} \pi_{sc}^{c} &= \pi_{s} + \sum_{i=1}^{n} \pi_{i} - \sum_{i=1}^{n} O_{i}(\vec{p},\vec{q}) - \sum_{i=1}^{n} U_{i}(\vec{p},\vec{q}) \\ &= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \right] \\ &+ \sum_{i=1}^{n} \left[\int_{\hat{x}_{i}-h}^{q_{i}} (p_{i} - w - c_{i}) x_{i} f(x_{i}) dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} (p_{i} - w - c_{i}) q_{i} f(x_{i}) dx_{i} - \int_{\hat{x}_{i}-h}^{q_{i}} (w - v) (q_{i} - x_{i}) f(x_{i}) dx_{i} - \int_{q_{i}}^{\hat{x}_{i}+h} k(x_{i} - q_{i}) f(x_{i}) dx_{i} \right] \\ &= \sum_{i=1}^{n} \left[\int_{\hat{x}_{i}-h}^{q_{i}} (p_{i} - c_{i} - v) x_{i} f(x_{i}) dx_{i} + v q_{i} \int_{\hat{x}_{i}-h}^{q_{i}} f(x_{i}) dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} (p_{i} - c_{i} + k) q_{i} f(x_{i}) dx_{i} - k \int_{q_{i}}^{\hat{x}_{i}+h} x_{i} f(x_{i}) dx_{i} \right] - c \sum_{i=1}^{n} q_{i} - e \left(\sum_{i=1}^{n} q_{i} \right)^{2} \\ &= \sum_{i=1}^{n} \left\{ \frac{(p_{i} - c_{i} - v)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{v q_{i}}{2h} (q_{i} - \hat{x}_{i} + h) + \frac{q_{i} (p_{i} - c_{i} + k)}{2h} (\hat{x}_{i} + h - q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i} + h)^{2} - q_{i}^{2} \right] \right\} - c \sum_{i=1}^{n} q_{i} - e \left(\sum_{i=1}^{n} q_{i} \right)^{2}. \end{aligned}$$

$$\tag{A.1}$$

The detail of equation (9) is as follows: $\pi_i^{d_1} = \pi_i - O_i(\overrightarrow{p}, \overrightarrow{q}) - U_i(\overrightarrow{p}, \overrightarrow{q})$

$$= \int_{\hat{x}_{i}-h}^{q_{i}} (p_{i}-w-c_{i})x_{i}f(x_{i})dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} (p_{i}-w-c_{i})q_{i}f(x_{i})dx_{i} - \int_{\hat{x}_{i}-h}^{q_{i}} (w-v)(q_{i}-x_{i})f(x_{i})dx_{i} - \int_{q_{i}}^{\hat{x}_{i}+h} k(x_{i}-q_{i})f(x_{i})dx_{i}$$

$$= \int_{\hat{x}_{i}-h}^{q_{i}} (p_{i}-c_{i}-v)x_{i}f(x_{i})dx_{i} + vq_{i}\int_{\hat{x}_{i}-h}^{q_{i}} x_{i}f(x_{i})dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} (p_{i}-c_{i}+k)q_{i}f(x_{i})dx_{i} - k\int_{q_{i}}^{\hat{x}_{i}+h} x_{i}f(x_{i})dx_{i} - wq_{i}$$

$$= \frac{(p_{i}-c_{i}-v)}{4h} \left[q_{i}^{2}-(\hat{x}_{i}-h)^{2}\right] + \frac{vq_{i}}{2h}(q_{i}-\hat{x}_{i}+h) + \frac{q_{i}(p_{i}-c_{i}+k)}{2h}(\hat{x}_{i}+h-q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i}+h)^{2}-q_{i}^{2}\right] - cq_{i} - 2eq_{i}\sum_{j=1}^{j=n} q_{j}.$$
(A.2)

The detail of equation (11) is as follows:

$$\pi_{s}^{d2} = \underbrace{w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]}_{\pi_{s}} - \sum_{i=1}^{n} \left[O_{i}(\overrightarrow{p}, \overrightarrow{q}) + U_{i}(\overrightarrow{p}, \overrightarrow{q})\right]$$

$$= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]$$

$$- \sum_{i=1}^{n} \left[\int_{\overrightarrow{x}_{i}-h}^{q_{i}} (w - v)(q_{i} - x_{i})f(x_{i})dx_{i} + \int_{q_{i}}^{\overrightarrow{x}_{i}+h} k(x_{i} - q_{i})f(x_{i})dx_{i}\right]$$

$$= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]$$

$$- \sum_{i=1}^{n} \left[\frac{(w - v)q_{i}}{2h}(q_{i} - \overrightarrow{x}_{i} + h) - \frac{(w - v)}{4h}\left[q_{i}^{2} - (\overrightarrow{x}_{i} - h)^{2}\right] + \frac{k}{4h}\left[(\overrightarrow{x}_{i} + h)^{2} - q_{i}^{2}\right] - \frac{kq_{i}}{2h}(\overrightarrow{x}_{i} + h - q_{i})\right].$$
(A.3)

The detail of equation (12) is as follows:

$$\frac{d\pi_s^{d2}}{dq_i} = w - \left[c + 2e\sum_{j=1}^n q_j\right] - \left[\frac{(w-v)}{2h}\left(q_i - \hat{x}_i + h\right) - \frac{k}{2h}\left(\hat{x}_i + h - q_i\right)\right]$$

$$= w - \left[c + 2e\sum_{j=1}^n q_j\right] - \left[\frac{(w-v+k)}{2h}\left(q_i - \hat{x}_i\right) + \frac{w-v-k}{2}\right] = 0.$$
(A.4)

The detail of equation (14) is as follows:@

$$\begin{aligned} \pi_{i}^{d2} &= \int_{\hat{x}_{i}-h}^{q_{i}} \left(p_{i}-w-c_{i}\right) x_{i} f\left(x_{i}\right) \mathrm{d}x_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} \left(p_{i}-w-c_{i}\right) q_{i} f\left(x_{i}\right) \mathrm{d}x_{i} \\ &= \frac{\left(p_{i}-w-c_{i}\right)}{4h} \left[q_{i}^{2}-\left(\hat{x}_{i}-h\right)^{2}\right] + \frac{q_{i}\left(p_{i}-w-c_{i}\right)}{2h} \left(\hat{x}_{i}+h-q_{i}\right) \\ &= \frac{\left(p_{i}-w-c_{i}\right)}{4h} \left[-q_{i}^{2}-\left(q_{i}-Q-h\right)^{2}+2q_{i}\left(q_{i}-Q+h\right)\right] \\ &= \frac{1}{4h} \left[\frac{\lambda_{i}d+Q-q_{i}}{\alpha+\beta} + \frac{\sum_{i=1}^{n} q_{i}-nQ-d}{\left(\alpha+\beta\right)\left(n-1-\left(\alpha/\beta\right)\right)} - \frac{2h\left(c+2e\sum_{j=1}^{n} q_{j}\right)+\left(k-\nu\right)Q-h\left(k+\nu\right)}{h-Q} - c_{i}\right] \\ &\quad \cdot \left[-q_{i}^{2}-\left(q_{i}-Q-h\right)^{2}+2q_{i}\left(q_{i}-Q+h\right)\right]. \end{aligned}$$

The detail of equation (18) is as follows:

$$\begin{aligned} \pi_{i}^{rf1} &= \pi_{i} - O_{i}(\overrightarrow{p}, \overrightarrow{q}) - U_{i}(\overrightarrow{p}, \overrightarrow{q}) + \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \pi_{s}^{r1} \\ &= \int_{\vec{x}_{i}-h}^{q_{i}} (p_{i} - w - c_{i})x_{i}f(x_{i})dx_{i} + \int_{q_{i}}^{\vec{x}_{i}+h} (p_{i} - w - c_{i})q_{i}f(x_{i})dx_{i} \\ &- \int_{\vec{x}_{i}-h}^{q_{i}} (w - v)(q_{i} - x_{i})f(x_{i})dx_{i} - \int_{q_{i}}^{\vec{x}_{i}+h} k(x_{i} - q_{i})f(x_{i})dx_{i} + wq_{i} - \left(cq_{i} + eq_{i}\sum_{j=1}^{n} q_{j}\right) \\ &= \int_{\vec{x}_{i}-h}^{q_{i}} (p_{i} - c_{i} - v)x_{i}f(x_{i})dx_{i} + vq_{i}\int_{\vec{x}_{i}-h}^{q_{i}} x_{i}f(x_{i})dx_{i} + \int_{q_{i}}^{\vec{x}_{i}+h} (p_{i} - c_{i} + k)q_{i}f(x_{i})dx_{i} \\ &- k\int_{q_{i}}^{\vec{x}_{i}+h} x_{i}f(x_{i})dx_{i} - \left(cq_{i} + eq_{i}\sum_{j=1}^{n} q_{j}\right) \\ &= \frac{(p_{i} - c_{i} - v)}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2}\right] + \frac{vq_{i}}{2h}(q_{i} - \hat{x}_{i} + h) + \frac{q_{i}(p_{i} - c_{i} + k)}{2h}(\hat{x}_{i} + h - q_{i}) - \frac{k}{4h} \left[(\hat{x}_{i} + h)^{2} - q_{i}^{2}\right] - cq_{i} - eq_{i}\sum_{j=1}^{n} q_{j}. \end{aligned}$$
(A.6)

The detail of equation (21) is as follows:

$$\begin{aligned} \pi_{i}^{rr1} &= \pi_{i} - O_{i}\left(\vec{p}, \vec{q}\right) - U_{i}\left(\vec{p}, \vec{q}\right) + \gamma_{i}\pi_{s} \\ &= \int_{\hat{x}_{i}-h}^{q_{i}} \left(p_{i} - w - c_{i}\right)x_{i}f\left(x_{i}\right)dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} \left(p_{i} - w - c_{i}\right)q_{i}f\left(x_{i}\right)dx_{i} - \int_{\hat{x}_{i}-h}^{q_{i}} \left(w - v\right)\left(q_{i} - x_{i}\right)f\left(x_{i}\right)dx_{i} \\ &- \int_{q_{i}}^{\hat{x}_{i}+h} k\left(x_{i} - q_{i}\right)f\left(x_{i}\right)dx_{i} + \gamma_{i}\left\{w\sum_{i=1}^{n}q_{i} - \left[c\sum_{i=1}^{n}q_{i} + e\left(\sum_{i=1}^{n}q_{i}\right)^{2}\right]\right\} \\ &= \int_{\hat{x}_{i}-h}^{q_{i}} \left(p_{i} - c_{i} - v\right)x_{i}f\left(x_{i}\right)dx_{i} + vq_{i}\int_{\hat{x}_{i}-h}^{q_{i}} x_{i}f\left(x_{i}\right)dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} \left(p_{i} - c_{i} + k\right)q_{i}f\left(x_{i}\right)dx_{i} \\ &- k\int_{q_{i}}^{\hat{x}_{i}+h} x_{i}f\left(x_{i}\right)dx_{i} - wq_{i} + \gamma_{i}\left\{w\sum_{i=1}^{n}q_{i} - \left[c\sum_{i=1}^{n}q_{i} + e\left(\sum_{i=1}^{n}q_{i}\right)^{2}\right]\right\} \\ &= \frac{\left(p_{i} - c_{i} - v\right)}{4h}\left[q_{i}^{2} - \left(\hat{x}_{i} - h\right)^{2}\right] + \frac{vq_{i}}{2h}\left(q_{i} - \hat{x}_{i} + h\right) + \frac{q_{i}\left(p_{i} - c_{i} + k\right)}{2h}\left(\hat{x}_{i} + h - q_{i}\right) - \frac{k}{4h}\left[\left(\hat{x}_{i} + h\right)^{2} - q_{i}^{2}\right] \\ &- \left(c + 2e\sum_{j=1}^{n}q_{j}\right)q_{i} + \gamma_{i}\left\{\left(c + 2e\sum_{j=1}^{n}q_{j}\right)\sum_{i=1}^{n}q_{i} - \left[c\sum_{i=1}^{n}q_{i} + e\left(\sum_{i=1}^{n}q_{i}\right)^{2}\right]\right\}. \end{aligned}$$

The detail of equation (27) is as follows:

$$\begin{aligned} \pi_{i}^{rf2} &= \pi_{i} + \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \pi_{s}^{r2} \\ &= \int_{\hat{x}_{i}-h}^{q_{i}} (p_{i} - w - c_{i}) x_{i} f(x_{i}) dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} (p_{i} - w - c_{i}) q_{i} f(x_{i}) dx_{i} + wq_{i} - \left(cq_{i} + eq_{i} \sum_{j=1}^{n} q_{j}\right) \\ &= (p_{i} - w - c_{i}) \frac{\left[-q_{i}^{2} - \left(\hat{x}_{i} - h\right)^{2} + 2q_{i}\left(\hat{x}_{i} + h\right)\right]}{4h} + wq_{i} - \left(cq_{i} + eq_{i} \sum_{j=1}^{n} q_{j}\right) \end{aligned}$$
(A.8)
$$&= \left(\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h\left(c + 2e\sum_{j=1}^{n} q_{j}\right) + (k - v)Q - h\left(k + v\right)}{h - Q} - c_{i}\right) \\ &= \frac{\left[-q_{i}^{2} - \left(q_{i} - Q - h\right)^{2} + 2q_{i}\left(q_{i} - Q + h\right)\right]}{4h} + wq_{i} - \left(cq_{i} + eq_{i} \sum_{j=1}^{n} q_{j}\right). \end{aligned}$$

The detail of equation (30) is as follows:

$$\begin{aligned} \pi_{i}^{rr2} &= \pi_{i} + \gamma_{i} \pi_{s}^{r2} = \int_{\hat{x}_{i}-h}^{q_{i}} \left(p_{i} - w - c_{i}\right) x_{i} f\left(x_{i}\right) dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} \left(p_{i} - w - c_{i}\right) q_{i} f\left(x_{i}\right) dx_{i} + \gamma_{i} \left\{ w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] \right\} \\ &= \left(p_{i} - w - c_{i}\right) \frac{\left[-q_{i}^{2} - \left(\hat{x}_{i} - h\right)^{2} + 2q_{i}\left(\hat{x}_{i} + h\right)\right]}{4h} + \gamma_{i} \left\{ w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] \right\} \\ &= \left(\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j} - nQ - d}{(\alpha + \beta)\left(n - 1 - (\alpha/\beta)\right)} - \frac{2h\left(c + 2e\sum_{j=1}^{n} q_{j}\right) + (k - \nu)Q - h\left(k + \nu\right)}{h - Q} - c_{i}\right) \\ &= \frac{\left[-q_{i}^{2} - \left(q_{i} - Q - h\right)^{2} + 2q_{i}\left(q_{i} - Q + h\right)\right]}{4h} + \gamma_{i} \left\{ w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] \right\}. \end{aligned}$$
(A.9)

The detail of equation (34) is as follows:

$$\begin{split} \pi_{i}^{rs1} &= \pi_{i} - \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \sum_{j=1}^{n} \left[O_{i}(\vec{p},\vec{q}) + U_{i}(\vec{p},\vec{q}) \right] \\ &= \int_{\hat{x}_{i}-h}^{q_{i}} \left(p_{i} - w - c_{i} \right) x_{i} f\left(x_{i} \right) dx_{i} + \int_{q_{i}}^{\hat{x}_{i}+h} \left(p_{i} - w - c_{i} \right) q_{i} f\left(x_{i} \right) dx_{i} \\ &- \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \sum_{j=1}^{n} \left[\int_{\hat{x}_{j}-h}^{q_{j}} \left(w - v \right) \left(q_{j} - x_{j} \right) f\left(x_{j} \right) dx_{j} + \int_{q_{j}}^{\hat{x}_{j}+h} k\left(x_{j} - q_{j} \right) f\left(x_{j} \right) dx_{j} \right] \right] \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \\ &\sum_{j=1}^{n} \left\{ \frac{(w - v)q_{j}}{2h} \left(q_{j} - \hat{x}_{j} + h \right) - \frac{(w - v)}{4h} \left[q_{j}^{2} - \left(\hat{x}_{j} - h \right)^{2} \right] + \frac{k}{4h} \left[\left(\hat{x}_{j} + h \right)^{2} - q_{j}^{2} \right] - \frac{kq_{j}}{2h} \left(\hat{x}_{j} + h - q_{j} \right) \right] \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{2h} \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{4h} \right] \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{2h} \right] \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{2h} \right] \\ &= \frac{(p_{i} - c - 2e\sum_{j=1}^{n} q_{j} - c_{i})}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i}(p_{i} - c - 2e\sum_{j=1}^{n} q_{j} - c_{i}}}{2h} \left(\hat{x}_{i} + h - q_{i} \right) - \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \right) \\ &= \frac{\sum_{j=1}^{n} \left\{ \frac{(c + 2e\sum_{j=1}^{n} q_{j} - v + k)}{4h} q_{j}^{2} - q_{j} \left[\frac{(c + 2e\sum_{j=1}^{n} q_{j} - v)}{2h} + \frac{k(\hat{x}_{j} + h)}{2h} \right] \\ &+ \frac{(c + 2e\sum_{j=1}^{n} q_{j} - v)}{4h} \left(\hat{x}_{j} - h \right)^{2} + \frac{k}{4h} \left(\hat{x}_{j} + h \right)^{2} \right\}.$$

The detail of equation (37) is as follows:

$$\pi_{s}^{rs2} = \underbrace{w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] - \phi_{0} \sum_{i=1}^{n} \left[O_{i}\left(\overrightarrow{p}, \overrightarrow{q}\right) + U_{i}\left(\overrightarrow{p}, \overrightarrow{q}\right)\right]}_{\pi_{s}}$$

$$= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] - \phi_{0} \sum_{i=1}^{n} \left[\int_{\overrightarrow{x}_{i}-h}^{q_{i}} (w - v)(q_{i} - x_{i})f(x_{i})dx_{i} + \int_{q_{i}}^{\overrightarrow{x}_{i}+h} k(x_{i} - q_{i})f(x_{i})dx_{i}\right]$$

$$= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]$$

$$- \phi_{0} \sum_{i=1}^{n} \left\{\frac{(w - v)q_{i}}{2h}(q_{i} - \overrightarrow{x}_{i} + h) - \frac{(w - v)}{4h}\left[q_{i}^{2} - (\overrightarrow{x}_{i} - h)^{2}\right] + \frac{k}{4h}\left[\left(\overrightarrow{x}_{i} + h\right)^{2} - q_{i}^{2}\right] - \frac{kq_{i}}{2h}\left(\overrightarrow{x}_{i} + h - q_{i}\right)\right].$$
(A.11)

The detail of equation (40) is as follows:

$$\begin{aligned} \pi_{i}^{rs2} &= \pi_{i} - \phi_{i} \sum_{i=1}^{n} \left[O_{i}(\overrightarrow{p}, \overrightarrow{q}) + U_{i}(\overrightarrow{p}, t\overrightarrow{q}) \right] = \int_{\vec{x}_{i}-h}^{q_{i}} \left(p_{i} - w - c_{i} \right) x_{i} f(x_{i}) dx_{i} + \int_{q_{i}}^{\vec{x}_{i}+h} \left(p_{i} - w - c_{i} \right) q_{i} f(x_{i}) dx_{i} \\ &- \phi_{i} \sum_{j=1}^{n} \left\{ \int_{\vec{x}_{j}-h}^{q_{j}} \left(w - v \right) \left(q_{j} - x_{j} \right) f(x_{j}) dx_{j} + \int_{q_{j}}^{\vec{x}_{j}+h} k\left(x_{j} - q_{j} \right) f(x_{j}) dx_{j} \right\} \\ &= \frac{\left(p_{i} - w - c_{i} \right)}{4h} \left[q_{i}^{2} - \left(\hat{x}_{i} - h \right)^{2} \right] + \frac{q_{i} \left(p_{i} - w - c_{i} \right)}{2h} \left(\hat{x}_{i} + h - q_{i} \right) \\ &- \phi_{i} \sum_{j=1}^{n} \left\{ \frac{\left(w - v + k \right)}{4h} q_{j}^{2} - q_{j} \left[\frac{\left(w - v \right) \left(\hat{x}_{j} - h \right)}{2h} + \frac{k\left(\hat{x}_{j} + h \right)}{2h} \right] + \frac{\left(w - v \right)}{4h} \left(\hat{x}_{j} - h \right)^{2} + \frac{k}{4h} \left(\hat{x}_{j} + h \right)^{2} \right\} \\ &= \frac{\left(p_{i} - w - c_{i} \right)}{4h} \left[q_{i}^{2} - \left(q_{i} - Q - h \right)^{2} \right] + \frac{q_{i} \left(p_{i} - w - c_{i} \right)}{2h} \left(h - Q \right) \\ &- \phi_{i} \sum_{j=1}^{n} \left\{ \frac{\left(w - v + k \right)}{4h} q_{j}^{2} - q_{j} \left[\frac{\left(w - v \right) \left(q_{j} - w - c_{i} \right)}{2h} \left(h - Q \right) \\ &- \phi_{i} \sum_{j=1}^{n} \left\{ \frac{\left(w - v + k \right)}{4h} q_{j}^{2} - q_{j} \left[\frac{\left(w - v \right) \left(q_{j} - Q - h \right)}{2h} + \frac{k\left(q_{j} - Q + h \right)}{2h} \right] + \frac{\left(w - v \right)}{4h} \left(q_{j} - Q - h \right)^{2} + \frac{k}{4h} \left(q_{j} - Q + h \right)^{2} \right\}. \end{aligned}$$
(A.12)

The detail of equation (43) is as follows:

$$\pi_{s}^{rs3} = \underbrace{w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right]_{\pi_{s}} - \theta_{0} \sum_{i=1}^{n} O_{i}(\vec{p}, \vec{q}) - w \sum_{i=1}^{n} U_{i}(\vec{p}, \vec{q})}_{i} = w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{n} q_{i}\right)^{2}\right] - \theta_{0} \sum_{i=1}^{n} \int_{\vec{x}_{i}-h}^{q_{i}} (w - v)(q_{i} - x_{i})f(x_{i})dx_{i} - \omega_{0} \sum_{i=1}^{n} \int_{q_{i}}^{\vec{x}_{i}+h} k(x_{i} - q_{i})f(x_{i})dx_{i}$$

$$= w \sum_{i=1}^{n} q_{i} - \left[c \sum_{i=1}^{n} q_{i} + e\left(\sum_{i=1}^{i=n} q_{i}\right)^{2}\right] - \theta_{0} \sum_{j=1}^{n} \left\{\frac{(w - v)q_{j}}{2h}(q_{j} - \hat{x}_{j} + h) - \frac{(w - v)}{4h}\left[q_{j}^{2} - (\hat{x}_{j} - h)^{2}\right]\right\}$$

$$- \omega_{0} \sum_{j=1}^{n} \left\{\frac{k}{4h}\left[(\hat{x}_{j} + h)^{2} - q_{j}^{2}\right] - \frac{kq_{j}}{2h}(\hat{x}_{j} + h - q_{j})\right\}.$$
(A.13)

The detail of equation (46) is as follows:

$$\begin{aligned} \pi_{i}^{rs3} &= \pi_{i} - \theta_{i} \sum_{i=1}^{n} O_{i}\left(\overrightarrow{p}, \overrightarrow{q}\right) - \omega_{i} \sum_{i=1}^{n} U_{i}\left(\overrightarrow{p}, \overrightarrow{q}\right) = \int_{\vec{x}_{i}-h}^{q_{i}} (p_{i} - w - c_{i})x_{i}f(x_{i})dx_{i} + \int_{q_{i}}^{\vec{x}_{i}+h} (p_{i} - w - c_{i})q_{i}f(x_{i})dx_{i} \\ &- \theta_{i} \sum_{i=1}^{n} \int_{\vec{x}_{i}-h}^{q_{i}} (w - v)(q_{i} - x_{i})f(x_{i})dx_{i} - \omega_{i} \sum_{i=1}^{n} \int_{q_{i}}^{\vec{x}_{i}+h} k(x_{i} - q_{i})f(x_{i})dx_{i} \\ &= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - (\hat{x}_{i} - h)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} (\hat{x}_{i} + h - q_{i}) - \theta_{i} \sum_{j=1}^{n} \left\{ \frac{(w - v)q_{j}}{2h} (q_{j} - \hat{x}_{j} + h) - \frac{(w - v)}{4h} \left[q_{j}^{2} - (\hat{x}_{j} - h)^{2} \right] \right\} \\ &- \omega_{i} \sum_{j=1}^{n} \left\{ \frac{k}{4h} \left[(\hat{x}_{j} + h)^{2} - q_{j}^{2} \right] - \frac{kq_{j}}{2h} (\hat{x}_{j} + h - q_{j}) \right\} \end{aligned}$$
(A.14)
$$&= \frac{(p_{i} - w - c_{i})}{4h} \left[q_{i}^{2} - (q_{i} - Q - h)^{2} \right] + \frac{q_{i}(p_{i} - w - c_{i})}{2h} (h - Q) - \theta_{i} \sum_{j=1}^{n} \left\{ \frac{(w - v)q_{j}}{2h} (Q + h) - \frac{(w - v)}{4h} \left[q_{j}^{2} - (q_{j} - Q - h)^{2} \right] \right\} \\ &- \omega_{i} \sum_{j=1}^{n} \left\{ \frac{k}{4h} \left[(q_{j} - Q + h)^{2} - q_{j}^{2} \right] - \frac{kq_{j}}{2h} (h - Q) \right\}. \end{aligned}$$

Proof of Proposition 1. $\forall j \in N, j \neq i, p_j = 0$, and when $\lambda_i d - \alpha p_i > 0$, we can obtain $0 < p_i < (\lambda_i d/\alpha)$. Meanwhile, $q_i \in [\hat{x}_i - h, \hat{x}_i + h]$, so the decision variables are bounded and the continuity conditions are satisfied. According to the

axiom of choice and bounded continuity, the maximum value of the decision function can be determined [39, 40].

Solve the first and second derivatives of π_{sc}^c :

$$\begin{split} \frac{\partial n_{sc}^{c}}{\partial p_{i}} &= \frac{q_{i}^{2} - \left(\hat{x}_{i} - h\right)^{2}}{4h} + \frac{\alpha(p_{i} - c_{i} - \nu)\left(\hat{x}_{i} - h\right)}{2h} + \frac{\alpha vq_{i}}{2h} + \frac{q_{i}\left(\hat{x}_{i} + h - q_{i}\right)}{2h} - \frac{\alpha q_{i}\left(p_{i} - c_{i} + k\right)}{2h} + \frac{\alpha k\left(\hat{x}_{i} + h\right)}{2h} \\ &+ \sum_{\substack{j=1\\j\neq i}}^{n} \left[-\frac{\beta(p_{j} - c_{j} - \nu)\left(\hat{x}_{j} - h\right)}{2h} + \frac{\beta q_{j}\left(p_{j} - c_{j} + k - \nu\right)}{2h} - \frac{\beta k\left(\hat{x}_{j} + h\right)}{2h} \right] \\ &= -\frac{q_{i}^{2}}{4h} + \frac{q_{i}}{2h}\left(\hat{x}_{i} + h + \alpha \nu - \alpha p_{i} + \alpha c_{i} - \alpha k\right) - \frac{\left(\hat{x}_{i} - h\right)^{2}}{4h} + \frac{\alpha\left(p_{i} - c_{i} - \nu\right)\left(\hat{x}_{i} - h\right)}{2h} + \frac{\alpha k\left(\hat{x}_{i} + h\right)}{2h} \\ &+ \sum_{\substack{j=1\\j\neq i}}^{n} \left[-\frac{\beta(p_{j} - c_{j} - \nu)\left(\hat{x}_{j} - h\right)}{2h} + \frac{\beta q_{j}\left(p_{j} - c_{j} + k - \nu\right)}{2h} - \frac{\beta k\left(\hat{x}_{j} + h\right)}{2h} \right], \end{split}$$
(A.15)
$$\frac{\partial^{2} n_{sc}^{c}}{\partial p_{i}^{2}} &= -\frac{\alpha q_{i}}{h} + \frac{\alpha\left(\hat{x}_{i} - h\right)}{2h} + \frac{\alpha\left(\hat{x}_{i} - h\right)}{2h} - \frac{\alpha^{2}\left(p_{i} - c_{i} - \nu\right)}{2h} - \frac{\alpha^{2}k}{2h} + \sum_{\substack{j=1\\j\neq i}}^{n} \left[-\frac{\beta^{2}\left(p_{j} - c_{j} - \nu + k\right)}{2h} \right] \\ &= \frac{\alpha\left(\hat{x}_{i} - h - q_{i}\right)}{h} - \frac{\alpha^{2}\left(p_{i} - c_{i} - \nu + k\right)}{2h} + \sum_{\substack{j=1\\j\neq i}}^{n} \left[-\frac{\beta^{2}\left(p_{j} - c_{j} - \nu + k\right)}{2h} \right] \\ &< 0, \\ \frac{\partial n_{sc}^{c}}{\partial q_{i}} &= \frac{q_{i}\left(p_{i} - c_{i} - \nu\right)}{2h} + \frac{\nu\left(2q_{i} - \hat{x}_{i} + h\right)}{2h} + \frac{p_{i} - c_{i} + k + \nu}{2} - c - 2e\sum_{j=1}^{n} q_{j}, \\ \frac{\partial^{2} n_{sc}^{c}}{\partial q_{i}^{2}} &= -\frac{p_{i} - c_{i} - \nu + k}{2h} - 2e < 0. \end{aligned}$$

Therefore, $\forall i \in N$, and there is an optimal (p_i, q_i) to maximize π_{sc}^c .

Proof of Proposition 2. Solve the first and second derivatives of $\pi_i^{d_1}$:

$$\begin{split} \frac{\partial \pi_i^{d_1}}{\partial p_i} &= \frac{q_i^2 - \left(\hat{x}_i - h\right)^2}{4h} + \frac{\alpha \left(p_i - c_i - v\right)\left(\hat{x}_i - h\right)}{2h} + \frac{\alpha vq_i}{2h} + \frac{q_i\left(\hat{x}_i + h - q_i\right)}{2h} - \frac{\alpha q_i\left(p_i - c_i + k\right)}{2h} + \frac{\alpha k}{2h}\left(\hat{x}_i + h\right) \\ &= -\frac{q_i^2}{4h} + \frac{q_i\left(\hat{x}_i + h\right)}{2h} - \frac{\alpha q_i\left(p_i - c_i - v + k\right)}{2h} - \frac{\left(\hat{x}_i - h\right)^2}{4h} + \frac{\alpha \left(p_i - c_i - v\right)\left(\hat{x}_i - h\right)}{2h} + \frac{\alpha k}{2h}\left(\hat{x}_i + h\right), \\ \frac{\partial^2 \pi_i^{d_1}}{\partial p_i^2} &= -\frac{\alpha q_i}{2h} - \frac{\alpha q_i}{2h} + \frac{\alpha \left(\hat{x}_i - h\right)}{2h} + \frac{\alpha \left(\hat{x}_i - h\right)}{2h} - \frac{\alpha^2 \left(p_i - c_i - v\right)}{2h} - \frac{\alpha^2 k}{2h} \\ &= -\frac{\alpha \left(\frac{20}{q_i - \hat{x}_i + h}\right)}{h} - \frac{\alpha^2 \left(\frac{20}{p_i - c_i} - \frac{20}{v + k}\right)}{2h} < 0, \end{split}$$
(A.16)
$$\frac{\partial \pi_i^{d_1}}{\partial q_i} &= \frac{q_i \left(p_i - c_i - v\right)}{2h} + \frac{v \left(2q_i - \hat{x}_i + h\right)}{2h} + \frac{p_i - c_i + k + v}{2} - c - 2e \sum_{j=1}^n q_j - 2eq_i \\ &= -\frac{2e \sum_{j=1}^n q_j - 2eq_i}{2h} = -\frac{\frac{20}{p_i - c_i} + \frac{20}{k - v}}{2h} - 4e < 0. \end{split}$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 2.

Proof of Proposition 3. Solve the first and second derivatives of $\pi_i^{d_2}$:

$$\begin{aligned} \frac{\partial \pi_i^{d2}}{\partial q_i} &= p_i - w - c_i + \frac{-q_i^2 - (q_i - Q - h)^2 + 2q_i(q_i - Q + h)}{4h} \times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{h - Q} \right] \\ &= \frac{\lambda_i d + Q - q_i}{\alpha + \beta} + \frac{\sum_{j=1}^n q_j - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h\left(c + 2e\sum_{j=1}^n q_j\right) + (k - \nu)Q - h\left(k + \nu\right)}{h - Q} \\ &c_i + \frac{-q_i^2 - (q_i - Q - h)^2 + 2q_i(q_i - Q + h)}{4h} \times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{h - Q} \right], \end{aligned}$$
(A.17)
$$\frac{\partial^2 \pi_i^{d2}}{\partial q_i^2} = 2 \left[\frac{\leq 0}{-\frac{1}{\alpha + \beta}} + \frac{\leq 0}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{\leq 0}{h - Q} \right] < 0. \end{aligned}$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 3.

Proof of Proposition 4. Solve the first and second derivatives of π_i^{rf1} :

$$\frac{\partial \pi_{i}^{rf1}}{\partial p_{i}} = -\frac{q_{i}^{2}}{4h} + \frac{q_{i}(\hat{x}_{i}+h)}{2h} - \frac{\alpha q_{i}(p_{i}-c_{i}-v+k)}{2h} - \frac{(\hat{x}_{i}-h)^{2}}{4h} + \frac{\alpha(p_{i}-c_{i}-v)(\hat{x}_{i}-h)}{2h} + \frac{\alpha k}{2h}(\hat{x}_{i}+h),$$

$$\frac{\partial^{2}\pi_{i}^{rf1}}{\partial p_{i}^{2}} = -\frac{\alpha \left(\frac{>0}{q_{i}-\hat{x}_{i}+h}\right)}{h} - \frac{\alpha^{2} \left(\frac{>0}{p_{i}-c_{i}} - \frac{>0}{v+k}\right)}{2h} < 0,$$
(A.18)
$$\frac{\partial \pi_{i}^{rf1}}{\partial q_{i}} = \frac{(p_{i}-c_{i}+k-v)(\hat{x}_{i}-q_{i})}{2h} + \frac{p_{i}-c_{i}+k+v}{2} - c - e\sum_{j=1}^{n} q_{j} - eq_{i},$$

$$\frac{\partial^{2}\pi_{i}^{rf1}}{\partial q_{i}^{2}} = -\frac{\sum_{j=1}^{>0} + \sum_{j=1}^{>0} + \sum_{j=1}^{>0} - 2e < 0.$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 4.

Proof of Proposition 5. Solve the first and second derivatives of π_i^{rr1} :

$$\begin{aligned} \frac{\partial \pi_{i}^{rr1}}{\partial p_{i}} &= -\frac{q_{i}^{2}}{4h} + \frac{q_{i}(\hat{x}_{i}+h)}{2h} - \frac{\alpha q_{i}(p_{i}-c_{i}-v+k)}{2h} - \frac{(\hat{x}_{i}-h)^{2}}{4h} + \frac{\alpha (p_{i}-c_{i}-v)(\hat{x}_{i}-h)}{2h} + \frac{\alpha k}{2h}(\hat{x}_{i}+h), \\ \frac{\partial^{2} \pi_{i}^{rr1}}{\partial p_{i}^{2}} &= -\frac{\alpha \left(\frac{\geq 0}{q_{i}-\hat{x}_{i}+h}\right)}{h} - \frac{\alpha^{2} \left(\frac{\geq 0}{p_{i}-c_{i}} \xrightarrow{\geq 0}{-v+k}\right)}{2h} < 0, \\ \frac{\partial \pi_{i}^{rr1}}{\partial q_{i}} &= \frac{(p_{i}-c_{i}+k-v)(\hat{x}_{i}-q_{i})}{2h} + \frac{p_{i}-c_{i}+k+v}{2} - c - 2e \sum_{j=1}^{n} q_{j} - 2eq_{i} + 2\gamma_{i}e \sum_{j=1}^{n} q_{j}, \\ \frac{\partial^{2} \pi_{i}^{rr1}}{\partial q_{i}^{2}} &= -\frac{\sum_{j=1}^{2} \frac{2}{p_{i}-c_{i}} + \frac{\sum_{j=1}^{0} \frac{2}{p_{j}-c_{j}}}{2h} - \frac{2}{q_{i}+2\gamma_{i}e} < 0. \end{aligned}$$
(A.19)

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 5.

Proof of Proposition 6. Solve the first and second derivatives of π_i^{rf2} :

$$\frac{\partial \pi_{i}^{rf2}}{\partial q_{i}} = \left(-\frac{1}{\alpha+\beta} + \frac{1}{(\alpha+\beta)(n-1-(\alpha/\beta))} - \frac{4he}{h-Q}\right) \frac{\left[-q_{i}^{2} - (q_{i}-Q-h)^{2} + 2q_{i}(q_{i}-Q+h)\right]}{4h} \\
+ \left(\frac{\lambda_{i}d+Q-q_{i}}{\alpha+\beta} + \frac{\sum_{j=1}^{n}q_{j} - nQ-d}{(\alpha+\beta)(n-1-(\alpha/\beta))} - \frac{2h(c+2e\sum_{j=1}^{n}q_{j}) + (k-\nu)Q - h(k+\nu)}{h-Q} - c_{i}\right) \\
+ \frac{2h(c+2e\sum_{j=1}^{n}q_{j}) + (k-\nu)Q - h(k+\nu)}{h-Q} + \frac{4he}{h-Q}q_{i} - \left(c+eq_{i}+e\sum_{j=1}^{n}q_{j}\right), \quad (A.20)$$

$$\frac{\partial^2 \pi_i^{rf2}}{\partial q_i^2} = 2 \left[\frac{1}{-\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} \right] - 2e < 0.$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 6.

Proof of Proposition 7. Solve the first and second derivatives of $\pi_i^{rr^2}$:

$$\begin{aligned} \frac{\partial \pi_{i}^{rr2}}{\partial q_{i}} &= \left(-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{h - Q} \right) \frac{\left[-q_{i}^{2} - (q_{i} - Q - h)^{2} + 2q_{i}(q_{i} - Q + h) \right]}{4h} \\ &+ \left(\frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{j=1}^{n} q_{j} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h\left(c + 2e\sum_{j=1}^{n} q_{j}\right) + (k - \nu)Q - h(k + \nu)}{h - Q} - c_{i} \right) \\ &+ \gamma_{i} \frac{2h\left(c + 2e\sum_{j=1}^{n} q_{j}\right) + (k - \nu)Q - h(k + \nu)}{h - Q} + \gamma_{i} \frac{4he}{h - Q} \sum_{i=1}^{n} q_{i} - \gamma_{i} \left(c + 2e\sum_{j=1}^{n} q_{j}\right), \end{aligned}$$
(A.21)
$$\frac{\partial^{2}\pi_{i}^{rr2}}{\partial q_{i}^{2}} = 2 \left[\frac{-\frac{\langle 0 - 1}{\alpha + \beta} + \frac{\langle 0 - 1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}}{1 + (\alpha + \beta)(n - 1 - (\alpha/\beta))} \right] + \frac{\langle 0 - 1 - \frac{\langle 0 - 1}{\beta} + \frac{\langle 0 - 1 - \frac{\langle 0 - 1 - \frac{\langle 0 - 1}{\beta} + \frac{\langle 0 - 1 - \frac{\langle 0 - 1}{\beta} + \frac{\langle 0 - 1 - \frac{\langle 0 - 1}{\beta} + \frac{\langle 0 - 1 - \frac{\langle 0 - 1}{\beta} + \frac{\langle 0 - 1 - \frac{\langle 0 - 1 -$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 7.

Proof of Proposition 8. Solve the first and second derivatives of π_i^{rs1} :

$$\begin{split} \frac{\partial \vec{r}_{i}^{n^{2}}}{\partial q_{i}} &= \frac{e}{2\hbar} \left[q_{i}^{2} - (\ddot{x}_{i} - h)^{2} \right] + \frac{(\bar{p}_{i} - e - 2e\sum_{j=1}^{n} q_{j} - c_{i})}{2\hbar} (\ddot{x}_{i} + h - q_{i}) - \frac{eq_{i}}{\hbar} (\ddot{x}_{i} + h - q_{i}) - \frac{Eq_{i}}{2h} (\ddot{x}_{i} - h) + \frac{h}{2h} (\ddot{x}_{j} - h)^{2} \right] \\ &+ \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)}{4\hbar} (\dot{x}_{j} - h)^{2} + \frac{k}{4\hbar} (\ddot{x}_{j} - h)^{2} \right] \\ &+ \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)}{4\hbar} (\dot{x}_{j} - h)^{2} + \frac{k}{4\hbar} (\ddot{x}_{j} - h)^{2} \right] \\ &+ \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)}{4\hbar} (\dot{x}_{j} - h)^{2} + \frac{k}{4\hbar} (\ddot{x}_{j} - h)^{2} \right] \\ &- \frac{q_{i}}{\sum_{j=1}^{n} q_{j}} \left\{ \sum_{j=1}^{n} \left[\frac{e}{2h} q_{j}^{2} - \frac{q_{j}e(\ddot{x}_{j} - h)}{\hbar} + \frac{e}{2h} (\dot{x}_{j} - h)^{2} \right] + \frac{e + 2e\sum_{j=1}^{n} q_{j} - v + k}{2\hbar} \\ &- \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\ddot{x}_{i} - h)}{2h} - \frac{h(\ddot{x}_{i} - h)}{2h} \right] , \\ \frac{\partial \pi_{j}^{n^{1}}}{\partial q_{i}^{2}} = \frac{2e}{h} (\dot{x}_{i} + h - q_{i}) - \frac{p_{i} - e - 2e\sum_{j=1}^{n} q_{j} - e_{i}}{2h} \\ &- \frac{2\sum_{j=1}^{n} q_{j}(\nabla_{j} - q_{j})}{2h} \\ &\times \sum_{j=1}^{n} \left\{ \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v + k)}{4h} q_{j}^{2} - q_{j} \left[\frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\ddot{x}_{i} - h)}{2h} - \frac{2\sum_{j=1}^{n} q_{j}(\nabla_{j} - q_{j})}{2h} \right] \right\} \\ &- \frac{2\sum_{j=1}^{n} q_{j}(\nabla_{j} - q_{j})}{2h} \\ &- \frac{2\sum_{j=1}^{n} q_{j} - v + k}{4h} q_{j}^{2} - q_{j} \left[\frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\ddot{x}_{i} - h)}{2h} \right] \right\} \\ &- \frac{2\sum_{j=1}^{n} q_{j}(\nabla_{j} - q_{j})}{2h} \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v + k)}{2h} q_{j}^{2} - q_{j} \left[\frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\ddot{x}_{j} - h)}{2h} \right] \right\} \\ &- \frac{2\sum_{j=1}^{n} q_{j}}{\frac{\sum_{j=1}^{n} q_{j}}} \left\{ \sum_{j=1}^{n} \left[\frac{e}{2h} q_{j}^{2} - \frac{q_{j}e(\ddot{x}_{j} - h)}{h} + \frac{e}{2h} (\dot{x}_{j} - h)^{2} \right\} + \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\ddot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e + 2e\sum_{j=1}^{n} q_{j} - v)(\dot{x}_{j} - h)}{2h} q_{j} - \frac{(e +$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 8.

Proof of Proposition 9. Solve the first and second derivatives of π_i^{rs2} :

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$$\begin{split} \frac{\partial \pi_{i}^{rs2}}{\partial q_{i}} &= p_{i} - w - c_{i} + \frac{q_{i}^{2} - (q_{i} - Q - h)^{2} + 2(h - Q)q_{i}}{4h} \\ &\times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{(2 - \phi_{0})h - \phi_{0}Q} \right] - \phi_{i} \left[\frac{(h + Q)^{2}}{4h} \times \frac{4nhe}{(2 - \phi_{0})h - \phi_{0}Q} + \frac{k}{2h} + \frac{k(2q_{i} - Q + h)}{2h} \right] \\ &= \frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{i=1}^{n} q_{i} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}) + \phi_{0}(k - \nu)Q - \phi_{0}h(k + \nu)}{(2 - \phi_{0})h - \phi_{0}Q} - c_{i} \\ &+ \frac{q_{i}^{2} - (q_{i} - Q - h)^{2} + 2(h - Q)q_{i}}{4h} \\ &\times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{(2 - \phi_{0})h - \phi_{0}Q} \right] - \phi_{i} \left[\frac{(h + Q)^{2}}{4h} \times \frac{4nhe}{(2 - \phi_{0})h - \phi_{0}Q} + \frac{k}{2h} + \frac{k(2q_{i} - Q + h)}{2h} \right], \\ \frac{\partial^{2}\pi_{i}^{re2}}{\partial q_{i}^{2}} = 2 \left[\frac{\frac{<0}{1}}{\alpha + \beta} + \frac{<0}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{<0}{(2 - \phi_{0})h - \phi_{0}Q} \right] - \phi_{i} \frac{k}{h} < 0. \end{split}$$
(A.23)

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 9.

Proof of Proposition 10. Solve the first and second derivatives of π_i^{rs3} :

$$\begin{split} \frac{\partial \pi_{i}^{rs3}}{\partial q_{i}} &= p_{i} - w - c_{i} + \frac{q_{i}^{2} - (q_{i} - Q - h)^{2} + 2(h - Q)q_{i}}{4h} \\ & \times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{(2 - \theta_{0})h - \theta_{0}Q} \right] - \theta_{i} \times \frac{(Q + h)^{2}}{4h} \times \frac{4nhe}{(2 - \theta_{0})h - \theta_{0}Q} \\ &= \frac{\lambda_{i}d + Q - q_{i}}{\alpha + \beta} + \frac{\sum_{i=1}^{n} q_{i} - nQ - d}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{2h(c + 2e\sum_{j=1}^{n} q_{j}) - \theta_{0}v(Q + h) - \omega_{0}k(h - Q)}{(2 - \theta_{0})h - \theta_{0}Q} \\ &- c_{i} + \frac{q_{i}^{2} - (q_{i} - Q - h)^{2} + 2(h - Q)q_{i}}{4h} \\ & \times \left[-\frac{1}{\alpha + \beta} + \frac{1}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{4he}{(2 - \theta_{0})h - \theta_{0}Q} \right] - \theta_{i} \times \frac{(Q + h)^{2}}{4h} \times \frac{4nhe}{(2 - \theta_{0})h - \theta_{0}Q}, \end{split}$$
(A.24)
$$&\frac{\partial^{2} \pi_{i}^{rs3}}{\partial q_{i}^{2}} = 3 \left[-\frac{\frac{<0}{1}}{-\frac{1}{\alpha + \beta}} + \frac{\frac{<0}{(\alpha + \beta)(n - 1 - (\alpha/\beta))}}{(\alpha + \beta)(n - 1 - (\alpha/\beta))} - \frac{\frac{<0}{4he}}{(2 - \theta_{0})h - \theta_{0}Q} \right] < 0.$$

Similar to the centralized optimal solution, the implicit expression can be expressed in Proposition 10.

Data Availability

The data used to support the findings of this study are available from the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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