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Research Article

Robust H_∞ Control for the Nonlinear Cascade Systems with Passive and Nonpassive Subsystems

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In this paper, H_∞ control for the uncertain switched nonlinear cascade systems with passive and nonpassive subsystems is investigated. Based on the average dwell time method, for any given passivity rate, average dwell time, and disturbance attenuation level, the feedback controllers of the subsystems by predetermined constants are designed to solve the exponential stability and L_2-gain problems of H_∞ control for switched nonlinear cascade systems. Two examples are provided to demonstrate the effectiveness of the proposed design method.

1. Introduction

With the development of scientific computing technology, the research on H_∞ control problems of nonlinear systems has been greatly promoted, and the results of nonlinear control problems continue to emerge [1, 2]. However, these methods usually bring a difficulty that needs to solve the Hamilton–Jacobi equation.

The passivity, from the electrical network, becomes an extremely useful property in switched systems, and many results about the passivity of switched systems have been published [3–11]. Storage functions that characterize passivity can be used as Lyapunov functions to analyze stabilization problems [3]. And the passivity is closely related to the robust stability of systems under certain negative feedback disturbances [6]. Recently, the storage function method has been found to ensure a top limit of the minimum dwell time to keep the passivity of linear systems [5]. For switched nonlinear systems, stability was inferred from the passivity described by using multiple storage functions [10]. The necessary and sufficient conditions were obtained for the local passivity of discrete-time switched nonlinear systems which consisted of passive and nonpassive modes, and the passivity of the affine system was studied [9]. Using multiple barrier storage functions, sufficient conditions were derived for guaranteeing the regional passivity of the switched systems [8]. And literature [7] considered the stability of switched nonlinear systems with feedback incrementally passive subsystems via the average dwell time method.

With the systems becoming more complex in actual problems, the robustness caused by external disturbance becomes a source of trouble, and there are a few achievements on passivity of H_∞ control problem of switched systems [12–15]. The stability of two types of passive H_∞ control for discrete-time linear switched systems was considered by multiple storage functions [12, 13]. And combining the piecewise Lyapunov function and the average dwell time method, the literature [15] investigated the disturbance of time-controlled switched systems consisting of several linear time-invariant subsystems. The H_∞ control of uncertain switched nonlinear systems with passivity was researched, and this research avoided solving the Hamilton–Jacobi inequality problem [14].

Complex nonlinear systems can be transformed into cascaded systems through certain conditions, and the stability of the cascaded system is studied by the stability and cascade properties of the subsystems [16]. This not only reduces the complexity of the controller but also reduces the difficulty of stability analysis [17–20]. A natural question is how to study the stability of switching nonlinear cascade robust H_∞ control systems through passivity, and in this paper, we will work to solve these problems.

In this paper, based on the method of average dwell time, the robust H_∞ control problem for a class of passive uncertain cascade switched systems with passiveness is considered. For passive subsystems and nonpassive subsystems, we design controllers and apply the multiple storage functions method to solve the stability and L_2-gain of the nonlinear uncertain cascade switched system under the given conditions. Finally, two numerical simulations are illustrated to support our analytical results. Compared with the method of existing nonlinear cascade switched systems' H_∞ control problem, the advantages are that we adopt the parametric equation method to avoid the Lyapunov function construction and the Hamilton–Jacobi equation solution, which reduces the computational difficulty.

Notions: $\mathbb{R}^n$ is the n-dimensional real Euclidean space; T denotes the matrix transposition; $\lambda_{\min}\{Q_1, Q_2\}$ means the smallest eigenvalue of the matrices Q_1 and Q_2, and $\lambda_{\max}\{Q_1, Q_2\}$ is the largest; $\|\cdot\|$ is the Euclidean norm of vector; $L_f V(x)$ stands for $L_f V(x) = (\partial V(x)/\partial x^T) f(x)$, where $f(x), V(x) \in C^1[\mathbb{R}^n, \mathbb{R}]$; and $g(t) \in L_2[0, +\infty)$ means $\int_0^\infty |g(t)|^2 \mathrm{d}t < \infty$.

2. Problem Statement and Preliminaries

Consider the uncertain switched nonlinear cascade system with the form

$$\begin{aligned} \dot{z} &= p_{\sigma(t)}(z,x) + q_{\sigma(t)}(z,x)\omega, \\ \dot{x} &= f_{\sigma(t)}(z,x) + \Delta f_{\sigma(t)}(z,x) + g_{\sigma(t)}(z,x)u_{\sigma(t)} + c_{\sigma(t)}(z,x)\omega, \\ y &= h_{\sigma(t)}(z,x) + d_{\sigma(t)}(z,x)\omega, \end{aligned} \tag{1}$$

where $z \in \mathbb{R}^{n-m}$, $x \in \mathbb{R}^m$, and $X = (z^T, x^T)$, $\omega \in \mathbb{R}$ is the disturbance input and $\omega \in L_2[0, +\infty)$, $u_\sigma \in \mathbb{R}^m$ is the control input, and $y \in \mathbb{R}^p$ is the output, defining the right continuous function $\sigma(t)$: $\mathbb{R}^+ \longrightarrow \underline{l} = \{1, 2, \ldots, l\}$ is the switching law. For $\forall i \in \underline{l}$, $p_i(\cdot,\cdot)$, $f_i(\cdot,\cdot)$, $g_i(\cdot,\cdot)$, and $h_i(\cdot,\cdot)$ are smooth functions of appropriate dimensions, $q_i(\cdot,\cdot)$, $c_i(\cdot,\cdot)$, and $d_i(\cdot,\cdot)$ are bounded and smooth functions of appropriate dimensions, and $\Delta f_i(\cdot,\cdot)$ is uncertain nonlinear functions of appropriate dimensions. Especially, $p_i(0,0) = 0$, $f_i(0,0) = 0$, and $h_i(0,0) = 0$. In the ideal state, the subsystem switching signal $\sigma(t)$ is defined on the following switching sequence:

$$\begin{aligned} \sum = \Big\{ \left(z_0^T, x_0^T\right)^T; (t_0, \sigma(t_0)), (t_1, \sigma(t_1)), (t_2, \sigma(t_2)), \ldots, \\ (t_k, \sigma(t_k)), k \in \underline{l} \Big\}, \end{aligned} \tag{2}$$

where t_0 and $(z_0^T, x_0^T)^T$ are initial time and initial state, respectively, $(t_k, \sigma(t_k))$ means that the i_kth subsystem is activated at $t \in [t_k, t_{k+1})$. Without loss of generality, we assume $t_0 = 0$. In order to better understand the switching between subsystems, the block diagram of the switched system (1) is shown in Figure 1.

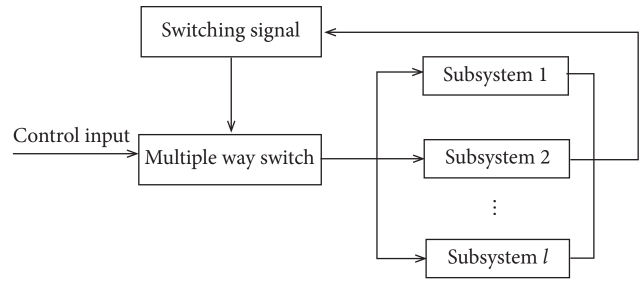

Figure 1: The block diagram of the switched system.

Assumption 1 (see [21]). For $z \in \mathbb{R}^{n-m}, x \in \mathbb{R}^m$, the constants $a_1, a_2, a_3 > 0$, and $\mu \geq 1$, there exist positive definite functions $W_i(z,x) \in C^1$, $i \in I$, such that the conditions

$$a_1 (\|z(t)\| + \|x(t)\|)^2 \leq W_i(z,x) \leq a_2 (\|z(t)\| + \|x(t)\|)^2, \tag{3}$$

$$\begin{aligned} \left\| \frac{\partial W_i(z,x)}{\partial x} \right\| &\leq a_3 \|x\|, \\ \left\| \frac{\partial W_i(z,x)}{\partial z} \right\| &\leq a_3 \|z\|, \end{aligned} \tag{4}$$

$$W_i(z,x) \leq \mu W_j(z,x), \quad \mu > 0,\ i, j = 1, \ldots, N, \tag{5}$$

hold.

For the subsystems of the switched system (1), we classify them into two groups: $i \in I_p \subset \underline{l}$ represents that the ith closed-loop subsystem is passive; $i \in I_n \subset \underline{l} - I_p$ represents nonpassive. Then, I_p and I_n satisfy Assumption 2.

Assumption 2. For $i \in I_p$,

$$\begin{aligned} L_{f_i} V_i(x) &\leq 0, \\ L_{p_i} U_i(z) &\leq 0, \\ L_{g_i} V_i(x) &= \frac{\partial V_i(x)}{\partial x} g_i = h_i^T(z,x). \end{aligned} \tag{6}$$

For $i \in I_n$, there exists a constant $\lambda > 0$ satisfying

$$L_{f_i} V_i(x) + L_{p_i} U_i(z) \leq \lambda W_i(z,x), \tag{7}$$

where U_σ and V_σ are smooth functions of appropriate dimensions.

Assumption 3 (see [6]). For uncertain function $\Delta f_i(z,x)$, it satisfies the bound $\|\Delta f_i(z,x)\| \leq \zeta(t)(\|z\| + \|x\|)$, $\forall i \in I$, where $\zeta(t)$ is a nonnegative function and satisfies $\int_{t_0}^t \zeta(\tau)\mathrm{d}\tau \leq \kappa(t - t_0) + \eta$ for nonnegative constants κ and η.

Definition 1 (see [22]). Let $N_\sigma(\tau, t)$ represent the number of switchings of $\sigma(t)$ in the interval (τ, t) for any switching signal $\sigma(t)$ and $0 < \tau < t$. If

$$N_\sigma(\tau, t) \leq N_0 + \frac{t - \tau}{\tau_a} \tag{8}$$

holds for $N_0, \tau_a > 0$. The constant τ_a is called average dwell time, and N_0 is the chatter bound. Without loss of generality, we choose $N_0 = 0$.

The notion of average dwell time is often used for identifying switching signals which have certain desirable properties.

Definition 2 (see [23]). For any $0 \le T_1 < T_2$, let $T_{p[T_1,T_2]}$ denote the total time when the passive subsystems are active on $[T_1, T_2]$. Then, the passivity rate of the switched system is recorded as $r_{p[T_1,T_2]} = (T_{p[T_1,T_2]}/T_2 - T_1)$. Clearly, $0 < r_{p[T_1,T_2]} \le 1$.

In this paper, we will study the following robust H_∞ control problem for system (1). For any constant $\gamma > 0$, define the control laws of each subsystems $u_i = u_i(z, x)$ and $u_i(0, 0) = 0$, $i = 1, \ldots, l$. Under the switching signal $\sigma(t)$, system (1) has the following properties [6, 19]:

(i) The closed-loop system (1) with $w(t) \equiv 0$ is globally robustly exponentially stable for all admissible uncertainties.

(ii) The closed-loop system (1) has a weighted L_2-gain level γ for some real-valued function with $\beta(z, x)$ and $\beta(0, 0) = 0$, that is, there exist a constant $\lambda > 0$ and $\omega(t) \in L_2[0, \infty)$, such that

$$\int_0^\infty e^{-\lambda s} y^T(s) y(s) \mathrm{d}s \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s) \mathrm{d}s + \beta(z_0, x_0), \tag{9}$$

holds.

Definition 3. In the nonlinear system,

$$\begin{aligned} \dot{z} &= p_{\sigma(t)}(z, x), \\ \dot{x} &= f_{\sigma(t)}(z, x), \\ y &= h_{\sigma(t)}(z, x), \end{aligned} \tag{10}$$

for degree $\bar{\lambda}$, it is exponentially small-time norm-observable if there exist positive constants $\delta > 0$ and $c > 0$, such that when $\|y(t+s)\| \le \delta$ holds for $t \ge t_0$ and $0 < s \le \tau$, $\tau > 0$, $\|z(t+\tau)\| + \|x(t+\tau)\| \le c e^{-\bar{\lambda}\tau}(\|z(t)\| + \|x(t)\|)$ is established.

Remark 1. The small-time norm-observability has been proposed for ensuring the asymptotical stability of switched systems [24]. In this paper, the exponential small-time norm-observability with degree $\bar{\lambda}$ is exponential form, and it is used to research global robust exponential stability of system (1).

Remark 2. A method is given to verify that system (10) is exponentially small-time norm-observable. Assume that there exist positive constants δ and $\bar{\lambda}$ and positive definite matrices Q_1 and Q_2, such that the following condition is satisfied:

$$\begin{aligned} &2z^T Q_1 p(z, x) + 2x^T Q_2 f(z, x) + (\delta + 2\bar{\lambda} - \|y(z, x)\|) \\ &\quad \cdot \left(z^T Q_1 z + x^T Q_2 x\right) \le 0. \end{aligned} \tag{11}$$

Let

$$\begin{aligned} W(t) &= z^T(t) Q_1 z(t) + x^T(t) Q_2 x(t), \\ l_1 &= \lambda_{\min}\{Q_1, Q_2\}, \\ l_2 &= \lambda_{\max}\{Q_1, Q_2\}. \end{aligned} \tag{12}$$

We can get

$$l_1(\|z\| + \|x\|)^2 \le z^T Q_1 z + x^T Q_2 x \le l_2(\|z\| + \|x\|)^2. \tag{13}$$

From (11), the time derivative of $W(t)$ along the trajectory of system (10) is

$$\begin{aligned} \frac{\mathrm{d}W(t)}{\mathrm{d}t} &= 2z^T Q_1 p(z, x) + 2x^T Q_2 f(z, x) \\ &\le (\|y(z, x)\| - \delta - 2\bar{\lambda}) W(t). \end{aligned} \tag{14}$$

When $\|y(z, x)\| \le \delta$ holds for $t \in [t^*, t^* + \tau)$ with length τ, we obtain

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} \le -2\bar{\lambda} W(t), \quad t \in [t^*, t^* + \tau]. \tag{15}$$

By (13) and (15), using the differential inequality theory, we obtain

$$W(t) \le e^{-2\bar{\lambda}(t - t^*)} W(t^*). \tag{16}$$

Hence,

$$\begin{aligned} l_1(\|z(t)\| + \|x(t)\|)^2 &\le l_2 e^{-2\bar{\lambda}(t - t^*)} \left(\|z(t^*)\| + \|x(t^*)\|\right)^2, \\ &\qquad t \in [t^*, t^* + \tau], \end{aligned} \tag{17}$$

which means

$$\begin{aligned} &\|z(t)\| + \|x(t)\| \le c e^{-\bar{\lambda}(t - t^*)} \left(\|z(t^*)\| + \|x(t^*)\|\right), \\ &c = \sqrt{\frac{l_2}{l_1}}, \quad t \in [t^*, t^* + \tau]. \end{aligned} \tag{18}$$

According to Definition 3, system (10) is exponentially small-time norm-observable.

Lemma 1. *If system (10) is exponentially small-time norm-observable with degree $\bar{\lambda}$, for any $k \ge 0$, it has*

$$\begin{aligned} (\|z(t+\tau)\| + \|x(t+\tau)\|)^2 &\le c_1 e^{-2(\bar{\lambda} - k)\tau} (\|z(t)\| + \|x(t)\|)^2 \\ &\quad - \int_t^{t+\tau} e^{-2(\bar{\lambda} - k)(t + \tau - \theta)} \\ &\quad \cdot \|y(z(\theta), x(\theta))\|^2 \mathrm{d}\theta, \end{aligned} \tag{19}$$

where $c_1 = k_0 + c^2$, $t \ge t_0$, and $\tau > 0$.

Proof. If system (10) is exponentially small-time norm-observable with degree $\bar{\lambda}$, there exists a constant $k_0 > 0$, such that

$$\|y(z(s),x(s))\|^2 \le \frac{k_0}{\tau}e^{-2\bar{\lambda}\tau}(\|z(t)\|+\|x(t)\|)^2, \quad \forall s\in[t,t+\tau], \tag{20}$$

holds. By (20), we have

$$\|y(z(s),x(s))\|^2 \le \frac{k_0}{\tau}e^{-2(\bar{\lambda}-k)(s-t)}(\|z(t)\|+\|x(t)\|)^2, \quad \forall s\in[t,t+\tau], \tag{21}$$

namely,

$$\tau e^{2s(\bar{\lambda}-k))}(\|y(z(s),x(s))\|)^2 \le k_0 e^{2t(\bar{\lambda}-k)}(\|z(t)\|+\|x(t)\|)^2, \quad \forall s\in[t,t+\tau]. \tag{22}$$

Apply the integral mean value theorem to the above formula, and there exists a constant s_0, and $t\le s_0\le t+\tau$, such as

$$\int_t^{t+\tau} e^{2(\bar{\lambda}-k)\theta}\|y(z(\theta),x(\theta))\|^2 d\theta = \tau e^{2(\bar{\lambda}-k)s_0}\|y(z(s_0),x(s_0))\|^2 \le k_0 e^{2t(\bar{\lambda}-k)}(\|z(t)\|+\|x(t)\|)^2. \tag{23}$$

Then,

$$-k_0 e^{-2\tau(\bar{\lambda}-k)}(\|z(t)\|+\|x(t)\|)^2 \le -\int_t^{t+\tau} e^{-2(\bar{\lambda}-k)(t+\tau-\theta)}\|y(z(\theta),x(\theta))\|^2 d\theta. \tag{24}$$

Because system (10) is exponentially small-time norm-observable, if $\|y(t+s)\|\le\delta$ can be given with $t_0\le t, 0<\tau$, and $0<s\le\tau$, we obtain

$$\|z(t)\|+\|x(t)\| \le ce^{-\bar{\lambda}\tau}\left(\|z(t^*)\|+\|x(t^*)\|\right), \quad t\in[t^*,t^*+\tau], \quad c=\sqrt{\frac{l_2}{l_1}}, \tag{25}$$

which means

$$\|z(t+\tau)\|+\|x(t+\tau)\| \le ce^{-\bar{\lambda}\tau}(\|z(t)\|+\|x(t)\|). \tag{26}$$

Then,

$$(\|z(t+\tau)\|+\|x(t+\tau)\|)^2 \le c^2e^{-2\bar{\lambda}\tau}(\|z(t)\|+\|x(t)\|)^2. \tag{27}$$

Then, the sum of (24) and (27) is

$$(\|z(t+\tau)\|+\|x(t+\tau)\|)^2 \le c_1e^{-2(\bar{\lambda}-k)\tau}(\|z(t)\|+\|x(t)\|)^2 - \int_t^{t+\tau} e^{-2(\bar{\lambda}-k)(t+\tau-\theta)} \cdot \|y(z(\theta),x(\theta))\|^2 d\theta, \tag{28}$$

where $c_1=k_0+c^2$. □

3. Main Results

In this section, we will discuss system (1) in two parts. Part I: when $\omega\equiv 0$, we will analyze the globally robustly exponentially stable of system (1) for all admissible uncertainties. Part II: when $\omega\ne 0$, the weighted L_2-gain level will be researched.

3.1. Part I: The Stability Analysis of $\omega\equiv 0$

Theorem 1. *Under the conditions of Assumptions 1 and 2, let the positive constants τ_a and r be any given average dwell time and passivity rate, respectively. For all admissible uncertainties, system (1) with $u_i=0$ is assumed to be exponentially small-time norm-observable with the positive constants $\bar{\lambda}, c$, and δ satisfying $\bar{\lambda}\ge(1/2)(\lambda^*-(a_3\kappa/a_1))$, $\xi=e^{(a_3\eta/a_1)}$, and $c\le\sqrt{(a_1/a_2)\xi}$, where*

$$\lambda^* = \frac{\lambda_1}{r}+\frac{\lambda}{r}+\frac{\ln\mu\xi}{r\tau_a}+\frac{a_3\kappa}{ra_1}-\lambda, \tag{29}$$

for a constant $\lambda>0$. Design the controllers

$$u_i(x) = \begin{cases} -k_i(W_i(z,x),\tau_a,r)\left(L_{g_i}V_i(x)\right)^T, & i\in I_p, \\ 0, & i\in I_n, \end{cases} \tag{30}$$

where

$$k_i(W_i(z,x),\tau_a,r) = \begin{cases} \lambda^*\left(\left\|L_{g_i}V_i(x)\right\|^2\right)^{-1}W_i(z,x), & \left\|L_{g_i}V_i(x)\right\|>\delta, \\ 0, & \left\|L_{g_i}V_i(x)\right\|\le\delta. \end{cases} \tag{31}$$

Then, the switched system (1) with $w\equiv 0$ is globally robustly exponentially stable under any switching signals with the average dwell time τ_a and passivity rate $r_{p[T_1,T_2]}\ge r$.

Proof. Let

$$W_\sigma(z(t),x(t)) = U_\sigma(z(t))+V_\sigma(x(t)), \tag{32}$$

where U_σ and V_σ are smooth functions of appropriate dimensions.

For $i\in I_p$, we make the set $S_i=\left\{t: \|L_{g_i}V_i(x(t))\|\le\delta\right\}$. Then, we divide the proof into two cases: one is $S_i=\varnothing$, and the other is $S_i\ne\varnothing$. □

Case 1: $S_i=\varnothing$.
Assume that the ith subsystem is active. For $\omega\equiv 0$, the time derivative of $W_i(z,x)$ along the trajectory of the switched system (1) is

$$\frac{\partial W_i(z(t),x(t))}{\partial t} = \frac{\partial U_i(z)}{\partial z}p_i+\frac{\partial V_i(x)}{\partial x}(f_i+\Delta f_i+g_iu_i). \tag{33}$$

Substituting controller (30) into (33), from (3), (4), and (7), for $i \in I_p$, we obtain that

$$\begin{aligned}\frac{\partial W_i(z(t),x(t))}{\partial t} &\le L_{p_i}U_i(z) + L_{f_i}V_i(x) + \frac{a_3}{a_1}W_i(z,x)\zeta(t) \\ &\quad - \frac{\partial V_i(x)}{\partial x}\frac{g_i\lambda^* W_i(z,x)\left(L_{g_i}V_i(x)\right)^T}{\left\|L_{g_i}V_i(x)\right\|^2} \\ &\le \frac{a_3}{a_1}W_i(z,x)\zeta(t) - \lambda^* W_i(z,x) \\ &= -\left(\lambda^* - \frac{a_3}{a_1}\zeta(t)\right)W_i(z,x),\end{aligned} \tag{34}$$

where $(\partial V_i(x)/\partial x)\Delta f_i \le a_3\Delta f_i\|x\| \le a_3\Delta f_i(\|x\|+\|z\|) \le a_3\zeta(t)(\|x\|+\|z\|)^2 \le (a_3/a_1)W_i(z,x)\zeta(t)$.

Similarly, for $i \in I_n$, it follows from (3), (4), and (6) that

$$\begin{aligned}\frac{\partial W_i(z(t),x(t))}{\partial t} &\le \lambda W_i(z,x) + \frac{a_3}{a_1}W_i(z,x)\zeta(t) \\ &= \left(\lambda + \frac{a_3}{a_1}\zeta(t)\right)W_i(z,x).\end{aligned} \tag{35}$$

For $t \in [t_k, t_{k+1})$, we apply the integral of (34) and (35) that

$$W_{i_k}(z(t),x(t)) \le \widetilde{\phi}_{i_k}(t,t_k)W_{i_k}(z(t_k),x(t_k)), \quad t \in [t_k,t_{k+1}), \tag{36}$$

where $\widetilde{\phi}_{i_k}(t,t_k) = \begin{cases} e^{-\lambda^*(t-t_k)+(a_3/a_1)\int_{t_k}^t \zeta(\tau)\mathrm{d}\tau}, & i_k \in I_p, \\ e^{\lambda(t-t_k)+(a_3/a_1)\int_{t_k}^t \zeta(\tau)\mathrm{d}\tau}, & i_k \in I_n. \end{cases}$

Define

$$\phi_{i_k}(t,t_k) := \begin{cases} \xi e^{-a^*(t-t_k)}, & i_k \in I_p, \\ \xi e^{a(t-t_k)}, & i_k \in I_n, \end{cases} \tag{37}$$

where $\phi_{i_k}(t,t_k)\text{:} = \begin{cases} \xi e^{-a^*(t-t_k)}, & i_k \in I_p, \\ \xi e^{a(t-t_k)}, & i_k \in I_n. \end{cases}$ From Assumption 3, then

$$W_{i_k}(z(t),x(t)) \le \phi_{ik}(t,t_k)W_{i_k}(z(t_k),x(t_k)), \quad t \in [t_k,t_{k+1}). \tag{38}$$

Choose the piecewise function:

$$W(z(t),x(t)) = W_{i_k}(z(t),x(t)), \tag{39}$$

where $W(z(t_0)x(t_0)) = W_{i_0}(z(t_0),x(t_0))$.

On the contrary, $\phi_{i_k}(t,\tau)\phi_{i_{k-1}}(\tau,s) = \xi\phi_{i_{k-1}}(t,s)$, for $t \in [t_k,t_{k+1})$, and we obtain

$$\begin{aligned}W(z(t),x(t)) &= W_{ik}(z(t),x(t)) \\ &\le \phi_{i_k}(t,t_k)W_{i_k}(z(t_k),x(t_k)) \\ &\le \phi_{i_k}(t,t_k)\mu W_{i_{k-1}}(z(t_k),x(t_k)) \\ &\le \phi_{i_k}(t,t_k)\mu\phi_{i_{k-1}}(t_k,t_{k-1})W_{i_{k-1}}(z(t_{k-1}),x(t_{k-1})) \\ &\cdots \\ &\le \phi_{i_k}(t,t_k),\ldots,\phi_{i_0}(t_1,t_0)W_{i_0}(z(t_0),x(t_0)) \\ &= \xi^k\phi(t,t_0)\mu^k W_{i_0}(z(t_0),x(t_0)) \\ &= \xi e^{(N_0-(t-t_0)/\tau_a)\ln\mu\xi - a^*T_{p[t_0,t]} + aT_{n[t_0,t]}} \\ &\quad \cdot W(z(t_0),x(t_0)).\end{aligned} \tag{40}$$

From $\lambda^* = (\lambda_1/r) + (\lambda/r) + (\ln\mu\xi/r\tau_a) + (a_3\kappa/ra_1) - \lambda$, we have $\lambda_1 = \lambda^* r - \lambda - (\ln\mu\xi/\tau_a) - (a_3\kappa/a_1) + \lambda r$; then,

$$\left(N_0 + \frac{t-t_0}{\tau_a}\right)\ln\mu\xi - a^*T_{p[t_0,t]} + aT_{n[t_0,t]} = N_0\ln\mu\xi - \lambda_1(t-t_0). \tag{41}$$

Taking (41) into (40), we obtain

$$W(z(t),x(t)) \le \xi^{N_0+1}\mu^{N_0}e^{-\lambda_1(t-t_0)}W(z(t_0),x(t_0)). \tag{42}$$

From (3), we get that

$$a_1(\|z(t)\| + \|x(t)\|)^2 \le \xi^{N_0+1}\mu^{N_0}a_2e^{-\lambda_1(t-t_0)}\left(\|z(t_0)\| + \|x(t_0)\|\right)^2, \tag{43}$$

which means

$$\|z(t)\| + \|x(t)\| \le \sqrt{\frac{a_2}{a_1}\xi^{N_0+1}\mu^{N_0}}e^{-(\lambda_1/2)(t-t_0)}\left(\|z(t_0)\| + \|x(t_0)\|\right). \tag{44}$$

Case 2: $S_i \neq \varnothing$.

In the case, for $i \in I_p$, we suppose that $\left\{t\text{: } \|L_{g_i}V_i(x(t))\| \le \delta\right\} = [t_{i_1},t_{i_1}'] \cup [t_{i_2},t_{i_2}'] \cup \cdots \subset [t_0,t]$.

From Definition 3, we have $\|z(t+\tau)\| + \|x(t+\tau)\| \le ce^{-\overline{\lambda}\tau}(\|z(t)\| + \|x(t)\|)$. Then,

$$\begin{aligned}\left\|z\left(t_{i_k}'\right)\right\| + \left\|x\left(t_{i_k}'\right)\right\| &\le ce^{-\overline{\lambda}\left(t_{i_k}'-t_{ik}\right)}\left(\|z(t_{ik})\| + \|x(t_{ik})\|\right) \\ &\le \sqrt{\frac{a_1\xi}{a_2}}e^{-(a^*/2)\left(t_{i_k}'-t_{ik}\right)}\left(\|z(t_{ik})\| + \|x(t_{ik})\|\right).\end{aligned} \tag{45}$$

From (3), we obtain

$$W_{ik}\left(z\left(t_{i_k}'\right),x\left(t_{i_k}'\right)\right) \le \xi e^{-a^*\left(t_{i_k}'-t_{ik}\right)}W_{ik}(z(t_{ik}),x(t_{ik})). \tag{46}$$

Similar to Case 1, we can achieve

$$\|z(t)\| + \|x(t)\| \le \sqrt{\frac{a_2}{a_1}\xi^{N_0+1}\mu^{N_0}}e^{-(\lambda_1/2)(t-t_0)} \cdot \left(\|z(t_0)\| + \|x(t_0)\|\right). \tag{47}$$

Therefore, the closed-loop switched system (1) is exponential stability for all admissible uncertainties. This completes the proof.

3.2. Part II: The L_2-Gain Analysis of $\omega \neq 0$. In this section, we investigate the H_∞ performance analysis of system (1) by the L_2-gain γ.

Theorem 2. *Assume the positive constants τ, r, and γ are average dwell time, passivity rate, and disturbance attenuation level, respectively. U_σ, V_σ, and W_σ still satisfy (3)–(7). For all admissible uncertainties and disturbance inputs, assume the passive subsystems are exponentially small-time norm-observability with positive constants $\overline{\lambda}$, c, δ, k_0, and k. All of these satisfy $k = \overline{\lambda} - (1/2)(\lambda^* - (a_3\kappa/a_1) - (a_3^2\rho^2/4a_1\gamma^2))$ and $a_2c^2 \le (a_1 - k_0)e^{(a_3\eta/a_1)}$. Then, we design the controllers:*

$$u_i(x) = \begin{cases} -k_i(W_i(z,x),\tau_a,r,\gamma)\left(L_{g_i}V_i(x)\right)^T, & i \in I_p, \\ -\left(L_{g_i}V_i(x)\right)^T, & i \in I_n, \end{cases} \tag{48}$$

where

$$k_i(W_i(z,x),\tau_a,r,\gamma) = \begin{cases} \dfrac{\lambda^* W_i(z,x) + y_i y_i^T}{\left\|L_{g_i}V_i(x)\right\|^2}, & \left\|L_{g_i}V_i(x)\right\| > \delta, \\ 0, & \left\|L_{g_i}V_i(x)\right\| \le \delta, \end{cases} \tag{49}$$

and λ^* is given by Theorem 1 with $\lambda_1 = \lambda_2 + (a_3^2\rho^2/4a_1\gamma^2)$ and $\lambda_2 > 0$. Then, the switched system (1) achieves a weighted L_2-gain from ω to y for all admissible uncertainties.

Proof. On the basis of Theorem 1, the time derivative of $W_i(z,x)$ along the trajectory of switched system (1) is

$$\frac{\partial W_i(z(t),x(t))}{\partial t} = \frac{\partial U_i(z)}{\partial z}(p_i + q_i\omega) + \frac{\partial V_i(x)}{\partial x} \cdot (f_i + \Delta f_i + g_i u_i + c_i\omega). \tag{50}$$

When $i \in I_p$, $S_i = \varnothing$, similarly the proof of Theorem 1, we have

$$\begin{aligned} \frac{\partial W_i(z(t),x(t))}{\partial t} &\le -\left(\lambda^* - \frac{a_3\xi}{a_1}\right)W_i(z,x) + \frac{\partial U_i(z)}{\partial z}q_i\omega + \frac{\partial V_i(x)}{\partial x}c_i\omega - y_i^T y_i \\ &\le -\left(\lambda^* - \frac{a_3}{a_1}\zeta(t)\right)W_i(z,x) + (a_3\|z\|q_i + a_3\|x\|c_i)\|\omega\| - y_i^T y_i \\ &\le -\left(\lambda^* - \frac{a_3}{a_1}\zeta(t)\right)W_i(z,x) + \frac{a_3^2\rho^2}{4\gamma^2}(\|z\| + \|x\|)^2 + \gamma^2\|\omega\|^2 - y_i^T y_i \\ &\le -\left(\lambda^* - \frac{a_3}{a_1}\zeta(t) - \frac{a_3^2\rho^2}{4a_1\gamma^2}\right)W_i(z,x) + \gamma^2\omega^T(t)\omega(t) - y_i^T y_i, \end{aligned} \tag{51}$$

where $\rho = \max\{q_i, c_i\}$.

When $i \in I_p, S_i \neq \varnothing$, on the interval $[t_{i_k}, t'_{i_k}]$, we have

$$\begin{aligned} \left(\left\|z(t'_{i_k}) + \|x(t'_{i_k})\right\|\right)^2 &\le c_1 e^{-2(\overline{\lambda}-k)\left(t'_{i_k}-t_{i_k}\right)}\left(\left\|z(t_{i_k})\right\| + \left\|x(t_{i_k})\right\|\right)^2 \\ &\quad - \frac{\xi}{a_2}\int_{t_{i_k}}^{t'_{i_k}} e^{-2(\overline{\lambda}-k)\left(t'_{i_k}-\theta\right)} y_i^T y_i \mathrm{d}\theta. \end{aligned} \tag{52}$$

From (3), we obtain

$$\begin{aligned} &W_{i_k}\left(z\left(t'_{i_k}\right), x\left(t'_{i_k}\right)\right) \\ &\quad \leq \left(c^2 a_2 + \xi k_0\right) e^{-2(\bar{\lambda}-k)\left(t'_{i_k}-t_{i_k}\right)} \left(\left\|z\left(t_{i_k}\right)\right\| + \left\|x\left(t_{i_k}\right)\right\|\right)^2 - \int_{t_{i_k}}^{t'_{i_k}} \xi e^{-2(\bar{\lambda}-k)\left(t'_{i_k}-\theta\right)} y_{i_k}^T y_{i_k} \mathrm{d}\theta \\ &\quad \leq \frac{c^2 a_2 + \xi k_0}{a_1} e^{-2(\bar{\lambda}-k)\left(t'_{i_k}-t_{i_k}\right)} W_{i_k}\left(z\left(t_{i_k}\right) x\left(t_{i_k}\right)\right) - \int_{t_{i_k}}^{t'_{i_k}} \xi e^{-2(\bar{\lambda}-k)\left(t'_{i_k}-\theta\right)} y_{i_k}^T y_{i_k} \mathrm{d}\theta. \end{aligned} \tag{53}$$

Due to $k = \bar{\lambda} - (1/2)(\lambda^* - (a_3\kappa/a_1) - (a_3^2\rho^2/4a_1\gamma^2))$, we get $\bar{\lambda} - k = (1/2)(\lambda^* - (a_3\kappa/a_1) - (a_3^2\rho^2/4a_1\gamma^2)) = (1/2)a^* - (a_3^2\rho^2/4a_1\gamma^2)$. Substituting $\bar{\lambda} - k$ into (53), we know

$$\begin{aligned} W_{i_k}\left(z\left(t'_{i_k}\right), x\left(t'_{i_k}\right)\right) \leq{}& \frac{c^2 a_2 + \xi k_0}{a_1} e^{-\left(a^* - \left(a_3^2\rho^2/4a_1\gamma^2\right)\right)\left(t'_{i_k}-t_{i_k}\right)} W_{i_k}\left(z\left(t_{i_k}\right), x\left(t_{i_k}\right)\right) \\ &+ \int_{t_{i_k}}^{t'_{i_k}} \xi e^{-\left(a^* - \left(a_3^2\rho^2/4a_1\gamma^2\right)\right)\left(t'_{i_k}-\theta\right)} \left(\gamma^2\omega^2(\theta) - y_{i_k}^T y_{i_k}\right) \mathrm{d}\theta. \end{aligned} \tag{54}$$

For $a_2 c^2 \leq (a_1 - k_0) e^{(a_3\eta/a_1)} = (a_1 - k_0)\xi = a_1\xi - k_0\xi$, which means $(c^2 a_2 + \xi k_0/a_1) \leq \xi$; then,

$$\begin{aligned} W_{i_k}\left(z\left(t'_{i_k}\right), x\left(t'_{i_k}\right)\right) \leq{}& \xi e^{-\left(a^* - \left(a_3^2\rho^2/4a_1\gamma^2\right)\right)\left(t'_{i_k}-t_{i_k}\right)} W_{i_k}\left(z\left(t_{i_k}\right), x\left(t_{i_k}\right)\right) \\ &+ \int_{t_{i_k}}^{t'_{i_k}} \xi e^{-\left(a^* - \left(a_3^2\rho^2/4a_1\gamma^2\right)\right)\left(t'_{i_k}-\theta\right)} \left(\gamma^2\omega^2(\theta) - y_{i_k}^T y_{i_k}\right) \mathrm{d}\theta. \end{aligned} \tag{55}$$

Similarly, when $i \in I_n$,

$$\begin{aligned} \frac{\partial W_i(z(t), x(t))}{\partial t} &\leq \left(\lambda + \frac{a_3\xi}{a_1}\right) W_i(z, x) + \frac{\partial U_i(z)}{\partial z} q_i \omega + \frac{\partial V_i(x)}{\partial x} c_i \omega - y_i^T y_i \\ &\leq \left(\lambda + \frac{a_3}{a_1}\zeta(t) + \frac{a_3^2\rho^2}{4a_1\gamma^2}\right) W_i(z, x) + \gamma^2\omega^T(t)\omega(t) - y_i^T y_i. \end{aligned} \tag{56}$$

Let $\tilde{a}^* = a^* - (a_3^2\rho^2/4a_1\gamma^2), \tilde{a} = a + (a_3^2\rho^2/4a_1\gamma^2)$, and $\Gamma_i(t) = \gamma^2\omega^T(t)\omega(t) - y_i^T y_i$, where $a^* = \lambda^* - (a_3\kappa/a_1)$, $a = \lambda + (a_3\kappa/a_1)$, and $\xi = e^{(a_3\eta/a_1)}$ $\int_{t_0}^{t} \zeta(\tau)\mathrm{d}\tau \leq \kappa(t - t_0) + \eta$. For (51), (55), and (56), the differential equation theory and the constant variable formula are used, respectively. When $i \in I_p$ and $S_i = \varnothing$,

$$W_{i_k}(z(t),x(t)) \le e^{\int_{t_k}^{t} -(\lambda^* - (a_3/a_1)\zeta(t) - (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s} \cdot \left[W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} \Gamma_{i_k}(\tau) e^{-\int_{t_k}^{\tau} -(\lambda^* - (a_3/a_1)\zeta(s) - (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s}\mathrm{d}\tau\right]$$
$$\le \xi e^{-\widetilde{a}^*(t-t_k)} W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} e^{-(\lambda^* - (a_3/a_1)\zeta(t) - (a_3^2\rho^2/4a_1\gamma^2))(t-\tau) + (a_3/a_1)\eta}\Gamma_{i_k}(\tau)\mathrm{d}\tau$$
$$\le \xi e^{-\widetilde{a}^*(t-t_k)} W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} \xi e^{-\widetilde{a}^*(t-\tau)}\Gamma_{i_k}(\tau)\mathrm{d}\tau. \quad (57)$$

When $i \in I_p$ and $S_i \neq \varnothing$,

$$W_{i_k}(z(t),x(t_k)) \le \xi e^{-\widetilde{a}^*(t-t_k)} W_{i_k}(t_k) + \int_{t_k}^{t} \xi e^{-\widetilde{a}^*(t-\tau)}\Gamma_{i_k}(\tau)\mathrm{d}\tau. \quad (58)$$

And when $i \in I_n$,

$$W_{i_k}(z(t),x(t)) \le e^{\int_{t_k}^{t} (\lambda + (a_3/a_1)\zeta(t) + (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s} \cdot \left[W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} \Gamma_{i_k}(\tau) e^{-\int_{t_k}^{\tau} (\lambda + (a_3/a_1)\zeta(t) + (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s}\mathrm{d}\tau\right]$$
$$\le \xi e^{\widetilde{a}(t-t_k)} W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} e^{(\lambda + (a_3/a_1)\zeta(t) + (a_3^2\rho^2/4a_1\gamma^2))(t-\tau) + (a_3/a_1)\eta}\Gamma_{i_k}(\tau)\mathrm{d}\tau$$
$$\le \xi e^{\widetilde{a}(t-t_k)} W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} \xi e^{\widetilde{a}(t-\tau)}\Gamma_{i_k}(\tau)\mathrm{d}\tau, \quad (59)$$

where

$$e^{\int_{t_k}^{t} -(\lambda^* - (a_3/a_1)\zeta(t) - (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s} \int_{t_k}^{t} \Gamma_{i_k}(\tau) e^{-\int_{t_k}^{\tau} -(\lambda^* - (a_3/a_1)\zeta(s) - (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s}\mathrm{d}\tau$$
$$= \int_{t_k}^{t} \Gamma_{i_k}(\tau) e^{\int_{\tau}^{t} -(\lambda^* - (a_3/a_1)\zeta(s) - (a_3^2\rho^2/4a_1\gamma^2))\mathrm{d}s}\mathrm{d}\tau$$
$$\le \int_{t_k}^{t} \Gamma_{i_k}(\tau) e^{-(\lambda^* - (a_3\kappa/a_1) - (a_3^2\rho^2/4a_1\gamma^2))(t-\tau) + (a_3/a_1)\eta}\mathrm{d}\tau$$
$$= \int_{t_k}^{t} \xi e^{-\widetilde{a}^*(t-\tau)}\Gamma_{i_k}(\tau)\mathrm{d}\tau. \quad (60)$$

Combining (57)–(59), we obtain

$$W_{i_k}(z(t),x(t)) \le \psi(t,t_k) W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^{t} \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau, \quad t \in [t_k, t_{k+1}), \quad (61)$$

where $\psi_{i_k}(t,\tau) = \begin{cases} \xi e^{-\widetilde{a}^*(t-\tau)}, & i_k \in I_p \\ \xi e^{\widetilde{a}(t-\tau)}, & i_k \in I_n \end{cases}$. Define the piecewise function $W(z(t),x(t)) = W_{i_k}(z(t),x(t))$ and $t \in [t_k, t_{k+1})$. When the time t satisfies $t_0 < t_1 < \cdots < t_n < t < t_{n+1} < \cdots$, by the property $\psi_{i_k}(t,\tau)\psi_{i_{k-1}}(\tau,s) = \xi\psi_{i_{k-1}}(t,s)$, $i \in I$, and from (61), we have

$$\begin{aligned} W(t) &= W_{ik}(z(t),x(t)) \\ &\le \psi_{i_k}(t,t_k)W_{i_k}(z(t_k),x(t_k)) + \int_{t_k}^t \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau \\ &\le \psi_{i_k}(t,t_k)\mu W_{i_{k-1}}(z(t_k),x(t_k)) + \int_{t_k}^t \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau \le \psi_{i_k}(t,t_k)\mu \\ &\le \left[\psi_{i_{k-1}}(t_k,t_{k-1})W_{i_{k-1}}(z(t_{k-1}),x(t_{k-1})) + \int_{t_{k-1}}^{t_k} \psi_{i_{k-1}}(t_k,\tau)\Gamma_{i_{k-1}}(\tau)\mathrm{d}\tau\right] + \int_{t_k}^t \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau \\ &\cdots \\ &\le \psi_{i_k}(t,t_k),\ldots,\psi_{i_0}(t_1,t_0)W_{i_0}(z(t_0),x(t_0))\mu^k \\ &\quad + \psi_{i_k}(t,t_k),\ldots,\psi_{i_1}(t_2,t_1)\mu^k \int_{t_0}^{t_1} \psi_{i_0}(t_1,\tau)\Gamma_{i_0}(\tau)\mathrm{d}\tau \\ &\quad + \cdots + \psi_{i_k}(t,t_k)\mu \int_{t_{k-1}}^{t_k} \psi_{i_{k-1}}(t_k,\tau)\Gamma_{i_{k-1}}(\tau)\mathrm{d}\tau + \int_{t_k}^t \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau \\ &= \psi_{i_k}(t,t_k),\ldots,\psi_{i_0}(t_1,t_0)W_{i_0}(z(t_0),x(t_0))\mu^k \\ &\quad + \sum_{n=1}^{k}\int_{t_{n-1}}^{t_n} \xi^{k-n+1}\mu^{k-n+1}\psi_{i_{n-1}}(t,\tau)\Gamma_{i_{n-1}}(\tau)\mathrm{d}\tau + \int_{t_k}^t \psi_{i_k}(t,\tau)\Gamma_{i_k}(\tau)\mathrm{d}\tau \\ &\le \xi^{k+1}\mu^k e^{-\widetilde{a}^* T_{p[t_0,t]} + \widetilde{a}T_{n[t_0,t]}} W(z(t_0),x(t_0)) + \xi\int_{t_0}^t (\mu\xi)^{N_\sigma(\tau,t)} e^{-\widetilde{a}^* T_{p[\tau,t]} + \widetilde{a}T_{n[\tau,t]}}\Gamma(\tau)\mathrm{d}\tau. \end{aligned} \tag{62}$$

Due to $\lambda_1 = \lambda_2 + (a_3^2\rho^2/4a_1\gamma^2)$, we have $-\widetilde{a}^* T_{p[t_0,t]} + \widetilde{a}T_{n[t_0,t]} = (-\lambda_2 - (\ln\mu\xi/\tau_a))(t-t_0)$.

Then,

$$\begin{aligned} 0 &\le \xi^k\mu^k e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-t_0)} W(z(t_0),x(t_0)) \\ &\quad + \int_{t_0}^t (\mu\xi)^{N_\sigma(\tau,t)} e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-\tau)}\Gamma(\tau)\mathrm{d}\tau, \end{aligned} \tag{63}$$

which means

$$\begin{aligned} 0 &\le e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-t_0) + N_\delta(t,t_0)\ln(\mu\xi)} W(z(t_0),x(t_0)) \\ &\quad + \int_{t_0}^t e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-\tau) + N_\delta(t,\tau)\ln(\mu\xi)}\Gamma(\tau)\mathrm{d}\tau. \end{aligned} \tag{64}$$

We multiply both sides of the above formula by $e^{-N_{\sigma(t_0,t)}\ln\mu\xi}$:

$$\begin{aligned} 0 &\le e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-t_0)} W(z(t_0),x(t_0)) \\ &\quad + \int_{t_0}^t e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-\tau) + N_\delta(t_0,\tau)\ln(\mu\xi)}\Gamma(\tau)\mathrm{d}\tau. \end{aligned} \tag{65}$$

Obviously, $-N_\delta(t_0,\tau) \le 0$, $\xi > 1$, $\mu > 1$, then $e^{-N_\delta(t_0,\tau)\ln(\mu\xi)} \le 1$, and putting $\Gamma(\tau)$, $N_\delta(t_0,\tau) = (\tau - t_0/\tau_a)$ into (65), we obtain

$$\begin{aligned} &\int_{t_0}^t e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-\tau) - (\ln\mu\xi/\tau_a)(\tau-t_0)} y^T(\tau)y(\tau)\mathrm{d}\tau \\ &\quad \le \gamma^2\int_{t_0}^t e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-\tau)}\omega^T(\tau)\omega(\tau)\mathrm{d}\tau + e^{(-\lambda_2 - (\ln\mu\xi/\tau_a))(t-t_0)} W(z(t_0),x(t_0)). \end{aligned} \tag{66}$$

For the trivial case of $\mu = 1$ and $\xi = 1$, we obtain

$$\int_{t_0}^{\infty} y^T(\tau)y(\tau)\mathrm{d}\tau \le \gamma^2 \int_{t_0}^{\infty} \omega^T(\tau)\omega(\tau)\mathrm{d}\tau + W(z(t_0), x(t_0)). \tag{67}$$

Next, we consider the nontrivial case of $\mu > 1$. Rearranging the double-integral area leads

$$\begin{aligned}&\int_{t_0}^{+\infty} e^{-(\ln\mu\xi/\tau_a)(\tau-t_0)} y^T(\tau)y(\tau)\mathrm{d}\tau \\ &\quad\le \gamma^2 \int_{t_0}^{+\infty} \omega^T(\tau)\omega(\tau)\mathrm{d}\tau + W(z(t_0), x(t_0)).\end{aligned} \tag{68}$$

Hence, the switched system (1) achieves a weighted L_2-gain from ω to y for all admissible uncertainties. This completes the proof. □

Remark 3. Under zero initial condition, we have $W(0,0) = 0$, and from (68), we can get the weighted L_2-gain level $\gamma^2 = \int_0^{+\infty} e^{-(\ln\mu\xi/\tau_a)(\tau-t_0)} y^T(\tau)y(\tau)\mathrm{d}\tau / \int_0^{+\infty}\omega^T(\tau)\omega(\tau)\mathrm{d}\tau$. The smaller the weighted L_2-gain level γ is, the better the performance of robust H_∞ control of system (1) is [25, 26].

Remark 4. The system in [19] is similar to system (1) in this paper, and it needs to satisfy these conditions in [19]: (i) $\gamma^2 - \gamma_3^2 - \gamma_d^2 > 0$; (ii) for unbounded positive definite differentiable functions $V_i(x_1), i = 1, \ldots, N$, constants $\gamma_1 > 0$ and $\lambda_0 > 0$, such that $(\partial V_i/\partial x_1) f_{1,i}(x_1, 0) + (1/4\gamma_1^2)(\partial V_i/\partial x_1) c_i(x_1,0) c_i^T(x_1,0)(\partial V_i^T/\partial x_1) + h_i^T(x_1,0)h_i(x_1,0) + \lambda_0 V_i \le 0$ holds. In our paper, we just need $\gamma > 0$, and the positive definite differentiable functions W_i do not need to satisfy condition (ii). So, this paper gets less conservative.

4. Numerical Example

In this section, we give two examples to demonstrate the effectiveness of the proposed method.

Example 1. Consider a switched continuous stirred tank reactor system with two modes feed stream [27, 28]:

$$\begin{aligned}\dot{\xi}_1 &= \frac{F_{\sigma(t)}}{V}\left(\xi_{1,\mathrm{in},\sigma(t)} - \xi_1\right) + K_{\sigma(t)}\varphi_{\sigma(t)}(\xi_1, \xi_2) - \mathrm{d}(t), \\ \dot{\xi}_2 &= \frac{F_{\sigma(t)}}{V}\left(\xi_{2,\mathrm{in},\sigma(t)} - \xi_2\right) - \Delta H_{\sigma(t)}(\xi_1, \xi_2)\varphi_{\sigma(t)}(\xi_1, \xi_2) \\ &\quad + \gamma(\xi_{2c} - \xi_2) + \mathrm{d}(t).\end{aligned} \tag{69}$$

In this paper, we ignore the influence of temperature on reaction speed, and only consider the disturbance $\mathrm{d}(t)$ on the concentration. And the physical meaning of the parameters in system (69) can be found in [28].

The control objective is to make the temperature to some constant reference ξ_1^* and ξ_2^*. And $u_k^* = \gamma\xi_{2c}^*$ is a steady-state control corresponding to the temperature set points ξ_1^* and ξ_2^*. Let $z = \xi_1 - \xi_1^*, x = \xi_2 - \xi_2^*, u_k = \gamma(\xi_{2c} - \xi_{2c}^*)$. System (69) can be expressed in the form with equilibrium point at the origin:

$$\begin{aligned}\dot{z} &= f_{1\sigma}(z, x) + \mathrm{d}(t), \\ \dot{x} &= f_{2\sigma}(z, x) + \Delta f_{2\sigma}(z, x) + u_\sigma + \mathrm{d}(t),\end{aligned} \tag{70}$$

where $f_{1\sigma} = (F_\sigma/V)(\xi_{1,\mathrm{in},\sigma} - \xi_1^* - z) + K_\sigma\varphi_\sigma(z + \xi_1^*, x + \xi_2^*)$, $f_{2\sigma} = (F_\sigma/V)(\xi_{2,\mathrm{in},\sigma} - \xi_2^* - x) + \gamma(\xi_{2c}^* - \xi_2^* - x)$ and $\Delta f_{2\sigma} = -\Delta H_\sigma\ (z + \xi_1^*, x + \xi_2^*)\varphi_\sigma(z + \xi_1^*, x + \xi_2^*)$.

Then, let the steady-state point $\xi_2^* = \xi_{2c}^* = 0K$, $\xi_1^* = 1$ mol/L, and parameters for the simulation $F_1 = 4$L/s, $\gamma = K_1 = 1$, $\xi_{2\mathrm{in}1} = \xi_{2\mathrm{in}2} = 0$ K, $V = 1L$, $\xi_{1\mathrm{in}1} = \xi_{1\mathrm{in}2} = 1$ mol/L, $\Delta H_1 = \Delta H_2 = -0.5$, $F_2 = 1$ L/s, $K_2 = 2$, $\varphi_1 = ze^{-x}$, $\varphi_2 = (z + x)e^{-2x}$, and $\mathrm{d}(t) = \omega(t)$. And defining the output $y = 2x$, we get two subsystems as follows:

$$\begin{cases}\dot{z} = -4z + ze^{-x} + \omega(t), \\ \dot{x} = -5x + 0.5ze^{-x} + u_1 + \omega(t), \\ y = 2x,\end{cases} \tag{71}$$

$$\begin{cases}\dot{z} = -z + 2(z + x)e^{-2x} + \omega(t), \\ \dot{x} = -2x + 0.5(z + x)e^{-2x} + u_2 + \omega(t), \\ y = 2x,\end{cases} \tag{72}$$

where $u_i, i = 1, 2$, are controllers and $\theta_i, i = 1, 2$, are unknown constants.

It is not difficult to know that $\|\Delta f_1(t, x)\| \le 0.5\|z\|$ and $\|\Delta f_2(t, x)\| \le 0.5(\|z\| + \|x\|)$. So, $\int_{t_0}^t \zeta(\tau)\mathrm{d}\tau \le \int_{t_0}^t 0.5\mathrm{d}\tau \le \kappa(t - t_0) + \eta$, where $\kappa = 1$ and $\eta = 0.5$.

Let $W_1 = z^2 + 0.5x^2$ and $W_2 = z^2 + x^2$. For system (71),

$$\begin{aligned}L_{p_1}U_1(z) &= \frac{\partial U_1}{\partial z}p_1 = 2z \times \left(-z + ze^{-x}\right) = -2z^2 + z^2e^{-x} \le 0, \\ L_{f_1}V_1(x) &= \frac{\partial V_1}{\partial x}f_1 = 2x \times (-5x) = -10x^2 \le 0, \\ L_{g_1}V_1(x) &= 2x.\end{aligned} \tag{73}$$

For system (72),

$$\begin{aligned}L_{p_2}U_2(z) &= 2z \times \left(-z + 2(z + x)e^{-2x}\right) \\ &= -2z^2 + 4z(z + x)e^{-2x}, \\ L_{f_2}V_2(x) &= 2x \times -x = -4x^2, \\ L_{g_2}V_2(x) &= 2x.\end{aligned} \tag{74}$$

Then,

$$\begin{aligned}L_{p_2}U_2(z) + L_{f_2}V_2(x) &= -2z^2 + 4z(z + x)e^{-2x} - 4x^2 \\ &\le 2z^2 + 4zx - 4x^2 \le 3z^2 \le 3W_2(z, x).\end{aligned} \tag{75}$$

It is easy to verify that system (71) is passive and system (72) is nonpassive.

A simple calculation shows that $a_1 = 0.5$, $a_2 = 2$, $a_3 = 2$, $\mu = 2$, $\rho = 1$, and $\lambda = 2$. In addition, we acquire that $c = 3$ and

$\xi = e^{(a_3\eta/a_1)}$. Let the average dwell time $\tau_a = 2$, the passivity rate $r = 1$, the disturbance attenuation level $\gamma = 2$, and $\lambda_2 = 1.2$. Then, $\lambda_1 = \lambda_2 + (a_3^2\rho^2/4a_1\gamma^2) = 1.7000$, $\lambda^* = (\lambda_1/r) + (\lambda/r) + (\ln(\mu\xi)/r\tau_a) + (a_3\kappa/ra_1) - \lambda = 7.0466$, $\overline{\lambda} = 10(\lambda^* - (a_3\kappa/a_1)) = 30.4657$, and $\overline{\lambda} - c\kappa - (1/2)(\lambda^* - (a_3\kappa/a_1) - (a_3^2/4a_1\gamma^2)) = 26.9424 > 0$. According to Theorem 1, when $\omega(t) \equiv 0$, we construct the controllers

$$u_i = \begin{cases} \dfrac{-14.0932\left(z^2 + 0.5x^2\right)x}{\|2x\|^2}, & i \in I_p, \|2x\| > 0.1, \\ 0, & i \in I_p, \|2x\| \le 0.1, \\ 0, & i \in I_n. \end{cases} \quad (76)$$

Figure 2 gives the control input of systems (71) and (72) and the switching signal $\sigma(t)$. Figure 3 is the simulation result with the initial states $z(1) = 2$ and $x(1) = 1$.

When $\omega(t) = e^{-t}$, we construct the controllers

$$n * u_i = \begin{cases} \dfrac{\left[-14.0932\left(z^2 + 0.5x^2\right) - 8x^2\right]x}{\|2x\|^2}, & i \in I_p, \|2x\| > 0.1, \\ 0, & i \in I_p, \|2x\| \le 0.1, \\ -2x, & i \in I_n. \end{cases} \quad (77)$$

From Remark 3, we define the function

$$\gamma^2(s) = \frac{\int_0^s e^{-(\ln \mu\xi/\tau_a)(\tau - t_0)} y^T(\tau) y(\tau) d\tau}{\int_0^s \omega^T(\tau)\omega(\tau) d\tau}. \quad (78)$$

In this example, we can get $\gamma^2(s) = \int_0^s e^{-26.9315} y^T(\tau)y(\tau)d\tau / \int_0^s \omega^T(\tau)\omega(\tau)d\tau$. Figure 4 gives the L_2-gain level with the initial states $z(1) = 0$ and $x(1) = 0$. And we can easily see the L_2-gain less than $\gamma = 2$.

Example 2. Consider the uncertain switched nonlinear cascade systems with two systems:

$$\begin{cases} \dot{z}_1 = -3z_1 - z_2x_1, \\ \dot{z}_2 = -z_2 + 5z_1x_1, \\ \dot{x}_1 = -x_1 - z_1x_2 + \theta_1x_1 - u_1, \\ \dot{x}_2 = -2x_2 + 0.3z_1x_1 + \theta_1x_2 - 3u_1, \\ y = -6x_1 - 6x_2, \end{cases} \quad (79)$$

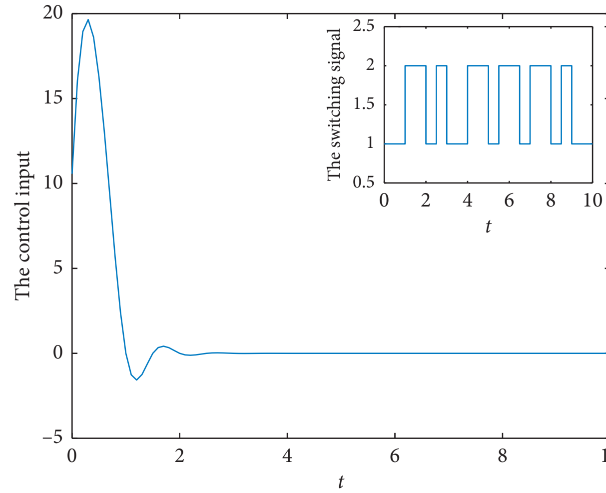

FIGURE 2: The control input u and the switching signal $\sigma(t)$.

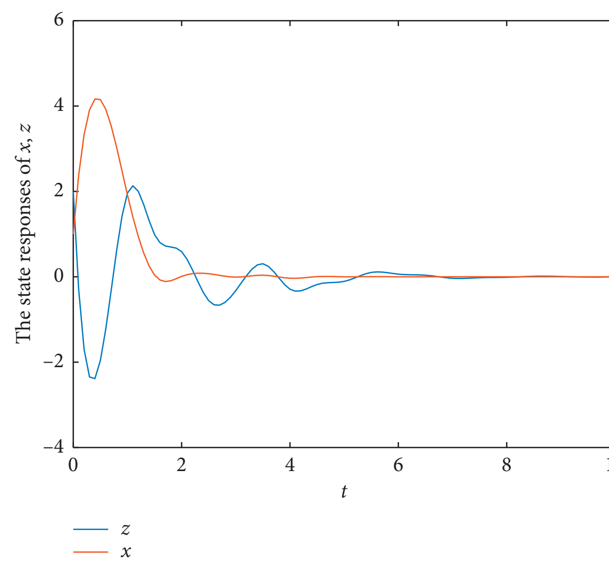

FIGURE 3: The system state response.

$$\begin{cases} \dot{z}_1 = z_1 - 0.2z_2x_1, \\ \dot{z}_2 = z_2 + 2z_1x_1, \\ \dot{x}_1 = x_1 + z_1x_2 + \theta_2x_1 + 0.1u_2 + \sin\left(x_1^2\right)\omega, \\ \dot{x}_2 = x_2 - z_1x_1 + \theta_2x_2 - 0.1u_2 + \sin\left(x_2^2\right)\omega, \\ y = 2x_1 + 2x_2 + \omega, \end{cases} \quad (80)$$

where $\theta_1 = -0.48$ and $\theta_2 = -0.06$ are generated constants by random numbers.

Let $W_1 = 0.5z_1^2 + 0.1z_2^2 + 3x_1^2 + x_2^2$, and $W_2 = z_1^2 + 0.1z_2^2 + x_1^2 + x_2^2$. For system (79),

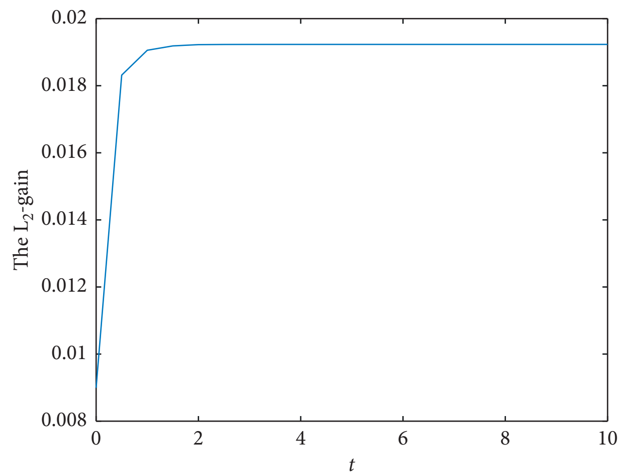

FIGURE 4: The L_2-gain level γ^2.

$$L_{p_1}U_1(z)=\begin{pmatrix} z_1 & 0.2z_2\end{pmatrix}\begin{pmatrix}-3z_1-z_2x_1\\ -z_2+5z_1x_1\end{pmatrix}=-3z_1^2-0.2x_2^2\le 0,$$
$$L_{f_1}V_1(x)=\begin{pmatrix} 6x_1 & 2x_2\end{pmatrix}\begin{pmatrix}-x_1-z_1x_2\\ -2x_2+0.3z_1x_1\end{pmatrix}=-6x_1^2-4x_2^2\le 0,$$
$$L_{g_1}V_1(x)=-6x_1-6x_2. \tag{81}$$

For system (80),

$$L_{p_2}U_2(z)=\begin{pmatrix} 2z_1 & 0.2z_2\end{pmatrix}\begin{pmatrix}z_1-0.2z_2x_1\\ z_2+2z_1x_1\end{pmatrix}=2z_1^2+0.2z_2^2,$$
$$L_{f_2}V_2(x)=\begin{pmatrix} 2x_1 & 2x_2\end{pmatrix}\begin{pmatrix}x_1+z_1x_2\\ x_2-z_1x_1\end{pmatrix}=2x_1^2+2x_2^2,$$
$$L_{g_2}V_2(x)=0.2x_1-0.2x_2. \tag{82}$$

Then,

$$L_{p_2}U_2(z)+L_{f_2}V_2(x)=2z_1^2+0.2z_2^2+2x_1^2+2x_2^2\le 2W_2(z,x). \tag{83}$$

It is easy to see that system (79) is passive and system (80) is nonpassive. A simple calculation shows that $a_1=0.1$, $a_2=3, a_3=2, \mu=10$, $\kappa=0$, $\eta=0.5$, $\rho=1$, and $\lambda=2$. In addition, we acquire that $c=20$ and $\xi=e^{(a_3\eta/a_1)}$. Let the average dwell time $\tau_a=2$, the passivity rate $r=6$, the disturbance attenuation level $\gamma=1$, and $\lambda_2=1.2$. Hence, $\lambda_1=\lambda_2+(a_3^2\rho^2/4a_1\gamma^2)=3.7000$, $\lambda^*=(\lambda_1/r)+(\lambda/r)+(\ln(\mu\xi)/r\tau_a)+(a_3\kappa/ra_1)-\lambda=0.3919$, $\overline{\lambda}=10(\lambda^*-(a_3\kappa/a_1))=3.9188$, and $\overline{\lambda}-c\kappa-(1/2)(\lambda^*-(a_3\kappa/a_1)-(a_3^2/4a_1\gamma^2))=4.0354>0$.

According to Theorem 1, we construct the controllers; when $\omega(t)\equiv 0$,

$$u_i=\begin{cases}\dfrac{0.3919\left(0.5z_1^2+0.1z_2^2+3x_1^2+x_2^2\right)(x_1+x_2)}{6\left\|x_1+x_2\right\|^2}, & i\in I_p, 6\left\|x_1+x_2\right\|>0.3,\\ 0, & i\in I_p, 6\left\|x_1+x_2\right\|\le 0.3,\\ 0, & i\in I_n.\end{cases} \tag{84}$$

Figure 5 shows the control input of systems (79) and (80) and the switching signal $\sigma(t)$. Figure 6 is the simulation result with the initial states $z_1(1)=1, z_2(1)=2, x_1(1)=-1$, and $x_2(1)=-1$.

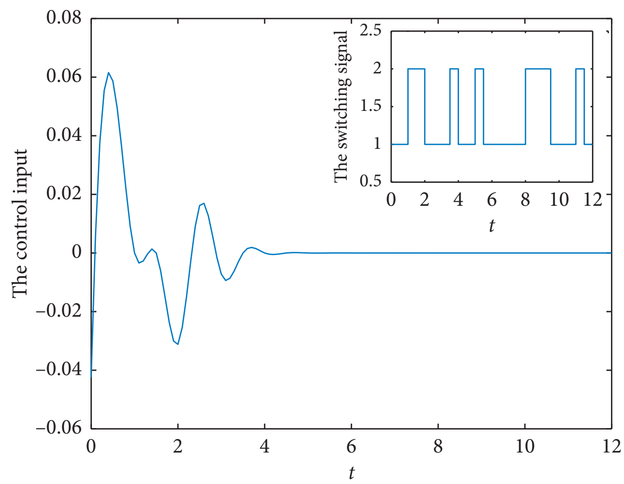

Figure 5: The control input u and the switching signal $\sigma(t)$.

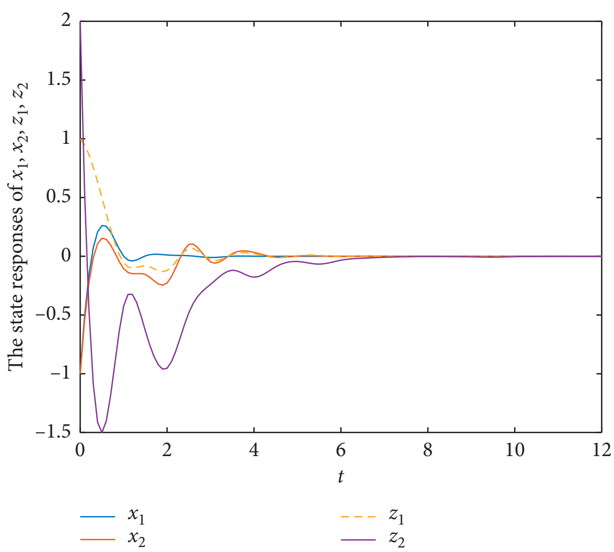

Figure 6: The system state response.

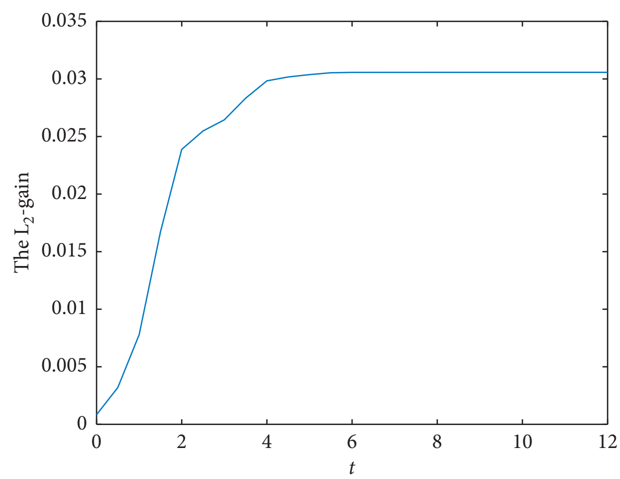

Figure 7: The L_2-gain level γ^2.

When $\omega(t) = \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}$, we construct the control functions:

$$u_i = \begin{cases} \dfrac{\left[0.3919\left(0.5z_1^2 + 0.1z_2^2 + 3x_1^2 + x_2^2\right) + y_1 y_1^T\right](x_1 + x_2)}{6\left\|x_1 + x_2\right\|^2} & i \in I_p, 6\left\|x_1 + x_2\right\| > 0.3, \\ 0, & i \in I_p, 6\left\|x_1 + x_2\right\| \leq 0.3, \\ -(0.2x_1 - 0.2x_2), & i \in I_n. \end{cases} \quad (85)$$

In this example, we can get $\gamma^2(s) = \int_0^s e^{-103.8155} y^T(\tau)y(\tau)d\tau / \int_0^s \omega^T(\tau)\omega(\tau)d\tau$. Figure 7 is the L_2-gain level with the initial states $z_1(1) = 0$, $z_2(1) = 0$, $x_1(1) = 0$, and $x_2(1) = 0$. And we can easily see the L_2-gain less than $\gamma = 1$.

5. Conclusion

Based on the method of average dwell time, we give sufficient conditions to ensure the solvability of the problem avoiding the Lyapunov function construction by the storage functions and reducing the computational complexity of the solution. For any switching signal, the system can achieve stability and have the weighted L_2-gain property under the action of the feedback controller designed by the given passivity rate, average dwell time, and interference attenuation level. The proposed scheme supplements the research methods of robust H_∞ control for the nonlinear cascade systems. In the future, we will extend the results of this paper to global stabilization of switched stochastic nonlinear robust H_∞ control systems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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