Research Article

Kinematic Analysis of a Novel System for Paper Rewinding Machines

Gianluca Gatti ¹, **Lorenzo Lupi**,² and **Giuseppe Lupi**²

¹*Department of Mechanical, Energy and Management Engineering, University of Calabria, Rende 87036, Italy*

²*United Converting Tissue Srl, Lucca 55025, Italy*

Correspondence should be addressed to Gianluca Gatti; gianluca.gatti@unical.it

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Rewinding machines are used in the paper converting industrial sector to unroll paper veils from large source reels and wind them around smaller rolls, for the production of paper logs to be used in household activities. Many different types of rewinding machine exist, and they are mostly based on stapling devices which automatically unroll a large coil of paper, roll it up around a new log, stop the veil allowing it to be torn off, unload the completed paper log, and reload a new paper core to be rolled up. Winding units generally include a primary and a secondary roller, supplying the desired rotational speed for winding, and a pressure unit consisting of a pressure roller in contact with the paper log being rolled up, which assures adequate containment and avoids the paper log to escape. All known systems have some mechanical limitations, mainly due to the contact of the pressure roller with the paper log, causing instabilities and vibrations, particularly when working with soft paper veils. This limits the maximum possible winding speed. The aim of this paper is to present a detailed kinematic analysis of a novel pressure unit which overcomes the aforementioned limitation.

1. Introduction

Rewinding machines are used in the paper converting industrial sector to unroll large reels of paper veils and wind them around smaller logs for the commercial sector, e.g., paper rolls for household keeping. In its most common configuration, a rewinding machine comprises a drive unit which supplies the paper veil, a winding unit for winding the paper veil supplied by the drive unit around a winding core, a primary and a secondary winding roller rotating in contact with the core being rolled up, and a pressure unit equipped with a pressure roller in contact with the paper log. During the winding phase, the peripheral speed of the rollers is substantially equal to the winding speed of the veil.

In its classical configuration, the pressure unit consists of one single pressure roller which is accomplishing a circular reciprocating motion around a fixed axis of rotation [1]. In [2], a discussion is presented on the application and limitations of reels in paper winding machines used for finished rolls. However, the interest of the scientific community

towards this topic has been mainly limited to the control system to feed the paper veil and assure adequate tension to avoid defects in the final product. In [3], a tension control system is proposed which realizes stable tension of the rewinding process overcoming the influence of external disturbances. In [4], a mechanical modification was introduced to improve the stability of a commercial rewinding machine. In [5], a control system for paper rewinding machine based on PLC is presented along with the control functions of unwinding tension and rewinding force. In [6], robust tension estimators were developed considering dynamic, friction, and inertial effects, to provide stable and precise sheet tension control without the use of load cells. In [7], the aim was to replace the signals from the load cell and the absolute encoder with their estimated values. In [8], a closed loop control system was proposed to overcome the effect of any disturbance during machine operations and improve the quality of the final product. In [9], a way of self-adjusting reel diameter calculation is presented providing superior reeling performance. In [10], a pressure-control scheme for a

hydraulic actuator is proposed to improve the tightness of the paper reel, and in [11], the influence of the alignment of the rolling elements on the product quality is concisely discussed.

As far as the authors are aware, except the work presented in [1], no significant contribution has been addressed in the scientific literature to the proposal of novel mechanical systems to improve at source the rewinding process in the paper industry. This paper thus proposes the kinematic analysis of a new pressure unit [12] which is able to improve the stability of the paper log being rolled up and thus increase the winding speed of the whole process.

2. Description of the System

Figure 1 illustrates a model of the system under study and shows the main components of the mechanism for winding a paper log. This is formed by the rotation of two winding rollers, referred to as the upper and lower winding roller, respectively. The paper log is kept in contact with these two rollers by means of a pressure unit, which is the proposed novelty in the machine. It consists of a main oscillating arm, rotating around a fixed axis A, coupled to a secondary arm (i.e., the triangular-shaped plate in Figure 1) via a rotational joint along axis B. The secondary arm supports three pressure rollers whose purpose is to apply the appropriate pressure to the paper log being rolled up, while keeping it in contact with the winding rollers. The pressure unit is controlled by means of two actuators: one revolute motor in B and one revolute motor in A or alternatively by means of a linear actuator as shown in Figure 1.

The different steps related to the paper log being rolled up are illustrated in Figure 2, where a sequence of snapshots is presented to clarify the motion of the system. Animation S0, provided in the Supplementary Material (Available here) for this paper, shows an animation of the proposed system and compares it to the traditional one. The different phases are characterized by the type of contact between the paper log being rolled up and the other rollers involved in the process, which assure the paper log stability during winding. The instability which may appear in a traditional pressure unit is pictorially emphasized in the animation S0, provided in the Supplementary Material (Available here). Figure 2(a) then shows the initial phase where the paper core is loaded and introduced between the winding rollers, and an initial paper veil is rolled up and glued around it. The paper log is thus pushed towards one of the pressure roller of the pressure unit, which holds its position ready for engagement. The critical issues during this phase are related to the geometric dimensions and constraints of the system, which may prevent the pressure rollers to get close enough to the log being rolled up without interfering with the other winding rollers. Figure 2(b) shows the case where the paper log has reached a certain diameter. In this case, the paper log is in contact with three rollers—two winding rollers and one pressure roller. To prevent the paper log to escape through the spaces between the rollers, the pressure roller must follow a prescribed trajectory. Such trajectory has to assure that the geometrical segment between the centres of rotation

FIGURE 1: Computer-aided model of the main components in a paper rewinding machine.

of the two winding rollers is always perpendicular to the line passing through the paper log centre and the centre of rotation of the pressure roller being in contact with the paper log. Figures 2(c) and 2(d) show two snapshots of the subsequent phases—the pressure unit rotates around axis B clockwise to assure the motion requirement discussed in the preceding phase and convey the paper log being rolled up. Such a rotation continues until a second pressure roller comes in contact with the paper log. This is a critical condition because it involves the transition from one specific trajectory to a different one. From this condition on, until the next phase, the paper log is constrained by four rollers (two pressure rollers and two winding rollers), which guarantee for the best condition of stability during the paper log winding. Figure 2(e) shows the situation where the paper log is about to be completed and it is led to ejection by losing contact with the upper winding roller. The paper log is then in contact with three rollers, two pressure rollers and one winding roller. During this phase, as shown in Figure 2(b), the motion of the pressure unit has to assure an optimal trajectory to avoid paper log instability and escapement. Figure 2(f) shows the final phase where the paper winding has been completed and paper log is ejected. The pressure unit goes back to its initial position and is ready for the next product engagement, after a rotation of 120° .

In the following analysis, it is assumed that the system has nominal geometry, i.e., no geometrical or assembly error is considered. Well-established methods for kinematic calibration [13] can be adopted on the physical system for parameters identification. Since this paper focuses on the kinematics of the device, other issues related to rotor dynamics and defective ball bearing [14] are not considered. Also, contact issues between the paper log and the machine rollers as well as the related dynamics are not considered in this study. This is motivated by the focus of the paper and the fact that the stiffness of the paper log is negligible respect to that of the machine rollers.

FIGURE 2: Snapshots of the system in motion: (a) initial phase involving the paper winding core, (b) paper log being rolled up in contact with one pressure roller, (c-d) paper log being rolled up in contact with two pressure rollers, (e) paper log being completed, and (f) paper log being ejected.

3. Trajectory of the Paper Log

From a kinematic point of view, the system in Figure 1 can be represented by the schematic drawing reported in Figure 3.

The aim of the present analysis is to determine the trajectory of the paper log being rolled up. This is then used in the next section to determine the trajectory of the pressure

unit according to the requirements for stability, mentioned in the previous section.

From a kinematic point of view, the motion of the system and its component trajectories are expressed as a function of the increasing radius of the paper log being rolled up, R_L , which is a function of the paper length wrapped per unit time, i.e., the winding speed, V , according to the following relation:

FIGURE 3: Schematic drawing of the system.

$$R_L = \sqrt{R_{Li}^2 + \frac{sVt}{\pi}}, \quad (1a)$$

where R_{Li} is the initial radius of the paper log, which coincides with the paper winding core, t is the time, and s is the theoretical paper thickness calculated as

$$s = \frac{\pi(R_{Lf}^2 - R_{Li}^2)}{L}, \quad (1b)$$

where R_{Lf} is the final radius of the completed paper log and L is the paper wrapped on each paper log.

With reference to the snapshots in Figure 2, the characteristic phases occurring during the system motion are summarized in Figure 4.

During Phase I in Figure 4(a), the pressure unit is in starting configuration, with pressure roller [C] waiting for the paper log [F] to initiate its winding. Such configuration is limited by the collision of the pressure rollers with the winding rollers [R1] and [R2].

During Phase II in Figure 4(b), the paper log reaches the engaging diameter, and the pressure unit begins to move, conveying the paper log being rolled up.

During Phase III in Figure 4(c), the paper log increases its diameter and the pressure unit moves and orients to assure an optimal trajectory. This can be defined as the one which guarantees that the line of pressure between the pressure roller [C] and the paper log [F] is perpendicular to the segment O_1O_2 in Figure 3. This can be cinematically achieved since the pressure unit is a two degrees-of-freedom mechanism.

During Phase IV in Figure 4(d), the pressure unit continues its motion, rotating both the main and secondary arms, until a second pressure roller [D] comes in contact with the paper log. This latter is then conveyed by two winding rollers and two pressure rollers until the final diameter is reached.

During Phase V in Figure 4(e), the paper log disengages from the upper winding roller, and it is kept in contact with

the lower one through the motion of two pressure rollers [C] and [D] until ejection.

In the final Phase VI in Figure 4(f), the product is ejected, and the pressure unit returns back to its initial configuration waiting for the next engagement. The secondary arms rotate 120° , and the pressure roller [D] replaces the pressure roller [C] in Phase I.

During Phases I to IV in Figure 4, the paper log is always in contact with both winding rollers.

To perform a mathematical analysis, the following parameters are defined, according to Figure 5. R_1 (R_2) is the radius of the lower (upper) winding roller; d is the constant length of the segment O_1O_2 between the rotational axes of the lower and upper winding rollers; L_1 (L_2) is the variable distance between the axis of rotation of the lower (upper) winding roller and the instantaneous axis of the rotation of the paper log, F ; ϕ is the constant angle between the horizontal axis and the segment O_1O_2 ; and ϕ_1 (ϕ_2) is the variable angle between the horizontal axis and the segment O_1F (O_2F). An orthogonal frame of reference xy is also defined with origin at the axis of rotation of the lower winding roller, O_1 .

According to the system geometry, the paper log is initially being moved along the line perpendicular to the segment O_1O_2 .

To ensure continuity of motion, it is required that the paper log is kept in contact with both winding rollers, from the initial diameter to the final diameter.

The following geometric constraints must thus be assured during motion:

$$\begin{aligned} L_1 \cos(\phi_1) - L_2 \cos(\phi_2) - d \cos(\phi) &= 0, \\ L_1 \sin(\phi_1) - L_2 \sin(\phi_2) - d \sin(\phi) &= 0, \end{aligned} \quad (2)$$

where $L_1 = R_1 + R_L$, $L_2 = R_2 + R_L$.

Squaring and adding equation (2), the following relation is obtained:

$$L_1^2 + L_2^2 - 2L_1L_2[\cos(\phi_1)\cos(\phi_2) + \sin(\phi_1)\sin(\phi_2)] = d^2, \quad (3)$$

which can be conveniently rewritten using trigonometric formulas as

$$\Delta\phi = \phi_1 - \phi_2 = \pm \arccos \frac{-d^2 + L_1^2 + L_2^2}{2L_1L_2}. \quad (4)$$

The ambiguity in the sign in equation (4) comes from the existence of two solutions to the problem—the one corresponding to the plus sign is where the paper log in Figure 5 moves along the positive x direction and the one with the minus sign is where the paper log in Figure 5 moves along the negative x direction. This latter is the one of interest in the specific problem under investigation.

Substituting equation (4) into equation (2), after some trigonometric manipulation, the following linear system of equations is obtained in the unknowns $\cos(\phi_1)$ and $\sin(\phi_1)$:

FIGURE 4: Characteristic phases of the system motion: (a) Phase I; (b) Phase II; (c) Phase III; (d) Phase IV; (e) Phase V; (f) Phase VI.

$$\begin{bmatrix} L_1 - L_2 \cos(\Delta\phi) & -L_2 \sin(\Delta\phi) \\ L_2 \sin(\Delta\phi) & L_1 - L_2 \cos(\Delta\phi) \end{bmatrix} \begin{bmatrix} \cos(\phi_1) \\ \sin(\phi_1) \end{bmatrix} = \begin{bmatrix} d \cos(\phi) \\ d \sin(\phi) \end{bmatrix}, \quad (5)$$

which can be solved to determine the expression of the trajectory of the paper log as a function of its instantaneous radius as

$$\phi_1 = \arctan\left(\frac{(R_1 + R_L)\sin(\phi) - (R_2 + R_L)\sin(\Delta\phi + \phi)}{(R_1 + R_L)\cos(\phi) - (R_2 + R_L)\cos(\Delta\phi + \phi)}\right) \quad \text{if } 2R_L > d - R_1 - R_2. \quad (6)$$

FIGURE 5: Geometric parameters of the system.

The condition in equation (6) is due to the fact that a real solution exists only in the case when the paper log diameter is greater than the distance between the two circles corresponding to the winding rollers.

The Cartesian trajectory of the paper log axis F, in the reference frame xy in Figure 5, is given by

$$\begin{aligned} F_x &= (R_1 + R_L)\cos(\phi_1), \\ F_y &= (R_1 + R_L)\sin(\phi_1). \end{aligned} \quad (7)$$

Once the final diameter of the paper log has been reached, the paper log is disengaged from the upper winding roller due to the proper motion of the pressure unit. Until the contact of the paper log with the lower winding roller is assured, the law of motion of the paper log will follow equation (7), in which $R_L = R_{Lf}$, and the trend of the angle ϕ_1 is selected based on how quickly the product is being ejected. It is worth to note that if it is necessary to temporarily stop the rotation of the pressure unit in the final winding phase, it is sufficient to keep ϕ_1 constant during the desired duration. As an example, Figure 6 shows as a dashed line, the trajectory followed by the axis of the paper log being rolled up.

It is also important to note that a kink is present in the trajectory of motion when the paper log disengages from the upper winding roller. Such kink is due to the different trajectory just before and after said disengagement.

4. Kinematics of the Pressure Unit

In this section, the variation of the joint angles in A and B is related to the paper log being rolled up, i.e., to its instantaneous radius and thus to the winding speed according to equations (1a) and (1b).

The required trajectory is designed so that the pressure unit is able to convoy the paper log roll-up and return back to the initial configuration, ready for the next paper log engagement.

The required motion must thus (1) guarantee a smooth trajectory for both the main and secondary arms of the pressure unit, (2) avoid collisions with the other rollers

FIGURE 6: Example of a trajectory (dashed line) of the paper log being rolled up.

located in the workspace, and (3) guarantee an optimal orientation of the line of pressure between the pressure roller(s) and the paper log.

In order to determine the kinematics of the pressure unit, i.e., the variation of the joints in A and B, it is important to identify two cases, according to the condition where one or two pressure rollers are in contact with the paper log. The first case is discussed in Section 4.1 and the second case in Section 4.2 as follows.

4.1. One Pressure Roller in Contact with the Paper Log.

The pressure unit and its main parameters for this case are illustrated in Figure 7. In particular, Figure 7(a) shows the overall view and geometry, while Figure 7(b) shows a close-up of the contact between the pressure roller and the paper log.

The main arm of the pressure unit is rotating around the fixed axis A in Figure 7(a), whose coordinates are A_x and A_y respect to the frame of reference xy . The secondary arms supporting the three pressure rollers are rotating around the axis B, which is at a constant distance k_1 from axis A. The distance between axis B and the axis of rotation of each of the pressure rollers is constant and labelled as k_2 in Figure 7(a).

The pressure unit has two degrees-of-freedom, and the corresponding generalized coordinates are labelled as the angles θ_1 and θ_2 in Figure 7(a), no matter the mechanical actuation system specifically adopted. The mobility of the system is such that an optimal direction for the line of pressure between the pressure roller and the paper log can be assured. As discussed in Section 3, such direction is perpendicular to the line passing through the centres of rotation of the two winding rollers.

An offset parameter a is conveniently introduced to allow varying the initial location of the pressure roller at first engagement, depending on the type of paper to be rolled up (e.g., thicker and soft; thinner and compact).

Being C the desired location of the centre of the first engaging pressure roller, as detailed in Figure 7(b), its coordinates are determined as

FIGURE 7: (a) Main geometry and (b) close-up of the pressure unit in case of one pressure roller in contact with the paper log.

$$\begin{aligned} C_x &= F_x + (R_P + R_L + a)\cos\left(\phi + \frac{\pi}{2}\right), \\ C_y &= F_y + (R_P + R_L + a)\sin\left(\phi + \frac{\pi}{2}\right), \end{aligned} \quad (8)$$

where R_P is the radius of each pressure roller, and the closure equations of the linkage in the xy reference plane can be thus written as

$$\begin{aligned} A_x + k_1 \cos \theta_1 + k_2 \cos(\theta_1 + \theta_2) &= C_x, \\ A_y + k_1 \sin \theta_1 + k_2 \sin(\theta_1 + \theta_2) &= C_y. \end{aligned} \quad (9)$$

Squaring and adding equation (9), the following expression is obtained for the second joint:

$$\theta_2 = \pm \arccos \frac{(C_x - A_x)^2 + (C_y - A_y)^2 - k_1^2 - k_2^2}{2k_1 k_2}. \quad (10)$$

As noted before, the sign ambiguity in equation (10) comes from the two different configurations of the linkage. The configuration corresponding to the minus sign is that of physical interest in this specific problem, where the secondary arms rotate clockwise.

Substituting equation (10) into equation (9), the following linear system of equations is determined:

$$\begin{bmatrix} k_1 + k_2 \cos(\theta_2) & -k_2 \sin(\theta_2) \\ k_2 \sin(\theta_2) & k_1 + k_2 \cos(\theta_2) \end{bmatrix} \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} = \begin{bmatrix} C_x - A_x \\ C_y - A_y \end{bmatrix}, \quad (11)$$

which can be first solved for the unknowns $\cos(\theta_1)$ and $\sin(\theta_1)$ and then solved in terms of the first joint angle yielding

$$\theta_1 = \arctan \left(\frac{(C_y - A_y)(k_1 + k_2 \cos(\theta_2)) - (C_x - A_x)k_2 \sin(\theta_2)}{(C_x - A_x)(k_1 + k_2 \cos(\theta_2)) + (C_y - A_y)k_2 \sin(\theta_2)} \right). \quad (12)$$

The Cartesian coordinates of point B and the centres D and E of the pressure rollers in Figure 7(b) can be determined by the following expressions, respectively:

$$\begin{aligned} B_x &= A_x + k_1 \cos(\theta_1), \\ B_y &= A_y + k_1 \sin(\theta_1), \end{aligned} \quad (13)$$

$$\begin{aligned} D_x &= B_x + k_2 \cos\left(\theta_1 + \theta_2 + \pi \frac{2}{3}\right), \\ D_y &= B_y + k_2 \sin\left(\theta_1 + \theta_2 + \pi \frac{2}{3}\right), \end{aligned} \quad (14)$$

$$\begin{aligned} E_x &= B_x + k_2 \cos\left(\theta_1 + \theta_2 + \pi \frac{4}{3}\right), \\ E_y &= B_y + k_2 \sin\left(\theta_1 + \theta_2 + \pi \frac{4}{3}\right). \end{aligned} \quad (15)$$

The following conditions are to be avoided.

FIGURE 8: Geometry for the condition of collision between the pressure roller and the winding rollers.

4.1.1. *Collision between Pressure Roller [C] and Upper Winding Roller [R2].* When the pressure unit is approaching the upper roller, the following condition is required:

$$\sqrt{(C_x - d \cos \phi)^2 + (C_y - d \sin \phi)^2} > R_2 + R_p. \quad (16)$$

Should equation (16) not being satisfied, the pressure unit should hold in place waiting for engagement at the following safety coordinates:

$$\begin{aligned} C_x &= d \cos(\phi) - (R_p + R_2 + \varepsilon_r) \cos(\phi - \beta_{\min}), \\ C_y &= d \sin(\phi) - (R_p + R_2 + \varepsilon_r) \sin(\phi - \beta_{\min}), \end{aligned} \quad (17)$$

which replaces equation (8), where ε_r is a safety parameter providing an adequate distance tolerance among the rollers, and

$$\beta_{\min} = \arccos\left(\frac{(d + R_2 - R_1)/2}{\varepsilon_r + R_2 + R_p}\right), \quad (18)$$

which is depicted in Figure 8.

4.1.2. *Collision between Pressure Roller [C] and Lower Winding Roller [R1].* When the pressure unit is approaching the lower roller, the following condition is required:

$$\sqrt{C_x^2 + C_y^2} > R_1 + R_p. \quad (19)$$

Should equation (19) not being satisfied, the pressure unit should hold in place waiting for engagement at the following safety coordinates:

$$\begin{aligned} C_x &= (R_p + R_1 + \varepsilon_r) \cos(\phi + \alpha_{\min}), \\ C_y &= (R_p + R_1 + \varepsilon_r) \sin(\phi + \alpha_{\min}), \end{aligned} \quad (20)$$

which again replaces equation (8), where

FIGURE 9: Configuration where the paper log is in contact with two pressure rollers.

$$\alpha_{\min} = \arccos\left(\frac{(d + R_1 - R_2)/2}{\varepsilon_r + R_1 + R_p}\right), \quad (21)$$

which is depicted in Figure 8.

4.1.3. *Collision between Pressure Roller [D] and Upper Winding Roller [R2].* Since the pressure unit is rotating clockwise, when the pressure roller [D] is approaching the engagement location, the following condition should be satisfied:

$$\sqrt{(D_x - d \cos \phi)^2 + (D_y - d \sin \phi)^2} > R_2 + R_p. \quad (22)$$

In this case, should equation (22) not being satisfied, the pressure unit cannot guarantee the optimal condition for the line of pressure between the pressure roller and the paper log.

4.2. *Two Pressure Rollers in Contact with the Paper Log.* The equations in Section 4.1 are valid until only the pressure roller [C] is in contact with the paper log being rolled up. It is worth noting that the pressure roller [C] remains tangent to the paper log at the point of contact between the paper log circle and the line orthogonal to the centres of the winding rollers, assuring the optimal line of pressure.

Due to the increase in the paper log diameter, the pressure unit rotates and the pressure roller [D] approaches the paper log. The contact between the pressure roller [D] and the paper log is achieved when

$$\sqrt{(D_x - F_x)^2 + (D_y - F_y)^2} < R_L + R_p + a. \quad (23)$$

In this case, the paper log is constrained by four rollers, two winding rollers and two pressure rollers. This is illustrated in Figure 9, and the motion of the pressure unit needs to be changed to accomplish such motion. This is the reason for the kink mentioned earlier in Section 3 with reference to Figure 6.

To assure the best possible containment for the paper log being rolled up, the configuration of the pressure unit is selected such as the pressure rollers [C] and [D] in Figure 9 are located symmetrically respect to the line passing through the centre of the paper log and point B. The configuration of the pressure unit is then determined through the Carnot theorem applied to the triangle FCB, in Figure 9, giving

$$(R_p + R_L)^2 = k_2^2 + k_3^2 - 2k_2k_3 \cos\left(\frac{\pi}{3}\right), \quad (24)$$

which can be rewritten as a quadratic equation in terms of k_3 , representing the varying distance between point B and the centre of the paper log, F , as

$$k_3^2 + k_3 \left(-2k_2 \cos\left(\frac{\pi}{3}\right) \right) + [k_2^2 - (R_p + R_L)^2] = 0. \quad (25)$$

Equation (25) can be thus solved to give

$$k_3 = k_2 \cos\left(\frac{\pi}{3}\right) \pm \frac{1}{2} \sqrt{4k_2^2 \cos^2\left(\frac{\pi}{3}\right) - 4[k_2^2 - (R_p + R_L)^2]} + a. \quad (26)$$

The closure equations of the linkage are then determined as

$$\begin{aligned} A_x + k_1 \cos \theta_1 + k_3 \cos\left(\theta_1 + \theta_2 + \frac{\pi}{3}\right) &= F_x, \\ A_y + k_1 \sin \theta_1 + k_3 \sin\left(\theta_1 + \theta_2 + \frac{\pi}{3}\right) &= F_y. \end{aligned} \quad (27)$$

As detailed in Section 4.1, the new expression of the second joint angle is determined as

$$\theta_2 = \pm \arccos \frac{(F_x - A_x)^2 + (F_y - A_y)^2 - k_1^2 - k_3^2}{2k_1k_3} - \frac{\pi}{3}, \quad (28)$$

where the configuration of interest is that corresponding to the minus sign.

Substituting equation (28) into equations (27), the following system of equations is obtained in terms of the unknowns $\cos(\theta_1)$ and $\sin(\theta_1)$:

$$\begin{bmatrix} k_1 + k_3 \cos\left(\theta_2 + \frac{\pi}{3}\right) & -k_3 \sin\left(\theta_2 + \frac{\pi}{3}\right) \\ k_3 \sin\left(\theta_2 + \frac{\pi}{3}\right) & k_1 + k_3 \cos\left(\theta_2 + \frac{\pi}{3}\right) \end{bmatrix} \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} = \begin{bmatrix} F_x - A_x \\ F_y - A_y \end{bmatrix}, \quad (29)$$

and it can be solved for the first joint angle as

$$\theta_1 = \arctan \left(\frac{(A_x - F_x)k_3 \cos((\pi/6) - \theta_2) - (A_y - F_y)(k_1 + k_3 \sin((\pi/6) - \theta_2))}{-(A_y - F_y)k_3 \cos((\pi/6) - \theta_2) - (A_x - F_x)(k_1 + k_3 \sin((\pi/6) - \theta_2))} \right). \quad (30)$$

4.3. Early Engagement of the Second Pressure Roller. In the case in which it is desired to anticipate the engagement of the second pressure roller with the paper log (e.g., to prevent paper log instability), equation (8) is replaced by

$$\begin{aligned} C_x &= F_x + (R_p + R_L + a) \cos\left(\phi + \frac{\pi}{2} + \gamma\right), \\ C_y &= F_y + (R_p + R_L + a) \sin\left(\phi + \frac{\pi}{2} + \gamma\right), \end{aligned} \quad (31)$$

where γ is an angular parameter varying from zero to a given final value, which depends on the desired conditions

for engagement. In particular, a value for the paper log radius, R_{La} , at which the early engagement begins, is to be defined, as well as a value for the paper log radius, R_{Lb} , at which the final engagement of the second pressure roller with the paper log occurs. It is worth noting that, during the phase from R_{La} to R_{Lb} , it is not possible to assure the optimal condition for the line of pressure between the pressure roller and the paper log, as described in Section 4.1.

Once the values for R_{La} and R_{Lb} have been specified, the value of γ varies from $\gamma_a = 0$, corresponding to the paper log radius R_{La} , to a final value of

TABLE 1: Geometric parameters used for simulations.

Parameter	Unit	Case 1	Case 2	Case 3
D	(mm)		253.5	
Φ	(rad)		$\pi/4$	
R_1	(mm)		97.5	
R_2	(mm)		110	
R_{Li}	(mm)		23.2	
R_{Lf}	(mm)	48.8	72.0	72.0
R_{La}	(mm)	—	—	39.5
R_{Lb}	(mm)	—	—	46.5
A_x	(mm)		-220.75	
A_y	(mm)		160.75	
k_1	(mm)		218	
k_2	(mm)		49.07	
R_p	(mm)		32.5	
ϵ_r	(mm)		10

FIGURE 10: Simulation results for Case 1.

FIGURE 11: Simulation results for Case 2.

FIGURE 12: Simulation results for Case 3.

$$\gamma_b = \arccos \left(\frac{(C_{xb} - F_{xb})\cos(\phi + (\pi/2)) + (C_{yb} - F_{yb})\sin(\phi + (\pi/2))}{\sqrt{(C_{xb} - F_{xa})^2 + (C_{yb} - F_{ya})^2}} \right), \quad (32)$$

which corresponds to the paper log radius R_{Lb} , where F_{xa} and F_{xb} are the coordinates of the position of the paper log axis at radius R_{Lb} , the coordinates C_{xb} and C_{yb} are determined on the base of the corresponding configuration achieved by the system with four rollers in contact, as detailed in Section 4.2.

4.4. Kinematics of the Pressure Unit after Log Ejection. The optimal condition for ejection of the paper log is achieved when the angle of the tangent to the pressure roller [C] and the paper log is less than that of the tangent to the lower winding roller and the paper log so that the following relation is satisfied:

$$\arctan \left(\frac{C_y - F_y}{C_x - F_x} \right) \leq \phi_1. \quad (33)$$

In this case, the paper log is smoothly released without the need to disengage the pressure unit, which is then free to return back to its initial position for the next engagement. Such a motion can be defined by a polynomial blending trajectory, which assures the required conditions for continuity and smoothness. For each joint angle θ_1 and θ_2 , the initial boundary conditions are given by equations (28) and (30) at the time of ejection. The final boundary conditions have to guarantee that the pressure unit returns back to its position with zero final velocity (at least) and with pressure roller [D] replacing [C]—this is achieved by an offset for θ_2 equal to -120° respect to the previous initial configuration.

5. Application

Applications are provided in this section to illustrate the device motion and to provide validation to the algorithm presented in the previous sections.

Three different simulations are performed, according to the geometric parameters listed in Table 1. For the three cases, the difference in the parameters is related to the final diameter of the paper log, which is smaller in Case 1 than in Case 2, and the early engagement of the second pressure roller, which is absent in Case 2 and present in Case 3.

Results of the simulations are reported in Figures 10–12 for the three cases, respectively. They are presented in terms of the joint angles of the pressure unit as functions of a nondimensional simulation time and are derived based on the inverse kinematic algorithm presented in the sections above. The various kinks in the joint trajectories denote a change from one contact phase to a different one. An additional kink in Figure 12 is due to the early engagement of the second pressure roller.

The results of the inverse kinematics shown in Figures 10–12 are verified by solving the direct kinematics of the system and combining it with the imposed trajectory of the paper log during roll-up. The results of such validation are presented as animation sequences in the video S1, S2, and S3, respectively, provided in the *Supplementary Material (Available here)*. It can be seen how the pressure unit follows the paper log during the different phases of the designed trajectories.

6. Conclusions

In this paper, a detailed kinematic analysis of a novel mechanical pressure unit integrated in a rewinding paper machine is presented. The system is based on a recent patent application, and it allows the creation of paper logs with different diameters by assuring adequate constraints during winding. The novelty of the system relies on the use of a pressure unit with three pressure rollers, respect to standard configurations with one roller only. The proposed system has two degrees-of-freedom, which need to be synchronized in order to adequately restrain and copy the paper roll motion. This allowed to increase the overall winding speed reducing machine downtime and motivated the introduced complexity respect to classical one degree-of-freedom pressure units. The trajectories of the two joint angles of the pressure unit are determined through an inverse kinematic analysis, and the conditions for collision are identified and taken into account in the algorithm. Numerical applications are presented which illustrate the motion of the pressure unit along with the paper log during formation. Some of the different possible scenarios are implemented. The algorithm presented in the paper is validated through the direct kinematics of the system.

Abbreviations

t :	Time
V :	Winding speed
s :	Paper thickness
R_L :	Radius of the paper log at time t
R_{Li} :	Initial radius of the paper log
R_{Lf} :	Final radius of the paper log
L :	Paper wrapped on each paper log
R_1 :	Radius of the lower winding roller
R_2 :	Radius of the upper winding roller
R_P :	Radius of the pressure rollers
d :	Constant distance between the lower and upper winding roller
ϕ :	Constant angle between horizontal axis and the direction of d
L_1 :	Variable distance between the lower winding roller and the paper log
ϕ_1 :	Variable angle between horizontal axis and the direction of L_1
L_2 :	Variable distance between the upper winding roller and the paper log
ϕ_2 :	Variable angle between horizontal axis and the direction of L_2
$\Delta\phi$:	Difference between ϕ_1 and ϕ_2
P_x :	Coordinate x and y of a generic point P
P_y :	
θ_1 :	Joint angle of the main arm
θ_2 :	Joint angle of the pressure unit
k_1 :	Constant length of the main arm
k_2 :	Constant length of second arms of the pressure unit
k_3 :	Variable distance between the centre of the paper log and secondary pivot joint of the pressure unit
a :	Offset of the pressure roller respect to the paper log at first engagement

ε_r :	Tolerance distance among the rollers
γ :	Angular parameter defining early engagement
γ_a :	Angular parameter where early engagement begins
γ_b :	Angular parameter where early engagement ends
R_{La} :	Radius of the paper log where early engagement begins
R_{Lb} :	Radius of the paper log where early engagement ends.

Data Availability

The data used to support this study are provided within the article.

Disclosure

This research work was performed under a research agreement between the Department of Mechanical, Energy and Management Engineering of the University of Calabria and the private company United Converting Tissue Srl.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

Supplementary Materials are provided for this paper as follows: S0: animation comparing the proposed system to a traditional one. S1: animation illustrating application Case 1. S2: animation illustrating application Case 2. S3: animation illustrating application Case 3. (*Supplementary Materials*)

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