Research Article

Curvature- and Torsion-Based Consensus for the Markovian Switching Multiagent System with Communication of Noise Disturbance and Time Delay

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Received 25 July 2020; Revised 1 March 2021; Accepted 5 March 2021; Published 25 March 2021

Academic Editor: Higinio Ramos

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This paper investigates the consensus problem for the distributed multiagent system (MAS), where the trajectory of each agent is displayed by curvature and torsion, and the communication behaviors among agents are influenced by time delay and corrupted by noises. According to the Frenet–Serret formulas, a class of consensus protocols is designed for all agents, and a closed-loop system is obtained. Based on the Lyapunov method, several consensus criteria are derived, where the consensus criteria are characterized by curvature functions and torsion functions. Finally, one example shows the reliability of the proposed methods.

1. Introduction

In recent decades, the consensus problem of the multiagent system (MAS) has attracted the increasing attention of a large number of scholars, and the results have been widely used for tracking [1–5], formation control [6], distributed optimization monitoring [7, 8], and other purposes [9, 10].

The consensus of MAS requires that the state of each node tends to be a common value. Generally, the consensus problem is to design the distributed protocol for MAS [11], and under the protocol, the states of all agents converge to a common value by the local communication. However, for the abrupt change of topology and stochastic disturbance, the local communication will encounter link failures in some real applications. It is well known that the switching topology is widely described by Markovian chain.

Recently, many consensus results with Markovian switching have been investigated for MAS [12–23]. In [12], transition rates are considered to be not completely known; by the stochastic technique, sufficient conditions are expressed to solve the consensus problem for nonlinear MAS under the semi-Markovian switching topologies. In order to reduce the effect of communication noises, a time-

varying gain in control protocol is utilized, and necessary and sufficient criteria are established to guarantee the mean square scaled consensus [13]. Considering the consensus control of grouped agents with linear or linearized nominal dynamics, a class of state-feedback controllers is proposed, and it shows that the controllers yield unbounded consensus region (see [14] and the references therein).

Group-couple consensus with the Markovian switching system is a duration-significant research field. In [18], by employing linear transformations, couple-group consensus is considered for a class of discrete-time stochastic multiagent systems, and the performance of the reduced-order systems is provided to get constructive approaches for the fixed and the Markovian switching topologies. Under a designed switching signal set, the group consensus problem of the second-order MAS is studied in [24], where the switching topology obeys the Markovian chain, and by utilizing a state transformation method, two consensus sufficient criteria are established. Based on a system transformation, the problem of group-couple is converted to a L2 control issue, and sufficient conditions are obtained for MAS, where the topology obeys a Markovian switching with unknown transition probabilities [17].

Event-triggered consensus problem with Markovian switching is another important issue on MAS, and it has attracted many scholars to study. In [25], in order to simplify information transmission, a distributed event-triggered mechanism is considered, and the proposed consensus protocol ensures MAS to achieve the consensus. By presenting an event-triggered control strategy, MAS achieves the fixed-time consensus [26–28].

Fault-tolerant tracking is another research field of MSA. In [29], an adaptive fault-tolerant tracking protocol is proposed, and it ensures each follower agent to track a leader agent. By considering the directed link failures or recoveries, a fault-tolerant control strategy with Markovian switching topology is provided for a nonlinear MAS, and some sufficient conditions of stochastic consensus are obtained, where MAS is in the presence of communication noises and actuator faults [30].

Notice that most of the results mentioned above were obtained via the eigenvalue of matrix or the LMI method. In order to get the criteria of the consensus for MAS, some inherent relations will be neglected because of the inequalities to be utilized, such as curvature function and torsion function of each node when we consider the track problems. According to the theory of classic differential geometry, all tracks of agents are decided by their curvature and torsion entirely except for the difference of their locations. If all agents keep the same curvature function and torsion function, then they can move with the same track, and they can keep a certain distance among them. Recently, the consensus problem of MAS in the sense of curvature and torsion with fixed topology was studied in literature [31]. However, the results may not be suitable to the switching system with stochastic disturbance. Motivated by this, in this paper, curvature- and torsion-based consensus problem with switching topology and noise disturbance is studied.

The remainder of this paper is organized as follows. Section 2 introduces some concepts. Section 3 proposes the consensus problem with Markovian switching. In Section 4, several consensus criteria are proposed for MAS. Section 5 provides a numerical example to show the reliability of the proposed methods. In Section 6, several conclusions are drawn.

Notations. The main notations which will be used in this paper are listed in the following. Z^+ is the positive integer set, R^n is the n -dimensional Euclidean space, C^3 is the 3-times continuously differentiable, I_n is the n times n unit matrix, and $\mathbf{0}$ is the zero vector.

2. Preliminaries

Let $\mathcal{V} = \{v_1, v_2, \dots, v_i, \dots, v_n\}$ be the agent set, where $i \in Z^+$, $1 \leq i \leq n$. Let $\mathcal{A} = (a_{ij})_{n \times n}$ be a weighted symmetric adjacent matrix of all agents, where a_{ij} is associated with an edge which is denoted by (i, j) . a_{ij} takes value in 1 or 0, and $a_{ij} = a_{ji}$. $a_{ij} = 1$ refers to the fact that there exists information flow between nodes v_j and v_i , or else $a_{ij} = 0$. Let $\mathcal{E} = \{(i, j) | a_{ij} = 1, i \neq j\}$ be the edge set, where (i, j) is the

edge between v_i and v_j . The neighbor set of v_i is denoted by N_i , where $N_i = \{v_j | (v_i, v_j) \in \mathcal{E}\}$.

Based on $\mathcal{V}$, $\mathcal{A}$, and $\mathcal{E}$, the topology of the multiagent system (MAS) can be presented by a weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$.

Let $\mathcal{G}(\mathbf{q}_l) = (\mathcal{V}, \mathcal{E}(\mathbf{q}_l), \mathcal{A}(\mathbf{q}_l))$ be the interaction topology of dynamic agents, where

$$\mathbf{q}(t) = \mathbf{q}_l, \quad t \in [t, t+a), a > 0, \quad (1)$$

and $\mathcal{G}(\mathbf{q}_l)$ refers to the time-varying topology which is driven by a random process $\mathbf{q}(t)$, and $\mathbf{q}_l \in \mathcal{T}$, where $\mathcal{T} = \{\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_i, \dots, \mathbf{q}_m\}$. The infinitesimal generator of the Markov chain $\{\mathbf{q}(t), t \geq 0\}$ is given by

$$P\{\mathbf{q}(t+a) = \mathbf{q}_j | \mathbf{q}_i\} = \begin{cases} \lambda_{ij}h + o(h), & \text{if } i \neq j, \\ 1 + \lambda_{ii}h + o(h), & \text{otherwise,} \end{cases} \quad (2)$$

where λ_{ij} is the transition rate from state $\mathbf{q}_i$ to $\mathbf{q}_j$; if $\mathbf{q}_i \neq \mathbf{q}_j$, then $\lambda_{ij} \geq 0$, $\sum_{i \neq j} \lambda_{ij} = 0$, and $\lim_{h \rightarrow 0} (o(h)/h) = 0$. $\forall v_i \in \mathcal{V}$, let

$$\mathbf{r}_i(t) = \{x_i(t), y_i(t), z_i(t)\}^T \in R^3 \quad (3)$$

be the state of agent v_i , where $t \in R$, and $x_i(t), y_i(t), z_i(t)$ are the C^3 functions; hence, the state of MAS is shown by

$$\mathbf{r}(t) = \{\mathbf{r}_1(t)^T, \mathbf{r}_2(t)^T, \dots, \mathbf{r}_i(t)^T, \dots, \mathbf{r}_n(t)^T\}^T. \quad (4)$$

Denote

$$\begin{aligned} \alpha_i(t) &= \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}, \\ \gamma_i(t) &= \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}, \\ \beta_i(t) &= \gamma_i(t) \times \alpha_i(t), \end{aligned} \quad (5)$$

where $\alpha_i(t)$ is the tangent unit vector, $\beta_i(t)$ is the normal unit vector, and $\gamma_i(t)$ is the binormal unit vector, and they satisfy the following Frenet–Serret formulas:

$$\begin{cases} \alpha_i'(t) = \kappa_i(t)\beta_i(t), \\ \beta_i'(t) = -\kappa_i(t)\alpha_i(t) + \tau_i(t)\gamma_i(t), \\ \gamma_i'(t) = -\tau_i(t)\beta_i(t), \end{cases} \quad (6)$$

where $\kappa_i(t)$ and $\tau_i(t)$ are the curvature function and torsion function of v_i , respectively.

As is known to all, the trajectory of v_i is decided entirely by the curvature function and the torsion function. Therefore, $\forall v_i$ and v_j , if

$$\begin{aligned} \kappa_i(t) &= \kappa_j(t), \\ \tau_i(t) &= \tau_j(t), \end{aligned} \quad (7)$$

then v_i and v_j have the same trajectory except for the difference of their locations.

For convenience, we denote $\hat{\tau} = \max\{\tau_i(t) | 1 \leq i \leq n\}$, $\hat{\kappa} = \max\{\kappa_i(t) | 1 \leq i \leq n\}$, $\tilde{\tau} = \min\{\tau_i(t) | 1 \leq i \leq n\}$, $\tilde{\kappa} = \min\{\kappa_i(t) | 1 \leq i \leq n\}$.

Assumption 1. We assume that

$$\begin{aligned}\kappa'_i(t) &= 0, \quad t \in [t, t+a), \\ \tau'_i(t) &= 0, \quad t \in [t, t+a).\end{aligned}\quad (8)$$

Consequently, based on the curvature function and torsion function, a consensus definition is proposed in the following.

Definition 1. $\forall v_i, v_j \in \mathcal{V}$, if

$$\begin{aligned}\lim_{t \rightarrow \infty} E\left(\|\kappa_i(t) - \kappa_j(t)\|\right) &= 0, \\ \lim_{t \rightarrow \infty} E\left(\|\tau_i(t) - \tau_j(t)\|\right) &= 0,\end{aligned}\quad (9)$$

then MAS is the consensus in a broad sense of curvature and torsion (BCT consensus).

Remark 1. In literature [31], CT consensus in the sense of curvature and torsion refers to the fact that

$$\begin{aligned}\lim_{t \rightarrow \infty} (\kappa_i(t) - \kappa_j(t)) &= 0, \\ \lim_{t \rightarrow \infty} (\tau_i(t) - \tau_j(t)) &= 0,\end{aligned}\quad (10)$$

and hence CT consensus is the special case of BCT consensus.

Remark 2. In the traditional literatures, e.g., see [8, 32], the consensus refers to the fact that $\forall v_i, v_j \in \mathcal{V}$,

$$\lim_{t \rightarrow \infty} [r_i(t) - r_j(t)] = \mathbf{0}, \quad (11)$$

and it follows that

$$\begin{aligned}\lim_{t \rightarrow \infty} [\kappa_i(t) - \kappa_j(t)] &= 0, \\ \lim_{t \rightarrow \infty} [\tau_i(t) - \tau_j(t)] &= 0,\end{aligned}\quad (12)$$

and the state error between every two agents converges to zero vector. However, in some applications, we need all

agents to keep nonzero distances while their trajectories possess the same shape.

According to the Definition 1, every two agents can keep the nonzero distance. Meanwhile, all trajectories of agents converge to an interest curve.

Remark 3. In Definition 1, if

$$\begin{aligned}\lim_{t \rightarrow \infty} E\left(\|\alpha_i(t) - \alpha_j(t)\|\right) &= 0, \\ \lim_{t \rightarrow \infty} E\left(\|\beta_i(t) - \beta_j(t)\|\right) &= 0, \\ \lim_{t \rightarrow \infty} E\left(\|\gamma_i(t) - \gamma_j(t)\|\right) &= 0,\end{aligned}\quad (13)$$

based on equation (6), it follows that

$$\beta_i(t) \cdot \alpha'_i(t) = \kappa_i(t), \quad (14)$$

and it holds that

$$\lim_{t \rightarrow \infty} E\left(\|\kappa_i(t) - \kappa_j(t)\|\right) = 0. \quad (15)$$

Similarly, it gets that

$$\lim_{t \rightarrow \infty} E\left(\|\tau_i(t) - \tau_j(t)\|\right) = 0. \quad (16)$$

3. Problem Statement

In this section, the protocols are provided for tangent vector, normal vector, and binormal vectors firstly, and then the dynamics of tangent vectors, normal vectors, and binormal vectors are described for MAS.

$\forall v_i \in \mathcal{V}$, the dynamics of $\alpha_i(t), \beta_i(t), \gamma_i(t)$ are described by

$$\begin{cases} \alpha'_i(t) = \kappa_i(t)\beta_i(t) + u_i^1(t), \\ \beta'_i(t) = -\kappa_i(t)\alpha_i(t) + \tau_i(t)\gamma_i(t) + u_i^2(t), \\ \gamma'_i(t) = -\tau_i(t)\beta_i(t) + u_i^3(t), \end{cases} \quad (17)$$

where $\forall t \in [s, s+a), \kappa'_i(t) = 0, \tau'_i(t) = 0, u_i^1(t), u_i^2(t),$ and $u_i^3(t)$ are the protocols of agent v_i , and they are defined by

$$\begin{cases} u_i^1(t) = \kappa_i(t-a) \sum_{j \in N_i} a_{ij} [\alpha_j(t-a) - \alpha_i(t-a) + \sigma(t)w_{ij}] - \frac{1}{d_i} \sum_{j \in N_i} a_{ij} [\kappa_i(t)\beta_j(t)], \\ u_i^2(t) = [\tau_i(t-a) - \kappa_i(t-a)] \sum_{j \in N_i} a_{ij} [\beta_j(t-a) - \beta_i(t-a) + \sigma(t)w_{ij}] + \frac{1}{d_i} \sum_{j \in N_i} a_{ij} [\kappa_i(t)\alpha_j(t) - \tau_i(t)\gamma_j(t)], \\ u_i^3(t) = \tau_i(t-a) \sum_{j \in N_i} a_{ij} [\gamma_j(t-a) - \gamma_i(t-a) + \sigma(t)w_{ij}] + \frac{1}{d_i} \sum_{j \in N_i} a_{ij} [\tau_i(t)\beta_j(t)], \end{cases} \quad (18)$$

where w_{ij} is the white noise, $\sigma(t) \geq 0$, and random press $\varrho(t)$ and w_{ij} are independent of each other.

From (17) and (18), we obtain that

$$\begin{aligned} \begin{bmatrix} \alpha'(t) \\ \beta'(t) \\ \gamma'(t) \end{bmatrix} &= \begin{bmatrix} 0 & \Pi_{12} & 0 \\ \Pi_{21} & 0 & \Pi_{23} \\ 0 & \Pi_{32} & 0 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \beta(t) \\ \gamma(t) \end{bmatrix} \\ &\quad - \begin{bmatrix} K(\varrho_l)L(\varrho_l) & 0 & 0 \\ 0 & -[T(\varrho_l) - K(\varrho_l)]L(\varrho_l) & 0 \\ 0 & 0 & T(\varrho_l)L(\varrho_l) \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \alpha(t-a) \\ \beta(t-a) \\ \gamma(t-a) \end{bmatrix} \\ &\quad + \begin{bmatrix} \Lambda_{11} & 0 & 0 \\ 0 & \Lambda_{22} & 0 \\ 0 & 0 & \Lambda_{33} \end{bmatrix} \sigma(t)CW, \end{aligned} \quad (19)$$

where

$$\begin{aligned} \alpha &= (\alpha_1^T(t), \alpha_2^T(t), \dots, \alpha_n^T(t))^T, \\ \beta &= (\beta_1^T(t), \beta_2^T(t), \dots, \beta_n^T(t))^T, \\ \gamma &= (\gamma_1^T(t), \gamma_2^T(t), \dots, \gamma_n^T(t))^T, \\ W_i^T &= \{w_{i0}, w_{i1}, \dots, w_{in}\}^T, \\ W &= \{W_1^T, \dots, W_n^T\}^T. \end{aligned} \quad (20)$$

Denote $X(t) = (\alpha^T(t), \beta^T(t), \gamma^T(t))^T$, and we have

$$X'(t) = \Pi(t)X(t) - \Sigma(t)X(t-a) + \Sigma(t)\sigma(t)CW, \quad (21)$$

where

$$\begin{aligned} \Pi &= \begin{bmatrix} 0 & \Pi_{12} & 0 \\ \Pi_{21} & 0 & \Pi_{23} \\ 0 & \Pi_{32} & 0 \end{bmatrix}, \\ \Sigma(t) &= \begin{bmatrix} \Sigma_1 & 0 & 0 \\ 0 & \Sigma_2 & 0 \\ 0 & 0 & \Sigma_3 \end{bmatrix}, \\ D(\varrho_l) &= \text{diag}\left\{\frac{1}{d_1}, \frac{1}{d_2}, \dots, \frac{1}{d_n}\right\}, \\ \Pi_{12} &= DK(\varrho_l)L(\varrho_l), \\ \Pi_{21} &= -D(\varrho_l)K(\varrho_l)L(\varrho_l), \\ \Pi_{23} &= D(\varrho_l)T(\varrho_l)L(\varrho_l), \\ \Pi_{32} &= -D(\varrho_l)T(\varrho_l)L(\varrho_l), \\ \Sigma_1 &= K(\varrho_l)L(\varrho_l), \\ \Sigma_2 &= -[K(\varrho_l) - T(\varrho_l)]L(\varrho_l), \\ \Sigma_3 &= TL(\varrho_l), \\ K(\varrho_l) &= \{\kappa_1(t), \kappa_2(t), \dots, \kappa_n(t)\}, \\ T(\varrho_l) &= \{\tau_1(t), \tau_2(t), \dots, \tau_n(t)\}, \end{aligned} \quad (22)$$

where C is a $n^2 \times n$ matrix, and

$$C = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & a_{21} & 0 & \cdots & a_{2n} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & a_{n1} & \cdots & 0 \end{bmatrix}. \quad (23)$$

In the following, the *BCT* consensus of system (21) will be studied.

4. The Main Results

In this section, the consensus problem is studied for system (21). Firstly, the state error of system (21) is discussed. Let $x_0(t) = (\alpha_0(t), \beta_0(t), \gamma_0(t))$ be the aim state of the MAS, where

$$\begin{aligned} \alpha_0(t) &= \frac{1}{n} \sum_{i=1}^n \alpha_i, \\ \beta_0(t) &= \frac{1}{n} \sum_{i=1}^n \beta_i, \\ \gamma_0(t) &= \frac{1}{n} \sum_{i=1}^n \gamma_i. \end{aligned} \quad (24)$$

Denote $e(t) = X(t) - x_0(t) \otimes I_{n+1} = (\delta_1(t)^T, \delta_2(t)^T, \delta_3(t)^T)^T$; then, it follows that

$$\begin{aligned} \delta_1(t) &= \alpha(t) - \alpha_0(t), \\ \delta_2(t) &= \beta(t) - \beta_0(t), \\ \delta_3(t) &= \gamma(t) - \gamma_0(t), \end{aligned} \quad (25)$$

and it holds that

$$e(t) = X(t) - x_0(t) \otimes I_{n+1} = (I - P)X(t), \quad (26)$$

where

$$P = \begin{bmatrix} P_{11} & 0 & 0 \\ 0 & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix}, \quad (27)$$

$$P_{11} = P_{22} = P_{33} = \frac{1}{n} \mathbf{1}_{n \times n}.$$

It follows that

$$\begin{aligned} e'(t) &= (I - P)X'(t) \\ &= (I - P)\Pi X(t) - (I - P)\Sigma(t)X(t-a) \\ &\quad + (I - P)\Sigma(t)\sigma(t)CW. \end{aligned} \quad (28)$$

Notice that $(\mathbf{1}_{n \times n}/n)L(\varrho_l) = 0$, and

$$\begin{aligned} \sum_{j \in N_i} [\alpha_j(t) - \alpha_i(t)] &= \sum_{j \in N_i} [\alpha_j(t) - \alpha_0(t) - \alpha_i(t) + \alpha_0(t)] \\ &= \sum_{j \in N_i} [e_j(t) - e_i(t)]. \end{aligned} \quad (29)$$

Hence, $L(\varrho_i)X(t) = L(\varrho_i)e(t)$, and system (28) can be shown by

$$e'(t) = (I - P)\Pi(t)e(t) + (I - P)\Sigma(t)e(t - a) + (I - P)\Sigma(t)CW, \quad (30)$$

$$de(t) = f(\varrho_i, e(t), e(t - a))dt + g(\varrho_i, e(t), e(t - a))dB(t), \quad (31)$$

where

$$\begin{aligned} f(\varrho_i, e(t), e(t - a)) &= (I - P)\Pi(t)e(t) \\ &\quad - (I - P)\Sigma(t)(t)e(t - a), \end{aligned} \quad (32)$$

$$g(\varrho_i, e(t), e(t - a)) = (I - P)\Sigma(t)\sigma(t)C.$$

Lemma 1 (see [33]). Let $a'(t) \leq \bar{a}(t > 0)$ hold. Assume that there is a function $V \in C^{2,1}(R^n \times R_+; R_+)$ and positive constants p, c_1, c_2, c_3, c_4 such that $c_3 > c_4/(1 - \bar{a})$,

$$\begin{aligned} c_1|e(t)|^p &\leq V(e(t), t, \varrho_i) \leq c_2|e(t)|^p, \\ LV(e(t), t, \varrho_i) &\leq -\lambda_1\|e(t)\|^p + \lambda_2\|e(t - a)\|^p, \end{aligned} \quad (33)$$

for all $e(t), e(t - a) \in R^n$, $t \geq 0$, and $\rho_i \in \mathcal{T}$. Then, equation (31) is asymptotically stable in p th moment.

Lemma 2 (see [33]). Let $a'(t) \leq \bar{a}(t > 0)$ hold and $p \geq 2$. Assume that for each $i \in \mathcal{T}$, there is a pair of real numbers ς_{ρ_i} and σ_{ρ_i} such that

$$\begin{aligned} e(t)^T f(e(t), e(t - a(t))) + \frac{p-1}{2} \|g(e(t), e(t - a(t)))\|^2 \\ \leq \varsigma_{\rho_i} \|e(t)\|^2 + \sigma_{\rho_i} \|e(t - a)\|^2, \end{aligned} \quad (34)$$

for all $(e(t), e(t - a), t) \in R^n \times R^n \times R_+$. Assume that

$$\mathcal{B} := -\text{diag}\{p\varsigma_{\rho_1}, \dots, p\varsigma_{\rho_m}\} - \Gamma \quad (35)$$

is a nonsingular M -matrix, so $(q_1, \dots, q_m)^T := \mathcal{B}^{-1}\vec{1} \gg 0$. If

$$pq_{\rho_i}\sigma_{\rho_i} < 1 - \bar{a}, \varsigma_{\rho_i} \in \mathcal{T}, \quad (36)$$

then equation (31) is asymptotically stable in p th moment.

Then, we discuss the stability of system (31). Let the Lyapunov candidate function be presented as

$$V(t) = \frac{1}{2} e(t)^T e(t), \quad (37)$$

and it follows that

$$\begin{aligned} e(t)^T [(I - P)\Pi(t)e(t)] &= [\delta_1(t)^T, \delta_2(t)^T, \delta_3(t)^T] (I - P)\Pi[\delta_1(t)^T, \delta_2(t)^T, \delta_3(t)^T]^T \\ &= -\delta_2(t)^T (I - P_{22})\Pi_{12}\delta_1(t) + \delta_1(t)^T (I - P_{11})\Pi_{12}\delta_2(t) \\ &\quad - \delta_3(t)^T (I - P_{33})\Pi_{23}\delta_2(t) + \delta_2(t)^T (I - P_{22})\Pi_{23}\delta_3(t) \\ &= 0, \end{aligned}$$

$$\begin{aligned} e(t)^T [(I - P)\Sigma(t)e(t - a)] &= [\delta_1(t)^T, \delta_2(t)^T, \delta_3(t)^T] (I - P)\Sigma(t)[\delta_1(t - a)^T, \delta_2(t - a)^T, \delta_3(t - a)^T]^T \\ &= \sum_{i=1}^3 \delta_i(t)^T (I - P_{ii})\Sigma_i(t)\delta_i(t - a) = \sum_{i=1}^3 \frac{1}{2} [\delta_i(t)^T (I - P_{ii})\Sigma_i(t)\delta_i(t) + \delta_i(t - a)^T (I - P_{ii})\Sigma_i(t)\delta_i(t - a) \\ &\quad - (\delta_i(t) - \delta_i(t - a))^T (I - P_{ii})\Sigma_i(t)(\delta_i(t) - \delta_i(t - a))]. \end{aligned} \quad (38)$$

To estimate the value of $e(t)^T [(I - P)\Sigma(t)e(t - a)]$, we need the following conditions and we also need the following assumptions.

Assumption 2. In this paper, we assume that

$$\hat{\kappa} \geq \check{\tau} \geq 0, \quad (39)$$

Assumption 3. Let $\tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3$ satisfy the following inequalities:

$$\begin{aligned} \|\tilde{\delta}_1\| &\leq \sqrt{k_1}\|\hat{\delta}_1\|, \\ \|\tilde{\delta}_2\| &\geq \sqrt{k_2}\|\hat{\delta}_2\|, \\ \|\tilde{\delta}_3\| &\leq \sqrt{k_3}\|\hat{\delta}_3\|, \end{aligned} \quad (40)$$

where $\tilde{\delta}_i = \delta_i(t) - \delta_i(t-a)$.

Under Assumption 3, it holds that

$$\begin{aligned}
 & \sum_{i=1}^3 \delta_i(t)^T (I - P_{ii}) \Sigma_i(t) \delta_i(t) \leq \sum_{i=1}^3 \|(I - P_{ii}) \Sigma_i(t)\| \|\delta_i(t)\|^T \delta_i(t), \\
 & e(t)^T [(I - P) \Sigma(t) e(t-a)] \\
 & = [\delta_1(t)^T, \delta_2(t)^T, \delta_3(t)^T] (I - P) \Sigma(t) [\delta_1(t-a)^T, \delta_2(t-a)^T, \delta_3(t-a)^T]^T \\
 & = \sum_{i=1}^3 \delta_i(t)^T (I - P_{ii}) \Sigma_i(t) \delta_i(t-a) \\
 & = \sum_{i=1}^3 \frac{1}{2} [\delta_i(t)^T (I - P_{ii}) \Sigma_i(t) \delta_i(t) + \delta_i(t-a)^T (I - P_{ii}) \Sigma_i(t) \delta_i(t-a) \\
 & \quad - (\delta_i(t) - \delta_i(t-a))^T (I - P_{ii}) \Sigma_i(t) (\delta_i(t) - \delta_i(t-a))].
 \end{aligned} \tag{41}$$

Notice that $\lambda(P_{ii}KL) = 0$; then, it follows that

$$\lambda[(I - P_{ii})KL] = \lambda(KL), \tag{42}$$

$$\lambda_2 \tilde{\kappa} \leq \lambda(KL) \leq \lambda_M \hat{\kappa}, \tag{43}$$

and we have

and it holds that

$$\begin{aligned}
 & -\delta_1(t)^T (I - P_1) \Sigma_1 \delta_1(t) = -\delta_1(t)^T KL \delta_1(t) \\
 & \leq -\tilde{\kappa} \lambda_2 \delta_1(t)^T \delta_1(t), \\
 & \delta_2(t)^T (I - P_2) \Sigma_2 \delta_2(t) = \delta_2(t)^T KL \delta_2(t) - \delta_2(t)^T TL \delta_2(t) \\
 & \leq [\hat{\kappa} \lambda_M - \tilde{\tau} \lambda_2] \delta_2(t)^T \delta_2(t), \\
 & -\delta_3(t)^T (I - P_3) \Sigma_3 \delta_3(t) = -\delta_3(t)^T KL \delta_3(t) \\
 & \leq -\tilde{\tau} \lambda_2 \delta_3(t)^T \delta_3(t).
 \end{aligned} \tag{44}$$

Similarly, it follows that

$$\begin{aligned}
 & -\delta_1(t-a)^T (I - P_1) \Sigma_1 \delta_1(t-a) \leq -\tilde{\kappa} \lambda_2 \delta_1(t-a)^T \delta_1(t-a), \\
 & \delta_2(t-a)^T (I - P_2) \Sigma_2 \delta_2(t-a) \leq \hat{\kappa} \lambda_M \delta_2(t-a)^T \delta_2(t-a) - \tilde{\tau} \lambda_2 \delta_2(t-a)^T \delta_2(t-a) \\
 & = (\hat{\kappa} \lambda_M t - n \tilde{\tau} q \lambda_2) \delta_2(t-a)^T \delta_2(t-a), \\
 & -\delta_3(t-a)^T (I - P_3) \Sigma_3 \delta_3(t-a) \leq -\tilde{\tau} \lambda_2 \delta_3(t-a)^T \delta_3(t-a).
 \end{aligned} \tag{45}$$

Assumption 4. We assume that

where $\tilde{\delta}_i(t) = \delta_i(t) - \delta_i(t-a)$, $h_i(t) \geq 0$ and $g_i(t) \geq 0$.

$$h_i(t) \delta_i^T(t) \delta_i(t) \leq \tilde{\delta}_i^T(t) \tilde{\delta}_i(t) \leq g_i(t) \delta_i^T(t) \delta_i(t), \tag{46}$$

In addition, $\tilde{\delta}_i(t)$, ($i = 1, 2, 3$) satisfy the following inequalities:

$$\begin{aligned}
& \tilde{\delta}_1(t)^T (I - P_1) \Sigma_1 \tilde{\delta}_1(t) = \tilde{\delta}_1(t)^T K L \tilde{\delta}_1(t) \\
& \leq \tilde{\kappa} \lambda_M \tilde{\delta}_1(t)^T \tilde{\delta}_1(t) \\
& \leq g_i(t) \tilde{\kappa} \lambda_M \delta_1(t)^T \delta_1(t), \\
& \tilde{\delta}_2(t)^T (I - P_2) \Sigma_2 \tilde{\delta}_2(t) = -\tilde{\delta}_2(t)^T K L \tilde{\delta}_2(t) + \tilde{\delta}_2(t)^T T L \tilde{\delta}_2(t), \\
& \leq -h_2(t) \tilde{\kappa} \lambda_2 \delta_2(t)^T \delta_2(t) + g_2(t) \tilde{\tau} \lambda_2 \delta_2(t)^T \delta_2(t) \\
& = (-\tilde{\kappa} \lambda_2 h_2(t) + \tilde{\tau} \lambda_M g_2(t)) \delta_2(t)^T \delta_2(t), \\
& \tilde{\delta}_3(t)^T (I - P_3) \Sigma_3 \tilde{\delta}_3(t) = \tilde{\delta}_3(t)^T K L \tilde{\delta}_3(t) \\
& \leq \tilde{\tau} \lambda_M g_3(t) \delta_3(t)^T \delta_3(t).
\end{aligned} \tag{47}$$

For convenience, denote

$$\begin{aligned}
f_1(\delta_1) &= [-\tilde{\kappa} \lambda_2 + g_i(t) \tilde{\kappa} \lambda_M] \delta_1(t)^T \delta_1(t), \\
f_2(\delta_1) &= [\tilde{\kappa} \lambda_M - h_2(t) \tilde{\kappa} \lambda_2 - \tilde{\tau} \lambda_2 + \tilde{\tau} \lambda_M g_2(t)] \delta_2(t)^T \delta_2(t), \\
f_3(\delta_1) &= [-\tilde{\tau} \lambda_2 + \tilde{\tau} \lambda_M g_3(t)] \delta_3(t)^T \delta_3(t).
\end{aligned} \tag{48}$$

On the other hand, it holds that

$$\begin{aligned}
& \text{trace}[(I - P) \Sigma(t) C]^T (I - P) \Sigma(t) C \sigma(t) \\
& \leq \text{trace}[C^T \Sigma(t)^T \Sigma(t) C \sigma(t)] \\
& = 2\sigma(t)^2 \sum_{i=1}^s d_i(\kappa_i^2 + \tau_i^2 - \kappa_i \tau_i) \\
& \leq 2s\sigma(t)^2 d_M(\tilde{\kappa}^2 + \tilde{\tau}^2 - \tilde{\kappa} \tilde{\tau}), \\
& e(t)^T f(\mathbf{q}_i, e(t), e(t-a)) + \frac{1}{2} |g(\mathbf{q}_i, e(t), e(t-a))|^2 \\
& \leq e(t)^T [(I - P) \Pi(t) e(t)] + e(t)^T [(I - P) \Sigma(t) e(t-a)] + \\
& \text{trace}[(I - P) \Sigma(t) C]^T (I - P) \Sigma(t) C \sigma(t) \\
& \leq f_1(\delta_1) + f_2(\delta_2) + f_3(\delta_3) + 2n\sigma(t)^2 d_M(\tilde{\kappa}^2 + \tilde{\tau}^2 - \tilde{\kappa} \tilde{\tau}),
\end{aligned} \tag{49}$$

and hence it gets the following result.

Theorem 1. *If $\mathcal{G}(\mathbf{q}_i)$ is connected and if*

$$\begin{aligned}
g_1(t) &= \frac{\tilde{\tau} \lambda_M}{\tilde{\kappa} \lambda_M} g_3(t), \\
g_3(t) &= \frac{\tilde{\kappa}}{\tilde{\tau}} - \frac{\tilde{\kappa} \lambda_2}{\tilde{\tau} \lambda_M} h_2(t) + g_2(t), \\
\sigma(t)^2 &< \frac{\nu + \tilde{\tau} \lambda_2 - \tilde{\kappa} \lambda_M}{2nd_M(\tilde{\kappa}^2 + \tilde{\tau}^2 - \tilde{\kappa} \tilde{\tau})} e(t-a)^T e(t-a),
\end{aligned} \tag{50}$$

where ν is a real number, and $\nu < 1$, $2q_i \sigma_i \leq 1$, then system (31) is asymptotically stable, where λ_2 is the second largest

eigenvalue of L , $A - 2 = \text{diag}\{\omega_1, \omega_2, \dots, \omega_m\} - \Gamma$ is a non-singular M -matrix, and $(q_1, q_2, \dots, q_m)^T = A^{-1} \vec{1}$. Therefore, system (31) is the BCT consensus.

Proof. Define the function $V: R^n \times R_+ \times S \rightarrow R_+$ by

$$V(\delta, \mathbf{q}_i, t) = q_i \sum_{j=1}^3 V_j = q_i \sum_{j=1}^3 \delta_j^T \delta_j, \tag{51}$$

with $c_1 = \min_{i \in \mathcal{I}} q_i$ and $c_2 = \max_{i \in \mathcal{I}} q_i$. Applying the generalised Itô formula to $e^{s^t} V_j$, it holds that

$$\begin{aligned}
LV_j &\leq 2q_i [\delta_j(t)^T f(\mathbf{q}_i, \delta_j(t), \delta_j(t-a)) \\
&+ \frac{1}{2} |g(\mathbf{q}_i, \delta_j(t), \delta_j(t-a))|^2] + \sum_{r=1}^m \gamma_{ir} q_i \delta_j(t)^T \delta_j(t),
\end{aligned} \tag{52}$$

and hence

$$\begin{aligned}
LV &\leq 2q_i [e(t)^T f(\mathbf{q}_i, e(t), e(t-a)) + \frac{1}{2} |g(\mathbf{q}_i, e(t), e(t-a))|^2] \\
&+ \sum_{r=1}^m \gamma_{ir} q_i e(t)^T e(t) \\
&\leq 2q_i [-\tilde{\kappa} \lambda_2 + g_i(t) \tilde{\kappa} \lambda_M] \delta_1(t)^T \delta_1(t) - \tilde{\kappa} \lambda_2 \delta_1(t-a)^T \delta_1(t-a) \\
&+ \sum_{r=1}^m \gamma_{ir} q_i \delta_1(t)^T \delta_1(t) + 2q_i [\tilde{\kappa} \lambda_M - h_2(t) \tilde{\kappa} \lambda_2 - \tilde{\tau} \lambda_2 \\
&+ \tilde{\tau} \lambda_M g_2(t)] \delta_2(t)^T \delta_2(t) \\
&+ (\tilde{\kappa} \lambda_M - \tilde{\tau} \lambda_2) \delta_2(t-a)^T \delta_2(t-a) + 2n\sigma(t)^2 d_M(\tilde{\kappa}^2 + \tilde{\tau}^2 - \tilde{\kappa} \tilde{\tau}) \\
&+ \sum_{r=1}^m \gamma_{ir} q_i \delta_2(t)^T \delta_2(t) \\
&+ [-\tilde{\tau} \lambda_2 + \tilde{\tau} \lambda_M g_3(t)] \delta_3(t)^T \delta_3(t) - \tilde{\tau} \lambda_2 \delta_3(t-a)^T \delta_3(t-a) \\
&+ \sum_{r=1}^m \gamma_{ir} q_i \delta_3(t)^T \delta_3(t) \\
&\leq -\delta_1(t)^T \delta_1(t) - \tilde{\kappa} \lambda_2 \delta_1(t-a)^T \delta_1(t-a) - \delta_2(t)^T \delta_2(t) \\
&+ (\tilde{\kappa} \lambda_M - \tilde{\tau} \lambda_2) \delta_2(t-a)^T \delta_2(t-a) - \delta_3(t)^T \\
&\delta_3(t) - \tilde{\tau} \lambda_2 \delta_3(t-a)^T \delta_3(t-a) + 2n\sigma(t)^2 d_M(\tilde{\kappa}^2 + \tilde{\tau}^2 - \tilde{\kappa} \tilde{\tau}) \\
&\leq -e(t)^T e(t) + (\tilde{\kappa} \lambda_M - \tilde{\tau} \lambda_2) e(t-a)^T e(t-a) \\
&\leq -e(t)^T e(t) + \nu e(t-a)^T e(t-a),
\end{aligned} \tag{53}$$

and according to Lemma 1, system (31) is asymptotically stable.

So, it holds that

$$\lim_{t \rightarrow \infty} E\|e(t)\| = 0, \tag{54}$$

and that system (31) is the BCT consensus. $\square$

FIGURE 1: The topologies of MAS (21).

TABLE 1: The values of curvature functions and torsion functions with different topologies.

κ and τ values of κ and τ topologies	ϱ_1	ϱ_2	ϱ_3
(κ_1, τ_1)	(0.1, 0.21)	(0.2, 0.1)	(0.25, 0.23)
(κ_2, τ_2)	(0.2, 0.21)	(0.33, 0.22)	(0.31, 0.2)
(κ_3, τ_3)	(0.23, 0.2)	(0.2, 0.1)	(0.25, 0.21)

FIGURE 2: Component coordinates of vectors α_1, α_2 , and α_3 converging to a same value.

FIGURE 3: Component coordinates of vectors β_1, β_2 , and β_3 converging to a same value.

Theorem 2. If $\mathcal{G}$ is connected and if

$$\begin{aligned} g_1(t) &= \frac{\hat{\tau}\lambda_M}{\hat{\kappa}\lambda_M} g_3(t), \\ g_3(t) &= \frac{\hat{\kappa}}{\hat{\tau}} - \frac{\check{\kappa}\lambda_2}{\hat{\tau}\lambda_M} h_2(t) + g_2(t), \\ \sigma(t)^2 &< \frac{\nu + \check{\tau}\lambda_2 - \hat{\kappa}\lambda_M}{2nd_M(\hat{\kappa}^2 + \hat{\tau}^2 - \check{\kappa}\check{\tau})} e(t-a)^T e(t-a), \end{aligned} \quad (55)$$

where ν is a real number, and $\nu < 1$, $\nu e^{a\varsigma_1} = 1$, then system (31) is asymptotically stable. Therefore, system (31) is the BCT consensus.

Proof. Since

$$\begin{aligned} \lambda e^{\varsigma_j t} V_j + e^{\varsigma_j t} L V_j &\leq \varsigma_j e^{\varsigma_j t} V_j + e^{\varsigma_j t} \\ &\cdot [f_j(t) - \check{\kappa}\lambda_2 \delta_1(t-a)^T \delta_1(t-a) + 2n\sigma(t)^2 c_3], \end{aligned} \quad (56)$$

where $c_3 = d_M(\hat{\kappa}^2 + \hat{\tau}^2 - \check{\kappa}\check{\tau})$, then

$$\begin{aligned} c_1 e^{\varsigma_j t} E|e(t)|^2 &\leq c_2 E[\xi_j(0)]^2 \\ &+ E\left[\int_0^t e^{\varsigma_j s} (-e(s)^T e(s) + \nu e(s-a)^T e(s-a)) ds\right], \end{aligned} \quad (57)$$

where

FIGURE 4: Component coordinates of vectors γ_1, γ_2 , and γ_3 converging to a same value.

$$\begin{aligned}
 & \int_0^t e^{\varsigma_1 s} \left(\nu e(s-a)^T e(s-a) \right) ds \\
 & \leq \nu e^{a\varsigma_1} \left[\int_{-a}^t e^{\varsigma_1 (s-a)} \left(\nu e(s-a)^T e(s-a) \right) d(s-a) \right] \\
 & = \nu e^{a\varsigma_1} \int_0^t e^{\varsigma_1 s} \left(e(s)^T e(s) \right) ds + \nu e^{a\varsigma_1} \int_{-a}^0 e^{\varsigma_1 s} \left(e(s)^T e(s) \right) ds.
 \end{aligned} \tag{58}$$

Since it holds that $\nu e^{a\varsigma_1} = 1$,

$$c_1 e^{\varsigma_1 t} E|e(t)|^2 \leq c_2 E|\xi_j(0)|^2 + \nu e^{a\varsigma_1} E\|\xi_1\|^2, \tag{59}$$

and ς_1 satisfies that

$$-\frac{\lambda_1}{2} \tilde{\kappa} + \frac{1}{2} e^{a\varsigma_1} (\lambda_M k_1 \tilde{\kappa} - \lambda_1 \tilde{\kappa}) = 0, \tag{60}$$

and according to Lemma 2, system (31) is asymptotically stable.

So, it holds that

$$\lim_{t \rightarrow \infty} E\|e(t)\| = 0, \tag{61}$$

and that system (31) is the *BCT* consensus. $\square$

5. A Numerical Example

To illustrate the effect of the methods in the main results, an example is proposed in the following.

Example 1. Consider a connected MAS (21) with three agents, and the topologies of (21) are shown in Figure 1, where $\mathcal{V} = \{v_1, v_2, v_3\}$. According to the different topologies, the related curvature functions and torsion functions are provided in Table 1.

$$\begin{aligned}
X(t) &= \{r_1(t), r_2(t), r_3(t)\}^T, \\
r_1(0) &= \{17, 0, 0\}^T, \\
r_2(0) &= \{50, 0, 0\}^T, \\
r_3(0) &= \{20, 0, 0\}^T.
\end{aligned} \tag{62}$$

The related matrices of equation (21) are listed below.

$$\begin{aligned}
\Pi_{12}(\varrho_1) &= \begin{bmatrix} 0.1 & -0.1 & 0 \\ -0.1 & 0.2 & -0.1 \\ 0 & -0.25 & 0.25 \end{bmatrix} = -\Pi_{21}(\varrho_1), \\
\Pi_{23}(\varrho_1) &= \begin{bmatrix} 0.21 & -0.21 & 0 \\ -0.05 & 0.1 & -0.05 \\ 0 & -0.22 & 0.22 \end{bmatrix} = -\Pi_{32}(\varrho_1), \\
\Pi_{12}(\varrho_2) &= \begin{bmatrix} 0.2 & 0 & -0.2 \\ 0 & 0.33 & -0.33 \\ -0.31 & -0.31 & 0.62 \end{bmatrix} = -\Pi_{21}(\varrho_2), \\
\Pi_{23}(\varrho_2) &= \begin{bmatrix} 0.21 & 0 & -0.21 \\ 0 & 0.22 & -0.22 \\ -0.11 & -0.11 & 0.22 \end{bmatrix} = -\Pi_{32}(\varrho_2), \\
\Pi_{12}(\varrho_3) &= \begin{bmatrix} 0.23 & -0.115 & -0.115 \\ -0.2 & 0.2 & 0 \\ -0.25 & 0 & 0.25 \end{bmatrix} = -\Pi_{21}(\varrho_3), \\
\Pi_{23}(\varrho_3) &= \begin{bmatrix} 0.2 & -0.1 & -0.1 \\ -0.1 & 0.1 & -0 \\ -0.21 & 0 & 0.21 \end{bmatrix} = -\Pi_{32}(\varrho_3), \\
\Sigma_1(\varrho_1) &= \begin{bmatrix} 0.1 & -0.1 & 0 \\ -0.2 & 0.4 & 0.2 \\ 0 & -0.25 & 0.25 \end{bmatrix}, \\
\Sigma_2(\varrho_1) &= \begin{bmatrix} 0.01 & -0.01 & 0 \\ 0.1 & -0.2 & 0.1 \\ 0 & 0.03 & -0.03 \end{bmatrix}, \\
\Sigma_3(\varrho_1) &= \begin{bmatrix} 0.21 & -0.21 & 0 \\ -0.1 & 0.2 & -0.1 \\ 0 & -0.22 & 0.22 \end{bmatrix}, \\
\Sigma_1(\varrho_2) &= \begin{bmatrix} 0.2 & 0 & -0.2 \\ 0 & 0.33 & -0.33 \\ -0.31 & -0.31 & 0.62 \end{bmatrix}, \\
\Sigma_2(\varrho_2) &= \begin{bmatrix} 0.01 & 0 & -0.01 \\ 0 & -0.11 & 0.11 \\ 0.11 & 0.11 & -0.22 \end{bmatrix}, \\
\Sigma_3(\varrho_2) &= \begin{bmatrix} 0.21 & 0 & -0.21 \\ 0 & 0.22 & -0.22 \\ -0.2 & -0.2 & 0.4 \end{bmatrix}, \\
\Sigma_1(\varrho_3) &= \begin{bmatrix} 0.46 & -0.23 & -0.23 \\ -0.2 & 0.2 & 0 \\ -0.25 & 0 & 0.25 \end{bmatrix}, \\
\Sigma_2(\varrho_3) &= \begin{bmatrix} -0.06 & 0.03 & 0.03 \\ 0.1 & -0.1 & 0 \\ 0.04 & 0 & -0.04 \end{bmatrix}, \\
\Sigma_3(\varrho_3) &= \begin{bmatrix} 0.4 & -0.2 & -0.2 \\ -0.1 & 0.1 & 0 \\ -0.21 & 0 & 0.21 \end{bmatrix}.
\end{aligned} \tag{63}$$

From the following Laplacian matrices:

$$\begin{aligned}
L(\varrho_1) &= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \\
L(\varrho_2) &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \\
L(\varrho_3) &= \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix},
\end{aligned} \tag{64}$$

it obtains that $\lambda_2 = 1, \lambda_M = 3$. Suppose that $\nu = 0.8$, $h_2(t) = 0.5$,

$$\begin{aligned}
\sigma(t, \varrho_1)^2 &< 0.2e(t-a)^T e(t-a), \\
\sigma(t, \varrho_2)^2 &< \frac{1}{72}e(t-a)^T e(t-a), \\
\sigma(t, \varrho_3)^2 &< 0.12e(t-a)^T e(t-a), \\
g_1((t, \varrho)) &= \frac{1}{3}, \\
g_2((t, \varrho)) &= \frac{\hat{\kappa}}{6\hat{\tau}}, \\
g_3((t, \varrho)) &= \frac{\hat{\kappa}}{\hat{\tau}},
\end{aligned} \tag{65}$$

$$\Gamma = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

According to Theorem 1, under the control of protocol (17), MAS (21) achieves the *BCT* consensus (see the simulation results of Figures 2–4).

Figures 2–4 show that under protocol (18), every component coordinate of vectors $\alpha_i(t), \beta_i(t), \gamma_i(t)$ converges to a common value, respectively, curvature functions and torsion functions converge to a value, respectively, and all agents converge to the *BCT* consensus. In some applications, the protocol ensures that all agents keep the same trajectories; in the meanwhile, agents can also keep nonzero distances.

6. Conclusion

In this paper, we have studied the consensus problem in the sense of torsion and curvature for stochastic MAS with Markovian switching topology and time delay. A kind of distributed consensus protocol has been designed for MAS, and we have gotten the MAS which is updating upon torsion function and curvature function, where the protocol is based on the Frenet–Serret formulas. By employing the Lyapunov functional method and the stochastic approaching, several

consensus criteria have been obtained, the results have shown that the protocol can ensure the system achieves the consensus, and the criteria have been analysed and illustrated by simulations. However, some challenging problems have not been considered in this paper, such as MAS in the small-world framework or systems with unknown partial Markovian transformation rate, and the related consensus problems will be studied by us in the future.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study was partially supported by the National Natural Science Foundation of China (61673257), Fund of Zhejiang Provincial Education Department (Y201941722), Cultivation Fund of Taizhou University (2019PY07), Research Initiation Fund of Taizhou University (0104010004), and Fund of Zhejiang College Student of Science and Technology Innovation Activity Program and New Talent Program (2019R436008).

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