

Research Article

The Plastic Zone of Tunnel Surrounding Rock under Unequal Stress in Two Directions Based on the Unified Strength Theory

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For the deficiencies that the existing calculation theory for the Plastic Zone of Tunnel Surrounding Rock (PZTSR) does not consider the effect of the intermediate principal stress σ_2 and interaction between the surrounding rock and support structure on the PZTSR under unequal stress, the Unified Strength Theory (UST) for the rock is adopted to replace the often used Mohr-Coulomb (M-C) strength criterion to consider the effect of σ_2 on the PZTSR. Meanwhile, the interaction mechanism between the surrounding rock and support structure is also considered in the proposed model. Finally, the effect of the initial elastic displacement of the surrounding rock, stiffness of the support structure, and the coefficient b of the intermediate principal stress on the plastic zone is discussed. The results show that the PZTSR will increase nonlinearly with increasing the initial elastic displacement of the surrounding rock, and when it increases to a certain value, its increase extent will be much obvious. With increasing the stiffness of the support structure, the PZTSR will gradually decrease nonlinearly, but the decrease extent is not very much. With increasing b , the PZTSR will decrease; namely, σ_2 can improve the stress condition of the surrounding rock and reduce the PZTSR.

1. Introduction

The tunnel construction will inevitably lead to some plastic zone in the surrounding rock, which is vital for the tunnel stability and design of the support. Up to now, it has been an important issue attracting more and more attention [1–5]. For a long time, Kastner's method [6] is often adopted to calculate the surrounding rock stress and plastic zone, and many improvements have been made to it [7], which greatly promotes the development of the tunnel engineering. However, the existing studies mainly focus on the circular tunnel under hydrostatic stress. In practice, the natural geostress field of the tunnel is often nonhydrostatic, and the horizontal stress is often larger than the vertical one with the increased buried depth of the tunnel and intensive geological tectonic action. Therefore, the study on the mechanical behavior

of the tunnel under unequal stress in two directions is of more practical significance. Kirsch [8] firstly obtained the elastic solution of the circular tunnel in hydrostatic stress, and thereafter, many researchers investigated the PZTSR with different rock strength criteria on basis of Kirsch's solution, such as M-C strength criterion [9–11], UST [12], and Hoek-Brown criterion [13], to determine the PZTSR. In sum, it can be seen that the profound research has been made on the PZTSR under unequal stress in two directions; however, the studies above are only suitable for the situation that the support is constructed immediately when the tunnel is excavated, and meanwhile, the support stress is invariable for all the time. In fact, the construction of the support always has some delay, some ground deformation has always occurred, and accordingly, the support stress will vary with it. Therefore, the previous researches do not agree with

the actual condition and cannot reflect the interaction between the support and ground.

On the basis of the practical tunnel construction process, Hou et al. [14] assumed that there were three deficiencies in the existing researches. Firstly, they assumed that the existing researches did not fit with the practical condition which assumes that the support stress was applied on the tunnel at the same time with the ground stress. In fact, the tunnel is always firstly loaded with the ground stress, then excavated, and finally supported. Second, the support stress is seen as the initiative one and applied for one time, which does not agree with the practice. In fact, except that the stress supplied by the prestressed bolt can be almost seen as the initiative one, all the other support stresses are passive which vary with the ground deformation towards the tunnel [15, 16]. Thirdly, after the tunnel is excavated, the instantaneous initial ground displacement towards the tunnel center will occur before the support is constructed. Therefore, it can be found that these three aspects are all related to the interaction mechanism between the ground and support. Now Convergence Confinement Analysis (CCA) or Characteristic Curve Analysis (CCA) proposed by many researchers [6, 17, 18] is an effective tool to analyze the interaction mechanism between the ground and support. Thereafter, many studies on CCA have been conducted to study the tunnel mechanical behavior, which can be classified into two categories, that is, the effect of construction

timing and stiffness of the support on the PZTSR. For the former, some researchers [19–22] found that the support delay would increase the scope of the PZTSR. For the latter, other researchers [23–25] assumed that the tunnel mechanical behavior is much related to the support stiffness.

However, although many studies have been conducted on this issue and much progress has been achieved, nearly no researches focus on the effect of the intermediate principal stress and the initial ground elastic displacement before the support construction and support stiffness on the PZTSR at the same time. Therefore, on basis of the calculation method of the PZTSR under unequal stress in two directions proposed by Ruppney [11], this study proposes a new model that can consider the abovementioned three factors at the same time, which will agree with the practical condition much more.

2. Elasticity Solution of the Circular Tunnel under Unequal Stress in Two Directions

Assume that the ground is homogeneous and isotropic, the mechanical model of the circular tunnel is shown in Figure 1. By superposing the stress components obtained from the models in Figures 1(b) and 1(c), the elastic stress and displacement of the model in Figure 1(a) are as follows [26]:

$$\left\{ \begin{array}{l} \sigma_r = \frac{1}{2} (1 + \lambda) p_0 \left(1 - \frac{r_0^2}{r^2} \right) - \frac{1}{2} (1 - \lambda) p_0 \left(1 - 4 \frac{r_0^2}{r^2} + 3 \frac{r_0^4}{r^4} \right) \cos 2\theta + p_s \frac{r_0^2}{r^2}, \\ \sigma_\theta = \frac{1}{2} (1 + \lambda) p_0 \left(1 + \frac{r_0^2}{r^2} \right) + \frac{1}{2} (1 - \lambda) p_0 \left(1 + 3 \frac{r_0^4}{r^4} \right) \cos 2\theta - p_s \frac{r_0^2}{r^2}, \\ \tau_{r\theta} = \frac{1}{2} (1 - \lambda) p_0 \left(1 + 2 \frac{r_0^2}{r^2} - 3 \frac{r_0^4}{r^4} \right) \sin 2\theta, \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} u = \frac{r}{2G} \left[p_s + \frac{1}{2} (1 + \lambda) p_0 \frac{r_0^2}{r^2} - \frac{1}{2} (1 - \lambda) p_0 \left(4 \frac{r_0^2}{r^2} (1 - \nu) - \frac{r_0^4}{r^4} \right) \cos 2\theta \right], \\ v = -\frac{r p_s \theta}{G} + \frac{r}{2G} \cdot \frac{1}{2} (1 - \lambda) p_0 \left[\frac{r_0^4}{r^4} + 2(1 - 2\nu) \frac{r_0^2}{r^2} \right] \sin 2\theta, \end{array} \right.$$

where σ_r and σ_θ are the radial and circumferential normal stress components, respectively; $\tau_{r\theta}$ is the shear stress component; θ is the polar angle; r is the polar radius; u and v are the radial and tangential displacement components, respectively; E , G , and ν are the rock elastic modulus, shear modulus, and Poisson's ratio, respectively.

3. The PZTSR under Unequal Stress in Two Directions Based on the UST

3.1. The Theoretical Model. The field test of the ground stress indicates that the horizontal stress is not equal to the vertical one in most cases [27, 28], and so the surrounding rock plastic

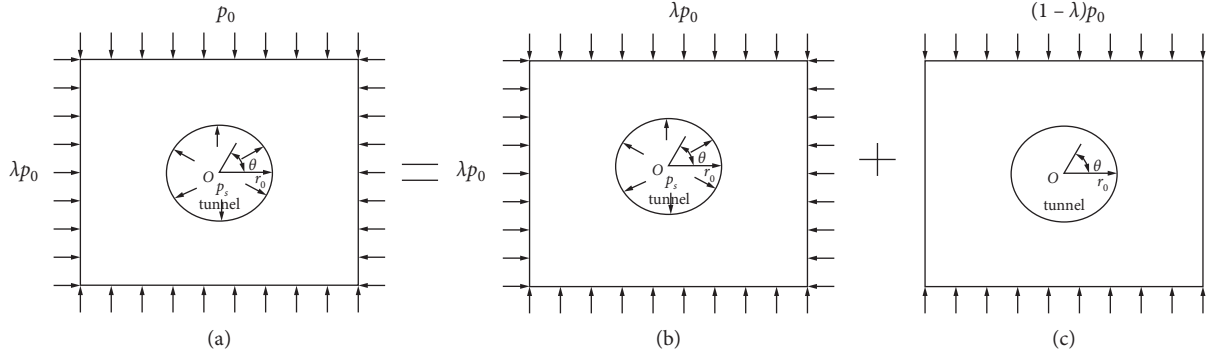


FIGURE 1: The mechanical model of the circular tunnel under unequal stress in two directions. r_0 is the tunnel radius, p_s is the support stress evenly acting on the tunnel inner wall, $p_0 (= \gamma h$, where γ is the rock gravitational density and h is the depth from the ground surface to the tunnel center) is the vertical ground self-gravity stress, λ is the lateral pressure coefficient, and θ is the polar angle. According to the superposition principle of the linear elasticity theory, the elasticity solution of the model, shown in (a), can be seen as the sum of that of the model in uniform stress condition, shown in (b), and that of the model in one-single direction stress condition, shown in (c).

zone in the circular tunnel is not circular anymore. As stated above, many researches have been conducted on this issue; for instance, Ruppneyt [11] proposed the calculation method of the

plastic zone radius r_p of the circular tunnel based on the M-C criterion, which is

$$r_p = r_0 \left\{ \frac{[(1 + \lambda)p_0 + 2c \cot \varphi] (1 - \sin \varphi)}{2p_s + 2c \cot \varphi} \right\}^{((1 - \sin \varphi)/(2 \sin \varphi))} \times \left\{ 1 + \frac{(1 - \lambda)p_0 (1 - \sin \varphi) \cos 2\theta}{[(1 + \lambda)p_0 + 2c \cot \varphi] \sin \varphi} \right\}, \quad (2)$$

where c and φ are the cohesion and internal friction angle of the rock, respectively, and the other parameters are stated as above.

If $\lambda = 1$, namely, the stress field is uniform, equation (2) is identical to that obtained by Kastner [6].

Although it is widely used in the soil and rock mechanics, the M-C criterion cannot reflect the effect of the intermediate principal stress on the rock failure, and then it cannot explain the failure phenomenon of the rock under high confining pressure and hydrostress. However, although the Drucker-Prager strength criterion proposed in the 1950s can consider the effect of the intermediate principal stress on

the rock failure, it cannot reflect the difference in the rock strength of different meridians [28]. And then Yu et al. [28, 29] proposed the UST for the rock and introduced it into the tunnel mechanical analysis to take into account the effect of the intermediate principal stress on the tunnel mechanical behavior [30].

Assume that these three principal stresses are σ_1 , σ_2 , and σ_3 , respectively, there is $\sigma_1 > \sigma_2 > \sigma_3$. According to the UST proposed by Yu et al. [28, 29], when the element is in the critical failure state, these three principal stresses should satisfy

$$\begin{cases} \frac{1 - \sin \varphi}{1 + \sin \varphi} \sigma_1 - \frac{1}{1 - b} (b\sigma_2 + \sigma_3) = \frac{2c \cos \varphi}{1 + \sin \varphi}, & \sigma_2 \leq \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin \varphi, \\ \frac{1 - \sin \varphi}{(1 + b)(1 + \sin \varphi)} (\sigma_1 + b\sigma_2) - \sigma_3 = \frac{2c \cos \varphi}{1 + \sin \varphi}, & \sigma_2 \geq \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin \varphi, \end{cases} \quad (3)$$

where b is the intermediate principal stress coefficient, which reflects the effect of the intermediate principal stress on the rock failure.

For a plane strain issue, when the material goes into the plastic state, the longitudinal axial stress is the intermediate principal stress σ_2 . It is approximately the mean of σ_1 and σ_3 ; namely,

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}. \quad (4)$$

Substituting equation (4) into (3) yields

$$\frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \frac{2(1 + b) \sin \varphi}{2 + b(1 + \sin \varphi)} = \frac{2(1 + b)c \cos \varphi}{2 + b(1 + \sin \varphi)}. \quad (5)$$

Let $\sin \varphi_t = ((2(1 + b) \sin \varphi)/(2 + b(1 + \sin \varphi)))$ and $c_t = ((2(1 + b)c \cos \varphi)/(2 + b(1 + \sin \varphi))) \cdot (1/\cos \varphi_t)$, and then equation (5) can be written as

$$\frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \sin \varphi_t = c_t \cos \varphi_t. \quad (6)$$

Equation (6) is completely similar to the M-C criterion in the form; therefore, it is assumed that the calculation method of the plastic zone radius r_p of the circular tunnel under unequal stress in two directions based on the UST can

$$r_p = r_0 \left\{ \frac{[(1 + \lambda)p_0 + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))] \cdot ((2 + b)(1 - \sin \varphi)/(2 + b(1 + \sin \varphi)))]^{((2+b)(1-\sin \varphi)/(4(1+b)\sin \varphi)}}{2p_s + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))]} \right\} \times \left\{ 1 + \frac{(1 - \lambda)p_0((2 + b)(1 - \sin \varphi)/(2 + b(1 + \sin \varphi))) \cos 2\theta}{[(1 + \lambda)p_0 + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))] \cdot ((2 + b)\sin \varphi)/(2 + b(1 + \sin \varphi))]} \right\}. \quad (7)$$

3.2. The Analysis of the Calculation Example. Here, the calculation model in Figure 1 is adopted. According to the field test in the buried depth of 205.65~583.15 m, the ground stress components S_H , S_h , and S_V of Sangzhuling tunnel, which is the key engineering of Sichuan-Tibet railway of China, are 9.41~17.72, 5.61~13.10, and 5.34~15.13 MPa, respectively. Therefore, the parameters in Table 1 are adopted, and the tunnel surrounding rock relative plastic zone (here, it is denoted by r_p/r_0) with different intermediate principal stress coefficient b is obtained with equation (7), which is shown in Figure 2.

It can be seen that the surrounding rock relative plastic zone of the circular tunnel under unequal stress in two directions is not a circle anymore, but an oval. It indicates that the coefficient λ has much effect on the PZTSR. In this calculation model, $\lambda = 0.6$; namely, the vertical stress is larger than the horizontal one, and accordingly, the size of the plastic zone in the vertical direction is less than that in the horizontal direction, which agrees with the research result obtained by Simanjuntak [13]. Meanwhile, the tunnel surrounding rock relative plastic zone decreases with increasing b , which is a group of ovals with the same center. Because b reflects the effect of the intermediate principal stress on the rock failure, it is assumed that the intermediate principal stress has much effect on the PZTSR. The UST with $b = 0$ is identical to the M-C criterion, and then it is assumed that the PZTSR is the largest when the intermediate principal stress is not considered, while for other cases, the effect of b is all considered, and the surrounding rock plastic zone decreases with increasing b ; namely, when b increases from 0 to 1, the surrounding rock relative plastic zone gradually decreases from 1.295 to 1.254, 1.227, 1.209, and 1.195. The largest decrease extent is 8.37%, which indicates that the intermediate principal stress has some effect on the PZTSR.

Here, the parametric sensitivity analysis is adopted to discuss the effect of the calculation parameters on the PZTSR, in which $b = 0.5$ and only one parameter in Table 1 is changed for one time.

3.2.1. Effect of p_0 on the PZTSR. Here, p_0 is assumed to be 8, 12, and 16 MPa, respectively, and the variation of the PZTSR with p_0 is shown in Figure 3. It can be found that the

TABLE 1: The calculation parameters of the tunnel.

r_0 (m)	E (GPa)	ν	p_0 (MPa)	p_s (MPa)	φ (°)	c (MPa)	λ
3	10	0.25	8	0.4	40	1.0	0.6

be obtained if c_t and φ_t are adopted to replace c and φ in equation (2). Then, we obtain

surrounding rock relative plastic zones are all oval. It indicates that increasing p_0 does not change the shape of the PZTSR but only affects its size. The larger the initial ground stress p_0 is, the larger the size of the PZTSR is. When p_0 increases from 8 MPa to 12 and 16 MPa, respectively, the corresponding surrounding rock relative plastic zone in the horizontal direction will increase from 1.227 to 1.339 and 1.426, respectively, whose increase extent is 9.13% and 6.50%, respectively. It indicates that the increase extent gradually becomes gentle.

3.2.2. Effect of p_s on the PZTSR. Here, p_s is assumed to be 0, 0.2, and 0.4 MPa, respectively, and the variation of the PZTSR with p_s is shown in Figure 4. It can also be found that the surrounding rock relative plastic zones are all oval. It indicates that increasing p_s does not change the shape of the PZTSR but only affects its size. With increasing p_s , the size of the PZTSR induced by the tunnel excavation becomes less and less. This is because the increase in the support stress will lead to an increase in the confining pressure of the rock, and therefore, the PZTSR decreases. Although the support stress can effectively reduce the PZTSR, the decrease extent is different. When p_s increases from 0 MPa to 0.2 and 0.4 MPa, respectively, the corresponding surrounding rock relative plastic zone in the horizontal direction gradually decreases from 1.321 to 1.270 and 1.227, respectively, whose decrease extent is 3.86% and 3.39%, respectively. Namely, the decrease extent becomes gentle. It indicates that the effect of the support stress on the PZTSR becomes not much when it increases to some degree.

3.2.3. Effect of λ on the PZTSR. Here, λ is assumed to be 0.6, 1, and 1.4, respectively, and the variation of the PZTSR with λ is shown in Figure 5. It can also be found that the surrounding rock relative plastic zones are all oval except for $\lambda = 1$, and its long axis direction changes with increasing λ . When $\lambda < 1$, its long axis is in the horizontal direction, which indicates that the PZTSR is larger in this direction. When $\lambda = 1$, it is circular, and when $\lambda > 1$, its long axis is in the vertical direction, which indicates that the PZTSR is larger in

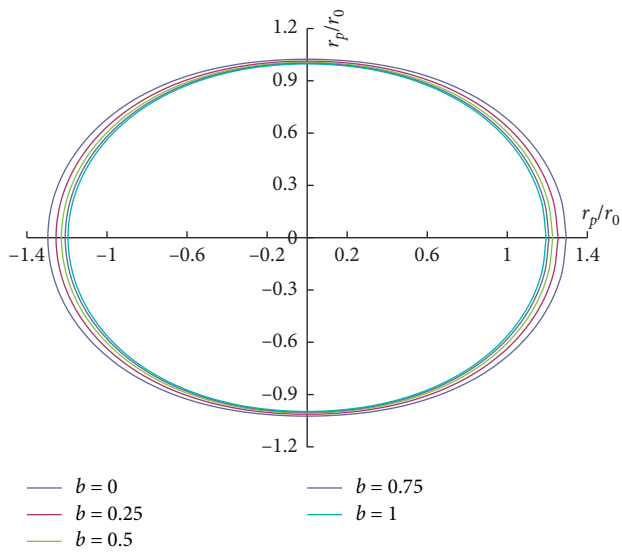


FIGURE 2: The effect of the intermediate principal stress coefficient b on the tunnel surrounding rock relative plastic zone.

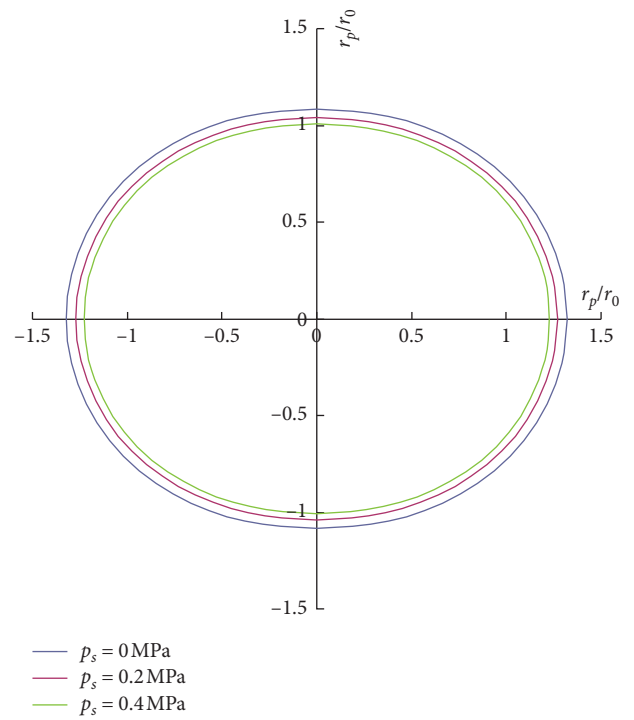


FIGURE 4: The variation of the tunnel surrounding rock relative plastic zone with p_s .

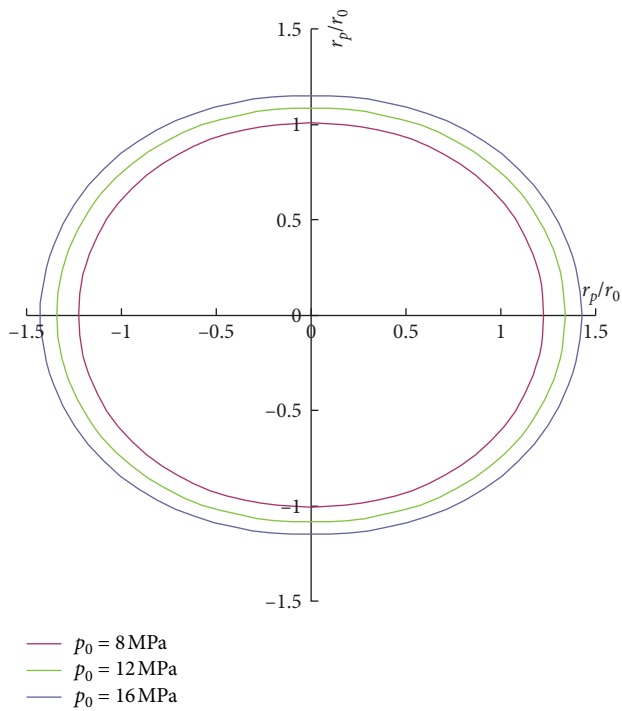


FIGURE 3: The variation of the tunnel surrounding rock relative plastic zone with p_0 .

this direction. Therefore, it is assumed that the lateral pressure coefficient λ has much effect on the shape of the PZTSR. In practice, the field ground stress test should be conducted in order to obtain the reliable lateral pressure

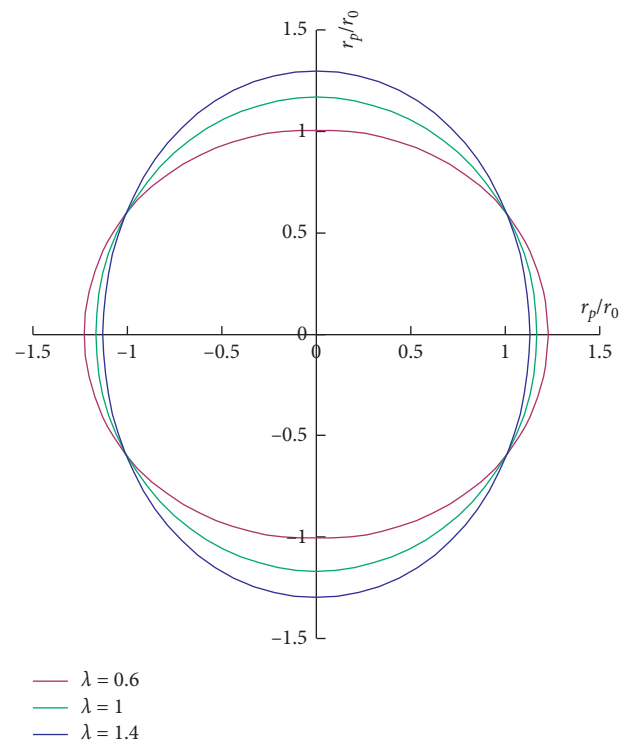


FIGURE 5: The variation of the tunnel surrounding rock relative plastic zone with λ .

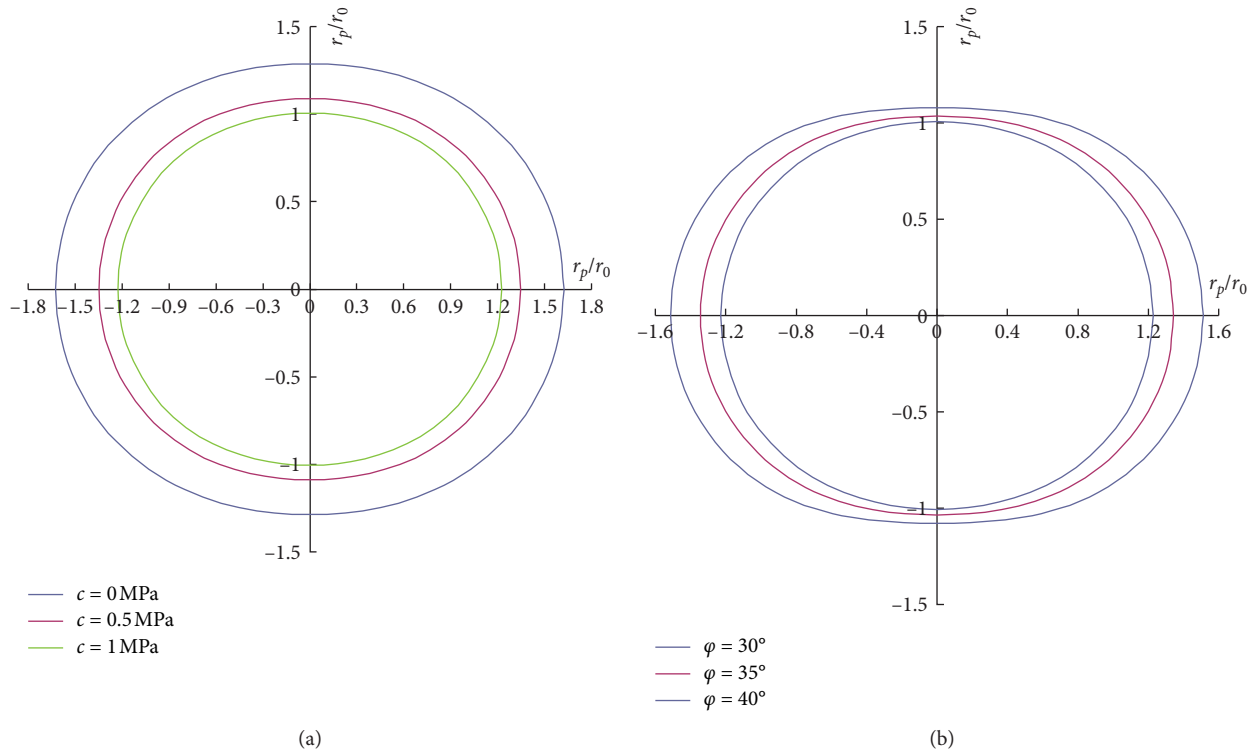


FIGURE 6: The variation of the tunnel surrounding rock relative plastic zone with c and ϕ .

coefficient, which provides the basis for the tunnel surrounding rock support design.

3.2.4. Effect of c and ϕ on the PZTSR. Here, the rock cohesion c is assumed to be 0, 0.5, and 1 MPa, respectively, and the variation of the PZTSR with c is shown in Figure 6(a). It can be seen that the surrounding rock relative plastic zones are all oval. It indicates that increasing c does not change the shape of the PZTSR and only affects its size. With increasing the rock strength, the PZTSR will become less and less. Therefore, some engineering reinforcement measures such as grouting can be adopted to reduce the PZTSR. Meanwhile, the decrease extent of the PZTSR becomes less and less with increasing c . The effect of rock internal friction angle ϕ on the PZTSR is shown in Figure 6(b), which is similar to that of the rock cohesion c , and we do not state it again.

4. The PZTSR by considering the Interaction between the Ground and the Support

Most of the existing tunnel mechanical models are based on Kastner's theory [6], which assumes that the radial support stress p_s induced by the support on the tunnel inner wall is invariable and applied immediately. However, as stated above, Hou et al. [14] made a detailed analysis on the deficiencies of Kastner's theory and assumed that p_s is not constant anymore, and it will vary with the interaction between the ground and the support structure. Next, p_s will

be solved from viewpoint of the interaction between the ground and the support structure.

Now, the often used support types in tunnel engineering are shotcrete, reinforced concrete liner, bolt, and their combination. Here, the reinforced concrete liner is taken as an example, and comparing with the rock mechanical property, it can be seen to be linear elastic. Therefore, it is assumed that the radial support stress p_s produced by the support is proportional to the radial displacement $(u_s)_{r=r_0}$ at the tunnel inner wall [31].

$$p_s = k_s (u_s)_{r=r_0}, \quad (8)$$

where k_s is the support stiffness. Because only the radial support stress is considered here, k_s only refers to the tensile or compressive stiffness.

Because the support is mostly constructed after the tunnel excavation, the initial ground radial displacement u_0 inevitably occurs. Therefore, the relationship between the support stress p_s and u_0 can be expressed as

$$p_s = k_s (u_s - u_0)_{r=r_0}. \quad (9)$$

Because the plastic deformation cannot be recovered, the support should be constructed before the maximum ground elastic radial displacement $(u_e)_{\max}$ happens, which can be calculated as follows [32]:

$$(u_e)_{\max} = \frac{r_0 p_0}{G}. \quad (10)$$

Therefore, $0 \leq u_0 \leq (u_e)_{\max}$.

Then, the calculation of the radial displacement $(u_s)_{r=r_0}$ at the tunnel inner wall is discussed. According to the elastoplastic theory [33], the calculation method of the ground displacement in the plastic condition is the same as that in the elastic condition when satisfying the plain strain and volume incompressibility conditions. Therefore, $(u_s)_{r=r_0}$ can be derived.

$$(u_s)_{r=r_0} = \frac{r_0}{4G} [2p_s + (1 + \lambda)p_0 - (1 - \lambda)p_0(3 - 4\nu)\cos 2\theta]. \quad (11)$$

Substituting equations (9) into (11) and assuming that no relative circumferential displacement occurs between the support and ground during the whole tunnel deformation, the radial displacement u_s of the support structure equals $(u_s)_{r=r_0}$ at the tunnel inner wall. After simplifying, we obtain

$$(u_s)_{r=r_0} = \frac{(1 + \lambda)p_0 - (1 - \lambda)p_0(3 - 4\nu)\cos 2\theta - 2k_s u_0}{4G/(r_0 - 2k_s)}. \quad (12)$$

Substituting equations (9) and (12) into (7) yields

$$r_p = r_0 \left\{ \frac{[(1 + \lambda)p_0 + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))] \cdot ((2 + b)(1 - \sin \varphi)/(2 + b(1 + \sin \varphi)))]}{((2k_s[(1 + \lambda)p_0 - (1 - \lambda)p_0(3 - 4\nu)\cos 2\theta - 2k_s u_0])/(4G/(r_0 - 2k_s))) - 2k_s u_0 + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))]} \right\}^{((2+b)(1-\sin \varphi)/(4(1+b)\sin \varphi))} \times \left\{ 1 + \frac{(1 - \lambda)p_0((2 + b)(1 - \sin \varphi)/(2 + b(1 + \sin \varphi))\cos 2\theta}{[(1 + \lambda)p_0 + 2c \cot[\arcsin((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi)))] \cdot ((2(1 + b)\sin \varphi)/(2 + b(1 + \sin \varphi))]} \right\}. \quad (13)$$

Therefore, the tunnel surrounding rock plastic zone radius r_p can be obtained with equation (13) by considering the intermediate principal stress, support stiffness, and construction timing at the same time.

5. Analysis of the Calculation Examples

The calculation model of the deeply buried circular tunnel and its calculation parameter are shown in Figure 1 and Table 1 with $b = 0.5$. Meanwhile, the support stiffness $k_s = 50$ MPa/m is adopted here. From equation (10), the maximum ground elastic radial displacement $(u_e)_{\max}$ at the tunnel inner wall can be calculated to be 0.015 m; accordingly, here, $u_0 = 0.001$ m is adopted. Then, the calculation results of the surrounding rock relative plastic zone and support stress obtained with equation (13) are shown in Figure 7. The following findings can be obtained. First of all, the PZTSR is still oval with a long axis in the horizontal direction because $\lambda = 0.6$. Second, the support stress is the least in the horizontal direction and is about 0.02 MPa, which is about 0.265 MPa in the vertical direction and about 13.25 times that in the horizontal direction. It indicates that the different support stress will produce in different directions because of unequal stress in two directions even if for the same support structure. Therefore, it is suggested to design the workload of the support structure according to the proposed method, which will not only ensure the safety of the tunnel but also reduce its cost.

In order to perfectly investigate the effect of p_0 , u_0 , k_s , and b on the PZTSR, the parametric sensitivity analysis is made.

5.1. Effect of p_0 on the PZTSR. Here, p_0 is assumed to be 8, 12, and 16 MPa, $b = 0.5$, $k_s = 50$ MPa/m, $u_0 = 0.001$ m, and the other parameters are shown in Table 1. The following findings can be obtained from Figure 8(a). First of all, the size of the PZTSR increases with increasing p_0 , but its shape is basically the same, which are oval with a long axis in the horizontal direction. It indicates that the initial ground stress

does not change the shape of the PZTSR but only affects its size. However, the effect extent of p_0 on the PZTSR is different. When p_0 increases from 8 MPa to 12 and 16 MPa, the size of the relative plastic zone in the horizontal direction only increases from 1.315 to 1.417 and 1.491, respectively, whose increase extent degrees are 7.76% and 5.22%, respectively. It shows that its increase extent becomes less and less, which can be explained with the support stress shown in Figure 8(b). Because the support stiffness is fixed in this case, when p_0 is little, the support stress is also little, and some plastic zone will occur in the surrounding rock. While with increasing p_0 , the increase extent of the support stress is larger, which indicates that the increase in p_0 fully motivates the capacity of the support structure and reduces the increase extent of the plastic zone of the surrounding rock. So, it indicates the interaction between the surrounding rock and support structure.

5.2. Effect of u_0 on the PZTSR. Here, u_0 is assumed to be 0, 0.005, and 0.01 m, $b = 0.5$, $k_s = 50$ MPa/m, and the other parameters are shown in Table 1. The following findings can be obtained from Figure 9. First of all, the size of the PZTSR increases with increasing u_0 , but its shape is basically the same, which are oval with a long axis in the horizontal direction. It indicates that the initial ground radial displacement does not change the shape of the PZTSR but only affects its size. The main factor affecting its shape is still the lateral pressure coefficient λ . What is more, the effect extent of u_0 on the PZTSR is different. When u_0 increases from 0 m to 0.005 and 0.01 m, the size of the relative plastic zone in the horizontal direction only increases from 1.288 to 1.466 and 2.019, respectively, whose increase extent degrees are 13.82% and 37.72%, respectively. It shows that its increase extent becomes larger and larger, which indicates that the size of the PZTSR will dramatically increase when u_0 increases to a certain value. Therefore, the support should be constructed as soon as

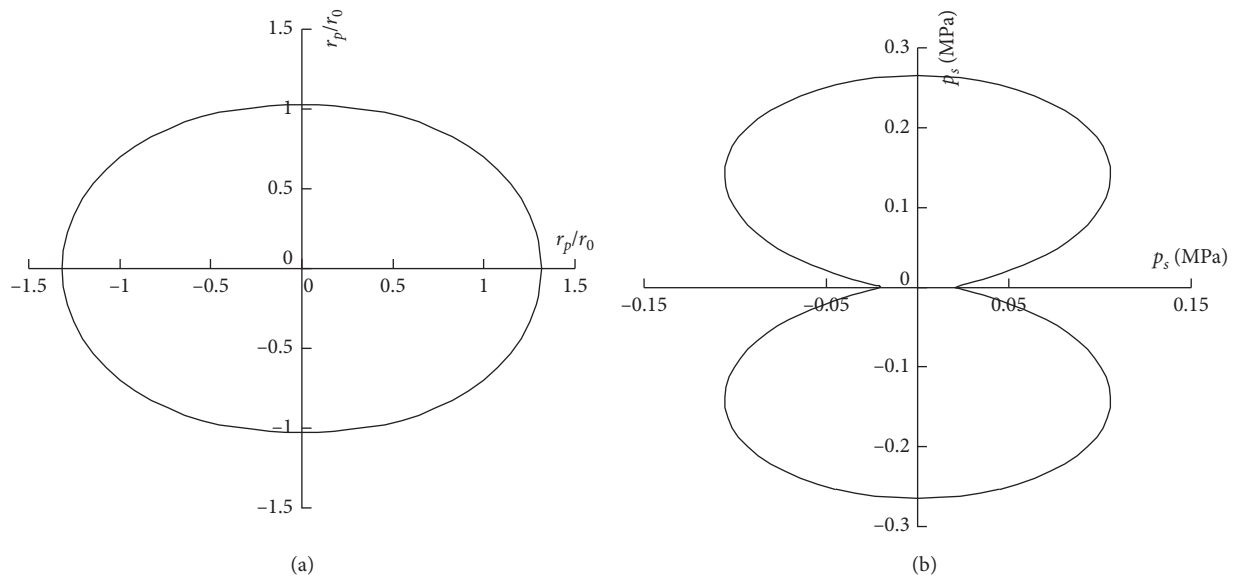


FIGURE 7: The calculation results of the PZTSR and support stress by considering the interaction between the surrounding rock and support structure.

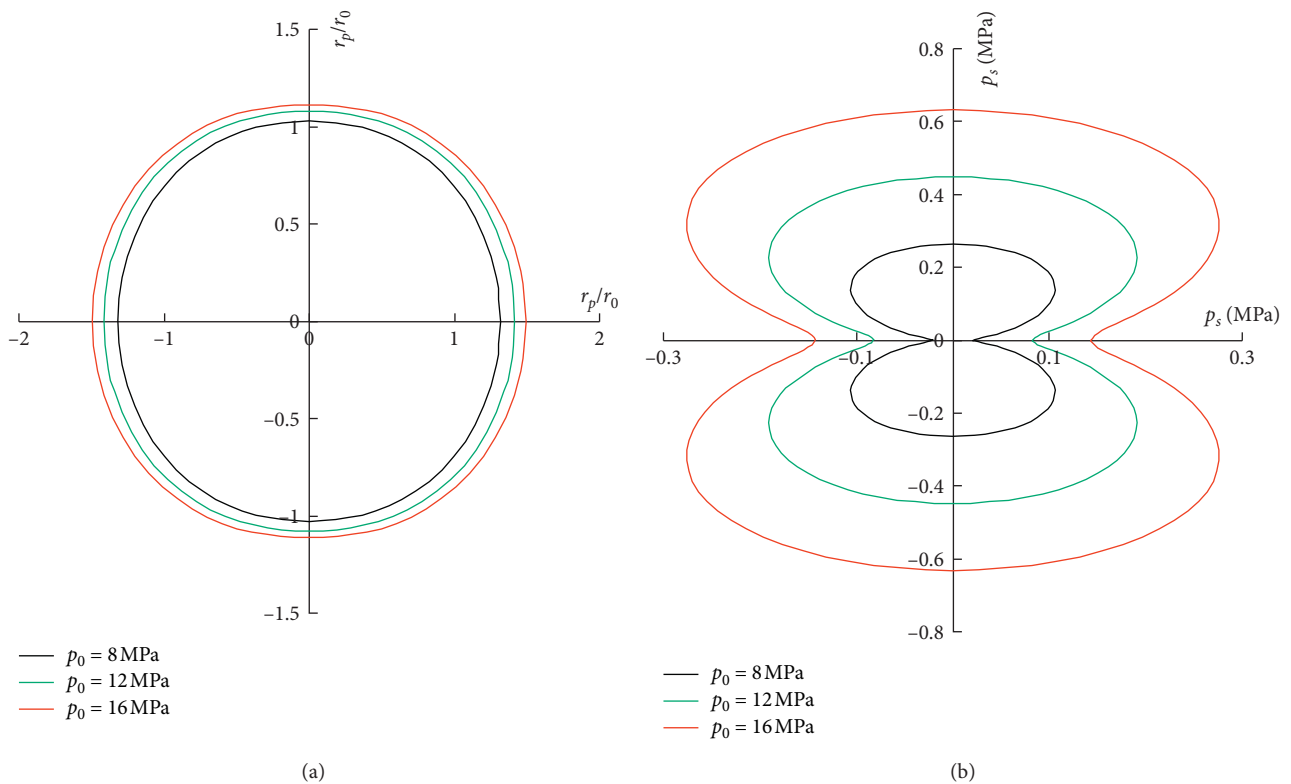


FIGURE 8: Effect of p_0 on the surrounding rock plastic zone and the support stress.

possible in the practical engineering in order to avoid the excessive surrounding rock failure.

5.3. *Effect of k_s on the PZTSR.* Here, k_s is assumed to be 100, 200, and 300 MPa/m, $u_0 = 0.001$ m, $b = 0.5$, and the other

parameters are shown in Table 1. The following findings can be obtained from Figure 10. First of all, when k_s varies, the PZTSR is the concentric oval, which indicates that k_s affects only the size of the PZTSR, but not its shape. Second, when k_s increases from 100 MPa/m to 200 and 300 MPa/m, respectively, the maximum size of the

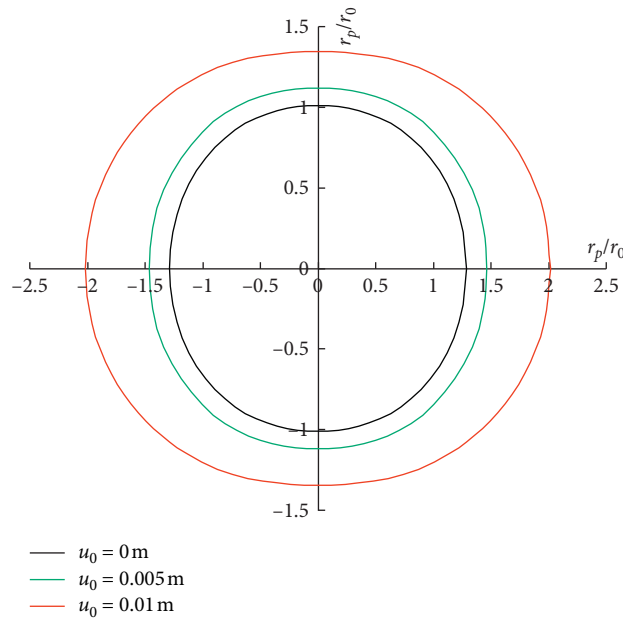


FIGURE 9: Effect of u_0 on the PZTSR.

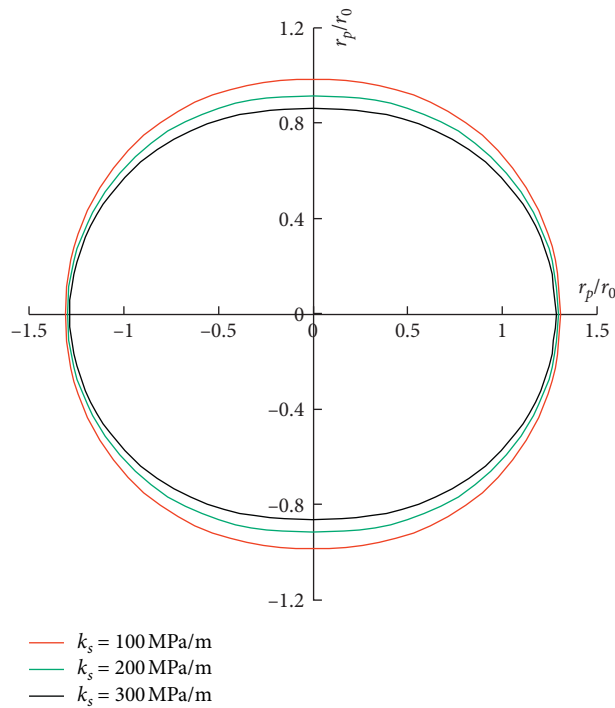


FIGURE 10: Effect of k_s on the PZTSR.

relative plastic zone decreases from 1.310 to 1.298 and 1.285, respectively, which indicates that the relative plastic zone gradually decreases with increasing k_s . It shows that increasing the support stiffness can reduce the plastic zone of the tunnel surrounding rock. Therefore, the support structure with large stiffness can reduce the

scope of the plastic zone of the tunnel surrounding rock. Accordingly, it will increase the cost of the support engineering, so in the practical engineering, we should comprehensively consider the relationship between the economy and safety to gain the maximum comprehensive benefits.

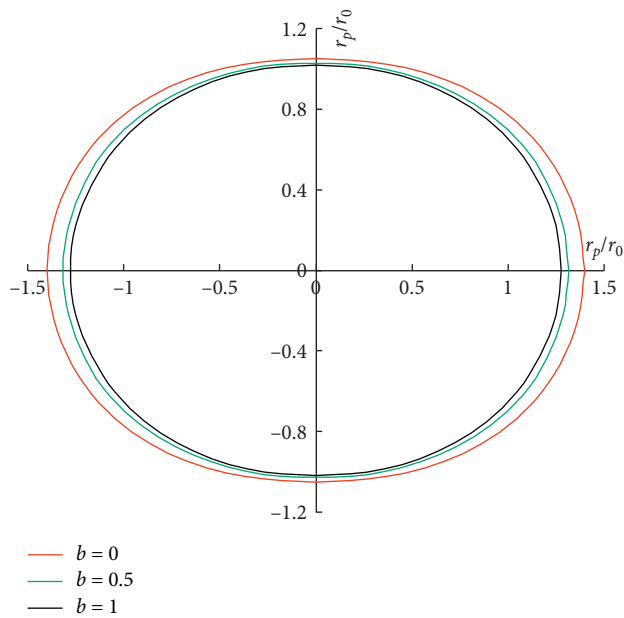


FIGURE 11: Effect of b on the PZTSR.

5.4. Effect of the Intermediate Principal Stress b on the PZTSR. Here, b is assumed to be 0, 0.5, and 1, $u_0 = 0.001$ m, $k_s = 50$ MPa/s, and the other parameters are shown in Table 1. The following findings can be obtained from Figure 11. First of all, when b varies, the PZTSR is the concentric oval. It indicates that b affects only the size of the PZTSR, but not its shape. Its size gradually decreases with increasing b , which indicates that the intermediate principal stress can strengthen the stability of the surrounding rock and reduce the PZTSR. Finally, as far as the decrease extent of b on the PZTSR is concerned, the decrease extent for b from 0 to 0.5 is much larger than that for b from 0.5 to 1, which indicates that the effect of the intermediate principal stress on the PZTSR is nonlinear.

6. Conclusions

- (1) In order to consider the effect of σ_2 on the PZTSR, UST for the rock is adopted to replace the M-C strength criterion.
- (2) To take into account the effect of the interaction mechanism between the surrounding rock and support structure on the PZTSR, p_0 is assumed to be linear with the radial displacement at the tunnel inner wall, which is not constant anymore. So, u_0 and k_s can be both considered in the proposed model.
- (3) The calculation examples show that b , u_0 , k_s , and rock strength all have an effect on the PZTSR, which agrees with the existing research conclusions.

Meanwhile, this study mainly discusses the effect of the rock strength criterion on the PZTSR and does not involve the rock mechanical behavior after yield such as strain softening or strengthening behavior. Therefore, this proposed method is more suitable for the rock with the ideal elastic-plastic mechanical behavior.

Data Availability

The data used to support the findings of this study are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All the authors contributed equally to this work.

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