Research Article

Layered Synchronization and Identification of Uncertain Delayed Hierarchical Networks

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This paper investigates the layered synchronization and identification of uncertain delayed hierarchical networks, featured by highlights in three aspects: first, the presentation of a multilayered delayed network model, which can be regarded as a 3D stereochemical structure in compliance with the ever-developing trend of hierarchical network modeling structure; second, the design of a multi-identification scheme combined with uncertain delayed hierarchical networks, which matches the unknown and hierarchical properties of the proposed networks; and third, the construction of a layered control method of Lyapunov function, which synthesizes the factors of the unknown, time delay and hierarchy. Based on these points, the laws of layered synchronization between drive hierarchical networks (DHNs) and response hierarchical networks (RHNs) are derived, the dual identification criteria for monitoring topological structures and identifying system parameters, respectively, are provided, and the hierarchical multiformity of the proposed method is validated through abundant numerical simulations in the end.

1. Introduction

In nature, a lot of practical complex networks such as intelligent networks, food webs, and neural networks can be boiled down to the interaction between nodes and edges, which has aroused many researchers' interests in interdisciplinary sciences and applied sciences [1–3]. Thus, modeling and controlling of complex networks have become burning issues due to their potential demand in broad fields [4–6]. Many network models have been established for explaining the nonlinear dynamic property of the real world [7–9], such as small-world network [10] and scale-free network [11]. Constantly left in varied conditions, many complex networks comprise multilevel structures and multicomponent systems [12], and stereoscopic hierarchical network is one of them. Characterized by hierarchy and diversity, hierarchical networks can meet the modeling requirements of layered neural networks and large-scale data transmission networks.

In the newest production, a new theory of event-triggered synchronization control in discrete-time neural networks is

proposed [13]. Intermittent quasisynchronization control, an optimal algorithm, is designed for delayed discrete-time neural networks [14]. The motivations of modeling and control based on practical needs are threefold: atmosphere's vertical stratification is an interesting novel feature but lacks a general topology framework for statistical research [15]; dynamical neural activity across cortex related to hierarchical levels of cortical organization should be developed to control the dynamical interactions [16]; and there are still a couple of limitations as to the identification of human brain hierarchical architecture [17]. These are realistic demands in modeling and control studies for uncertain delayed hierarchical networks. Due to the complexity and uncertainty of networks, it is difficult to know beforehand their node dynamic states and topological structures from various theoretical models [18]. Meanwhile, during the information transmission and variable coupling process, the state variables of network nodes are vulnerable to time delay, which is a universally existed phenomenon in biological neural networks, epidemiological models, etc., because of the memory effects and different transmission speeds of coupling. In many situations, network identification is more important than its

synchronization. For instance, the structures of a cell network change with protein-DNA interactions, DNA replication, etc., and the basic elements of a neural network are hierarchical structures and multiple neurons, so the monitoring of neural network structure and the identification of protein-DNA have become an important study in cell networks with significant values for life sciences. Therefore, network modeling, monitoring, and identification could be a sign of deeper problems and should be further investigated, which is of great significance conceptually.

A lot of research into the hierarchical network has been achieved in recent years on the exploration of the specific structures and interactional ways of multilevel, multidimensional, and multiplex networks, which reflects the transitional trend of network modeling structure from the plane to the stereoscopic. The self-structuring algorithm has been proposed to solve the high-dimensional problems in the hierarchical neural network [19]; the asynchronous and intermittent sampled-data controllers have been studied to control a kind of hierarchical time-varying neural networks [20]; the finite-time synchronization in coupled hierarchical hybrid neural networks has been concerned [21]; the synchronization control on heterogeneous FitzHugh–Nagumo networks with hierarchical topologies has been studied [22]; the new reinforcement learning algorithm has been designed to realize hierarchical optimal synchronization [23]; the real brain hierarchical modular organization has been found during the brain processing with diverse functional interactions [24]; however, the hierarchical network of combining chain structure with global structure is not considered. Likewise, plenty of identification methods are confined to the identification of parameters or network topology [25–27]. The parameter identification and synchronization of chaotic neural networks with time-varying delay have been introduced via combining two control laws [28]; the new delay-dependent laws to ensure the passivity analysis of neural networks have been derived in terms of tractable linear matrix inequalities [29]; the identification laws to tackle the uncertain parameters between uncertain time-delayed networks with multilink and different topologies have been investigated [30]. However, the identification of hierarchical networks with time-delayed coupling is not designed correspondingly.

In many practical models, complex networks are faced with far more complex structures and unknown factors, such as multilayer, nonlinearity, and time delay, and it is no easy task to conjecture structures and parameters beforehand and to identify the topology types of hierarchical structures by using previous methods, which necessitates a new multiobjective layered control method. Therefore, it is meaningful to build a fire-new network model in a multiobjective layered control way to understand, predict, and identify the dynamic characteristics of multinodes and multilayers constituting networks, which lays a theoretical basis for the network world of unsolved mysteries.

Compared with prior work, a new method based on the Lyapunov theory is created in this paper to monitor the topological structures and identify the system parameters of hierarchical networks regardless of the number of unknown

parameters and hierarchy types, which is featured by uncertainties, time-delayed coupling, and hierarchical structures. The new model not only provides a basic framework for understanding the mechanism of internal coupling in biological, physical, and engineering sciences but can also be applied to various types of cell neural networks.

The paper is organized as follows: besides introduction and conclusion, a model of the delayed hierarchical network and its preliminaries are illustrated in Section 2; the theoretical design of the new model is tested by a numerical experiment with desired results in Section 3, where the simulation results are discussed from three aspects via detailed instances to verify the above theoretical analysis. Section 4 provides the conclusion.

2. A New Model and Its Preliminaries

2.1. Hierarchical Network Model. Hierarchy denotes multilayers, i.e., the two topological structures of intrastratal and interbedded coupling connections, while diversity signifies multisystems, i.e., different dynamical systems in different layers. In different layers and channels of complex networks, there might exist more complex coupling. The structure diagram of a hierarchical network is shown in Figure 1. This paper assumes there are intrastratal chain coupling and interbedded global coupling in the hierarchical network. As can be seen from the above figure, a hierarchical network equates to a vast network comprising multiple superimposed coupling layers. Each layer has its exclusive topology structure and dynamic systems. Hierarchical networks have three advantages of multilayers, multiple topologies, and multinodes over complex networks, which can better satisfy the actual need of networks. Multilayers refer to the layers constituting the stereoscopic hierarchical network by mutual coupling in which each layer can be seen as an isolated network; multiple topologies are the two topological structures of internal structures among nodes and external structures among layers; and multinodes mean different layers have different nodes with nonidentical dynamic properties. The hierarchical network in Figure 1 contains m layers, and each layer has N nodes, which has $M = m \times N$ nodes.

2.2. Preliminaries. Consider the uncertain hierarchical dynamical target system

$$\dot{x}_i^l(t) = F_l(x_i^l(t), \beta_i^l, t), \quad (1)$$

and split system (1) into the following two parts:

$$\dot{x}_i^l(t) = f_l(x_i^l(t), t) + p_l(x_i^l(t), t)\beta_i^l. \quad (2)$$

Then, consider the hierarchical response system

$$\dot{y}_i^l(t) = f_l(y_i^l(t), t) + p_l(y_i^l(t), t)\beta_i^l, \quad (3)$$

where $x_i^l(t), y_i^l(t) \in R^n$ are state vectors and $\beta_i^l \in R^{n_i}$ with $l = 1, 2, \dots, m$ and $i = 1, \dots, N$ being unknown system parameter vectors, in which n_i are nonnegative integers. For

FIGURE 1: Structure diagram of the hierarchical network.

each i and l , $f_l(x_i^l(t), t)$ is a $n \times 1$ matrix and $p_l(x_i^l(t), t)$ is a $n \times n_i$ matrix.

Assumption 1. Suppose that time-varying lag variable $\tau(t)$ is differentiable and satisfies $\dot{\tau}(t) \leq 1$.

Assumption 2. Suppose there exist nonnegative constants $H_i^l (l = 1, 2, \dots, m)$ and $i = 1, 2, \dots, N$ satisfying

$$|F_l(y_i^l(t), \beta_i^l, t) - F_l(x_i^l(t), \beta_i^l, t)| \leq H_i^l |y_i^l(t) - x_i^l(t)|, \quad (4)$$

where $x_i^l(t), y_i^l(t) \in R^n$ are time-varying vectors and β_i^l is the parameter vector of $F_l(x_i^l(t), \beta_i^l, t)$.

Consider the hierarchical network comprising m layers and $m \times N$ nodes

$$\begin{aligned} \dot{x}_i^l(t) = & f_l(x_i^l(t), t) + p_l(x_i^l(t), t)\beta_i^l + c_1 \sum_{j=1}^N a_{ij}\Gamma x_j^l(t) \\ & + c_2 \sum_{k=1}^m b_{lk}\Gamma x_N^k(t - \tau(t)), \end{aligned} \quad (5)$$

where $x_i^l(t) = (x_{i1}^l(t), x_{i2}^l(t), \dots, x_{in}^l(t))^T \in R^n$ is the state vector of the i -th node in the l -th layer ($l = 1, 2, \dots, m, i = 1, 2, \dots, N$) and delay $\tau(t)$ is time-varying. $A = (a_{ij})_{N \times N} \in R^{N \times N}$ is an intralayer matrix and $B = (b_{lk})_{m \times m} \in R^{m \times m}$ is an unknown interbedded coupling matrix. If there exists a link from layers l to k ($k \neq l$), then $b_{lk} \neq 0$ and b_{lk} is the coupling strength between layers; otherwise, $b_{lk} = 0$; if there exists a link from nodes i to j ($j \neq i$), then $a_{ij} \neq 0$ and a_{ij} is the coupling strength; otherwise, $a_{ij} = 0$. $\Gamma: R^n \rightarrow R^n$ is an inner-coupling matrix.

Consider the response hierarchical network for identifying the hierarchical complex network

$$\begin{aligned} \dot{y}_i^l(t) = & f_l(y_i^l(t), t) + p_l(y_i^l(t), t)\hat{\beta}_i^l + c_1 \sum_{j=1}^N a_{ij}\Gamma y_j^l(t) \\ & + c_2 \sum_{k=1}^m \hat{b}_{lk}\Gamma y_N^k(t - \tau(t)) + u_i^l, \end{aligned} \quad (6)$$

where $y_i^l(t) = (y_{i1}^l(t), y_{i2}^l(t), \dots, y_{in}^l(t))^T \in R^n$ is the response state vector of the i -th node in the l -th layer, $u_i^l \in R^n$ is

its controllers, and $\hat{b}_{lk}$ and $\hat{\beta}_i^l$ are used to estimate the values of b_{lk} and β_i^l .

Define $\hat{\beta}_i^l(t) = \hat{\beta}_i^l - \beta_i^l$, $e_{lk}^b(t) = \hat{b}_{lk} - b_{lk}$, and $e_i^l(t) = y_i^l(t) - x_i^l(t)$. If $e_i^l(t) \rightarrow 0$ as $t \rightarrow \infty$, the two hierarchical complex networks (5) and (6) realize layered synchronization. The layered error dynamic equation is defined as follows:

$$\begin{aligned} \dot{e}_i^l(t) = & f_l(y_i^l(t), t) - f_l(x_i^l(t), t) + p_l(y_i^l(t), t)\hat{\beta}_i^l \\ & - p_l(x_i^l(t), t)\beta_i^l + c_1 \sum_{j=1}^N a_{ij}\Gamma e_j^l(t) \\ & + c_2 \sum_{k=1}^m \hat{b}_{lk}\Gamma y_N^k(t - \tau(t)) - c_2 \sum_{k=1}^m b_{lk}\Gamma x_N^k(t - \tau(t)) + u_i^l. \end{aligned} \quad (7)$$

Equation (7) can be further rewritten as

$$\begin{aligned} \dot{e}_i^l(t) = & f_l(y_i^l(t), t) - f_l(x_i^l(t), t) + p_l(y_i^l(t), t)\hat{\beta}_i^l \\ & + p_l(y_i^l(t), t)\beta_i^l - p_l(x_i^l(t), t)\beta_i^l + c_1 \sum_{j=1}^N a_{ij}\Gamma e_j^l(t) \\ & + c_2 \sum_{k=1}^m \hat{b}_{lk}\Gamma y_N^k(t - \tau(t)) - c_2 \sum_{k=1}^m b_{lk}\Gamma x_N^k(t - \tau(t)) + u_i^l, \end{aligned} \quad (8)$$

which leads to

$$\begin{aligned} \dot{e}_i^l(t) = & F_l(y_i^l(t), \beta_i^l, t) - F_l(x_i^l(t), \beta_i^l, t) + p_l(y_i^l(t), t)\hat{\beta}_i^l \\ & + c_1 \sum_{j=1}^N a_{ij}\Gamma e_j^l(t) + c_2 \sum_{k=1}^m \hat{b}_{lk}\Gamma y_N^k(t - \tau(t)) \\ & - c_2 \sum_{k=1}^m b_{lk}\Gamma y_N^k(t - \tau(t)) + c_2 \sum_{k=1}^m b_{lk}\Gamma y_N^k(t - \tau(t)) \\ & - c_2 \sum_{k=1}^m b_{lk}\Gamma x_N^k(t - \tau(t)) + u_i^l. \end{aligned} \quad (9)$$

According to equation (9), it can be obtained that

$$\begin{aligned} \dot{e}_i^l(t) = & F_l(y_i^l(t), \beta_i^l, t) - F_l(x_i^l(t), \beta_i^l, t) + p_l(y_i^l(t), t)\hat{\beta}_i^l \\ & + c_1 \sum_{j=1}^N a_{ij}\Gamma e_j^l(t) + c_2 \sum_{k=1}^m e_{lk}^b(t)\Gamma y_N^k(t - \tau(t)) \\ & + c_2 \sum_{k=1}^m b_{lk}\Gamma e_N^k(t - \tau(t)) + u_i^l. \end{aligned} \quad (10)$$

Theorem 1. The interbedded coupling matrix B and uncertain parameter β_i^l of the delayed hierarchical network (5) are identified by the predicted values $\hat{b}_{lk}$ and $\hat{\beta}_i^l$ in the response hierarchical network (6) when Assumption 2 holds

$$\begin{cases} \dot{\hat{\beta}}_i^l = -p_l(y_i^l(t), t)^T e_i^l(t), \\ \dot{\hat{b}}_{lk} = -\sum_{i=1}^N \gamma_{lk} e_i^l(t)^T \Gamma y_N^k(t - \tau(t)), \\ u_i^l = -s_i e_i^l(t) - d e_i^l(t) - c_2 \sum_{k=1}^m b_{lk} \Gamma e_N^k(t - \tau(t)), \end{cases} \quad (11)$$

where $i, j \in \{1, 2, \dots, N\}$, $l \in \{1, 2, \dots, m\}$, and γ_{lk} are any positive constants.

In the existing designs of synchronization [25, 26, 28, 30], only the Lyapunov functions are used, while a new layered Lyapunov function is constructed in the study, bearing the following three advantages.

- (1) Equation (12) synthesizes more factors since its derivative is associated with unknown parameters, time delay and hierarchical structures. Compared with the Lyapunov function in [25, 26, 28, 30], the layered Lyapunov function further enlarges the identification ranges among different hierarchies.

- (2) Compared with the Lyapunov function in [25, 26, 28], terminal error is considered in building the layered Lyapunov function in order to implement the layered synchronization and identification through simple control input.
- (3) In contrast with the Lyapunov function in [30], layered synchronization, and identification downgrade control difficulty and cost, a benefit only can be achieved by adjusting two control gains.

Proof. Construct the Lyapunov candidate

$$\begin{aligned} V(t) = & \frac{1}{2} \sum_{l=1}^m \sum_{i=1}^N e_i^l(t)^T e_i^l(t) + \frac{1}{2} \sum_{l=1}^m \sum_{k=1}^m \frac{1}{\gamma_{lk}} (e_{lk}^b(t))^2 \\ & + \frac{1}{2} \sum_{l=1}^m \sum_{i=1}^N \left(\hat{\beta}_i^l \right)^T \hat{\beta}_i^l + \int_{t-\tau(t)}^t \sum_{l=1}^m e_N^l(\alpha)^T e_N^l(\alpha) d\alpha, \end{aligned} \quad (12)$$

which leads to

$$\begin{aligned} \dot{V}(t) = & \sum_{l=1}^m \sum_{i=1}^N e_i^l(t)^T \dot{e}_i^l(t) + \sum_{l=1}^m \sum_{k=1}^m \frac{1}{\gamma_{lk}} e_{lk}^b(t) \dot{e}_{lk}^b(t) \\ & + \sum_{l=1}^m \sum_{i=1}^N \left(\hat{\beta}_i^l \right)^T \dot{\hat{\beta}}_i^l + \sum_{l=1}^m e_N^l(t)^T \dot{e}_N^l(t) - (1 - \dot{\tau}(t)) \\ & \cdot \sum_{l=1}^m e_N^l(t - \tau(t))^T \dot{e}_N^l(t - \tau(t)) \\ = & \sum_{l=1}^m \sum_{i=1}^N e_i^l(t)^T \left[F_l(y_i^l(t), \hat{\beta}_i^l, t) - F_l(x_i^l(t), \beta_i^l, t) + p_l(y_i^l(t), t) \hat{\beta}_i^l(t) \right. \\ & + c_1 \sum_{j=1}^N a_{ij} \Gamma e_j^l(t) + c_2 \sum_{k=1}^m e_{lk}^b(t) \Gamma y_N^k(t - \tau(t)) \\ & \left. + c_2 \sum_{k=1}^m b_{lk} \Gamma e_N^k(t - \tau(t)) + u_i^l \right] \\ & + \sum_{l=1}^m \sum_{k=1}^m \frac{1}{\gamma_{lk}} e_{lk}^b(t) \dot{e}_{lk}^b(t) + \sum_{l=1}^m \sum_{i=1}^N \left(\hat{\beta}_i^l \right)^T \dot{\hat{\beta}}_i^l + \sum_{l=1}^m e_N^l(t)^T \dot{e}_N^l(t) \\ & + (\dot{\tau}(t) - 1) \sum_{l=1}^m e_N^l(t - \tau(t))^T \dot{e}_N^l(t - \tau(t)). \end{aligned} \quad (13)$$

According to Assumption 2, it can be obtained that

$$\begin{aligned} \dot{V}(t) \leq & \sum_{l=1}^m \sum_{i=1}^N H_i^l e_i^l(t)^T e_i^l(t) + \left[\sum_{l=1}^m \sum_{i=1}^N e_i^l(t)^T p_l(y_i^l(t), t) \tilde{\beta}_i^l(t) + \sum_{l=1}^m \sum_{i=1}^N \left(\tilde{\beta}_i^l \right)^T \tilde{\beta}_i^l \right] + \left[c_2 \sum_{l=1}^m \sum_{i=1}^N \sum_{k=1}^m e_{lk}^b e_i^l(t)^T \Gamma_N^k(t - \tau(t)) \right. \\ & + \sum_{l=1}^m \sum_{k=1}^m \frac{1}{\gamma_{lk}} e_{lk}^b(t) \hat{b}_{lk}(t) \left. \right] + \left[c_1 \sum_{l=1}^m \sum_{i=1}^N \sum_{j=1}^N e_i^l(t)^T a_{ij} \Gamma e_j^l(t) + c_2 \sum_{l=1}^m \sum_{i=1}^N \sum_{k=1}^m b_{lk} e_i^l(t)^T \Gamma e_N^k(t - \tau(t)) + \sum_{l=1}^m \sum_{i=1}^N e_i^l(t)^T u_i^l \right] \\ & + \sum_{l=1}^m e_N^l(t)^T e_N^l(t) + (\dot{\tau}(t) - 1) \sum_{l=1}^m e_N^l(t - \tau(t))^T e_N^l(t - \tau(t)). \end{aligned} \quad (14)$$

From Theorem 1, setting $H = \max \{1 \leq i \leq N, 1 \leq l \leq m | H_i^l\}$ would lead to

$$\begin{aligned} \dot{V}(t) \leq & \sum_{l=1}^m \sum_{i=1}^N H e_i^l(t)^T e_i^l(t) + c_1 \sum_{l=1}^m \sum_{i=1}^N \sum_{j=1}^N e_i^l(t)^T a_{ij} \Gamma e_j^l(t) \\ & - \sigma \sum_{l=1}^m e_N^l(t)^T e_N^l(t) - \sum_{l=1}^m \sum_{i=1}^N d \|e_i^l(t)\|^2 \\ & + \sum_{l=1}^m e_N^l(t)^T e_N^l(t) + (\dot{\tau}(t) - 1) \sum_{l=1}^m e_N^l(t - \tau(t))^T e_N^l(t - \tau(t)). \end{aligned} \quad (15)$$

With $e^l(t) = (e_1^l(t)^T, e_2^l(t)^T, \dots, e_N^l(t)^T)^T \in R^{nN \times 1}$, $S = \text{diag}(s_1, s_2, \dots, s_N) = \text{diag}(0, 0, \dots, \sigma)$, and $Q = A \otimes \Gamma$, $\otimes$ being a Kronecker product, the following equation is obtained:

$$\begin{aligned} \dot{V}(t) = & \sum_{l=1}^m H e^l(t)^T e^l(t) + c_1 \sum_{l=1}^m e^l(t)^T Q e^l(t) \\ & - \sum_{l=1}^m d \|e^l(t)\|^2 + (1 - \sigma) \sum_{l=1}^m e_N^l(t)^T e_N^l(t) \\ & + (\dot{\tau}(t) - 1) \sum_{l=1}^m e_N^l(t - \tau(t))^T e_N^l(t - \tau(t)). \end{aligned} \quad (16)$$

Then, the following inequation can be deduced:

$$\begin{aligned} \dot{V}(t) \leq & \sum_{l=1}^m (H + c_1 \lambda_{\max}(Q) - d) e^l(t)^T e^l(t) \\ & + (1 - \sigma) \sum_{l=1}^m e_N^l(t)^T e_N^l(t) \\ & + (\dot{\tau}(t) - 1) \sum_{l=1}^m e_N^l(t - \tau(t))^T e_N^l(t - \tau(t)). \end{aligned} \quad (17)$$

Therefore, $(\dot{\tau}(t) - 1) \leq 0$ obtained from Assumption 1 would lead to

$$\begin{aligned} \dot{V}(t) \leq & \sum_{l=1}^m (H + c_1 \lambda_{\max}(Q) - d) e^l(t)^T e^l(t) \\ & + (1 - \sigma) \sum_{l=1}^m e_N^l(t)^T e_N^l(t). \end{aligned} \quad (18)$$

Setting $d = H + c_1 \lambda_{\max}(Q) + 1$ and $\sigma = 2$ results in $\dot{V}(t) < 0$.

3. Numerical Results

A numerical example shows that the theoretical design of the new hierarchical model of double coupling provides accurate results. Suppose a hierarchical network has four layers with a hundred nodes on each and each layer can send a hundred chaotic signals. Two different systems are loaded on the first and third layers and the second and fourth layers, respectively. Chain coupling is chosen as the interaction style among intrastratal nodes in each layer and global coupling between layers. The Lorenz system and the Lü system are loaded on the odd-numbered layers and the even-numbered layers in the four-layer network in turn as follows [31, 32]:

$$\begin{aligned} \dot{x}_i^l(t) = & F_l(x_i^l(t), \beta_i^l, t) + c_1 \sum_{j=1}^N a_{ij} \Gamma x_j^l(t) \\ & + c_2 \sum_{k=1}^m b_{lk} \Gamma x_N^k(t - \tau(t)), \end{aligned} \quad (19)$$

where $F_l = (\beta_{l1}^l(-x_{l1}^l(t) + x_{l2}^l(t)), \beta_{l2}^l x_{l1}^l(t) - x_{l2}^l(t) - x_{l1}^l(t) x_{l3}^l(t), x_{l1}^l(t) x_{l2}^l(t) - \beta_{l3}^l x_{l3}^l(t))^T$ with $l=1, 3$ and $F_l = (\beta_{l1}^l(-x_{l1}^l(t) + x_{l2}^l(t)), \beta_{l2}^l x_{l1}^l(t) - x_{l2}^l(t) x_{l3}^l(t), x_{l1}^l(t) x_{l2}^l(t) - \beta_{l3}^l x_{l3}^l(t))^T$ with $l=2, 4$.

The internal and external coupling configuration matrices are described by chain matrix A and global coupling matrix B , respectively, as follows:

$$A = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{100 \times 100}, \quad (20)$$

$$B = \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix},$$

FIGURE 2: Algorithm flow of uncertain delayed hierarchical networks.

$x_{i1}^l(t), x_{i2}^l(t), x_{i3}^l(t)$ ($l=1, 2, \dots, 4$ and $i=1, 2, \dots, 100$). $i=100$ is the cutoff point between the first and second layers, the second and third layers, and the third and fourth layers, drawing different chaotic evolution trajectories. Sharing the same systems, the odd-numbered layers and the even-numbered layers have similar but quite distinct evolutions.

In Figure 2, $\hat{b}_{11}$ proportionally follows the changes of layer number, i.e., when the number of layers increases, $\hat{b}_{11}$ shifts from a small value to a large value, particularly in switching topology hierarchical networks. Once the hierarchical topology of the network is changed, corresponding monitoring is adjusted. Meanwhile, the parameter identifier designed by layered Lyapunov function receives nonlinear feedbacks from chaotic systems on different network layers, through which the actual number of nodes on each layer can be predicted by the number of curves.

Remark 1. Extensive as the research in network modeling, synchronization control, and parameter identification is, there is still much to explore [13–19]. The advantage of combining the factors of the unknown, time delay and hierarchy, lies not only in the high-capacity structures and hierarchical attributes displayed by the hierarchical networks but also in the different nonlinear nodes among different layers in the drive-response hierarchical networks. Layered control and dual identification scheme are the novelties of the proposed theory.

Remark 2. Previous research results focus on the innovation of synchronization schemes while overlooking the more practical combined design of hierarchical control and dual identification [20–24]. Hierarchical control is employed in 3D networks, while dual identification is applied when dynamics nodes and hierarchical topology need to be identified during the process of layered control.

Remark 3. A novel layered control and dual identification design between DHN and RHN is proposed in Theorem 1. By splitting the state variables of DHN and RHN into four parts, an improved layered Lyapunov function is constructed, in which the parameters and topology information of hierarchical networks can be fully captured. Combined with Assumptions 1 and 2, the three layered controllers

FIGURE 3: State evolution of the first errors.

FIGURE 4: State evolution of the second errors.

FIGURE 5: State evolution of the third errors.

presented for synchronization and identification are different from those in [25, 26, 28, 30]. In this case, two control gains d and σ can be chosen and adjusted freely to achieve better control effects.

FIGURE 6: Monitoring of layer structures in four layers.

FIGURE 7: Identification of uncertain parameters in the first variable.

FIGURE 8: Identification of uncertain parameters in the second variable.

FIGURE 9: Identification of uncertain parameters in the third variable.

Firstly, the state evolution of $e_{i1}^l(t)$, $e_{i2}^l(t)$, $e_{i3}^l(t)$ ($l=1, 2, \dots, 4$ and $i=1, 2, \dots, 100$) with t is depicted in Figures 3–5. The gradation green, cyan, and magenta curves with 100 in each signify the error evolutions of the four layers in turn. The gradient colors change every 100 nodes, and the lighter the color, the further back the nodes. All the curves approach zero after 7 seconds, which means the entire hierarchical network is under control by the designed controllers. Secondly, the monitoring of hierarchical network structure is shown in Figure 6, where $\hat{b}_{11}$ ($\hat{b}_{22}$, $\hat{b}_{33}$, $\hat{b}_{44}$) switches from -7 (-8 , -9 , -10) to -3 , $\hat{b}_{12}$ ($\hat{b}_{13}$, $\hat{b}_{14}$) from -2 (3 , 14) to 1 , $\hat{b}_{21}$ ($\hat{b}_{23}$, $\hat{b}_{24}$) from 0.5 (6 , 17) to 1 , $\hat{b}_{31}$ ($\hat{b}_{32}$, $\hat{b}_{34}$) from -4 (5 , 10) to 1 , and $\hat{b}_{41}$ ($\hat{b}_{42}$, $\hat{b}_{43}$) from -11 (4 , 13) to 1 after 7 seconds.

Finally, Figures 7–9 present the identification of unknown chaotic parameters of $\beta_i^1 = (\beta_{i1}^1, \beta_{i2}^1, \beta_{i3}^1)^T = \beta_i^3 = (\beta_{i1}^3, \beta_{i2}^3, \beta_{i3}^3)^T = (10, 28, 8/3)^T$ and $\beta_i^2 = (\beta_{i1}^2, \beta_{i2}^2, \beta_{i3}^2)^T = \beta_i^4 = (\beta_{i1}^4, \beta_{i2}^4, \beta_{i3}^4)^T = (36, 20, 3)^T$ ($l=1, 2, \dots, 4$ and $i=1, 2, \dots, 100$) from four different layers, in which the identification of uncertain chaotic parameters from the first and third layers is denoted by $\hat{\beta}_i^1 = (\hat{\beta}_{i1}^1, \hat{\beta}_{i2}^1, \hat{\beta}_{i3}^1)^T = ([-3, 7], [-4, 6], [-5, 5])^T$ and $\hat{\beta}_i^3 = (\hat{\beta}_{i1}^3, \hat{\beta}_{i2}^3, \hat{\beta}_{i3}^3)^T = ([-2, 18], [0, 40], [-8, 12])^T$ ($l=1, 3$ and $i=1, 2, \dots, 100$), and the second and fourth layers by $\hat{\beta}_i^2 = (\hat{\beta}_{i1}^2, \hat{\beta}_{i2}^2, \hat{\beta}_{i3}^2)^T = ([-2, 8], [-5, 5], [-5, 5])^T$ and $\hat{\beta}_i^4 = (\hat{\beta}_{i1}^4, \hat{\beta}_{i2}^4, \hat{\beta}_{i3}^4)^T = ([5, 45], [-12, 28], [-17, 23])^T$ ($l=2, 4$ and $i=1, 2, \dots, 100$). The gradation green, cyan, and magenta curves with 100 in each signify the identification evolutions of the four layers in Figures 7–9. The darkness of the color indicates the layers: the progressive depth of the color varies inversely with the level of a layer: the highest color depth marks the lowest layer, and vice versa. In particular, in Figure 9, the $\hat{\beta}_{i3}^1$ and $\hat{\beta}_{i3}^3$ in the odd layers are identified as $8/3$ on the left ordinates, while the $\hat{\beta}_{i3}^2$ and $\hat{\beta}_{i3}^4$ in the even layers are identified as 3 corresponding to the values on the right ordinate, respectively. Speediness and preciseness are well demonstrated in the monitoring and identification of hierarchical topology and unknown parameters.

Remark 4. After continuous application and optimization in control methods in actual nonlinear areas, significant innovative results meeting realistic needs have been obtained. The extended dissipativity analysis is studied in the Markovian jump neural networks with time-varying delay [33]. The event-triggered controller and the switching state-feedback controller are designed for the synchronization of semi-Markov switching dynamical networks in fixed time [34]. The problem of H_∞ performance state estimation is investigated leading to performance improvement and difficulty reduction [35]. Based on the above research, switching dynamical hierarchical networks with time-varying delay, analyzing dissipativity, and estimating H_∞ performance state are the direction of our further research.

4. Conclusion

In this paper, a new model and layered control design were proposed to identify the layered topology and unknown parameters of the hierarchical network affected by uncertainties and time-delayed factors. Multiobjective layered control policies were obtained by the Lyapunov theory, which are accurate and reliable for identifying the hierarchical network no matter how many nodes with different nonlinear systems and layers with multicoupled matrices it has. The intrastatal parameters and interbedded coupling matrix were predicted through adaptive multiobjective control by numerical simulations accurately. The model and multiobjective layered control conform to the actual application demand in various engineering domains.

Data Availability

The data that support the findings of this study are available within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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