Research Article

Finite-Time Stability and Stabilization of Switched Linear Time-Varying Systems with Time-Varying Delay

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This paper deals with the finite-time stability (FTS) of switched linear time-varying (SLTV) systems with time-varying delay. Firstly, based on Lyapunov–Krasovskii functional technique and average dwell time (ADT) approach, a sufficient criterion on FTS for SLTV systems with time-varying delay is obtained. For the SLTV system with delay and control input, based on the criterion, a state feedback controller is designed such that the closed-loop system is finite-time stable (FTS). Finally, an example is employed to illustrate the validity of our results.

1. Introduction

Switched systems are a special kind of hybrid dynamical systems, which consist of several subsystems and a switching law that orchestrates the switching between the subsystems [1]. It has been widely applied in intelligent traffic control [2], formation flying [3], sensor networks [4, 5], robotics [6, 7], and so on. In practice, time delays widely exist in many switched systems, which may cause some unexpected errors or even lead to crash; therefore, stability analysis for switched systems with time delay is particularly important and has been extensively studied. Most of the existing results focus on Lyapunov asymptotic stability (LAS). As we all know, LAS is defined over infinite-time interval. However, due to large state amplitude in the transient process, some asymptotic stable systems may be useless. Thus, it is more meaningful to study the stability in a fixed time interval. The concept of finite-time stability (FTS) was first introduced in 1960s [8, 9]. A system is said to be finite-time stable (FTS) if its state does not exceed a certain bound during a specified time interval for a given bound on the initial condition. For such stability, there are many practical applications such as networked control systems [10] and network congestion control [11].

Up to now, many important results on FTS have been reported [12–15]. For instance, Ren et al. proposed a suitable

event-driven communication scheme for the networked switched systems, and finite-time boundedness and input-output FTS are simultaneously considered [16]. Yu et al. investigated FTS of switched positive linear systems with time-varying delays using the ADT method, and a sufficient condition on FTS is provided for the case of time-invariant systems [17].

We can see that the majority of the existing literatures related to finite-time stability analysis are mainly on time-invariant systems. However, many systems in practical applications are time-varying. For the active suspension system in auto diving, the vehicle sprung and unsprung masses vary with the loading conditions. To increase the control accuracy, this system should be represented by a time-varying system [18]. Therefore, the stability analysis for time-varying systems is very important and necessary. In the past few decades, the stability theory of linear time-invariant systems has developed considerably [19, 20], while that of linear time-varying (LTV) systems is comparatively slow. This is because the analysis of LTV systems is much more complicated than that of linear time-invariant systems. The state transition matrixes of LTV systems, which are generally needed to ascertain the properties of stability, are impossible to be derived in addition to particular cases [21–23]. So, we must find another way to analyze the stability of SLTV

systems with time-varying delays. With the help of Lyapunov functions, a series of differential Lyapunov-inequality-based sufficient conditions is derived for FTS analysis. And, as is well known, general Lyapunov-Krasovskii functionals with more information on time delay are helpful for reducing conservatism [24, 25]. Therefore, in this paper, we analyze FTS of SLTV systems with time-varying delays using the Lyapunov-Krasovskii functional method.

In this paper, the main contributions of this paper can be summarized as follows: (1) sufficient conditions on FTS of the switched system will be given under the case of LTV systems; (2) a state feedback controller will be designed for the SLTV system with time-varying delays.

The paper is organized as follows. The problem to be considered is formulated in Section 2. In Section 3, FTS criteria are derived in terms of linear matrix inequalities (LMIs). In Section 4, a state feedback controller will be designed which can guarantee the FTS of the systems. In Section 5, a numerical example will be given to show the effectiveness of the proposed method. Section 6 concludes the paper.

2. Preliminaries

$\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, n -dimensional real space and $n \times m$ -dimensional real matrix space. $\lambda_{\min}(R)$ denotes the minimal of all eigenvalues of matrix R and $\lambda_{\max}(R)$ denotes the maximum of all eigenvalues of matrix R . R^T denotes the transpose of matrix R . I denotes identity matrix with an appropriate dimension.

Consider the following SLTV systems:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t-d(t)), & t \in [0, T_f], \\ x(s) = \varphi(s), & \forall s \in [-d, 0], \end{cases} \quad (1)$$

where $x(\cdot) \in \mathbb{R}^n$ is the system state and T_f is a positive constant which represents the end time of system operation; switching signal $\sigma(t)$ is a piecewise constant function that maps from $[0, T_f]$ into the index set $M = \{1, 2, \dots, m\}$; $A_i, B_i (i \in M)$ are system matrices with appropriate dimensions; $d(t)$ is the bounded time delay with $0 \leq d(t) \leq d$ and $\dot{d}(t) \leq \rho < 1$ (d and ρ are positive constants). The system state is denoted by $x(t, \sigma_0, \varphi)$ with initial mode $\sigma(0) = \sigma_0$ and initial state $\varphi(s), s \in [-d, 0]$. When clear from the context, we write $x(t)$ instead of $x(t, \sigma_0, \varphi)$.

For convenience, we shall introduce the following definitions.

Definition 1. System (1) is said to be FTS with respect to (c_1, c_2, T_f) if

$$\sup_{-d \leq s \leq 0} \varphi^T(s)\varphi(s) \leq c_1 \implies x^T(t)x(t) \leq c_2, \quad \forall t \in [0, T_f], \quad (2)$$

where c_1 and c_2 are given positive constants with $c_1 \leq c_2$.

Remark 1. Definition 1 implies that if system (1) is FTS, the state must be within the prescribed bound in the fixed time

interval, and constants c_1 and c_2 are usually assigned according to the actual situation of the specific problem. In engineering applications, there are many systems that work on a finite-time interval, such as the climb process of hypersonic aircraft, and it must reach supersonic speed within the fixed time interval; in the process of spacecraft rendezvous and docking, the two spacecrafts must be combined in space orbit and structurally connected into a whole within the fixed time interval.

Definition 2. Let $N_\sigma(t, T)$ denote the switching number of $\sigma(t)$ over $[t, T)$, $\forall T > t \geq 0$. If $N_\sigma(t, T) \leq N_0 + (T - t/\tau_D)$ holds for some $\tau_D > 0$ and $N_0 \geq 0$, then τ_D and N_0 are called ADT and chattering bound, respectively.

3. Finite-Time Stability Analysis

In this section, using Lyapunov-Krasovskii functional technique and ADT approach, a sufficient condition on FTS of system (1) will be given as follows.

Theorem 1. System (1) is FTS with respect to (c_1, c_2, T_f) if, for any $i \in M$, there are positive definite matrix-valued functions $P_i(t), Q_i(t), Z_i(t) \in \mathbb{R}^{n \times n}$, positive constant λ , and constant $\beta > 1$, such that the following inequalities hold:

$$\begin{pmatrix} \Psi_i(t) & P_i(t)B_i(t) \\ B_i^T(t)P_i(t) & -e^{\lambda d}Q_i(t)(1-\rho) \end{pmatrix} < 0, \quad (3)$$

$$P_i(t) \leq \beta P_j(t), Q_i(t) \leq \beta Q_j(t), Z_i(t) \leq \beta Z_j(t), \quad \forall i, j \in M, \quad (4)$$

$$\dot{Q}_i(t) \leq 0, \quad (5)$$

$$\dot{Z}_i(t) \leq 0, \quad (6)$$

and the ADT satisfies

$$\tau_D > \frac{T_f \ln \beta}{\ln c_2 + \ln \lambda_p - \lambda T_f - \ln \eta_1 + N_0 \ln \beta}, \quad (7)$$

where

$$\begin{aligned} \Psi_i(t) &= A_i^T(t)P_i(t) + \dot{P}_i(t) + P_i(t)A_i(t) + Q_i(t) \\ &\quad + dZ_i(t) - \lambda P_i(t), \\ \lambda_p &= \min_{i \in M} (\lambda_{\min}(P_i(t))), \\ \eta_1 &= \alpha_1 c_1 + d e^{\lambda d} \alpha_2 c_1 + d^2 e^{\lambda d} \alpha_3 c_1, \\ \alpha_1 &= \max_{i \in M} \{\lambda_{\max}(P_i(0))\}, \\ \alpha_2 &= \max_{i \in M} \{\lambda_{\max}(Q_i(0))\}, \\ \alpha_3 &= \max_{i \in M} \{\lambda_{\max}(Z_i(0))\}. \end{aligned} \quad (8)$$

Proof. Take the following Lyapunov-Krasovskii functional candidate:

$$V_{\sigma(t)}(t, x_t) = V_{\sigma(t),1}(t, x_t) + V_{\sigma(t),2}(t, x_t) + V_{\sigma(t),3}(t, x_t), \quad (9)$$

Calculating the derivatives of $V_{\sigma(t),1}(t)$, $V_{\sigma(t),2}(t)$, and $V_{\sigma(t),3}(t)$ with respect to t along the trajectories of system (1), we can obtained

where

$$\begin{aligned} V_{\sigma(t),1}(t, x_t) &= x^T(t)P_{\sigma(t)}(t)x(t), \\ V_{\sigma(t),2}(t, x_t) &= \int_{t-d(t)}^t x^T(s)e^{\lambda(t-s)}Q_{\sigma(t)}(t)x(s)ds, \\ V_{\sigma(t),3}(t, x_t) &= \int_{-d}^0 \int_{t+\theta}^t x^T(s)e^{\lambda(t-s)}Z_{\sigma(t)}(t)x(s)dsd\theta. \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{V}_{\sigma(t),1}(t, x_t) &= x^T(t)(A_{\sigma(t)}^T(t)P_{\sigma(t)}(t) + \dot{P}_{\sigma(t)}(t) + P_{\sigma(t)}(t)A_{\sigma(t)}(t))x(t) \\ &\quad + x^T(t-d(t))B_{\sigma(t)}^T(t)P_{\sigma(t)}(t)x(t) \\ &\quad + x_{\sigma(t)}^T(t)P_{\sigma(t)}(t)B_{\sigma(t)}(t)x(t-d(t)), \\ \dot{V}_{\sigma(t),2}(t, x_t) &= \lambda V_{\sigma(t),2}(t) + x^T(t)Q_{\sigma(t)}(t)x(t) \\ &\quad - e^{\lambda d(t)}x^T(t-d(t))Q_{\sigma(t)}(t)x(t-d(t))(1-\dot{d}(t)) \\ &\quad + \int_{t-d(t)}^t e^{\lambda(t-s)}x^T(s)\dot{Q}_{\sigma(t)}(t)x(s)ds, \\ \dot{V}_{\sigma(t),3}(t, x_t) &= \lambda V_{\sigma(t),3}(t) - \int_{t-d}^t e^{\lambda(t-s)}x^T(s)Z_{\sigma(t)}(t)x(s)ds \\ &\quad + \int_{-d}^0 \int_{t+\theta}^t e^{\lambda(t-s)}x^T(s)\dot{Z}_{\sigma(t)}(t)x(s)dsd\theta + dx^T(t)Z_{\sigma(t)}(t)x(t). \end{aligned} \quad (11)$$

Furthermore,

$$\begin{aligned} &\dot{V}_{\sigma(t)}(t, x_t) - \lambda V_{\sigma(t)}(t, x_t) \\ &= x^T(t)(A_{\sigma(t)}^T(t)P_{\sigma(t)}(t) + \dot{P}_{\sigma(t)}(t) + P_{\sigma(t)}(t)A_{\sigma(t)}(t) + Q_{\sigma(t)}(t) + dZ_{\sigma(t)}(t) \\ &\quad - \lambda P_{\sigma(t)}(t))x(t) + x^T(t-d(t))(B_{\sigma(t)}^T(t)P_{\sigma(t)}(t)x(t) \\ &\quad - e^{\lambda d(t)}Q_{\sigma(t)}(t)x(t-d(t))(1-\dot{d}(t))) + x^T(t)P_{\sigma(t)}(t)B_{\sigma(t)}(t)x(t-d(t)) \\ &\quad - \int_{t-d}^t e^{\lambda(t-s)}x^T(s)Z_{\sigma(t)}(t)x(s)ds + \int_{t-d(t)}^t e^{\lambda(t-s)}x^T(s)\dot{Q}_{\sigma(t)}(t)x(s)ds \\ &\quad + \int_{-d}^0 \int_{t+\theta}^t e^{\lambda(t-s)}x^T(s)\dot{Q}_{\sigma(t)}(t)x(s)dsd\theta \\ &\leq x^T(t)(A_{\sigma(t)}^T(t)P_{\sigma(t)}(t) + \dot{P}_{\sigma(t)}(t) + P_{\sigma(t)}(t)A_{\sigma(t)}(t) + Q_{\sigma(t)}(t) + dZ_{\sigma(t)}(t) \\ &\quad - \lambda P_{\sigma(t)}(t))x(t) + x^T(t-d(t))(B_{\sigma(t)}^T(t)P_{\sigma(t)}(t)x(t) \\ &\quad - e^{\lambda d}Q_{\sigma(t)}(t)x(t-d(t))(1-\rho)) + x^T(t)P_{\sigma(t)}(t)B_{\sigma(t)}(t)x(t-d(t)) \\ &\quad - \int_{t-d}^t e^{\lambda(t-s)}x^T(s)Z_{\sigma(t)}(t)x(s)ds + \int_{t-d(t)}^t e^{\lambda(t-s)}x^T(s)\dot{Q}_{\sigma(t)}(t)x(s)ds \\ &\quad + \int_{-d}^0 \int_{t+\theta}^t e^{\lambda(t-s)}x^T(s)\dot{Q}_{\sigma(t)}(t)x(s)dsd\theta. \end{aligned} \quad (12)$$

Inequality (12) can be rewritten as follows:

$$\begin{aligned} & \dot{V}_{\sigma(t)}(t, x_t) - \lambda V_{\sigma(t)}(t, x_t) \\ & \leq \begin{pmatrix} x(t) \\ x(t-d(t)) \end{pmatrix}^T \begin{pmatrix} \Psi_{\sigma(t)}(t) & P_{\sigma(t)}(t)B_{\sigma(t)}(t) \\ B_{\sigma(t)}^T(t)P_{\sigma(t)}(t) & -e^{\lambda d}Q_{\sigma(t)}(t)(1-\rho) \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d(t)) \end{pmatrix} \\ & \quad - \int_{t-d}^t x^T(s)e^{\lambda(t-s)}Z_{\sigma(t)}(t)x(s)ds + \int_{t-d}^t x^T(s)e^{\lambda(t-s)}\dot{Q}_{\sigma(t)}(t)x(s)ds \\ & \quad + \int_{-d}^0 \int_{t+\theta}^t x^T(s)e^{\lambda(t-s)}\dot{Z}_{\sigma(t)}(t)x(s)dsd\theta. \end{aligned} \quad (13)$$

From conditions (3), (5), and (6), we can obtain

$$\dot{V}_{\sigma(t)}(t, x_t) - \lambda V_{\sigma(t)}(t, x_t) < 0. \quad (14)$$

Since $(d/dt)(e^{-\lambda t}V_i(t, x_t)) = e^{-\lambda t}\{\dot{V}_i(t, x_t) - \lambda V_i(t, x_t)\}$, inequality (14) can be transformed into

$$\frac{d}{dt}(e^{-\lambda t}V_{\sigma(t)}(t, x_t)) < 0. \quad (15)$$

Let t_k stand for the instant of k th switching and t_{k-} denote the instant just before t_k , $\sigma(t) = \sigma(t_k)$, $\forall t \in [t_k, t_{k+1})$. Integrating both sides of (15) from t_k to t , it follows that

$$V_{\sigma(t_k)}(t, x_t) \leq e^{\lambda(t-t_k)}V_{\sigma(t_k)}(t_k, x_{t_k}), \quad t \in [t_k, t_{k+1}). \quad (16)$$

From (4) and the continuity of $x(t)$, the following inequalities hold:

$$\begin{aligned} V_{\sigma(t_k)}(t, x_t) & \leq \beta e^{\lambda(t-t_k)}V_{\sigma(t_k-)}(t_k, x_{t_k}), \\ V_{\sigma(t_k-)}(t_k, x_{t_k}) & \leq e^{\lambda(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1}, x_{t_{k-1}}). \end{aligned} \quad (17)$$

So,

$$V_{\sigma(t)}(t, x_t) \leq \beta e^{\lambda(t-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1}, x_{t_{k-1}}). \quad (18)$$

Iteratively, we can obtain

$$V_{\sigma(t)}(t, x_t) \leq \beta^{N_{\sigma}(0,t)}e^{\lambda t}V_{\sigma_0}(0, x_0). \quad (19)$$

It is obvious that $V_{\sigma_0}(0, x_0)$ can be written as follows:

$$\begin{aligned} V_{\sigma_0}(0, x_0) & = x^T(0)P_{\sigma_0}(0)x(0) + \int_{-d(0)}^0 x^T(s)e^{\lambda(t-s)}Q_{\sigma_0}(0)x(s)ds \\ & \quad + \int_{-d}^0 \int_{\theta}^0 x^T(s)e^{\lambda(t-s)}Z_{\sigma_0}(0)x(s)dsd\theta, \end{aligned} \quad (20)$$

$$\leq \alpha_1 c_1 + de^{\lambda d}\alpha_2 c_1 + d^2 e^{\lambda d}\alpha_3 c_1, \quad (21)$$

where $\alpha_1 = \max_{i \in M}\{\lambda_{\max}(P_i(0))\}$, $\alpha_2 = \max_{i \in M}\{\lambda_{\max}(Q_i(0))\}$, and $\alpha_3 = \max_{i \in M}\{\lambda_{\max}(Z_i(0))\}$. By (19) and (21),

$$V_{\sigma(t)}(t, x_t) \leq \beta^{N_{\sigma}(0,t)}e^{\lambda t}(\alpha_1 c_1 + de^{\lambda d}\alpha_2 c_1 + d^2 e^{\lambda d}\alpha_3 c_1). \quad (22)$$

According to (9),

$$\begin{aligned} V_{\sigma(t)}(t, x_t) & > x^T(t)P_{\sigma(t)}(t)x(t) \geq \lambda_{\min}(P_{\sigma(t)}(t))x^T(t)x(t) \\ & \geq \lambda_p x^T(t)x(t). \end{aligned} \quad (23)$$

Then, assuming that the total switching number of $\sigma(t)$ over $[0, T_f]$ is N . From (22) and (23),

$$\begin{aligned} x^T(t)x(t) & < \beta^{N_{\sigma}(0,t)}e^{\lambda t}(\alpha_1 c_1 + de^{\lambda d}\alpha_2 c_1 + d^2 e^{\lambda d}\alpha_3 c_1)\lambda_p^{-1} \\ & = \beta^{N_{\sigma}(0,t)}e^{\lambda t}\eta_1 \lambda_p^{-1} \\ & < \beta^N e^{\lambda T_f} \eta_1 \lambda_p^{-1}. \end{aligned} \quad (24)$$

From the definition of ADT, $N = N_{\sigma}(0, T_f) \leq N_0 + T_f/\tau_D$. By inequality (7), we have

$$\frac{T_f \ln \beta}{\ln c_2 + \ln \lambda_p - \lambda T_f - \ln \eta_1 + N_0 \ln \beta} < \tau_D. \quad (25)$$

It can be transformed into

$$N_0 + \frac{T_f}{\tau_D} < \frac{\ln c_2 + \ln \lambda_p - \lambda T_f - \ln \eta_1}{\ln \beta}, \quad (26)$$

i.e.,

$$N < \frac{\ln c_2 + \ln \lambda_p - \lambda T_f - \ln \eta_1}{\ln \beta}. \quad (27)$$

So, $N \ln \beta < \ln(c_2 \lambda_p / e^{\lambda T_f} \eta_1)$, and

$$\beta^N e^{\lambda T_f} \eta_1 \lambda_p^{-1} < c_2. \quad (28)$$

Consequently, from (24), $x^T(t)x(t) < c_2$ for $t \in [0, T_f]$. System (1) is FTS with respect to (c_1, c_2, T_f) . $\square$

4. Finite-Time Stability via State Feedback

In this section, the LMIs conditions, introduced in Theorem 1, will be exploited to design a state feedback controller for the SLTV control system with time-varying delay. The resultant closed-loop system could be proved to be FTS. The problem is described concretely as follows.

Given the time interval $[0, T_f]$, let us consider the SLTV control system:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t-d(t)) + G_{\sigma(t)}u(t), & t \in [0, T_f], \\ x(s) = \varphi(s), & \forall s \in [-\tau, 0], \end{cases} \quad (29)$$

where $u(\cdot)$ represents the control input. The aim of this section is to find a memoryless, state feedback control law

$u(t) = K_{\sigma(t)}(t)x(t)$, such that the resultant closed-loop system

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)}(t) + G_{\sigma(t)}(t)K_{\sigma(t)}(t))x(t) + B_{\sigma(t)}(t)x(t-d(t)), & t \in [0, T_f], \\ x(s) = \varphi(s), & \forall s \in [-\tau, 0], \end{cases} \quad (30)$$

is FTS with respect to (c_1, c_2, T_f) .

The following theorem states the sufficient conditions for the FTS of the closed-loop system (30) with respect to (c_1, c_2, T_f) .

Theorem 2. *System (30) is FTS with respect to (c_1, c_2, T_f) if there exist piecewise continuously differentiable positive definite matrix-functions $D_i(\cdot)$, $H_i(\cdot)$, and $J_i(\cdot)$ and a piecewise continuous matrix-function $L_i(\cdot)$ with appropriate dimensions such that the following LMIs hold, for $t \in [0, T_f]$,*

$$\begin{pmatrix} \Theta_i(t) & B_i(t) \\ B_i^T(t) & \Pi_i(t) \end{pmatrix} < 0, \quad (31)$$

$$\beta D_i(t) \geq D_j(t), H_i(t) \leq \beta H_j(t), J_i(t) \leq \beta J_j(t), \quad \forall i, j \in M, \quad (32)$$

$$\dot{H}_i(t) \leq 0, \quad (33)$$

$$\dot{J}_i(t) \leq 0, \quad (34)$$

and the ADT satisfies

$$\tau_D > \frac{T_f \ln \beta}{\ln c_2 + \ln \gamma_p - \lambda T_f - \ln \delta_1 + N_0 \ln \beta}, \quad (35)$$

where

$$\begin{aligned} \Theta_i(t) &= D_i(t)A_i^T(t) - \dot{D}_i(t) + A_i(t)D_i(t) + H_i(t) \\ &\quad + dJ_i(t) - \lambda D_i(t) \\ &\quad + G_i(t)L_i(t) + L_i^T(t)G_i^T(t), \\ \Pi_i(t) &= -e^{\lambda d}H_i(t)(1-\rho), \\ \mu_1 &= \max_{i \in M} \{\lambda_{\max}(D_i^{-1}(0))\}, \\ \mu_2 &= \max_{i \in M} \{\lambda_{\max}(D_i^{-1}(0)H_i(0)D_i^{-1}(0))\}, \\ \mu_3 &= \max_{i \in M} \{\lambda_{\max}(D_i^{-1}(0)J_i(0)D_i^{-1}(0))\}, \\ \gamma_p &= \min_{i \in M} (\lambda_{\min}(D_i^{-1}(t))), \end{aligned} \quad (36)$$

and the state feedback controller gain $K_i(t) = L_i(t)D_i^{-1}(t)$, $\forall i \in M$.

Proof. To prove the finite time stability of system (30), we only need to prove that system (30) satisfies all the conditions of Theorem 1. Concretely, we only need to prove conditions (3)–(6) are equivalent to (31)–(34), separately.

Now, we prove the equivalence of (3) and (31). For system (30), from (3), we have

$$\begin{pmatrix} \Phi_i(t) & P_i(t)B_i(t) \\ B_i^T(t)P_i(t) & -e^{\lambda d}Q_i(t)(1-\rho) \end{pmatrix} < 0, \quad (37)$$

where $\Phi_i(t) = (A_{\sigma(t)}(t) + G_{\sigma(t)}(t)K_{\sigma(t)}(t))^T P_i(t) + \dot{P}_i(t) + P_i(t)(A_{\sigma(t)}(t) + G_{\sigma(t)}(t)K_{\sigma(t)}(t)) + Q_i(t) + dZ_i(t) - \lambda P_i(t)$. Let $D_i(t) = P_i^{-1}(t)$; we pre- and postmultiply (37) by the block tridiagonal matrix $(D_i(t), I)$, and we obtain

$$\begin{pmatrix} Y_i(t) & B_i(t) \\ B_i^T(t) & \Psi_i(t) \end{pmatrix} < 0, \quad (38)$$

where

$$\begin{aligned} Y_i(t) &= D_i(t)A_i^T(t) + D_i(t)G_i^T(t)K_i^T(t) - \dot{D}_i(t) \\ &\quad + A_i(t)D_i(t) \\ &\quad + G_i(t)K_i(t)D_i(t) + D_i(t)Q_i(t)D_i(t) \\ &\quad + dD_i(t)Z_i(t)D_i(t) - \lambda D_i(t), \end{aligned} \quad (39)$$

$$\Psi_i(t) = -e^{\lambda d}D_i(t)Q_i(t)D_i(t)(1-\rho).$$

Then, from $K_i(t) = L_i(t)D_i^{-1}(t)$, $H_i(t) = D_i(t)Q_i(t)D_i(t)$, and $J_i(t) = D_i(t)Z_i(t)D_i(t)$, it can be obtained that $\Theta_i(t) = Y_i(t)$ and $\Pi_i(t) = \Psi_i(t)$, and further, (31) and (3) are equivalent. In the same way, equivalence between (4) and (32) is straightforward. Similarly, (5) and (33) are equivalent, (6) and (34) are equivalent. Therefore, from Theorem 1, the closed-loop system (30) is FTS. $\square$

Remark 2. Here, it is assumed that there is no delay between the controllers and the subsystems, i.e., the control act on the subsystems without delay.

5. Numerical Example

In this section, a numerical example will be given to illustrate the validity of our results.

Example 1. Consider the following SLTV system:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t - d(t)) \\ \quad + G_{\sigma(t)}u(t), & t \in [0, T_f], \\ x(s) = \varphi(s), & \forall s \in [-\tau, 0), \end{cases} \quad (40)$$

where $\sigma(t) \in M = \{1, 2\}$, $d(t) = 0.0001t$,

$$\begin{aligned} A_1 &= \begin{pmatrix} 0.01t & 0.01t \\ 0.1 \cos(t) & 0.1 \end{pmatrix}, \\ B_1 &= \begin{pmatrix} 0.01 & -0.01t \\ 0.1 \sin(t) & 0.01 \end{pmatrix}, \\ G_1 &= \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \\ A_2 &= \begin{pmatrix} 0.01 & -0.01t \\ 0.1 \sin(t) & 0.01 \end{pmatrix}, \\ B_2 &= \begin{pmatrix} 0.01t & 0.01t \\ 0.1 \cos(t) & 0.1 \end{pmatrix}, \\ G_2 &= \begin{pmatrix} 4 \\ 1 \end{pmatrix}. \end{aligned} \quad (41)$$

Let $c_1 = 1$, $c_2 = 8$, and $T_f = 10$ be given. The switching law of system (40) is shown in Figure 1, and it satisfies the ADT parameters $\tau_D = 1.25$ and $N_0 = 1$. With this switching law, Figure 2 shows the system state of system (40) without control and with initial state $(-0.1 \ 0.5)^T$. From Figure 2, it can be seen that the system state is unbounded and is not FTS.

From inequality (7), we let $\lambda = 0.1$ and $\beta = 1.05$. Applying Theorem 2, we design a finite time stable state feedback controller $u(t) = K_{\sigma(t)}(t)x(t)$. Figure 3 shows the time traces for the two elements of state feedback controller gain $K_{\sigma(t)}(t)$. It can be seen from Figure 4 that, with the state feedback controller, the system state with initial state $(-0.1 \ 0.5)^T$ satisfies $x^T(t)x(t) < c_2 = 8$ over $[0, 10]$.

Remark 3. Since $P(t)$, $Q(t)$, and $Z(t)$ are continuously time varying, LMIs cannot be solved directly. In order to

FIGURE 1: Switching law.

FIGURE 2: State of system (40) with $u \equiv 0$.

get a computationally tractable problem, we discretize the time interval into equally spaced subintervals. If the length of the subintervals is sufficient small, the continuous functions can be approximated by piecewise functions. Then, the LMIs can be solved. This approximating method can also be found in [26, 27]. Concretely, discretize $[0, T_f]$ into equally spaced time instances t_i ($i = 0, 1, 2, \dots, N$) with $t_0 = 0$, $t_N = T_f$, and $t_k - t_{k-1} \triangleq \varepsilon = (T_f/N)$. Thus, on the time interval $(t_{k-1}, t_k]$ with small enough ε , the time-varying matrix-valued functions $P(t)$, $Q(t)$, and $Z(t)$ can be denoted as $P(t) = P(t_k)$, $Q(t) = Q(t_k)$, and $Z(t) = Z(t_k)$, for $t \in (t_{k-1}, t_k]$. Then, $P(t)$ can be denoted as $(P(t_k) - P(t_{k-1}))/\varepsilon$ with sufficiently small ε . Similarly, $Q(t)$ and $\dot{Z}(t)$ can be replaced by $(Q(t_k) - Q(t_{k-1}))/\varepsilon$ and $(Z(t_k) - Z(t_{k-1}))/\varepsilon$.

FIGURE 3: Time traces of the elements in state feedback controller gain $K_i(t) = (k_1(t), k_2(t))$.

FIGURE 4: State of the resultant closed-loop system of system (40).

6. Conclusions

This paper focuses on the FTS analysis of SLTV systems with time-varying delays. For this kind of systems, based on the Lyapunov–Krasovskii functional technique and ADT method, a sufficient criterion on FTS is obtained. Based on this criterion, using the LMI method, a state feedback controller is designed such that the system is FTS. Finally, a numerical example illustrates the validity of the obtained results. The innovation mainly lies in the studying of FTS for the switched system on the basis of the LTV system, which has become more and more important now. And, a time-varying state feedback controller is designed. Future efforts will be devoted to obtain less conservative criteria for the

finite-time stability of linear time-varying system and extend the FTS theory to the neural network context.

Data Availability

No data were used to support this study. The authors only used MATLAB for simulation. Therefore, simulation programming can be obtained from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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