

Research Article

Optimal Strategies of Retailers Facing Potential Crisis in an Online-to-Offline Supply Chain

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We confine our interest to the O2O (online-to-offline) supply chain system consisting of an online retailer and an offline retailer. Given that the brand they sell may encounter a brand crisis that will damage the goodwill, we formulate an O2O supply chain model with the impact of random crisis to explore the countermeasures of retailers when facing a potential crisis. After analysis, we find the following: (1) The crisis happened earlier with the increase of hazard rate and retailers should lower their investment in the precrisis stage. (2) The existence of crisis divides the whole planning period into two phases and make retailers have different phase preference in different scenarios. In a word, retailers will pay more attention to the postcrisis stage with the increase of hazard rate and damage rate and therefore invest more in the postcrisis stage. (3) Crisis will decrease the investment level of retailers and therefore make the goodwill and profits lower than when there is no crisis.

1. Introduction

It is evitable for any company to encounter a brand crisis in the operation process. On the one hand, the crisis may be related to quality issues. For example, Volkswagen recalled almost four hundred thousand defective vehicles in China for its potential risk in DSG gearbox in 2013. Samsung suffered a tremendous loss in sales due to many reported explosion events of Galaxy note 7 for a fatal defect in the battery. On the other hand, the brand crisis may be triggered by illegal behavior or failing to fulfill social responsibility. See what happened to GE when their financial scandal was disclosed. Sometimes, the crisis is not due to the company's certain behavior but, for example, caused by the negative image of the endorser of the brand. This kind of event that occurs randomly and unexceptionally will inflict damage on the goodwill, sale, and profitability of the company.

Hence, farsighted companies cannot afford to ignore the likelihood and possible consequence of the crisis when making their decisions especially when the O2O pattern is gaining prevalence and the crisis may cause wider influence. O2O supply chain is a novel business pattern integrating

both online and offline channels to provide consumers with a product as well as a superior shopping experience and cater for various channel preferences. Hence, in the O2O pattern, the products are sold simultaneously through online and offline channels and both channels will suffer from the crisis. In this sale system, the online retailer will popularize the products by means of advertisement on the online platform and the offline retailer will offer service to further win customers. However, the goodwill accumulated by their marketing tools may be damaged by the crisis mentioned and therefore make the investments of retailers less efficient. Hence, the retailers are forced to consider the following questions: (1) what they should do to respond to the crisis and (2) how should they adjust their investments when the hazard rate or the damage rate rises. To answer these questions, we formulate and analyze an O2O supply chain model with a random brand crisis in order to provide insights to retailers envisioning crisis.

Three streams of literature are closely related to our research: (1) study of dual-channel or O2O supply chain and showrooming; (2) study of stochastic differential game; and (3) study of brand crisis. The first related stream concerns

the study of dual-channel and O2O supply chain. Dumrongisiri et al. [1] incorporate the service into the study of dual-channel supply as a marketing tool to gain the preference of customers and their analyses show that the service in the offline channel can benefit both online and offline channels. Yan and Pei [2] study the influence of offline service strategy on the decision-making of supply chain members and their results show that the elevation of offline service can help to lower wholesale price and enhance the total performance of the supply chain system. Dan et al. [3] concentrate on the service and pricing strategies of supply chain members and their results show that offline service exerts tremendous influence on pricing strategy and customers' channel loyalty also dictates. The studies mentioned above all note that offline service has a crucial role in the dual-channel supply chain but they fail to incorporate the showrooming effect into their study. He et al. [4] consider the phenomenon into their modeling of the channel demand. Showrooming, by definition, is the phenomenon that customers use the offline store to browse and experience the products but finally buy online according to [5–7]. Essentially, the existence of this effect forces the demand generated by offline service to transfer from offline channel toward the online channel. Given that this phenomenon is ubiquitous, we incorporate this effect into our model. In this paper, we assume that the online and offline channels sell homogeneous products and we mainly focus on the decision-making problem of retailers when they envision the brand crisis. The goodwill established by advertising and service will be damaged in the crisis and the dynamic of goodwill is different from that before the crisis. Hence, we introduce the stage state variable to depict pre- and postcrisis stages and use the modified Nerlove-Arrow model to describe the evolution of goodwill during the whole planning period. Based on the studies above, we formulate the basic O2O model with the showrooming effect as a foundation to further study the effect of the crisis.

In the study of the stochastic differential game, Prasad and Sethi [8] focus on the stochastic cooperative advertising strategies. Jiang et al. [9] and Fu et al. [10] tackle the pollution control problems based on the stochastic differential game. All of these researches study the uncertainty by introducing the Wiener process to explore and analyze the optimal strategies. However, the Wiener process is desirable to depict small and discrete shocks whose net impacts, on average, are zero. But the crisis occurs randomly and discontinuously and its average impact on goodwill is not zero. Hence, the Wiener process is undesirable to represent the discrete crisis event. Hence, in this paper, we use the random occurrence process to depict this kind of uncertainty caused by a rarely happening crisis. We then incorporate this process into the dynamic O2O pattern to study the optimal strategies of retailers when facing the crisis.

Another stream of literature is about the study of brand crisis. Heerde et al. [11] find that the crisis not only damages the reputation and brand equity of the company but also will attenuate the effectiveness of its marketing effort. This is because the occurrence of the crisis may let the customers feel betrayed and may have the image of customers formed

before the change. Meanwhile, his opponents may take advantage of the crisis and add fuel to the fire. Similarly, MacKenzie and Lutz [12] argue that if the customers lose faith in the brand, they will also become dubious to its advertising. Ahluwalia [13] also holds that once the negative publicity of the brand is exposed to customers, they will become more resistant to the advertising trying to convince them. Furthermore, unsatisfactory customers may share their unpleasant experiences with other customers and make them less friendly to the brand according to Ouardighi et al. [14]. Zhao et al. [15] prove that the sensitivity of customers to the brand will be lowered after the crisis. In a word, the crisis will make it hard for the company to retain and win customers. Hence, we take this point into consideration when formulating the model by making the influence parameters after the crisis lower than those before the crisis.

Based on the researches mentioned above, we formulate a dynamic O2O supply chain model with a stochastic crisis to explore the decision problems of the retailers when facing a potential crisis. The rest of the paper is arranged as follows: we formulate the model and offer corresponding assumptions in Section 2; we solve and analyze the optimal strategies in Section 3; we conduct comparison analysis in Section 4; we use a numerical example to analyze and exhibit the outcomes deduced before; Section 5 concludes the whole paper.

2. Model Formulation

2.1. Channel Demand with Showrooming Effect. This paper studies an O2O supply chain system composed of an online retailer and an offline retailer. The online retailer utilizes the e-commercial platform to advertise the products through nationwide advertising, the level of which is represented by $A_E(t)$. While the offline retailer is closer to consumers and therefore will offer local service, the level of which is represented by $S_R(t)$, to further win consumers' preference. Hence, the brand goodwill is affected by both online retailer's advertising and offline retailer's service strategies and the dynamics of brand goodwill can be described as follows:

$$\begin{aligned} \dot{G}(t) &= \gamma A_E(t) + \zeta S_R(t) - \delta G(t), \\ G(0) &= G_0 > 0, \end{aligned} \quad (1)$$

where $G(t)$ represents the goodwill of the brand and $G_0 > 0$ represents the initial goodwill level. $\gamma > 0$ and $\zeta > 0$ measure the influence of advertisement and service on goodwill, respectively. $\delta > 0$ represents the decay rate of goodwill. Equation (1) depicts the dynamics of goodwill and manifests the influence of supply chain members' strategies. The reason why we use the dynamic model is that it can capture the dynamic process of recognition of customers to a certain brand. It takes time for consumers to finally accept and purchase the product, while the static model fails to depict the process. Furthermore, the other important reason for using the dynamic model is that it can also model the impact of the crisis. The goodwill may drop and evolve in a different way after the crisis.

Then, we will establish the demand for online and offline channels based on the showrooming phenomenon. According to the analysis above, the showrooming effect refers to the situation that consumers browse and experience goods in brick-and-mortar stores but buy them online afterward. Inspired by Mehra et al. [7], we divide the consumers into three groups to analyze the demand of both channels. The first group, denoted by $D_1(t)$, refers to the consumers who choose to buy products online directly. The second group denoted by $D_2(t)$ refers to the consumers who experience the products in offline stores but finally purchase them online. The third group, denoted by $D_3(t)$, refers to the consumers who choose to buy products in the offline store after experiencing the products. The significance of categorizing consumers is to reveal the essence of the showrooming effect—a critical factor affecting the division of demand between channels. The more influential the effect is, the more consumers choose to browse offline and buy online. The amount of three groups combines to determine the channel demand.

Since the first group of consumers do not visit the offline stores, the quantity of them is only related to advertising and the level of goodwill. However, since not all consumers are attracted by advertising and goodwill buys products online directly, we assumed that the first group of consumers account for α of the total numbers caused by advertising and goodwill. The other $1 - \alpha$ proportion of consumers choose to experience products offline first and then make decisions. The second and third groups will first go to offline stores to experience products, so their quantities are also affected by the service efforts of offline retailers. However, the second type of consumers ultimately chooses to buy online, while the third type of consumers chooses to buy in the offline store. Assume that the proportion of the second type and the third type of consumers is π and $1 - \pi$, respectively. Therefore, according to the above analysis, the number of three groups of consumers is

$$\begin{aligned} D_1(t) &= \alpha[\beta A_E(t) + \theta G(t)], \\ D_2(t) &= \pi(1 - \alpha)[\beta A_E(t) + \theta G(t)] + \pi\eta S_R(t), \\ D_3(t) &= (1 - \alpha)(1 - \pi)[\beta A_E(t) + \theta G(t)] + (1 - \pi)\eta S_R(t), \end{aligned} \tag{2}$$

where $\beta > 0$, $\eta > 0$, and $\theta > 0$ measure the efficacy of advertising, service, and goodwill on demand, respectively. α denotes the channel preference parameter and π denotes the showrooming parameter. The specific composition of channel demand is shown in the following figure.

Adding D_1 and D_2 , we can get the demand of online channel D_E .

$$D_E = [1 - (1 - \alpha)(1 - \pi)][\beta A_E(t) + \theta G(t)] + \pi\eta S_R(t). \tag{3a}$$

Then the demand for offline retail channel D_R is

$$D_R = (1 - \alpha)(1 - \pi)[\beta A_E(t) + \theta G(t)] + (1 - \pi)\eta S_R(t). \tag{3b}$$

We make $\mu = (1 - \alpha)(1 - \pi)$ in the following analysis for clarity.

In addition, by observing the expressions of $D_E(t)$ and $D_R(t)$, we can find that the total demand of the market is determined by three aspects which are goodwill, advertising, and service. We can also find that the value of online channel preference and showrooming parameters jointly determines the distribution of the total demand between online and offline channels and the change of the parameters actually reflects the transfer of demand between channels. Obviously, the demand for the online channel will increase if the two parameters increase.

2.2. Occurrence Process of Crisis. Next, we will incorporate the crisis occurrence into our study. For the sake of simplicity, we assume that the crisis just happens once at a random moment T with hazard rate χ . The occurrence process of crisis can be described by the following model:

$$\lim_{\Delta t \rightarrow 0} \frac{P\{t \leq T < t + \Delta t | T \geq t\}}{\Delta t} = \chi. \tag{4}$$

The conditional probability in equation (4) means the probability that a crisis does not occur before time t but occurs within the time interval $[t, t + \Delta t)$. Let $f(t)$ and $F(t)$ be probability density function and probability distribution function, respectively. Then the conditional probability in equation (4) can be written as

$$\begin{aligned} &\lim_{\Delta t \rightarrow 0} \frac{P\{t \leq T < t + \Delta t | T \geq t\}}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{((F(t + \Delta t) - F(t)) / [1 - F(t)])}{\Delta t} = \frac{\dot{F}(t)}{1 - F(t)} = \chi. \end{aligned} \tag{5}$$

By solving the differential equation above, we can get $f(t) = \chi e^{-\chi t}$ and $F(t) = 1 - e^{-\chi t}$. Hence, we can find that the probability of the crisis occurring before time T turns out to be $P\{t \leq T\} = 1 - e^{-\chi T}$. Apparently, the greater the hazard rate χ is, the greater the $P\{t \leq T\}$ is, indicating that the crisis occurs earlier. Meanwhile, we can also find that the expected value of occurrence time is $E(t) = (1/\chi)$. In addition, we can infer from equation (4) that the probability of crisis is objective. Once certain products are launched, supply chain members cannot control their value but only have to accept them passively. This assumption is consistent with the research of Thirumalai and Sinha [16] which believes that the potential risk of serious events such as brand crisis actually existed before the product was launched. Therefore, supply chain members must estimate the possibility of crisis objectively and make the corresponding adjustment to properly respond to the potential crisis.

The existence of crisis divides the whole planning period into two phases which are precrisis and postcrisis phases. Since the crisis will damage the goodwill at the occurrence time, the goodwill will be no longer continuous during the planning period.

$$G(T^+) = (1 - \phi)G(T^-), \quad (6)$$

where ϕ represents the damage rate of goodwill. $G(T^+)$ and $G(T^-)$ represent the goodwill levels just after and right before the crisis event, respectively. A higher damage rate implies more loss in goodwill accumulated before crisis. Furthermore, the occurrence of the crisis may weaken the

influence of marketing activities on the goodwill and therefore make them less efficient on the establishment of the goodwill. Thus, the dynamics of goodwill evolve in different ways in precrisis and postcrisis phases which can be described as follows:

$$\begin{cases} \dot{G}_1(t) = \gamma_1 A_E(t) + \zeta_1 S_R(t) - \delta_1 G_1(t), & G_1(0) = G_0, 0 \leq t < T, \\ \dot{G}_2(t) = \gamma_2 A_E(t) + \zeta_2 S_R(t) - \delta_2 G_2(t), & G_2(T^+) = (1 - \phi)G_1(T^-), T \leq t, \end{cases} \quad (7)$$

where $\gamma_1 \geq \gamma_2$, $\zeta_1 \geq \zeta_2$, and $\delta_1 \leq \delta_2$ which indicate that, in the postcrisis phase, the influences of advertising and service on goodwill are lower than those before the crisis and the degree of consumers' forgetting is higher than that before the crisis. According to the research of [11, 17, 18] on crisis and its impact, the consumers tend to be dubious toward the brand advertising after the crisis and companies are more difficult to gain the potential buyers due to bad propaganda, indicating that the efficacy of marketing efforts will be lower after

the crisis, based on which, we suppose that $\gamma_1 \geq \gamma_2$ and $\zeta_1 \geq \zeta_2$. Meanwhile, their opponents may take advantage of the crisis by negative propaganda to canvass consumers originally loyal to the business-facing crisis. Hence, the consumers may become easier to abandon the brand and we assume that $\delta_1 < \delta_2$.

Therefore, the expected profits of online retailers and offline retailers in the planning period are given by

$$J_{E,R}[A_{E1}(S_{R1}), A_{E2}(S_{R2})] = E \left[\int_0^T e^{-rt} (\rho_{E,R} D_{E,R} - C_{E,R}) dt + e^{-rT} V_{E,R2}(G) \right]. \quad (8)$$

Calculating the expectations above, we can get

$$J_{E,R}[A_{E1}(S_{R1}), A_{E2}(S_{R2}), \chi] = \int_0^\infty e^{-(r+\chi)t} [(\rho_{E,R} D_{E,R} - C_{E,R}) + \chi V_{E,R2}(G)] dt, \quad (9)$$

where $V_{E2}(G)$ and $V_{R2}(G)$ are the value functions of supply chain members in the postcrisis stage. $A_{E1}(S_{R1})$ is the advertising (service) level before the crisis and $A_{E2}(S_{R2})$ is the advertising (service) level after the crisis. $\rho_{E,R}$ and $D_{E,R}$ represent the marginal profits and channel demand of both players. We assume that the marginal profits of both members are constant for the reason that, in the perfect competitive market, the companies are only price takers instead of price settlers, making the marginal profit constant. Previous literature, such as Lu et al. [19] and Saha et al. [20], also takes this assumption. $C_{E,R}$ represent advertising and service cost and the cost are assumed to be of quadratic form which should be $(1/2)k_E A_E^2$ and $(1/2)k_R S_R^2$, where k_E and k_R are positive constant cost parameters.

According to equation (9), the expected profits of online and offline retailers are not only functions of the advertising and service strategies but also functions of the hazard rate χ . It is also worth noting that the discount rate changes from r to $r + \chi$, indicating that the retailers pay more attention to the current rather than the future profits since according to Jørgensen and Zaccour [21], the discount rate measures the time preference of players and the larger the discount rate is,

the more impatient the players are. Thus, a differential game model of online and offline retailers with the brand crisis is constructed by (4)–(9).

3. Equilibrium Strategies

3.1. Equilibrium Strategies in Postcrisis Regime. According to the analysis above, the brand crisis not only damages the goodwill of the brand but also makes the efficacy of advertising and service strategies on goodwill lower than before. Therefore, the problems to be solved in precrisis and postcrisis regimes are different from the other. In order to maximize the expected profits in the whole planning period, retailers have to set the optimal strategies at every moment in the precrisis and postcrisis regimes according to Rubel et al. [22]. We use superscript *IM* to represent this pattern. According to the method provided in [23–27], we will first solve the optimization problem in the postcrisis stage. Let $V_E^{IM}(G)$ and $V_R^{IM}(G)$ be the value functions of both players, respectively. Hence, the optimization problem of the online retailer in the postcrisis stage is as follows:

$$\begin{aligned} \max_{A_{E2}^{IM}} J_{E2}^{IM} &= \int_0^\infty e^{-(r+\lambda)t} \left\{ \rho_E [(1-\mu)(\beta A_{E2}^{IM} + \theta G) + \pi \eta S_{R2}^{IM}] - \frac{1}{2} k_E (A_{E2}^{IM})^2 \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_2 A_{E2}(t) + \zeta_2 S_{R2}(t) - \delta_2 G(t), G(T^+) = (1-\phi)G(T^-). \end{aligned} \tag{10}$$

The optimization problems of offline retailer in the postcrisis stage are as follows:

$$\begin{aligned} \max_{S_{R2}^{IM}} J_{R2}^{IM} &= \int_0^\infty e^{-(r+\lambda)t} \left\{ \rho_R [\mu(\beta A_E^{IM} + \theta G) + (1-\pi)\eta S_{R1}^{IM}] - \frac{1}{2} k_R (S_{R2}^{IM})^2 \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_2 A_{E2}(t) + \zeta_2 S_{R2}(t) - \delta_2 G(t), G(T^+) = (1-\phi)G(T^-). \end{aligned} \tag{11}$$

Proposition 1. *The optimal strategies of online and offline retailers in the postcrisis regime are*

$$\begin{aligned} A_{E2}^{IM*} &= \frac{\rho_E \beta (1-\mu) + \gamma_2 n_1}{k_E}, \\ S_{R2}^{IM*} &= \frac{\rho_R \eta (1-\pi) + \zeta_2 m_1}{k_R}. \end{aligned} \tag{12}$$

The optimal value functions of both players in the postcrisis regime are

$$\begin{aligned} V_{E2}^{IM}(G) &= n_1 G + n, \\ V_{R2}^{IM}(G) &= m_1 G + m_2. \end{aligned} \tag{13}$$

where

$$\begin{cases} n_1 = \frac{\rho_E \theta (1-\mu)}{(r + \delta_2)}, \\ n_2 = \frac{(n_1 \zeta_2 + \rho_E \eta \pi) S_{R2}^{IM*}}{r} + \frac{[\rho_E \beta (1-\mu) + \gamma_2 n_1]^2}{2r k_E}, \\ m_1 = \frac{\rho_R \theta \mu}{(r + \delta_2)}, \\ m_2 = \frac{(\rho_R \beta \mu + m_1 \gamma_2) A_{E2}^{IM*}}{r} + \frac{[\rho_R \eta (1-\pi) + \zeta_2 m_1]^2}{2r k_R}. \end{cases} \tag{14}$$

The proof of Proposition 1 is shown in Appendix.

We can find from Proposition 1 that the optimal advertising strategy of online retailers consists of two parts which are $(\rho_E \beta (1-\mu)/k_E)$ and $((\gamma_2 \rho_E \theta (1-\mu))/k_E (r + \delta_2))$. The first term derives from the fact that advertising strategies can directly increase the online channel demand. The second term is due to its indirect influence on demand by contributing to the goodwill. Similarly, the optimal strategy of offline retailers is also composed of two parts. By analyzing the influence of important parameters on the optimal

advertising and service strategy, we can have the following results.

Corollary 1. *The impacts of key parameters on the optimal strategies can be given by*

$$\begin{aligned} \frac{\partial A_{E2}^{IM*}}{\partial \pi} &> 0, \\ \frac{\partial A_{E2}^{IM*}}{\partial \alpha} &> 0, \\ \frac{\partial A_{E2}^{IM*}}{\partial \rho_E} &> 0, \\ \frac{\partial A_{E2}^{IM*}}{\partial \beta} &> 0, \\ \frac{\partial A_{E2}^{IM*}}{\partial \theta} &> 0, \\ \frac{\partial S_{R2}^{IM*}}{\partial \pi} &< 0, \\ \frac{\partial S_{R2}^{IM*}}{\partial \alpha} &< 0, \\ \frac{\partial S_{R2}^{IM*}}{\partial \rho_R} &> 0, \\ \frac{\partial S_{R2}^{IM*}}{\partial \eta} &> 0, \\ \frac{\partial S_{R2}^{IM*}}{\partial \theta} &> 0. \end{aligned} \tag{15}$$

We can find the following from Corollary 1: (1) The increase of π or α indicates that some part of demand is transferred from offline channel to online channel. Hence, when π or α increases, the offline retailer should reduce the

optimal service strategy while online retailers should increase the advertising strategy. Actually, the showrooming effect parameter π and online channel preference parameter α jointly determine the distribution of demand between channels and the changes of the two parameters reflect the transfer of demand between channels. This phenomenon is due to the definition of demand as illustrated in Figure 1 and consistent with intuition. (2) The marginal profit can stimulate retailers to invest more in advertising and service that contribute to improving the goodwill and demand. Therefore, the marginal profit, an indicator of profitability, is crucial in determining the optimal strategies. Both players should pay attention to the improvement of marginal profit. (3) The influence parameters of advertising and service on demand also affect the optimal strategies. The increase of influence parameters implies the increase of efficiency of advertising and service investment. Hence, when the

parameters increase, both online and offline retailers should enhance their investment. (4) Since both online and offline retailers' strategies are able to boost goodwill which is directly relevant to demand, the increase of influence parameter of goodwill on demand will stimulate investment of both players. Actually, the increase of this parameter manifests the elevation of the efficacy of their efforts which underlies their motivations in augmenting investment.

3.2. Equilibrium Strategies in Precrisis Regime. When online and offline retailers decide the optimal strategy before the crisis, they should consider the situation after the crisis so as to maximize the expected profits during the whole planning period. Therefore, the optimization problem that the online retailer should solve before the crisis is as follows:

$$\begin{aligned} \max_{A_{E1}^{IM}} J_{E1}^{IM} &= \int_0^\infty e^{-(r+\chi)t} \left\{ \rho_E [(1-\mu)(\beta A_{E1}^{IM} + \theta G) + \pi \eta S_{R1}^{IM}] - \frac{1}{2} k_E (A_{E1}^{IM})^2 + \chi V_{E2}^{IM} [(1-\phi)G] \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_1 A_{E1}(t) + \zeta_1 S_{R1}(t) - \delta_1 G(t), G(0) = G_0. \end{aligned} \tag{16}$$

That of offline retailers is

$$\begin{aligned} \max_{S_{R1}^{IM}} J_{R1}^{IM} &= \int_0^\infty e^{-(r+\chi)t} \left\{ \rho_R [\mu(\beta A_{E1}^{IM} + \theta G) + (1-\pi)\eta S_{R1}^{IM}] - \frac{1}{2} k_R (S_{R1}^{IM})^2 + \chi V_{R2}^{IM} [(1-\phi)G] \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_1 A_{E1}(t) + \zeta_1 S_{R1}(t) - \delta_1 G(t), G(0) = G_0. \end{aligned} \tag{17}$$

Proposition 2. *The optimal strategies of online and offline retailers in the precrisis regime are*

$$\begin{aligned} A_{E1}^{IM*} &= \frac{\rho_E \beta (1-\mu) + \gamma_1 k_1}{k_E}, \\ S_{R1}^{IM*} &= \frac{\rho_R \eta (1-\pi) + \zeta_1 h_1}{k_R}. \end{aligned} \tag{18}$$

The optimal value functions of online and offline retailers in the precrisis regime are

$$\begin{aligned} V_{E1}^{IM}(G) &= k_1 G + k_2, \\ V_{R1}^{IM}(G) &= h_1 G + h_2. \end{aligned} \tag{19}$$

The dynamic of goodwill during the whole planning period is

$$G^{IM}(t) = \begin{cases} \left[G_0 - \frac{\gamma_1 A_{E1}^{IM*} + \zeta_1 S_{R1}^{IM*}}{\delta_1} \right] e^{-\delta_1 t} + \frac{\gamma_1 A_{E1}^{IM*} + \zeta_1 S_{R1}^{IM*}}{\delta_1}, & t \in [0, T], \\ \left[(1-\phi)G(T) - \frac{\gamma_2 A_{E2}^{IM*} + \zeta_2 S_{R2}^{IM*}}{\delta_2} \right] e^{-\delta_2(t-T)} + G_\infty^{IM}(t), & t \in (T, \infty), \end{cases} \tag{20}$$

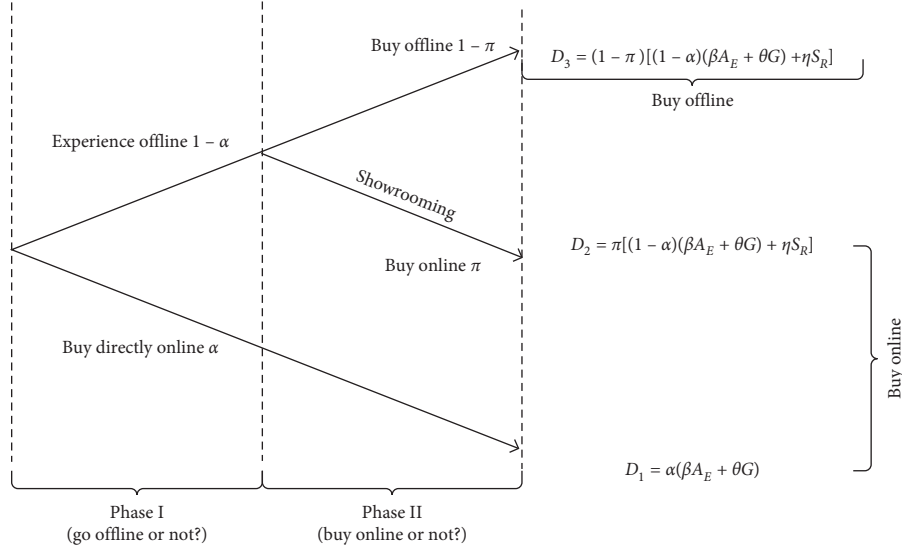


FIGURE 1: Demand for online and offline channels.

where

$$G_{\infty}^{IM}(t) = \frac{\gamma_2 A_{E2}^{IM*} + \zeta_2 S_{R2}^{IM*}}{\delta_2},$$

$$\left\{ \begin{array}{l} k_1 = \frac{[\rho_E \theta (1 - \mu) + \chi n_1 (1 - \phi)]}{(r + \chi + \delta_1)}, \\ k_2 = \frac{[\rho_E (1 - \mu) \beta + k_1 \gamma_1]^2}{2k_E (r + \chi)} + \frac{[(\rho_E \eta \pi + k_1 \zeta_1) S_{R1}^{IM*} + \chi n_2]}{(r + \chi)}, \end{array} \right. \quad (21)$$

$$\left\{ \begin{array}{l} h_1 = \frac{[\rho_R \theta \mu + \chi m_1 (1 - \phi)]}{(r + \chi + \delta_1)}, \\ h_2 = \frac{[\rho_R \eta (1 - \pi) + \zeta_1 h_1]^2}{2k_R (r + \chi)} + \frac{[(\rho_R \beta \mu + h_1 \gamma_1) A_{E1}^{IM*} + \chi m_2]}{(r + \chi)}. \end{array} \right.$$

The proof of Proposition 2 is shown in Appendix.

It can be seen from Propositions 1 and 2 that the online and offline retailers should set their optimal strategies according to the different business conditions before and after the crisis. Therefore, the optimal strategies in the precrisis for both sides are also a function of the crisis occurrence rate χ and the goodwill loss rate ϕ , showing that if online and offline retailers should evaluate the probability of crisis and its negative impact on goodwill at the beginning of making strategies to get the maximum expected profit in the whole planning period. The optimal strategies of both sides are composed of two parts, the structures and some properties of which are similar to those in postcrisis. For example, precrisis and postcrisis advertising strategies are both decreasing functions of the decay rate δ and the cost coefficient k_E but increasing functions of the influence coefficient β and

θ . By analyzing the influence of parameters on the optimal strategies of online and offline retailers, we can have the following conclusion.

Corollary 2. *The impacts of key parameters on the optimal strategies can be given by*

$$\frac{\partial A_{M1}^{IM*}}{\partial \delta_1} < 0,$$

$$\frac{\partial A_{M1}^{IM*}}{\partial \delta_2} < 0,$$

$$\frac{\partial A_{M1}^{IM*}}{\partial \chi} < 0,$$

$$\begin{aligned}
\frac{\partial A_{M1}^{IM*}}{\partial \phi} &< 0, \\
\frac{\partial S_{R1}^{IM*}}{\partial \delta_1} &< 0, \\
\frac{\partial S_{R1}^{IM*}}{\partial \delta_2} &< 0, \\
\frac{\partial S_{R1}^{IM*}}{\partial \chi} &< 0, \\
\frac{\partial S_{R1}^{IM*}}{\partial \phi} &< 0.
\end{aligned}
\tag{22}$$

We can find the following from Corollary 2: (1) The precrisis strategies are not only decreasing functions of the precrisis decay rate δ_1 but also decreasing functions of the postcrisis decay δ_2 , which indicates that both retailers should take into consideration the impact of the crisis on the business environment when deciding strategies at the beginning of the period. (2) The precrisis optimal advertising and service strategies are decreasing function of hazard rate χ and goodwill loss rate ϕ . Hence, retailers should adjust their decisions according to the variation of the crisis. When the hazard rate increases, retailers should reduce their investment before the crisis happens. According to the analysis in Section 2.2, the greater the hazard rate is, the greater the probability $P\{t \leq T\} = 1 - e^{-\chi T}$ (probability of occurring before the time T) will be, which means that the crisis occurs earlier, and the accumulated goodwill will not be kept for a long time. Since the goodwill will suffer due to the crisis, part of the investment before the crisis will disappear with the loss of goodwill. Hence, if the crisis is closer or more destructive, the retailers should reduce the investment before the crisis and leave more funds for the recovery of goodwill after the crisis. In addition, although the goodwill level decreases only after the crisis, the expectation of the crisis affects the decision-making before the crisis. This is because the reverse induction principle requires both parties to anticipate the situation after the crisis and then reversely deduce the global optimal dynamic strategies. Therefore, when confronting the possible crisis, retailers should fully consider the impact of the crisis and make the corresponding actions to maximize the profit maximization based on the global consideration of both pre- and postcrisis stages.

4. Comparisons and Analyses

This section will compare and analyze the optimal strategies of members to further reveal the impact of the potential crisis on member's decisions. Firstly, comparing the strategies with no crisis with those with a potential crisis, we can have Proposition 3. And comparing the strategies of retailers before and after the crisis, we will have Proposition 4. Obviously, the situation without a crisis is the situation with a hazard rate $\chi = 0$. Let $\chi = 0$ in Proposition 2; we will have

the optimal strategies with no crisis represented by superscript AVG.

Proposition 3. *The comparison results of the optimal strategies with or without crisis are as follows:*

$$\begin{aligned}
A_E^{AVG*} &> A_{E1}^{IM*}, \\
A_E^{AVG*} &> A_{E2}^{IM*}, \\
S_R^{AVG*} &> S_{R1}^{IM*}, \\
S_R^{AVG*} &> S_{R2}^{IM*}.
\end{aligned}
\tag{23}$$

According to Proposition 3, retailers tend to lower their investment when facing a potential crisis. According to the analysis of Corollary 2, the precrisis advertising and service strategies are all the decreasing functions of hazard rate χ and when $\chi = 0$; we will have $A_E^{AVG*} = A_{E1}^{IM*}$ and $S_R^{AVG*} = S_{R1}^{IM*}$. Hence, if $\chi > 0$, there must be $A_E^{AVG*} > A_{E1}^{IM*}$ and $S_R^{AVG*} > S_{R1}^{IM*}$. Their envision of the crisis will reduce their precrisis advertising and service investment. Moreover, the occurrence of the crisis will also change the business environment ($\gamma_1 \geq \gamma_2$, $\zeta_1 \geq \zeta_2$, and $\delta_1 \leq \delta_2$), which will make retailers lower their investment after the crisis compared with the situation with no crisis.

Proposition 4. *The comparison results of the optimal strategies of online and offline retailers before and after the crisis are as follows:*

$$\begin{aligned}
A_{E1}^{IM*} - A_{E2}^{IM*} &= \begin{cases} \geq 0, & \frac{\gamma_2}{\gamma_1} \leq \omega, \\ < 0, & \frac{\gamma_2}{\gamma_1} > \omega, \end{cases} \\
S_{R1}^{IM*} - S_{R2}^{IM*} &= \begin{cases} \geq 0, & \frac{\zeta_2}{\zeta_1} \leq \omega, \\ < 0, & \frac{\zeta_2}{\zeta_1} > \omega, \end{cases}
\end{aligned}
\tag{24}$$

where $\omega = ((r + (1 - \phi)\chi + \delta_2)/(r + \chi + \delta_1))$.

From Proposition 4, we can find that retailers should consider their influence parameters of their strategies when deciding how to allocate their investment between pre- and postcrisis regimes. There is a threshold ω . If $(\gamma_2/\gamma_1) \leq \omega$, online retailers should reduce their postcrisis advertising investment, while if $(\gamma_2/\gamma_1) > \omega$, online retailers should increase their postcrisis investment. The service strategy of offline retailer shares similar property.

This result reflects the regime reference in decision-making, which is essentially consistent with the conclusion of the previous analysis. According to the analysis in Section 2.2, we can find that when the hazard rate χ is small, retailers tend to pay more attention to the precrisis stage but if the crisis rate χ is large, they should put more emphasis on the postcrisis stage. Derive ω with regard to χ and we can get

$$\frac{\partial \omega}{\partial \chi} = \frac{r + \delta_2 - (1 - \phi)(r + \delta_1)}{(r + \chi + \delta_1)^2} < 0. \quad (25)$$

When the hazard rate χ decreases, the threshold ω will increase and the probability that (γ_2/γ_1) is less than ω will increase. Hence, the probability that the retailers reduce their investment after the crisis will increase and the precrisis stage is more important for retailers. Particularly, if $\chi \rightarrow 0$, then $\omega \rightarrow ((r + \delta_2)/(r + \delta_1)) \geq 1$ and $(\gamma_2/\gamma_1) \leq \omega$ is definite, which means that retailers do not have to consider the postcrisis stage. This because $\chi \rightarrow 0$ means that the crisis time is infinitely pushed back and the precrisis stage duration is infinite. When the hazard rate χ increases, ω will decrease, indicating that the probability that (γ_2/γ_1) is greater than ω will increase and the retailers will be more likely to increase their investment after the crisis. Further, we can get the value range of the threshold ω .

$$1 - \phi < \omega < \frac{r + \delta_2}{r + \delta_1}. \quad (26)$$

Essentially, the stage preference of retailers is due to the different efficiency of investment in different stages. If the hazard rate χ is very large, the crisis will occur very early. Then the input of players before the crisis will be wasted soon because of the loss of goodwill. Hence, it is better to reduce the investment. However, if the crisis seriously affects the influence parameters of retailers' strategies on goodwill, then it is necessary to weigh the relationship between (γ_2/γ_1) and ω . Particularly, if $\gamma_2(\zeta_2) \rightarrow 0$, indicating that the influence of member's strategies on goodwill can be ignored after the crisis, then more resources should be channeled to the precrisis regime to improve the level of goodwill even if the crisis will damage goodwill. In a word, the potential crisis will make the retailers have stage preference, and the hazard rate, goodwill loss rate, and the influence of members' strategies on goodwill determine the efficiency of investment in different regimes. The following special scenarios can be obtained by further analysis.

4.1. Scenario 1. If $\gamma_1 = \gamma_2$, $\zeta_1 = \zeta_2$, and $\delta_1 = \delta_2$, then $A_{E1}^{IM} \leq A_{E2}^{IM}$ and $S_{R1}^{IM} \leq S_{R2}^{IM} \forall \chi > 0$.

In this scenario, even if the crisis will immediately damage the goodwill, it will not cause other long-term effects (such as reduce the influence parameters of strategies on the goodwill or increase consumers' forgetting effect). Hence, the goodwill has the identical evolution rules before and after the crisis. In this situation, retailers should increase their investment after the crisis so that the damaged goodwill can be recovered as soon as possible. As a consequence, retailers' investment in the second stage is higher than that in the precrisis stage regardless of the hazard rate and the degree of goodwill loss. Furthermore, the investment in the postcrisis stage is equal to that with no crisis; namely, $A_{E2}^{IM} = A_E^{AVG}$ and $S_{R2}^{IM} = S_R^{AVG}$.

4.2. Scenario 2. If $(\gamma_2/\gamma_1) \leq 1 - \phi$ and $(\zeta_2/\zeta_1) \leq 1 - \phi$, then $A_{E1}^{IM} \geq A_{E2}^{IM}$ and $S_{R1}^{IM} \geq S_{R2}^{IM} \forall \chi > 0$.

Different from Scenario 1, the crisis will not only reduce the influence of advertising and service but also increase the forgetting effect consumers, and in this situation, the long-term impact of the crisis outweighs its direct impact, which makes the investment of retailers in postcrisis regime less efficient than in the precrisis regime. Hence, the retailers need to pay more attention to the precrisis stage and allocate more resources before the crisis. In addition, it is also worth noting that the impact of the reduction of the impact of advertising and services is greater than that of the increase of the forgetting effect (because in this scenario, we have $(\gamma_2/\gamma_1) \leq \omega$, but in situation 1, when $\gamma_1 = \gamma_2$ and $\delta_1 = \delta_2$, we will have $\omega < 1$).

4.3. Scenario 3. If $1 - \phi < (\gamma_2/\gamma_1) \leq ((r + \delta_2)/(r + \delta_1))$ and $1 - \phi < (\zeta_2/\zeta_1) \leq ((r + \delta_2)/(r + \delta_1))$, then $A_{E1}^{IM} \geq A_{E2}^{IM}$ for $\chi \leq \hat{\chi}$, $S_{R1}^{IM} \geq S_{R2}^{IM}$ for $\chi \leq \tilde{\chi}$, $A_{E1}^{IM} < A_{E2}^{IM}$ for $\chi > \hat{\chi}$, $S_{R1}^{IM} < S_{R2}^{IM}$ for $\chi > \tilde{\chi}$, where

$$\begin{aligned} \hat{\chi} &= \frac{\gamma_1(r + \delta_2) - \gamma_2(r + \delta_1)}{\gamma_2 - (1 - \phi)\gamma_1}, \\ \tilde{\chi} &= \frac{\zeta_1(r + \delta_2) - \zeta_2(r + \delta_1)}{\zeta_2 - (1 - \phi)\zeta_1}. \end{aligned} \quad (27)$$

Similar to Scenario 2, the crisis will make it more difficult for advertisement and service to attract consumers and make the goodwill decay faster detrimental to the long-term profits of retailers. But the direct impact of the crisis is greater than its long-term impact ($((\gamma_1 - \gamma_2)/\gamma_1) < \phi$). Hence, in this scenario, the investment allocation issues depend on the value of the hazard rate. If $\chi < \hat{\chi}$, an online retailer will increase the advertising investment in the precrisis stage, and when $\chi < \tilde{\chi}$, offline retailers will increase the service investment in the precrisis stage. Since the hazard rate is relatively small, the crisis occurs later and the precrisis stage will be longer. Generally, the direct impact of the crisis will make retailers tend to invest more in the postcrisis stage and the long-term impact of the crisis will make retailers tend to invest more in the precrisis stage. Therefore, when making decisions, retailers should consider the role of their strategies in the whole planning period.

5. Numerical Analysis

In this section, we use numerical analysis to further verify and exhibit former conclusions. We analyze the impact of random crisis on optimal strategies and profits of both retailers. The basic parameters are set as follows: $G_0 = 0$; $r = 0.1$; $\gamma_1 = 1$; $\gamma_2 = 0.8$; $\zeta_1 = 0.2$; $\zeta_2 = 0.15$; $\delta_1 = 0.2$; $\delta_2 = 0.25$; $\alpha = 0.3$; $\pi = 0.4$; $k_E = 1$; $k_R = 1$; $\rho_E = 6$; $\rho_R = 6$; $\beta = 0.2$; $\theta = 0.2$; $\eta = 0.2$; $T = 5$; $\chi = 0.5$; $\phi = 0.5$.

5.1. Impact of Crisis on Retailers' Strategies. Firstly, we analyze the impact of the crisis on the strategies of both retailers at the beginning of the period. Figures 2–4 manifest the impact of hazard rate and goodwill loss rate on online retailers' advertising strategies.

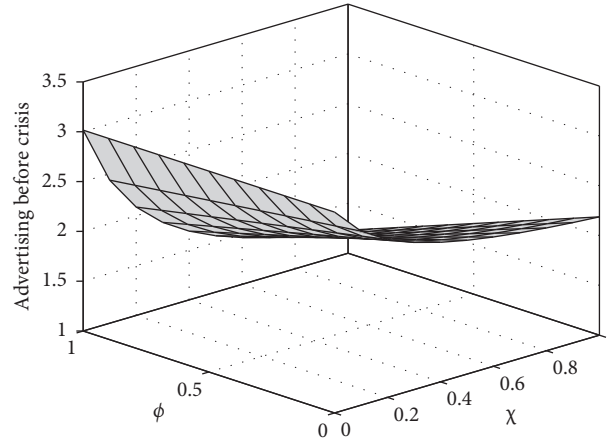


FIGURE 2: Advertising before crisis.

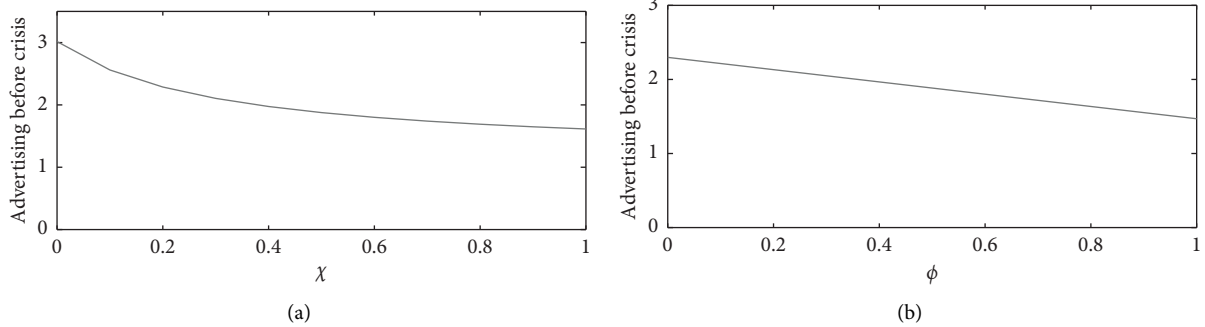


FIGURE 3: Changes with regard to (a) χ and (b) ϕ .

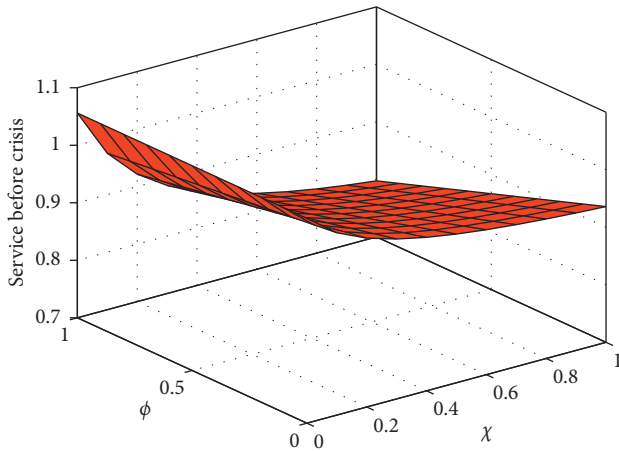


FIGURE 4: Service before crisis.

Figures 2 and 3 verify the conclusion of Corollary 2. When the crisis will occur, the hazard rate and the loss rate are crucial factors that should be taken into consideration when the online retailer makes the advertising strategy. And the retailer should properly respond to the changes of the parameters and make corresponding adjustments to achieve the maximum expected profit in the planning period. It can be seen from Figures 2 and 3 that when the hazard rate

increases, online retailers should reduce their advertising investment, and when the loss rate of on goodwill increases, online retailers should also reduce their advertising investment. Figures 4 and 5 analyze the impact of the crisis on offline retailer’s service strategy.

Similar to the online retailer’s advertising strategy, the offline retailer should also reduce their service investment no matter the hazard rate or loss rate increases. Hence, when the crisis may occur, the envision of the crisis will affect the determination of the retailers’ strategies. This is because the purpose of both advertising and service is to increase goodwill and expand sales, but the occurrence of the crisis will make the accumulated goodwill suffer and therefore waste the investment of retailers. This point has important management insight. In managerial practice, if the crisis will not happen, the retailers’ strategies are only based on their own situation and market environment. But in actual operational activity, the brands sold by retailers have odds to confront the brand crisis and therefore have the goodwill injured. Hence, retailers should have certain knowledge of the products they sold and the manufacturers of products to analyze the probability of crisis and the impact on goodwill. For example, if the manufacturer does not do well in quality control and often receives consumer’s complaints or the manufacturer has business behaviors that does not conform to business ethics, the retailers really need to know about

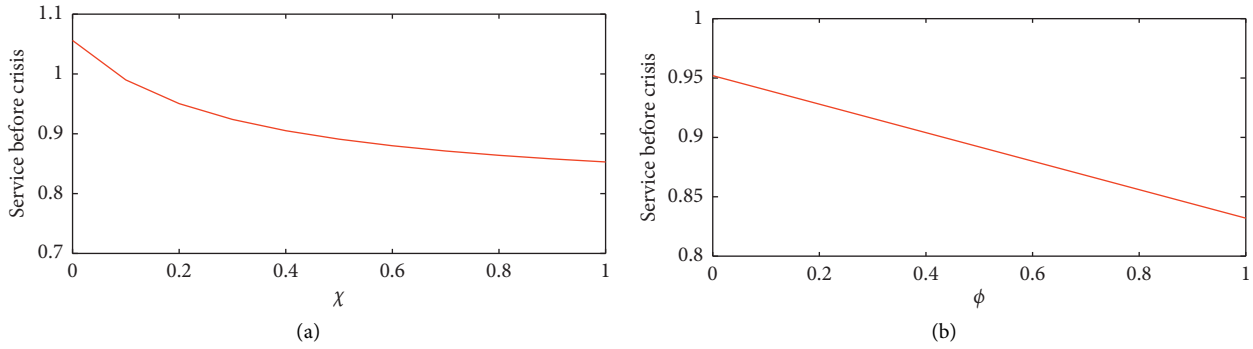


FIGURE 5: Changes with regard to (a) χ and (b) ϕ .

these situations. Because these hidden dangers may occur at any time which will lead to a crisis against the brand. The more the hidden dangers in the operation process are, the greater the probability of crisis will be. Retailers should adjust their advertising and service inputs to respond to the potential risk.

5.2. Impact of Crisis on Goodwill. The impact of the crisis on goodwill is shown in Figures 6–8. Figure 6 compares the goodwill with crisis to that without crisis. Figure 7 analyzes the impact of hazard rate on goodwill and Figure 8 analyzes the impact of loss rate.

According to the analysis of Corollary 2, when the severity of the crisis increases, both online and offline retailers should reduce their precrisis advertising and service investment. In Figure 6, even if the crisis has not yet occurred $T < 5$, the goodwill is still lower than that without the crisis. When the crisis happened $T = 5$, the goodwill suddenly loses. In addition, according to the analysis of Proposition 3, the advertising and service strategies of retailers after the crisis are also lower than those without crisis and the growth rate of goodwill in the postcrisis stage is still lower than that without crisis. Figure 7 analyzes the impact of hazard on goodwill. It can be seen that the higher the hazard rate is, the lower the level of goodwill will be. This is because the investment will decrease with the increase of hazard rate, leading to a lower level of goodwill. In addition, since the strategies of retailers after the crisis are no longer affected by the crisis but only related to the retailers themselves and the market, the strategies after the crisis are the same regardless of the situation of the crisis. Hence, in Figure 7, the goodwill in three cases converges to the same level. Figure 8 analyzes the impact of loss rate on goodwill. We can also find that the higher the loss rate is, the lower the level of goodwill will be, not only because the loss rate will reduce the investment of both sides but also because of the higher damage of the crisis.

5.3. Impact of Crisis on Retailers' Profits. Figures 9–11 analyze the impact of the crisis on the profits of online retailers, and Figures 12–14 analyze the impact of the crisis on the profits of offline retailers.

The impact of the crisis on profits is complex. On the one hand, according to the previous analysis, the envision of the

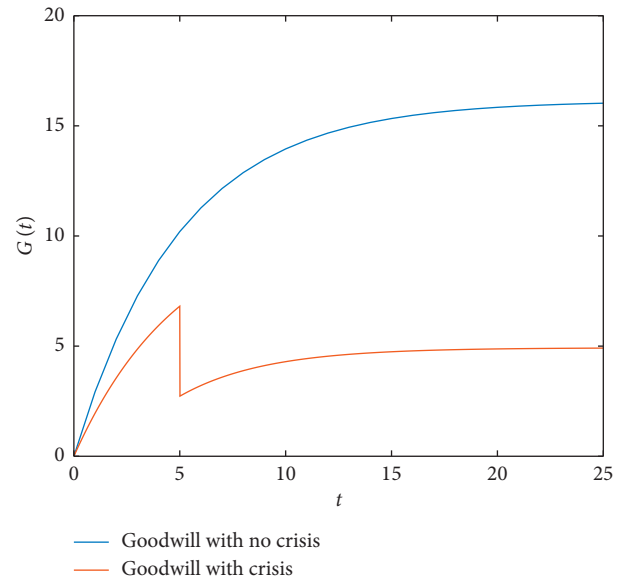


FIGURE 6: Comparison of goodwill.

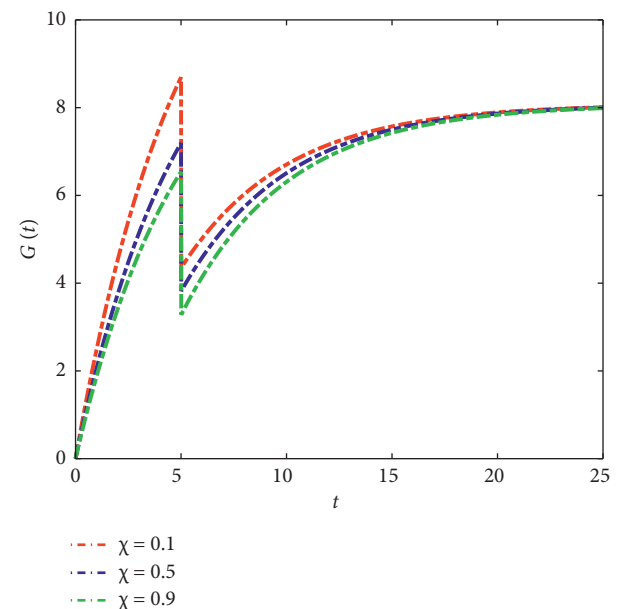


FIGURE 7: Influence of hazard rate.

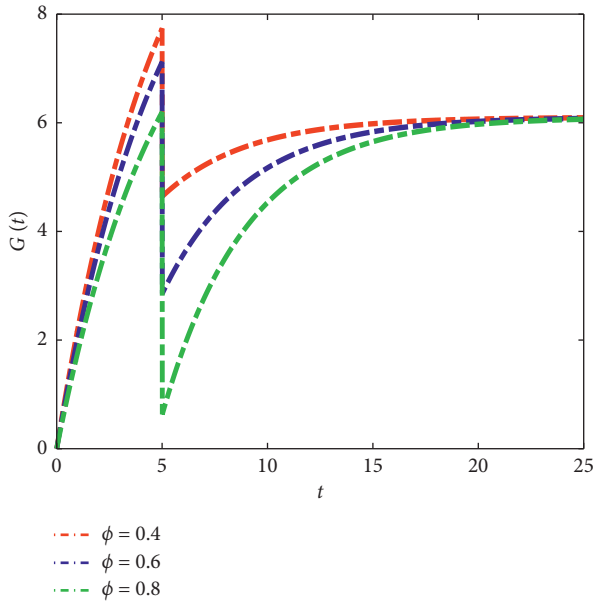


FIGURE 8: Influence of loss rate.

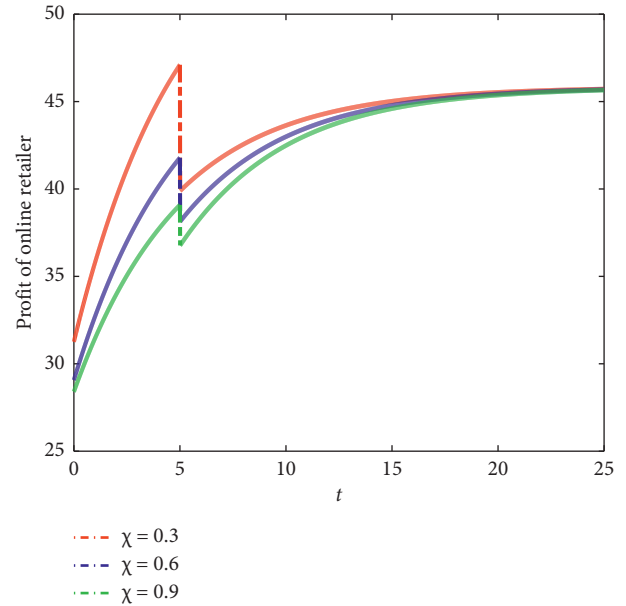


FIGURE 10: Influence of hazard rate.

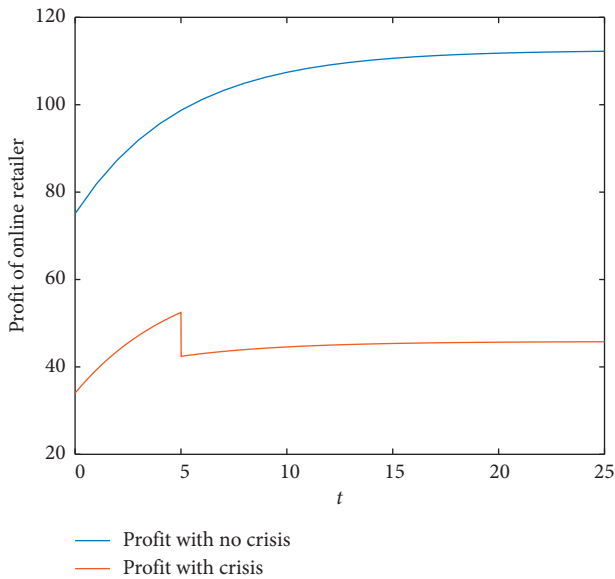


FIGURE 9: Comparison of online profit.

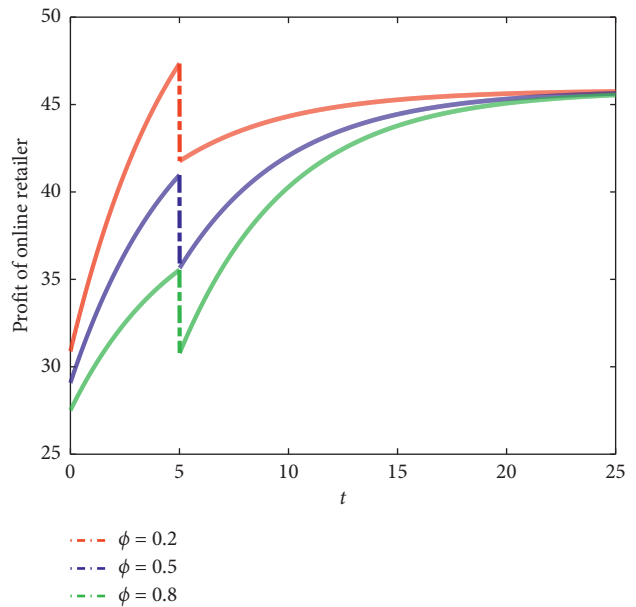


FIGURE 11: Influence of loss rate.

crisis will make both retailers reduce their advertising and service investment, which will not only lead to the reduction of goodwill but also directly affect demand. On the other hand, the occurrence of the crisis will lead to changes in market conditions, which will jointly lead to lower profit levels before and after the crisis than when there is no crisis. Specifically, both of the hazard rate and the loss rate will have an impact on the profits. If the loss rate is fixed and the hazard rate increases, the profits of both sides will decrease with the increase of the hazard rate. Similarly, if the hazard rate remains unchanged, the profits of both sides will decrease with the increase of the loss rate. Hence, the impact of the crisis is ultimately reflected in the impact on profits.

Therefore, when the brand may encounter brand crisis, retailers should consider the overall profit before and after the crisis to maximize the expected profit.

5.4. Strategies before and after Crisis. Proposition 4 compares and analyzes the strategies of retailers before and after the crisis and considers three special situations which will be further presented and analyzed in the following figures.

Figures 15–17 show the advertising strategies before and after the crisis in three scenarios and Figures 18–20 show the service strategies before and after the crisis in three scenarios. In the first scenario, although the crisis will reduce

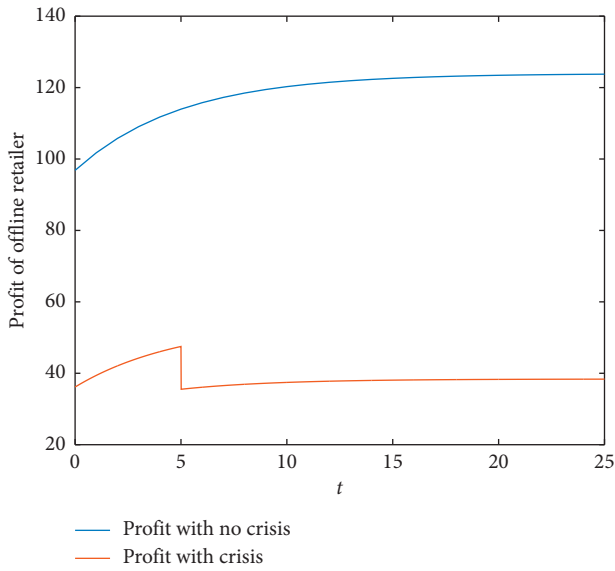


FIGURE 12: Comparison of offline profit.

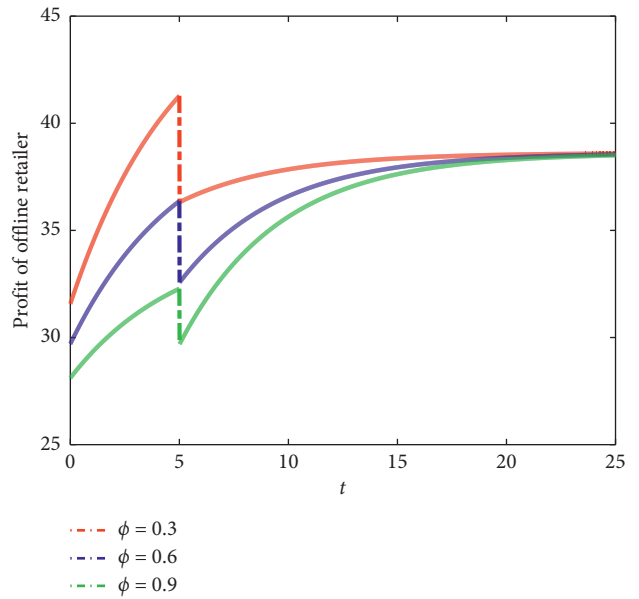


FIGURE 14: Influence of loss rate.

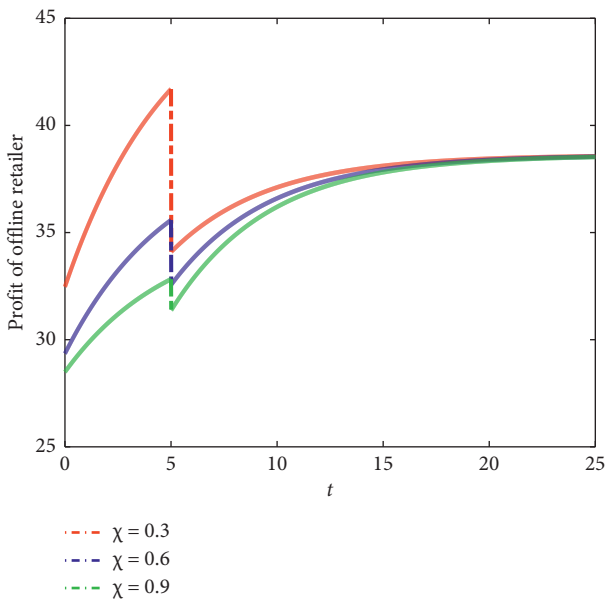


FIGURE 13: Influence of hazard rate.

the goodwill, it will not affect the long-term profitability of the retailers. For example, the occurrence of the crisis is not due to the responsibility of the brand. Hence even though the crisis will cause short-term loses, it will not affect the trust of consumers in the brand. We assume that $\gamma_1 = \gamma_2$, $\zeta_1 = \zeta_2$, and $\delta_1 = \delta_2$ in this scenario. And according to Propositions 1 and 2, the advertising strategy in the post-crisis stage is the same as that with no crisis; hence, in Figure 15, the curves of A_E and A_{E2} coincide. But the crisis will happen and will damage the goodwill, which causes the investment before the crisis to be lower than that after the crisis.

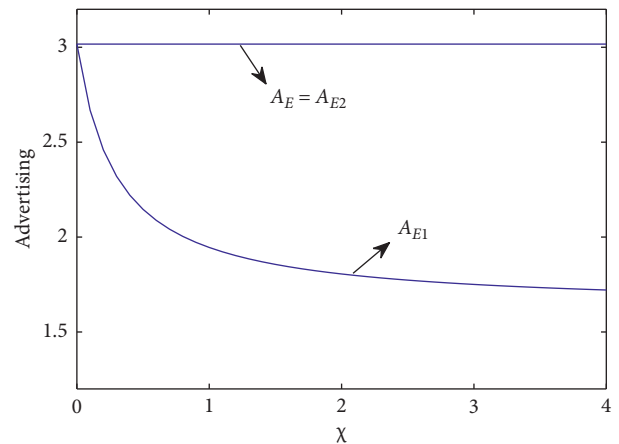


FIGURE 15: Comparison of advertising (Scenario 1).

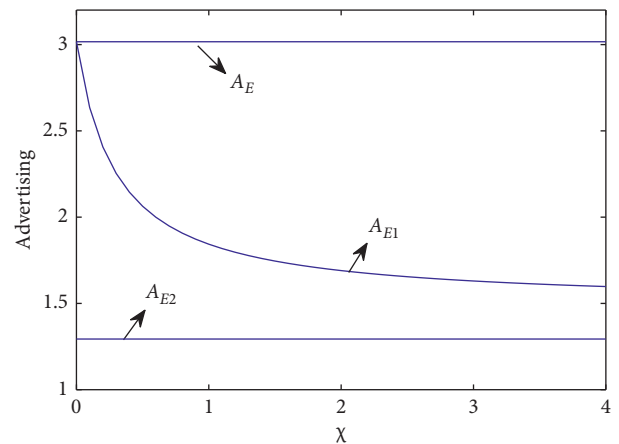


FIGURE 16: Comparison of advertising (Scenario 2).

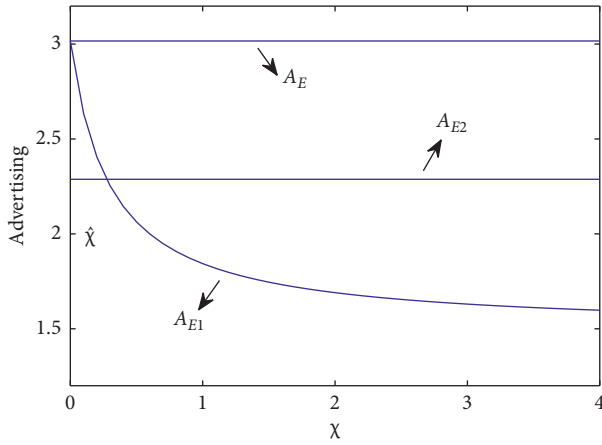


FIGURE 17: Comparison of advertising (Scenario 3).

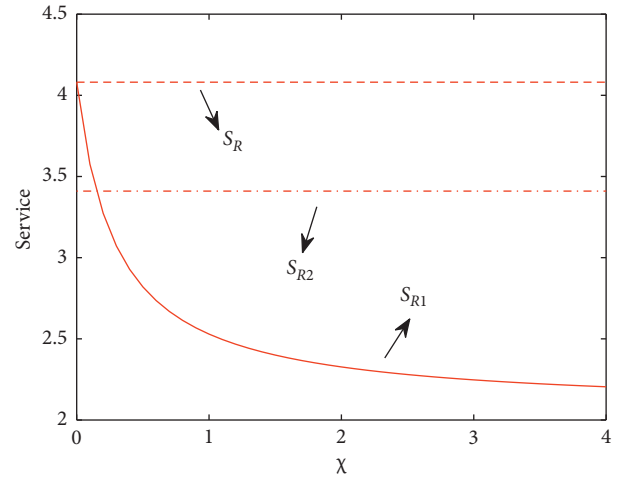


FIGURE 20: Comparison of service (Scenario 3).

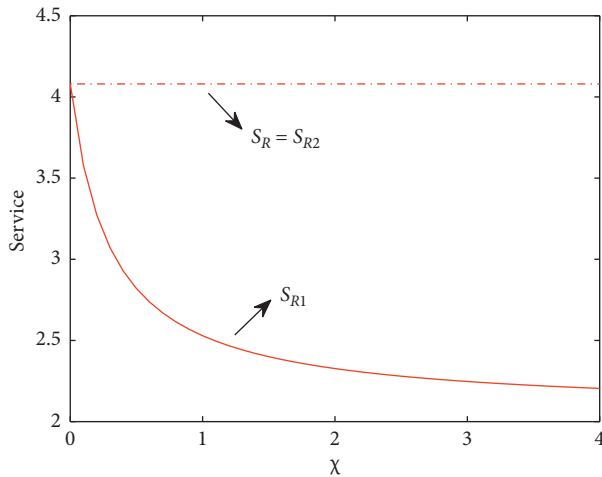


FIGURE 18: Comparison of service (Scenario 1).

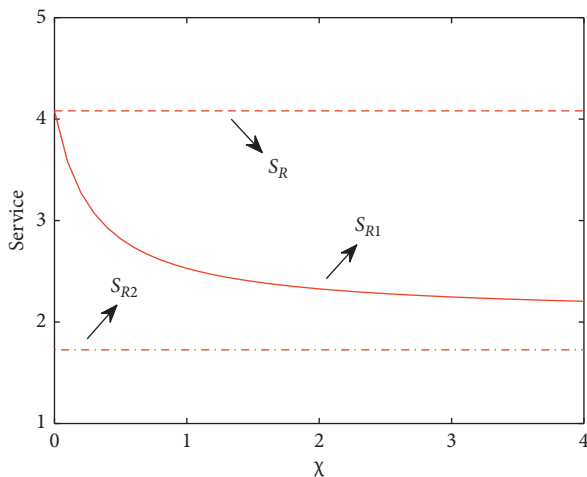


FIGURE 19: Comparison of service (Scenario 2).

Figure 16 depicts the scenario of $(\gamma_2/\gamma_1) \leq 1 - \phi$. At this time, even if $\chi \rightarrow \infty$, online retailers still need to invest more before the crisis ($\partial\omega/\partial\chi < 0$), and $\lim_{\chi \rightarrow \infty} \omega = 1 - \phi$. $(\gamma_2/\gamma_1) \leq 1 - \phi$ shows that the crisis severely weakens the

influence of advertisement. Hence, the long-term impact of the crisis outweighs its short-term impact and the crisis will make the investment in the postcrisis stage less efficient than that in the precrisis stage. So, the retailer should increase the investment before the crisis and reduce the investment after the crisis. Figure 17 depicts the scenario of $(\gamma_2/\gamma_1) > 1 - \phi$. Compared with Scenario 2, the short-term impact of the crisis is greater than the long-term impact. Retailer should reduce the investment before the crisis unless the hazard rate is less than the threshold $\hat{\chi}$. This indicates that although the crisis will have a greater short-term impact and waste part of the precrisis investment, the retailer should still pay more attention to the precrisis stage as long as the hazard rate is small enough and the time of the crisis is late enough. Similarly, the relationship of service strategies before and after the crisis in the three scenarios presents similar conclusions as shown in Figures 18–20. We can find from the figures above that, in any of the scenarios, the level of strategy without crisis is the highest resulting in the highest level of goodwill as illustrated in Figure 6. Moreover, when making decisions before and after the crisis, retailers should always consider which stage of investment is more effective and therefore determine the allocation strategy.

6. Conclusions

In the O2O supply chain system, products are sold simultaneously through online and offline retail channels. To improve the level of goodwill and expand the market demand, online retailers will carry out advertising activity relying on their online platforms. The offline retailer is closer to consumers and therefore offers a good service experience to win consumers. However, the brand they sold may encounter brand crisis and the occurrence of the crisis will lead to the loss of goodwill accumulated by advertising and services. Hence, the purpose of this paper is to study the strategies of both online and offline retailers when facing a potential crisis and analyze the impact of the crisis. Firstly, we describe the dynamics of goodwill under the influence of crisis by introducing stage state variables. Secondly, we

construct the benchmark model of O2O supply chain with consideration of the showrooming effect. After solving and analyzing the optimal strategies and profits with and without crisis, we find the following:

- (1) When facing a possible crisis, online and offline retailers should anticipate the risk in advance and take the hazard rate and the loss rate into account when deciding the advertising and service strategies. And when the hazard rate and loss rate increase, both retailers should reduce their precrisis investment. Moreover, the increase of hazard rate implies that the crisis is closer and the increase of the loss rate indicates a higher damage degree of the goodwill. Hence, if the hazard rate increases, the goodwill accumulated due to retailers' investment will be kept for a shorter time which is equivalent to wasting retailers' investment and retailers are unable to fully benefit from the goodwill. This why retailers should reduce their investment when the hazard rate increases. Similarly, when the loss rate of goodwill increases, retailers should also reduce the precrisis investment.
- (2) The existence of crisis divides the planning period into two stages: precrisis stage and postcrisis stage and the efficiency of retailers' investment in different stages is different. If the efficiency of investment in the precrisis stage is high, retailers should increase the investment before the crisis and reduce the investment after the crisis otherwise. This forces retailers to decide the allocation of investment in two stages according to different situations which results in stage preference.

- (3) Due to the envision of crisis, online and offline retailers should reduce their advertising and service input before the crisis, and due to changes in the market environment in the postcrisis stage, the advertising and service levels are still lower than those with no crisis. Hence, if the crisis is likely to occur, the goodwill level will be lower in the whole planning period, and so are the profits. Retailers should consider the overall profit before and after the crisis when making strategies at the beginning of the period to maximize the expected profit. Besides, when the severity of the crisis increases, they should reduce investment rather than increase investment. In a word, in managerial practice, retailers need to have a certain understanding of the products they sold and the manufacturers of the products so as to know the probability of crisis and the impact on goodwill. If there are many hidden dangers in the operation process, the probability of a crisis is large and then retailers should make corresponding countermeasures to reduce the loss when selling products.

Appendix

Proof of Propositions 1 and 2

Proof. According to the reverse induction principle, we should firstly solve the optimal strategies and profits of retailers in the postcrisis stage. Hence, the optimization problem to be solved by the online retailer after the crisis is

$$\begin{aligned} \max_{A_{E2}^{IM}} J_{E2}^{IM} &= \int_0^{\infty} e^{-rt} \left\{ \rho_E \left[(1 - \mu)(\beta A_{E2}^{IM} + \theta G) + \pi \eta S_{R2}^{IM} \right] - \frac{1}{2} k_E (A_{E2}^{IM})^2 \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_2 A_{E2}^{IM}(t) + \zeta_2 S_{R2}^{IM}(t) - \delta_2 G(t), G(T^+) \neq (1 - \phi)G(T^-), \end{aligned} \tag{A.1}$$

and the optimal problems to be solved by the offline retailer after the crisis are

$$\begin{aligned} \max_{S_{R2}^{IM}} J_{R2}^{IM} &= \int_0^{\infty} e^{-rt} \left\{ \rho_R \left[\mu(\beta A_{E2}^{IM} + \theta G) + (1 - \pi) \eta S_{R2}^{IM} \right] - \frac{1}{2} k_R (S_{R2}^{IM})^2 \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_2 A_{E2}^{IM}(t) + \zeta_2 S_{R2}^{IM}(t) - \delta_2 G(t), G(T^+) \neq (1 - \phi)G(T^-). \end{aligned} \tag{A.2}$$

Then, the HJB equations of both retailers are

$$\begin{aligned}
rV_{E2}^{IM}(G) &= \max_{A_{E2}^{IM}} \left\{ \rho_E \left[(1-\mu)(\beta A_{E2}^{IM} + \theta G) + \pi \eta S_{R2}^{IM} \right] - \frac{1}{2} k_E (A_{E2}^{IM})^2 + \frac{\partial V_{E2}^{IM}(G)}{\partial G} (\gamma_2 A_{E2}^{IM} + \zeta_2 S_{R2}^{IM} - \delta_2 G) \right\}, \\
rV_{R2}^{IM}(G) &= \max_{S_{R2}^{IM}} \left\{ \rho_R \left[\mu(\beta A_{E2}^{IM} + \theta G) + (1-\pi) \eta S_{R2}^{IM} \right] - \frac{1}{2} k_R (S_{R2}^{IM})^2 + \frac{\partial V_{R2}^{IM}(G)}{\partial G} (\gamma_2 A_{E2}^{IM} + \zeta_2 S_{R2}^{IM} - \delta_2 G) \right\}.
\end{aligned} \tag{A.3}$$

By solving the optimization problems on the right side of the HJB equation, we can have

$$\begin{aligned}
A_{E2}^{IM} &= \frac{\rho_E (1-\mu) \beta}{k_E} + \frac{\gamma_2}{k_E} \frac{\partial V_{E2}^{IM}(G)}{\partial G}, \\
S_{R2}^{IM} &= \frac{\rho_R (1-\pi) \eta}{k_R} + \frac{\zeta_2}{k_R} \frac{\partial V_{R2}^{IM}(G)}{\partial G}.
\end{aligned} \tag{A.4}$$

By substituting the results above into the HJB equations, we can get

$$\begin{aligned}
rV_{E2}^{IM}(G) &= \max_{A_{E2}^{IM}} \left\{ \rho_E (1-\mu) \theta G - \delta_2 \frac{\partial V_{E2}^{IM}(G)}{\partial G} G + \frac{1}{2k_E} \left[\rho_E (1-\mu) \beta + \gamma_2 \frac{\partial V_{E2}^{IM}(G)}{\partial G} \right]^2 \right. \\
&\quad \left. + \left[\rho_E \pi \eta + \zeta_2 \frac{\partial V_{E2}^{IM}(G)}{\partial G} \right] \left[\frac{\rho_R (1-\pi) \eta}{k_R} + \frac{\zeta_2}{k_R} \frac{\partial V_{R2}^{IM}(G)}{\partial G} \right] \right\}, \\
rV_{R2}^{IM}(G) &= \max_{S_{R2}^{IM}} \left\{ \frac{1}{2k_R} \left[\rho_R (1-\pi) \eta + \zeta_2 \frac{\partial V_{R2}^{IM}(G)}{\partial G} \right]^2 + \rho_R \mu \theta G - \delta_2 \frac{\partial V_{R2}^{IM}(G)}{\partial G} G \right. \\
&\quad \left. + \left[\rho_R \mu \beta + \gamma_2 \frac{\partial V_{R2}^{IM}(G)}{\partial G} \right] \left[\frac{\rho_E (1-\mu) \beta}{k_E} + \frac{\gamma_2}{k_E} \frac{\partial V_{E2}^{IM}(G)}{\partial G} \right] \right\}.
\end{aligned} \tag{A.5}$$

According to the form of the HJB equation, we assume the value functions of retailers to be

$$\begin{aligned}
V_{E2}^{IM}(G) &= n_1 G + n_2, \\
V_{R2}^{IM}(G) &= m_1 G + m_2.
\end{aligned} \tag{A.6}$$

Hence, we can get the value of the coefficients of the value function

$$\begin{cases}
n_1 = \frac{\rho_E \theta (1-\mu)}{(r + \delta_2)}, \\
n_2 = \frac{(n_1 \zeta_2 + \rho_E \eta \pi) S_{R2}^{IM*}}{r} + \frac{[\rho_E \beta (1-\mu) + \gamma_2 n_1]^2}{2rk_E}, \\
m_1 = \frac{\rho_R \theta \mu}{(r + \delta_2)}, \\
m_2 = \frac{(\rho_R \beta \mu + m_1 \gamma_2) A_{E2}^{IM*}}{r} + \frac{[\rho_R \eta (1-\pi) + \zeta_2 m_1]^2}{2rk_R}.
\end{cases} \tag{A.7}$$

The optimization problem of both retailers after the crisis is solved. Then, the optimization problem to be solved by the online retailer before the crisis is

$$\begin{aligned} \max_{A_{E1}^{IM}} J_{E1}^{IM} &= \int_0^{\infty} e^{-(r+\chi)t} \left\{ \rho_E [(1-\mu)(\beta A_{E1}^{IM} + \theta G) + \pi \eta S_{R1}^{IM}] - \frac{1}{2} k_E (A_{E1}^{IM})^2 + \chi V_{E2}^{IM} [(1-\phi)G] \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_1 A_{E1}(t) + \zeta_1 S_{R1}(t) - \delta_1 G(t), \quad G(T^+) = (1-\phi)G(T^-), \end{aligned} \quad (\text{A.8})$$

and the optimization problem to be solved by the offline retailer before the crisis is

$$\begin{aligned} \max_{S_{R1}^{IM}} J_{R1}^{IM} &= \int_0^{\infty} e^{-(r+\chi)t} \left\{ \rho_R [\mu(\beta A_{E1}^{IM} + \theta G) + (1-\pi)\eta S_{R1}^{IM}] - \frac{1}{2} k_R (S_{R1}^{IM})^2 + \chi V_{R2}^{IM} [(1-\phi)G] \right\} dt \\ \text{s.t. } \dot{G}(t) &= \gamma_1 A_{E1}(t) + \zeta_1 S_{R1}(t) - \delta_1 G(t), \quad G(T^+) = (1-\phi)G(T^-). \end{aligned} \quad (\text{A.9})$$

Through the same method, we can have

where

$$\begin{aligned} V_{E1}^{IM}(G) &= k_1 G + k_2, \\ V_{R1}^{IM}(G) &= h_1 G + h_2, \end{aligned} \quad (\text{A.10})$$

$$\begin{cases} k_1 = \frac{[\rho_E \theta (1-\mu) + \chi n_1 (1-\phi)]}{(r + \chi + \delta_1)}, \\ k_2 = \frac{[\rho_E (1-\mu)\beta + k_1 \gamma_1]^2}{2k_E (r + \chi)} + \frac{[(\rho_E \eta \pi + k_1 \zeta_1) S_{R1}^{IM*} + \chi n_2]}{(r + \chi)}, \\ h_1 = \frac{[\rho_R \theta \mu + \chi m_1 (1-\phi)]}{(r + \chi + \delta_1)}, \\ h_2 = \frac{[\rho_R \eta (1-\pi) + \zeta_1 h_1]^2}{2k_R (r + \chi)} + \frac{[(\rho_R \beta \mu + h_1 \gamma_1) A_{E1}^{IM*} + \chi m_2]}{(r + \chi)}. \end{cases} \quad (\text{A.11})$$

Then, we will have the trajectory of goodwill during the whole planning period

$$G^{IM}(t) = \begin{cases} \left[G_0 - \frac{\gamma_1 A_{E1}^{IM*} + \zeta_1 S_{R1}^{IM*}}{\delta_1} \right] e^{-\delta_1 t} + \frac{\gamma_1 A_{E1}^{IM*} + \zeta_1 S_{R1}^{IM*}}{\delta_1}, & t \in [0, T], \\ \left[(1-\phi)G(T) - \frac{\gamma_2 A_{E2}^{IM*} + \zeta_2 S_{R2}^{IM*}}{\delta_2} \right] e^{-\delta_2 (t-T)} + G_{\infty}^{IM}(t), & t \in (T, \infty). \end{cases} \quad (\text{A.12})$$

Data Availability

All the data included in this study are available upon request by contacting the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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