Research Article

Multiperiod Dynamic Pricing and Inventory Control Decisions for an Omnichannel BOPS Retailer with Reference Price Effects

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This paper utilizes the consumers' reference price in prospect theory to analyze an omnichannel retailer's multiperiod pricing and inventory management problem in which consumers can cancel their orders before payment and return the products after payment if the products do not meet their expectation. The omnichannel retailer's optimal equilibrium pricing and ending inventory level are derived under reference price effects by maximizing the discounted total profit over the infinite planning horizon, where the optimal decisions we discussed under two scenarios: loss neutrality and loss aversion. The analysis shows that the convergence of the pricing and ending inventory level toward their equilibrium is from above or below, depending on the relative location of the initial reference price with respect to the unique equilibrium price. Moreover, a set of sensitivity analyses is discussed to characterize the impacts of system parameters on the optimal decisions. This research fills the gap of behavioral operation in the field of omnichannel joint pricing and inventory management.

1. Introduction

With the vigorous development of mobile Internet technology, great changes have taken place in the communication between retailers and consumers. In order to improve the probability of successful marketing and increase market share, retailers have emerged a series of new retail modes, such as online, mobile, e-mail, and QQ, and a new retail model that opens up various channels, "omnichannel retail" came into being. For example, facing the competition from JD.com, both Suning and Gome, the two largest traditional physical home appliance retailers in China, launched their online businesses in 2009. Additionally, the world-famous e-commerce retailers, such as Amazon, Google, and eBay, have also built their tech-enabled physical stores. Omnichannel retail focuses on "a truly integrated approach across the whole retail operation that delivers a seamless response to the consumer experience through all available shopping channels" (see, for example, [1, 2]). Through channel integration, not only can the trust and satisfaction of consumers be improved but

also the competitive advantage and channel synergy of retailers can be improved, so as to better match the channels and consumers. Data show that the global e-commerce sales reached $1.86 trillion in 2016 and reached $3.88 trillion in 2018, with a growth rate of over 10% [3]. With the strong support for e-commerce, the innovative business model based on Internet plus has developed rapidly in China, domestic retail giants such as Suning, JD, and Taobao have emerged, and they are constantly exploring new omnichannel business models. At present, pay-and-buy-online-pick-up-in-store (BOPS) is a retail mode that is widely adopted by omnichannel retail. Taking Suning as an example, consumers can enjoy store pick up, store appraisal, and store return and exchange services at any physical store across the country after placing an order on the Suning cloud platform [4]. In addition, BOPS can also bring potential additional transactions for offline stores, which is called cross-selling profit [5]. A recent United Parcel Service (UPS) study shows that 45% of consumers will have new purchase when they pick up goods offline [6]. The implementation of the BOPS gives

full play to the offline advantages of traditional retailers and enhances retailers' competitive advantages via offline professional services, convenience, and experience advantages.

As an important practice of omnichannel retail, BOPS has been adopted by many retailers, but it also faces many technical challenges. These challenges mainly come from two levels: retailers and consumers. First of all, at the retailer level, omnichannel retail online and offline inventory sharing, in order to ensure the availability of goods purchased by consumers at any time, it is particularly important to formulate a reasonable inventory and a balanced price strategy, which will help improve consumers' loyalty and the reputation of retailers. Secondly, at the consumer level, it is mainly manifested in the purchase behavior of consumers. Consumers' purchase decision will be affected by the reference price. Behavioral science pointed out that consumers will remember the past price information of a product in their mind when they repeatedly purchase a product, thereby establishing the concept of "fair price," which is called reference price [7]. When a new purchase decision is made for the same product, consumers will take the reference price as the benchmark of the current selling price of the product. If the current selling price is lower (higher) than the reference price, they will think that they have "earned" ("lost") and are, therefore, more likely (unwilling) to buy. This phenomenon is called reference price effects. Research shows that the joint pricing and inventory strategies based on the reference price effects can greatly increase the profits of retailers [8]; thirdly, the common phenomenon of consumer return under the BOPS mode will also affect the decision making of retailers [5]. Thus, the following technical problems will arise: (1) How will the BOPS mode affect an omnichannel retailer' market demand and profitability? (2) What pricing and inventory decisions should the retailer make to realize the product price advantage, save inventory cost, and improve profit? These are the practical problems that need to be solved urgently when the retailer implements the BOPS mode. The solution of these problems meets the current development needs and has important practical significance.

In order to solve the abovementioned problems, this paper studies the pricing and inventory strategies of an omnichannel retailer with BOPS mode based on the influence of consumers' reference price. By analyzing the mechanism of the influence of consumer reference price on the pricing and inventory of the omnichannel retailer, it provides theoretical suggestions for omnichannel retailers to implement the pricing and inventory management of the BOPS mode, whereas recent research on joint pricing and inventory decisions of omnichannel retailers mainly focused on single ordering cycle and ignored the consumers' reference price effects (see, for example, [5, 9]). Besides, previous literature on multiperiod coordination pricing and inventory control problem with reference price effects mainly focused on the single-channel supply chain, and little attention has been paid to the omnichannel field (see, for example, [8, 10, 11]). This paper studies the multiperiod joint pricing and inventory strategies of omnichannel retailers considering the impact of consumers' reference price and

tries to fill the gap in the decision making of multiperiod joint pricing and inventory in an omnichannel environment.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents a theoretical model to formulate the omnichannel under reference price effects. Section 4 investigates several structural properties of the optimal strategies. Numerical results of sensitivity analysis are represented in Section 5. Some managerial insights are provided in Section 6. Section 7 concludes our paper. All the proofs are presented in Appendix.

2. Literature Review

This paper is related to two streams of literature: one delves into omnichannel retail operations, while the other discusses the multiperiod pricing-inventory model with reference price effects. We review the related areas below.

Studies on omnichannel retailing are emerging, most of which are exploratory. At present, the omnichannel BOPS operation management has attracted extensive attention from the academic community. According to Retail Systems Research (RSR), as of June 2013, 64% of the retailers provided consumers with the option to BOPS [12], and they benefit from allowing consumers to pick up their online orders in store. According to a recent UPS study, among those who have used an in-store pick up option, 45% of them have made a new purchase when picking up the product in store (UPS, 2015). Such an additional profit is called the cross-selling benefit [6, 13]. Actually, such cross selling can generate a substantial amount of store sales: it is estimated that, on average, when a consumer comes to the store intending to buy $100 worth of merchandise, he/she leaves with $120 to $125 worth of merchandise [14]. Some recent works explore the pricing strategies of the omnichannel retailer. Zhang et al. [15] analyze the pricing problem when retailers who implement the BOPS model compete with those who only implement network channel operation. Harsha et al. [16] consider the pricing strategy of online and offline inventory sharing. However, there are few studies that consider coordination of pricing and inventory decisions. Fan et al. [17] discuss the problem of pricing and inventory decision making where an online retailer and an offline retailer cooperate to implement the BOPS mode. Moreover, it is worth mentioning that the BOPS mode requires consumers to pay at the moment of ordering online, and they know the exact value of the product only after receiving/picking it, which may cause many product returns [18]. Thus, Liu and Xu [9] investigate an omnichannel BOPS retailer's pricing and ordering model considering online returns. Zhang et al. [5] consider the omnichannel BOPS mode that allows both online and offline consumers to return and cancel orders. Their mode allows a consumer who places an order online to choose to (i) pay online and wait for the package delivered via an express company or (ii) visit the physical stores to touch and feel the product before payment and then buy and pick up it. The BOPS mode in our paper follows the assumption of [5], i.e., a consumer who places an order online can choose to (i) or (ii) according to personal preference. The main differences between our's and [5] are as follows: First, we consider the

multiperiod joint pricing and inventory model, while they consider the single-period setting; second, we consider the consumers' behavior, i.e., reference price effects, which is ignored in their study. The abovementioned studies have a common characteristic that none of them consider the consumers' behavior. In reality, consumers' behaviour has a significant impact on retailers' operation and marketing strategies. Thus, it is necessary to consider the consumers' behaviour when making the joint pricing and inventory strategies.

The second stream of related research is on the periodic review joint pricing and inventory model with reference price effects. This line of research started with Gimpl-Heersink [19], who proves the optimality of the base-stock-list-price for the single-period and two-period models when the customers are loss neutral. However, the optimality of the base-stock-list-price is stricter for the multiperiod setting. Urban [20] analyzes a single-period joint pricing and inventory model with symmetric and asymmetric reference price effects and shows that the consideration of the reference price has a substantial impact on the firm's profitability. Taudes and Rudloff [21] provide an application of the two-period model from the work of Gimpl-Heersink [19] to electronic commodities. Zhang [22] uses a class of transformation techniques to prove the optimality of the base-stock-list-price policy, even if the single-period profit function is nonconcave. Güler et al. [10] extend the model of [19] to the concave demand function, and they address the nonconcavity of the revenue function by combining the transformation technique proposed by Zhang [22] and the inverse demand function. The optimality of the state-dependent order-up-to strategy is proved for the transformed concave revenue function model. Güler et al. [11] use the safety stock as a decision variable to characterize the steady state solution to the problem when the planning horizon is infinite. Chen et al. [8] introduce a new concave transform technique to ensure that the profit function is concave by using the preservation property of supermodularity in parameter optimization problems with the nonlattice structure proposed in [23] and then prove the optimality of the base-stock-list-price strategy. Li and Teng [24] investigate the multiperiod pricing and inventory decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks. For other related works in this stream of research, interested readers may refer to the review by Ren and Huang [25]. In summary, the abovementioned studies have shown that reference price effects have important effects on retailer decision making, but the retailer's joint pricing and inventory strategies considering reference price effects are still inadequate in an omnichannel retail environment. This gives a reason for us to investigate this gap.

3. Model Description

3.1. Modeling the Omnichannel under Reference Price Effects. We consider a retailer ("he"), initially an online retailer, who previously operated a single online channel that only allowed consumers to shop online directly. Currently, he has added a physical store and implemented the omnichannel strategy, which allows consumers to place orders online without paying immediately. With this allowance, a consumer ("she") who places an order online can choose to (i) pay online and wait for the package delivered or (ii) visit the physical stores to touch and feel the product before payment and then buy and pick up it (i.e., choose "BOPS"). The buying procedure of a consumer who orders online is depicted in Figure 1, where $1 - \sigma$ and σ are the fractions of consumers who place orders online by choosing to (i) and (ii), respectively.

As shown in Figure 1, after placing an order online, if a consumer chooses to (i), she knows the exact value of the product only after receiving it. If the actual value does not meet her expectation, she can return the product with a full refund [26], but she needs to pay the return shipping fee for each unit of the product [27]. Suppose the forward shipping fee is paid by the retailer while the return shipping fee is paid by the consumer and the express company charges the same shipping fee m for both of them. If an online consumer chooses to (ii), she needs to pay a travelling cost t ($t > 0$) to visit the store and then decide whether to keep the product or cancel the order in store. The cancellation of the orders does not pay anything. Moreover, assume that there is an additional cross-selling profit l from every consumer visiting the store [6].

The omnichannel retailer orders and sells a single item over an infinite planning horizon. Suppose the inventory periods are identical with equal length T and the shortages are not allowed. The inventory is shared across channels [28], and the replenishment rate is complete and instantaneous. Let $I_i(t)$ be the inventory level at time t in the ith period. Demand rate is proportional to $[I_i(t)]^a$, where $0 \le a < 1$ implies the diminishing marginal effect of inventory level on demand. The retailer charges the same price p_i ($\underline{p} \le p_i \le \overline{p}$) in the ith period over the physical and online channels. The cost of purchase is c per unit, and let h be the holding cost per unit and s be the salvage value per unit satisfying $p_i > c > s$. Let v_i be the consumer's valuation for the product in the ith period which is random and follows the cdf $G(\cdot)$ (also the pdf $g(\cdot)$). Without loss of generality, we assume that $v_i > c$. The reference price depends on past prices and the current price. A commonly used model for the evolution of the reference price is the exponential smoothing model (see, for example, [8, 10, 11]):

$$r_i = \alpha r_{i-1} + (1 - \alpha)p_{i-1} = \alpha^{i-1} r_1 + (1 - \alpha) \sum_{j=1}^{i-1} \alpha^{i-1-j} p_j,$$

$$i = 2, 3, 4, \ldots,$$

$$(1)$$

where the initial reference price r_1 is known and α ($0 \le \alpha < 1$) is the memory factor. The larger the α, the longer the memory. If α is high, then consumers have a long memory and the past price effect is larger. If α is small, then the current price has a greater effect than the past on the reference price. The initial reference price is given by $r_1 \in [\underline{p}, \overline{p}]$, and then, all r_t belong to the interval.

The demand rate at time t in the ith period, denoted by $X(p_i, r_i, t)$, is a multiplicative form of the selling price
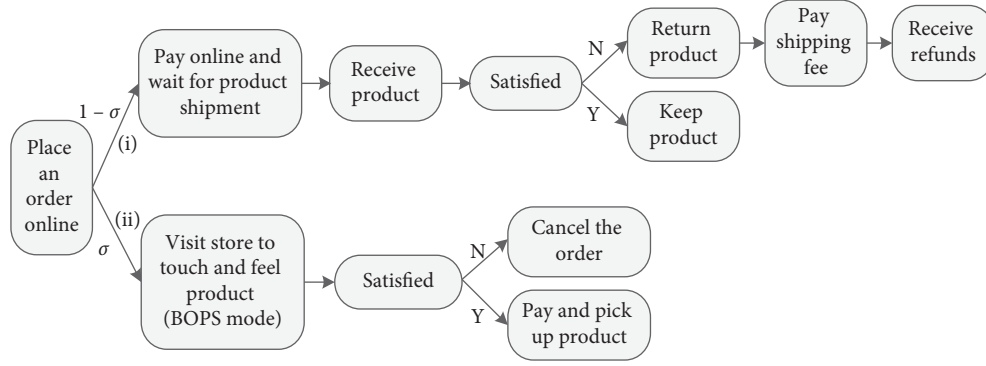
FIGURE 1: Buying procedure of a consumer who orders online under the omnichannel environment.

p_i, reference price r_i, and inventory level $I_i(t)$, which is given by

$$X(p_i, r_i, t) = \{\beta_0 - \beta_1 p_i - \lambda \times \max\{p_i - r_i, 0\} - \gamma \times \min\{p_i - r_i, 0\}\}[I_i(t)]^a, \tag{2}$$

where the nonnegative parameters λ and γ measure the sensitivities of demand associated with the perceived losses and gains, respectively. Demand is classified as loss averse, loss neutral, or loss seeking, depending on whether $\lambda > \gamma, \lambda = \gamma$ or $\lambda < \gamma$. Let $k\,(0 < k < 1)$ be the fraction of the demand served under the single online channel, i.e., the demand was $kX(p_i, r_i, t)$ at time t in the ith period when the retailer used to operate a single offline channel. However, the omnichannel strategy will lead to an incremental demand $(1 - k)X(p_i, r_i, t)$ for the retailer from both the online and offline channels at time t in the ith period [17]. Let ρ and $1 - \rho$ be the fractions of the incremental demand coming from the online and the

offline market, respectively. Then, at time t in the ith period, the online channel demand is increased from $kX(p_i, r_i, t)$ to $[k + \rho(1 - k)]X(p_i, r_i, t)$, and offline channel demand is increased from 0 to $(1 - \rho)(1 - k)X(p_i, r_i, t)$.

During the ith period, the depletion of the inventory due to the effect of demand, $I_i(t)$ can be expressed by the following differential equation:

$$I_i'(t) = -X(p_i, r_i, t) - \{\beta_0 - \beta_1 p_i - \lambda \times \max\{p_i - r_i, 0\} - \gamma \times \min\{p_i - r_i, 0\}\}[I_i(t)]^a, \tag{3}$$

with the boundary condition $I_i(T) = E_i$, where E_i is the ending inventory level in the ith period. Without loss of generality, we assume that $E_i > 0$. Solving differential equation (3) gives

$$I_i(t) = \{E_i^{1-a} + (1 - a)(T - t)[\beta_0 - \beta_1 p_i - \lambda \times \max\{p_i - r_i, 0\} - \gamma \times \min\{p_i - r_i, 0\}]\}^{(1/1-a)}, \tag{4}$$

$0 \le t \le T$. The time-weighted inventory during the ith period is as follows:

$$\int_0^T I_i(t)\mathrm{d}t = \int_0^T \{E_i^{1-a} + (1 - a)(T - t)[\beta_0 - \beta_1 p_i - \lambda \times \max\{p_i - r_i, 0\} - \gamma \times \min\{p_i - r_i, 0\}]\}^{(1/1-a)}. \tag{5}$$

In an omnichannel retail environment, the exact inventory information is available to the consumers when they place orders online, and they will not order the products for those out of stock. Hence, a consumer who places an order online does not suffer a utility loss from the stock-out risk. In the following sections, we start with the case of loss neutral demand (i.e., $\lambda = \gamma$) and then extend our analysis to the case of loss averse demand. Thus, we give the utility of a consumer at different trading times for both online and offline payment under the omnichannel strategy with the case of loss neutral demand in Table 1.

As can be seen from Table 1, the consumer utility of buy online directly and BOPS is $\max\{v_i - p_i + \lambda(r_i - p_i), -m\}$ and $\max\{v_i - p_i + \lambda(r_i - p_i) - t, -t\}$, respectively. According to the principle of utility maximization, if a consumer chooses the buy online directly channel, then the probabilities that he/she keeps and returns the product are $\overline{G}(p_i - m - \lambda(r_i - p_i))$ and $G(p_i - m - \lambda(r_i - p_i))$, respectively. If a consumer chooses the BOPS, the probabilities that he/she chooses to buy or not buy the product are $\overline{G}(p_i - \gamma(r_i - p_i))$ and $G(p_i - \gamma(r_i - p_i))$, respectively. Similarly, the probabilities that an offline consumer chooses

TABLE 1: Consumer utility under two different channels when demand is loss neutral.

Demand source	Purchase channels	At the time of placing the order	At the time of receiving the product	
		Place order	Keep (buy) product	Return product/cancel order/leave store
Online consumers	Buy online directly	$-p_i$	$v_i - p_i + \lambda(r_i - p_i)$	$-m$
	BOPS	0	$v_i - p_i + \lambda(r_i - p_i) - t$	$-t$
Offline consumers	Buy offline directly	0	$v_i - p_i + \lambda(r_i - p_i) - t$	$-t$

to buy or not buy are $\overline{G}(p_i - \gamma(r_i - p_i))$ and $G(p_i - \gamma(r_i - p_i))$, respectively.

Based on the abovementioned analysis, the omnichannel retailer's total profit during the ith period can be written as

$$
\begin{aligned}
\pi(p_i, r_i, E_i), \\
&= \Big[pK_1(1-\sigma)\overline{G}(p_i - m - \lambda(r_i - p_i)) + sK_1(1-\sigma)G(p_i - m - \lambda(r_i - p_i)) - mK_1(1-\sigma) \\
&\quad + pK_1\sigma\overline{G}(p_i - \lambda(r_i - p_i)) + sK_1\sigma G(p_i - \lambda(r_i - p_i)) + lK_1\sigma + pK_2\overline{G}(p_i - \lambda(r_i - p_i)) + sK_2 G(p_i - \lambda(r_i - p_i)) + lK_2 \Big] \\
&\quad \cdot \left[I_i(0) - E_i \right] - cI_i(0) + sE_i - h\int_0^T I_i(t)\mathrm{d}t \\
&= \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + s \right] \cdot \left[I_i(0) - E_i \right] - cI_i(0) + sE_i - h\int_0^T I_i(t)\mathrm{d}t,
\end{aligned}
\tag{6}
$$

where $K_1 = \rho(1-k) + k$, $K_2 = (1-\rho)(1-k)$, and $K_1 + K_2 = 1$. $\lambda_1 = (1-\sigma)K_1$, $\lambda_2 = \sigma K_1 + K_2$, and $\lambda_1 + \lambda_2 = 1$.

Due to the complexity of $h\int_0^T I_i(t)\mathrm{d}t$, following [24, 29], we use the average inventory holding cost ($hT[I_i(0) + E_i]/2$) to estimate the holding cost during the period. Then, the total profit (6) becomes

$$
\begin{aligned}
\pi(p_i, r_i, E_i) &= \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) \right] \cdot \left[\varphi^\lambda(p_i, r_i, E_i) \right]^{(1/1-a)} \\
&\quad - \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) \right] E_i - \frac{hT}{2}\left\{ \left[\varphi^\lambda(p_i, r_i, E_i) \right]^{(1/1-a)} + E_i \right\} \\
&= \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) - \frac{hT}{2} \right] \cdot \left[\varphi^\lambda(p_i, r_i, E_i) \right]^{(1/1-a)} \\
&\quad - \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + \frac{hT}{2} \right] E_i,
\end{aligned}
\tag{7}
$$

where $A(p_i, r_i), B(p_i, r_i)$ are defined in Section 3.2 and

$$
\phi^\lambda(p_i, r_i, E_i) = E_i^{1-a} + (1-a)T[\beta_0 - \beta_1 p_i - \lambda(p_i - r_i)].
\tag{8}
$$

We notice from (7) that $\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) - (hT/2)$ represents the gross unit profit, and we assume that $\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) - (hT/2) > 0$ throughtout the paper. Moreover,

$$\lim_{E_i \longrightarrow \infty} \frac{[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - (hT/2)] \cdot [\phi^\lambda(p_i, r_i, E_i)]^{(1/1-a)}}{E_i[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + (hT/2)]}$$

$$= \lim_{E_i \longrightarrow \infty} \frac{[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - (hT/2)] \cdot \{E_i^{1-a} + (1 - a)T[\beta_0 - \beta_1 p_i - \lambda(p_i - r_i)]\}^{(1/1-a)}}{E_i[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + (hT/2)]}$$

$$= \lim_{E_i \longrightarrow \infty} \frac{[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - (hT/2)] \cdot \{1 + ((1 - a)T[\beta_0 - \beta_1 p_i - \lambda(p_i - r_i)]/E_i^{1-a})\}^{(1/1-a)}}{[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + (hT/2)]}$$

$$= \frac{\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - (hT/2)}{\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + (hT/2)} < 1,$$

$$\text{(9)}$$

which implies that $\lim_{E_i \longrightarrow \infty} \pi(p_i, r_i, E_i) = -\infty$. Therefore, E_i is bounded, and we assume that $0 \leq E_i < \overline{E} < \infty$, where $\overline{E}$ is a large number.

Let $\Pi(P, E)$ denote the omnichannel retailer's discounted total profit over the infinite planning horizon. Given the initial reference price r_1 and discount factor $z \in (0, 1)$, the profit-maximizing problem is formulated as follows:

$$\max_{(P,E) \in \Omega} \Pi(P, E) = \sum_{i=1}^{\infty} z^{i-1} \pi(p_i, r_i, E_i), \quad (10)$$

where $P = \{p_1, p_2, p_3, \ldots\}$, $E = \{E_1, E_2, E_3, \ldots\}$ and

$$\Omega = \left\{(P, E): \underline{p} \leq p_i \leq \overline{p}_i^{2*}(r_i) < \overline{p}, 0 \leq E_i \leq \overline{E}\right\}. \quad (11)$$

It is worth mentioning that $p_i \leq \overline{p}_i^{2*}(r_i)$ follows from Assumption 1 and Lemma 2 in Section 3.2, where $\overline{p}_i^{2*}(r_i)$ is the solution of $L_2(p_i, r_i) = 0$. In addition, we can obtain the following result, which indicates that the omnichannel retailer's discount total profit increases as the initial reference price increases.

Lemma 1. $\Pi(P, E)$ *is increasing in* r_1.

3.2. Notations and Assumptions. The related parameters and variables used in this paper are summarized in Table 2; other notations will be defined as needed.

To facilitate the analysis, we define the following functions:

$$L_1(p_i, r_i) = \overline{G}(p_i - m - \lambda(r_i - p_i)) - (p_i - s)(1 + \lambda)g(p_i - m - \lambda(r_i - p_i)),$$

$$L_3(p_i, r_i) = \overline{G}(p_i - \lambda(r_i - p_i)) - (p_i - s)(1 + \lambda)g(p_i - \lambda(r_i - p_i)),$$

$$L_2(p_i, r_i) = \lambda_1 L_1(p_i, r_i) + \lambda_2 L_3(p_i, r_i), \quad (12)$$

$$A(p_i, r_i) = (p_i - s)\overline{G}(p_i - m - \lambda(r_i - p_i)) - m,$$

$$B(p_i, r_i) = (p_i - s)\overline{G}(p_i - \lambda(r_i - p_i)) + l,$$

where $\lambda_1 = (1 - \sigma)[1 - (1 - \rho)k]$ and $\lambda_2 = \sigma[1 - (1 - \rho)k] + (1 - \rho)k$.

Furthermore, we need the following assumptions.

Assumption 1. Suppose $\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - (hT/2) > 0$ and $L_2(p_i, r_i) > 0$.

Assumption 2. Suppose

$$-g(p_i - m - \lambda(r_i - p_i)) - (1 + \lambda)(p_i - s)g'(p_i - m - \lambda(r_i - p_i)) < 0$$

$$-g(p_i - \lambda(r_i - p_i)) - (1 + \lambda)(p_i - s)g'(p_i - \lambda(r_i - p_i)) < 0$$

$$\text{(13)}$$

throughout the paper.

Assumption 2 ensures the omnichannel retailer's discounted total profit (10) to be concave with respect to retail price p and ending inventory level E, and thus, the optimal p and E are unique. Moreover, the following lemmas can be obtained.

Lemma 2. $L_j(p_i, r_i)$, $j = 1, 2, 3$, *is decreasing in* p_i *and increasing in* r_i. *Thus, for each fixed reference price* r_i, $L_j(p_i, r_i) = 0$, $j = 1, 2, 3$ *has an unique solution* $\overline{p}_i^{j*}(r_i)$.

Lemma 3. $(\partial A(p_i)/\partial p_i) = L_1(p_i, r_i)$, $(\partial B(p_i)/\partial p_i) = L_3(p_i, r_i)$, *and* $(\partial[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i)]/\partial p_i) = L_2(p_i, r_i)$.

TABLE 2: Summary of notations.

Notation	Description
Decision	
p_i	The omnichannel retailer's retail price in the ith period
E_i	The ending inventory level in the ith period
Parameters	
D_i	The demand rate in the ith period
$\Pi(\cdot)$	The omnichannel retailer's discounted total profit over the infinite horizon
$\pi_i(\cdot)$	The omnichannel retailer's total profit in the ith period
z	The discount factor $(0 < z < 1)$
λ, γ	The reference price effects coefficient implies the sensitivity of consumers to the gap between the reference price and the retail price. The demand is called loss neutral if $\lambda = \gamma$, loss averse if $\lambda > \gamma$, and loss seeking if $\lambda < \gamma$
α	The memory factor $(0 \le \alpha < 1)$
v_i	The valuation of the product by the consumer in the ith period, random values that follow distribution $G(\cdot)$ and density $g(\cdot)$, and $\overline{G}(\cdot) = 1 - G(\cdot)$
c	The unit inventory and procurement cost
m	The unit shipping fee
t	The unit travelling cost of consumers to visit the store
s	The salvage price for a leftover unit
l	The cross-selling benefit
σ	The fraction of online consumers choosing to BOPS, and $1 - \sigma$ is the fraction of online consumers choosing to buy online directly
ρ	The fraction of the incremental demand (brought by the omnichannel strategy) coming from the online market, and $1 - \rho$ is the fraction of the incremental demand coming from offline market
k	The fraction of the market demand served under the single online channel strategy

4. Structural Analysis of the Optimal Omnichannel Strategies

In this section, we analyze structural properties of the omnichannel retailer's optimal pricing and ending inventory level. In what follows, we first analyze the case of loss neutral demand. Secondly, we extend the loss neutral demand to the other case of loss averse demand.

4.1. Loss Neutral Demand. In this subsection, we analyze the omnichannel retailer's optimal pricing and ending inventory level when demand is loss neutral. First, in order to prove the uniqueness of the optimal pricing and ending inventory level, the concavity of $\Pi(P, E)$ is needed.

Proposition 1. $\Pi(P, E)$ *is concave in p_i and E_i, and thus, the optimal p_i^* and E_i^* in the ith period are unique.*

Next, we investigate the initial reference price r_1 on the optimal pricing and ending inventory level decisions, and the following lemma is needed.

Lemma 4. *If $\beta_0 - \beta_1 \overline{p} - \lambda(\overline{p} - \underline{p}) > \max\{M_1, M_2\}$, where*

$$M_1 = \frac{a}{1-a}(\beta_1 + \lambda)\frac{\lambda_1 A(\overline{p}, r_i) + \lambda_2 B(\overline{p}, r_i) - (c-s) - (hT/2)}{L_2(\overline{p}, r_i)},$$

$$M_2 = \frac{(\beta_1 + \lambda)}{(1-a)T} \cdot \frac{[\lambda_1(\overline{p} - s)g(\overline{p} - m - \lambda(r_i - \overline{p})) + \lambda_2(\overline{p} - s)g(\overline{p} - \lambda(r_i - \overline{p}))] + aT[\lambda_1 A(\overline{p}, r_i) + \lambda_2 B(\overline{p}, r_i) - (c-s) - (hT/2)]}{L_2(\overline{p}r_i)},$$

(14)

then

$$\phi^\lambda(p_i, r_i, E_i) > a(\beta_1 + \lambda)T\frac{\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) - (hT/2)}{L_2(p_i, r_i)},$$

$$\phi^\lambda(p_i, r_i, E_i) > (\beta_1 + \lambda)\frac{\{[\lambda_1(p_i - s)g(p_i - m - \lambda(r_i - p_i)) + \lambda_2(p_i - s)g(p_i - \lambda(r_i - p_i))] + aT[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c-s) - (hT/2)/2]\}}{L_2(p_i, r_i)}.$$

(15)

Proposition 2. *When the demand is loss neutral, then* $\{p_i^*\}_{i=1}^{\infty}$ *and* $\{E_i^*\}_{i=1}^{\infty}$ *both increase as* r_1 *increases if* $\beta_0 - \beta_1\overline{p} - \lambda(\overline{p} - \underline{p}) > \max\{M_1, M_2\}$.

Proposition 2 indicates that both the optimal price and ending inventory level sequences $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ increase with the initial reference price r_1. This can be intuitively illustrated as follows: With the increase of consumers' initial reference price, i.e., when consumers' initial valuation of the product increases, the demand will increase and the optimal price will rise as well. The omnichannel retailer orders more to raise the inventory level so as to meet the demand as much as possible. As a result, the ending inventory level will increase. The following result shows the monotonicity behavior of the optimal decisions depending on the initial reference price.

Proposition 3. *When the demand is loss neutral, the following results hold:*

(i) *If* $p_1^* > r_1$, *then* $\{p_i^*\}_{i=1}^{\infty}$ *and* $\{E_i^*\}_{i=1}^{\infty}$ *are both increasing*

(ii) *If* $p_1^* < r_1$, *then* $\{p_i^*\}_{i=1}^{\infty}$ *and* $\{E_i^*\}_{i=1}^{\infty}$ *are both decreasing*

(iii) *If* $p_1^* = r_1$, *then* $r_1 = p_1^* = p_2^* = p_3^* = \cdots$ *and* $E_1^* = E_2^* = E_3^* = \cdots$

Proposition 3 shows that both the optimal price and ending inventory level sequences $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ are monotonic in the same direction depending on the initial reference price r_1. Thus, both $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ are convergent due to their boundness. Let p_e and E_e denote the equilibrium selling price and ending inventory level when demand is loss neutral. Then, p_e and E_e can be characterized via the following result.

Proposition 4. *If an interior equilibrium exists, pe and Ee can be simultaneously determined by*

$$
\begin{aligned}
L_2\left(p_e, p_e\right)&\left(\frac{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) + (hT/2)}{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - (hT/2)}\right)^{(1/a)} - 1 \\
&= \frac{\left(\beta_1 + \lambda(1 - z/1 - \alpha z)\right)\left[\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - (hT/2)\right]}{(1-a)\left(\beta_0 - \beta_1 p_e\right)} \\
&\quad \times \left[\left(\frac{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) + (hT/2)}{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - (hT/2)}\right)^{(1/a)} - \left(\frac{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) + (hT/2)}{\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - (hT/2)}\right)\right], \\
E_e &= \left[\frac{(1-a)T\left(\beta_0 - \beta_1 p_e\right)}{\left(\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) + (hT/2)/\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - (hT/2)\right)^{(1-a/a)} - 1}\right]^{(1/1-a)},
\end{aligned}
\tag{16}
$$

respectively.

The next proposition characterizes the impact of the strength of reference price effects coefficient (λ), memory factor (α), unit shipping fee m, and cross-selling benefit l on the optimal pricing and ending inventory level.

Proposition 5. *If an interior equilibrium exists, then*

(i) p_e *and* E_e *are both decreasing in* λ

(ii) p_e *and* E_e *are both decreasing in* α

(iii) p_e *and* E_e *are both increasing in* m

(iv) p_e *and* E_e *are both increasing in* l

Proposition 5 indicates that both equilibrium selling price p_e and ending inventory level E_e decrease with the initial reference price λ and α. This can be intuitively interpreted as follows: First, when λ increases, it implies that consumers are more sensitive to the gap between the selling price and the reference price. The retailer should reduce the selling price in order to make the selling price closer to the consumers' reference price and reduce the inventory level at the same time. Thus, p_e and E_e are both decreasing. Second, when α increases,

consumers have a long memory and the past price effect is larger. This means that consumers adapt to the new price information at a lower rate and behave less loyally; then, the retailer should decrease its selling price while reducing the inventory level. Hence, the equilibrium selling price p_e and ending inventory level E_e also decrease. Third, since the forward shipping fee m is paid by the retailer, if the forward shipping fee increases, the selling price will naturally increase. Moreover, the high return shipping fee m will reduce the consumers' desire to buy, which leads to an increase in the retailer's ending inventory. Fourth, when the cross-selling benefit l increases, it indicates that there are more consumers who patronize the physical stores (including offline consumers and BOPS consumers), and retailers can increase inventory to meet the needs of these consumers. Additionally, the retailer will increase the corresponding service cost for the extreme experience service provided by the physical store, which will make the retailer increase the sales price of goods. Combining Propositions 3–5, we can get the following conclusion.

Proposition 6. *If an interior equilibrium exists, then*

(i) *If* $p_1^* > r_1$, *then* $\{p_i^*\}_{i=1}^{\infty}$ *and* $\{E_i^*\}_{i=1}^{\infty}$ *both increase monotonically and converge to* p_e *and* E_e, *respectively*

(ii) If $p_1^* > r_1$, then $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ both decrease monotonically and converge to p_e and E_e, respectively

(iii) If $p_1^* = r_1$, then $p_e = p_1^* = p_2^* = p_3^* = \cdots$ and $E_e = E_1^* = E_2^* = E_3^* = \cdots$

4.2. Loss Averse Demand.

In this section, we extend the case of loss neutral demand to that of loss averse, i.e., $\lambda > \gamma$. Following the analysis similar to that in [24, 29], for $P = \{p_1, p_2, p_3, \ldots\}$, $E = \{E_1, E_2, E_3, \ldots\}$, and $\Omega = \{(P, E): \underline{p} \leq p_i \leq \overline{p}_i^{2*}(r_i) < \overline{p}, 0 \leq E_i \leq \overline{E}\}$, the profit-maximizing problem for the loss averse demand is formulated as follows:

$$\max_{(P,E)\in\Omega} \Pi(P, E) = \max_{(P,E)\in\Omega} \sum_{i=1}^{\infty} z^{i-1} \pi(p_i, r_i, E_i),$$

$$= \max_{(P,E)\in\Omega} \sum_{i=1}^{\infty} z^{i-1} \min\{\pi^\lambda(p_i, r_i, E_i), \pi^\gamma(p_i, r_i, E_i)\}, \quad (17)$$

$$= \min\left\{ \max_{(P,E)\in\Omega} \Pi^\lambda(P, E), \max_{(P,E)\in\Omega} \Pi^\gamma(P, E)\right\},$$

where $\Pi^x(P, E) = \sum_{i=1}^{\infty} z^{i-1} \pi^x(p_i, r_i, E_i)$, $x = \lambda$ or γ, and

$$\pi^x(p_i, r_i, E_i) = \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - \frac{hT}{2} \right] \cdot [\phi^x(p_i, r_i, E_i)]^{(1/1-a)}$$

$$- \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) + \frac{hT}{2} \right] E_i, \quad x = \lambda \text{ or } \gamma, \quad (18)$$

$$\phi^x(p_i, r_i, E_i) = E_i^{1-a} + (1 - a)T[\beta_0 - \beta_1 p_i - x(p_i - r_i)].$$

Similar to the analysis in Section 4.1, we first prove the uniqueness of the optimal pricing and ending inventory level.

Proposition 7. *The omnichannel retailer's discounted total profit (16) is concave in p_i and E_i, and thus, the optimal p_i^* and E_i^* in the ith period are unique.*

Since

$$\pi^\lambda(p_i, r_i, E_i) - \pi^\gamma(p_i, r_i, E_i),$$

$$= \begin{cases} \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - \frac{hT}{2} \right] \left\{ [\phi^\lambda(p_i, r_i, E_i)]^{(1/1-a)} - [\phi^\gamma(p_i, r_i, E_i)]^{(1/1-a)} \right\} < 0, & p_i > r_i, \\ 0, & p_i = r_i, \\ \left[\lambda_1 A(p_i, r_i) + \lambda_2 B(p_i, r_i) - (c - s) - \frac{hT}{2} \right] \left\{ [\phi^\lambda(p_i, r_i, E_i)]^{(1/1-a)} - [\phi^\gamma(p_i, r_i, E_i)]^{(1/1-a)} \right\} > 0, & p_i < r_i, \end{cases} \quad (19)$$

it follows from Proposition 3 that both the selling price $\{p_i^*\}_{i=1}^{\infty}$ and ending inventory level $\{E_i^*\}_{i=1}^{\infty}$ are monotonic in the same direction as the reference price path. Then, both $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ are convergent due to their boundness. Let p_e^x and E_e^x denote the equilibrium selling price and ending inventory level when demand is loss averse, $x = \lambda$ or γ. Moreover, since $\lambda > \gamma$, $p_e^\lambda < p_e^\gamma$ and $E_e^\lambda < E_e^\gamma$ by applying Proposition 6. We can, thus,

obtain the following structural properties of the optimal decisions for the case of loss averse demand.

Proposition 8. *If an interior equilibrium exists, then*

(i) If $p_e^\gamma < r_1$, then $\max_{(P,E)\in\Omega}\Pi(P, E) = \max_{(P,E)\in\Omega}\Pi^\gamma(P, E)$, and $\{p_i^*\}_{i=1}^{\infty}$ and $\{E_i^*\}_{i=1}^{\infty}$ both decrease

monotonically and converge to p_e^γ and E_e^γ, respectively

(ii) If $p_e^\lambda > r_1$, then $\max_{(P,E)\in\Omega} \Pi(P, E) = \max_{(P,E)\in\Omega} \Pi^\lambda(P, E)$, and $\{p_i^*\}_{i=1}^\infty$ and $\{E_i^*\}_{i=1}^\infty$ both increase monotonically and converge to p_e^λ and E_e^λ, respectively

(iii) If $p_e^\lambda \le r_1 \le p_e^\gamma$, then $\max_{(P,E)\in\Omega} \Pi(P, E) = \max_{(P,E)\in\Omega} \Pi^\lambda(P, E) = \max_{(P,E)\in\Omega} \Pi^\gamma(P, E)$, $r_1 = p_1^* = p_2^* = p_3^* = \cdots$, and

$$E_i = \left[\frac{(1-a)T(\beta_0 - \beta_1 r_1)}{(\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)/\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2))^{(1-a/a)} - 1} \right]^{(1/1-a)}, \quad i = 1,2,3,\dots. \tag{20}$$

5. Numerical Analysis

In this section, we carry out several numerical experiments to illustrate the abovementioned theoretical results and gain some managerial insights. The following analysis considers the loss of neutral consumers because, according to (16), the situation when consumers are loss aversion can be similarly analyzed. All experiments below are performed in MATLAB R2014b on a laptop with an Intel(R) Core (TM) i5-7200U central processing unit (CPU) (2.50 GHz, 2.70 GHz) and 8.0 GB of RAM running 64 bit Windows 10 Enterprise.

We consider an omnichannel inventory system with the following initial parameter values: $\beta_0 = 160$, $\beta_1 = 2$, $c = 10$, $h = 2$, $s = 1$, $m = 1$, $l = 2$, $\alpha = 0.8$, $a = 0.2$, $z = 0.6$, $\sigma = 0.3$, $k = 0.6$, $\rho = 0.6$, $T = 1$, $\underline{p} = 20$, $\overline{p} = 40$, and $v_i \sim U[10, 30]$. The optimal equilibrium price, ending inventory level, and optimal discounted total profit for the different values of λ and r_1 can be obtained by Proposition 4, which are presented in Table 3. Furthermore, the sensitivity analysis of the key system parameters, including the memory factor α, the ratio of BOPS consumers σ, the cross-selling profit l, and the shipping fee m, is presented on the optimal equilibrium price p_e and ending inventory level E_e, as well as the optimal discounted total profit $\Pi(P^*, E^*)$, and the corresponding computational results are shown in Tables 4–7.

As shown in Table 3, the optimal equilibrium price p_e and ending inventory level E_e both are decreasing in λ, which is consistent with Proposition 5 (i). Furthermore, the optimal discounted total profit, $\Pi(P^*, E^*)$, increases with the initial reference price r_1, which is also consistent with Lemma 1. This can be intuitively illustrated as follows. When λ is large, which means that the consumers are more sensitive to the gap between the consumers' reference price and the retailer's actual selling price is large, the retailer should decrease the selling price in order to reduce this gap, stimulating demand. This increases the retailer's profit $\Pi(P^*, E^*)$ and reduces the ending inventory level E_e.

Table 4 presents the impact of memory factor α on the optimal equilibrium price p_e, ending inventory level E_e and optimal discounted total profit $\Pi(P^*, E^*)$. As shown in Table 4, the optimal equilibrium price p_e and ending inventory level E_e both are decreasing in α, which is consistent with Proposition 5 (ii). We can also see from Table 4 that when the initial reference price r_1 is high, the retailer's profit

increases with α. The intuition is that when α is large, the consumers have a long-term memory of the initial reference price r_1. If the current price is high enough, consumers will not be willing to buy. Hence, the retailer should decrease the selling price below the initial reference price r_1 in order to reduce the gap between the consumers' reference price and the retailer's actual selling price, thus stimulating demand, which increases the retailer's total profit $\Pi(P^*, E^*)$ and reduces the ending inventory level E_e.

Table 5 provides the impact of the ratio of BOPS consumers' σ on the optimal equilibrium price p_e, ending inventory level E_e, and optimal discounted total profit $\Pi(P^*, E^*)$. It follows from Table 5 that the optimal equilibrium price p_e and ending inventory level E_e both are decreasing in σ. This implies that the increase of the ratio of consumers who choose BOPS will make the retailer reduce the selling price, which will not only attract more consumers to patronize the physical store but also make the retailer earn more cross-selling profit, thus increasing the total profit $\Pi(P^*, E^*)$.

Table 6 shows the impact of shipping fee m on the optimal equilibrium price p_e, ending inventory level E_e and optimal discounted total profit $\Pi(P^*, E^*)$. As shown in Table 6, the optimal equilibrium price p_e and ending inventory level E_e both are increasing in m, which is consistent with Proposition 5 (iii). The intuitive explanation is that when the shipping fee increases, the retailer will inevitably increase the selling price, which in turn will reduce the demand and increase the ending inventory level, which is not conducive to the retailer's profit $\Pi(P^*, E^*)$.

Table 7 presents the impact of cross-selling profit l on the optimal equilibrium price p_e, ending inventory level E_e and optimal discounted total profit $\Pi(P^*, E^*)$. As shown in Table 7, the optimal equilibrium price p_e and ending inventory level E_e both are increasing in l, which is consistent with Proposition 5 (iv). Table 5 implies that when the cross-selling profit increases, if the retailer increases the selling price, the demand will be restrained, resulting in the increase of ending inventory and the decrease of profit $\Pi(P^*, E^*)$.

6. Managerial Insights

In this section, some managerial insights are derived from the numerical analysis, which can be adopted by an

TABLE 3: The optimal solutions with respect to different values of λ and $r1$.

λ	r_1	$\Pi(P^*, E^*)$	p_e	E_e
1	35	108.66	30.8677	1.5071
1	40	1197.4	30.8677	1.5071
1	45	3622.7	30.8677	1.5071
1	48	5791.5	30.8677	1.5071
1	55	13111	30.8677	1.5071
1.25	35	247.87	30.8341	1.4344
1.25	40	2163.5	30.8341	1.4344
1.25	45	6295.4	30.8341	1.4344
1.25	48	9955.7	30.8341	1.4344
1.25	55	22234	30.8341	1.4344
1.50	35	443.80	30.8066	1.3755
1.50	40	3445.4	30.8066	1.3755
1.50	45	9794.8	30.8066	1.3755
1.50	48	15387	30.8066	1.3755
1.50	55	34072	30.8066	1.3755

TABLE 6: Effect of change in m for the dynamic model with $\lambda = \gamma = 1.25$.

r_1	m	$\Pi(P^*, E^*)$	p_e	E_e
35	1	247.87	30.8341	1.4344
35	2	149.63	31.1084	2.0644
35	3	66.290	31.3900	2.7163
35	4	1.1265	31.6790	3.4398
35	5	-42.368	31.9753	4.2151
45	1	6295.4	30.8341	1.4344
45	2	5812.8	31.1084	2.0644
45	3	5317.3	31.3900	2.7163
45	4	4820.2	31.6790	3.4398
45	5	4322.5	31.9753	4.2151
55	1	22234	30.8341	1.4344
55	2	21386	31.1084	2.0644
55	3	20492	31.3900	2.7163
55	4	19575	31.6790	3.4398
55	5	18632	31.9753	4.2151

TABLE 4: Effect of change in α for the dynamic model with $\lambda = \gamma = 1.25$.

r_1	α	$\Pi(P^*, E^*)$	p_e	E_e
35	0.750	244.59	30.8426	1.4527
35	0.775	246.21	30.8384	1.4437
35	0.800	247.87	30.8341	1.4344
35	0.825	249.57	30.8297	1.4249
35	0.850	251.35	30.8251	1.4151
45	0.750	6280.3	30.8426	1.4527
45	0.775	6287.7	30.8384	1.4437
45	0.800	6295.4	30.8341	1.4344
45	0.825	6303.2	30.8297	1.4249
45	0.850	6311.3	30.8251	1.4151
55	0.750	22208	30.8426	1.4527
55	0.775	22221	30.8384	1.4437
55	0.800	22234	30.8341	1.4344
55	0.825	22247	30.8297	1.4249
55	0.850	22262	30.8251	1.4151

TABLE 7: Effect of change in l for the dynamic model with $\lambda = \gamma = 1.25$.

r_1	l	$\Pi(P^*, E^*)$	p_e	E_e
40	2	2163.5	30.8341	1.4344
40	3	494.88	32.6087	1.4366
40	4	294.13	33.0113	7.1432
40	5	89.083	33.4010	8.3197
40	6	−64.894	33.7791	9.4956
45	2	6295.4	30.8341	1.4344
45	3	2983.4	32.6087	1.4366
45	4	2723.9	33.0113	7.1432
45	5	2184.4	33.4010	8.3197
45	6	1697.9	33.7791	9.4956
55	2	22234	30.8341	1.4344
55	3	17057	32.6087	1.4366
55	4	15386	33.0113	7.1432
55	5	14188	33.4010	8.3197
55	6	13042	33.7791	9.4956

TABLE 5: Effect of change in σ for the dynamic model with $\lambda = \gamma = 1.25$.

r_1	σ	$\Pi(P^*, E^*)$	p_e	E_e
35	0.1	102.47	31.0677	1.9531
35	0.3	247.87	30.8341	1.4344
35	0.5	459.16	30.5985	0.9473
35	0.7	744.14	30.3608	0.5031
35	0.9	1105.9	30.1208	0.1282
45	0.1	4104.3	31.0677	1.9531
45	0.3	6295.4	30.8341	1.4344
45	0.5	8735.9	30.5985	0.9473
45	0.7	11424	30.3608	0.5031
45	0.9	14346	30.1208	0.1282
55	0.1	15110	31.0677	1.9531
55	0.3	22234	30.8341	1.4344
55	0.5	29779	30.5985	0.9473
55	0.7	37731	30.3608	0.5031
55	0.9	46059	30.1208	0.1282

omnichannel retailer to formulate its pricing and ending inventory strategies with the reference price effects.

The parameter λ has a negative impact on selling price and positive impact on profit. When the reference price effects on demand are great, which means that the gap between the reference price and the retailer's actual selling price is large, the retailer should adopt low selling price to reduce this gap and stimulate demand so that the retailer can raise its profit and reduce the ending inventory level.

The memory parameter α has a negative impact on selling price and positive impact on profit. As α increases, consumers have a long-term memory of the initial reference price r_1. If the current selling price is high enough, consumers will not be willing to buy. Hence, the retailer should decrease the selling price in order to reduce the gap between the reference price and the retailer's selling price, thus stimulating demand, which helps to increase the retailer's profit and reduce the ending inventory level.

When the BOPS is considered, as the ratio of BOPS consumers σ increases, the retailer should adopt a low-price strategy, which will not only attract more consumers to patronize the physical store by means of the advantages of in-store experience but also make the retailer earn more cross-selling profit, thus increasing its profit. In addition, Table 5 indicates that the retailer should not increase the selling price when the cross-selling profit increases because this will restrain the demand. Finally, the shipping fee m has a negative impact on selling price and profit. The reason is that a higher unit shipping fee can decrease the consumers' willingness to pay for the product, which in turn will reduce the demand and increase the ending inventory level, which is not conducive to the retailer's profit.

7. Conclusions

Our research complements the existing research stream in coordinating pricing and inventory replenishment decisions under an omnichannel retail environmental by taking into the consideration of consumers' reference price effects. Specifically, this paper considers an omnichannel retailer's multiperiod pricing and inventory retail operations under reference price effects in which consumers can cancel their order before payment and return the product after payment if the product does not meet their expectation. The omnichannel retailer's optimal equilibrium pricing and ending inventory level are derived under the reference price effects by maximizing the discounted total profit over the infinite planning horizon, where the optimal decisions we discussed are under two scenarios: loss neutrality and loss aversion. In addition, the impact of system parameters on the optimal price and ending inventory level is studied. Our main results are summarized as follows. First, our analysis shows that the convergence of the pricing and ending inventory level toward their equilibrium is from above or below, depending on the relative location of the initial reference price with respect to the unique equilibrium price. Second, we investigate how key system parameters affect the optimal decisions. When consumers are more sensitive to the gap between the selling price and the reference price or

consumers have a long memory of the past price, both equilibrium selling price p_e and ending inventory level E_e decrease. Furthermore, when the shipping fee m or the cross-selling benefit l increases, both equilibrium selling price p_e and ending inventory level E_e increase. This research fills in the gap of behavioral operation management in the study of pricing and inventory in omnichannel retail operations.

Though this paper has identified the effects of reference price on the coordination of pricing and inventory decisions for an omnichannel retailer, there are still some shortcomings that can be investigated in the future. First, this paper analyzes the pricing and inventory decisions of an omnichannel retailer under consumers' reference price effects, unaware of the influence of loss aversion on the omnichannel retailer. An interesting future research topic is to examine the pricing and inventory decisions for considering the reference point and loss aversion of an omnichannel retailer. Second, in our study, the customers' reference price can be observed by retailers. However, the information on the reference price is difficult to obtain in reality. Therefore, demand learning can be incorporated into formulating pricing and inventory strategies in the presence of the reference price effects under the omnichannel environment. Third, the government subsidy is necessary because the high shipping fee will increase the selling price, thus restraining the increase of demand and profit (see, for example, [30]). Hence, government subsidy for the shipping fee is another factor that can be incorporated into formulating pricing and inventory strategies in the presence of the reference price effects under the omnichannel environment.

Appendix

Proof of Lemma 1. Since $\phi(p_i, r_i, E_i)$ is increasing in r_1, $\pi(p_i, r_i, E_i)$ is increasing in r_1. Thus, $\Pi(\mathrm{P}, \mathrm{E}) = \sum_{i=1}^{\infty} z^{i-1} \pi(p_i, r_i, E_i)$ is increasing in r_1. □

Proof of Lemma 2. Since

$$
\frac{\partial L_1(p, r)}{\partial p} = -(1 + \gamma)\left[g(p - m - \gamma(r - p)) + (p - s)(1 + \gamma)g'(p - m - \gamma(r - p))\right],
$$

$$
\frac{\partial L_3(p, r)}{\partial p} = -(1 + \gamma)\left[g(p - \gamma(r - p)) + (p - s)(1 + \gamma)g'(p - \gamma(r - p))\right],
$$

$$
\frac{\partial L_1(p, r)}{\partial r} = \gamma\left[g(p - m - \gamma(r - p)) + (p - s)(1 + \gamma)g'(p - m - \gamma(r - p))\right],
$$

$$
\frac{\partial L_3(p, r)}{\partial r} = \gamma\left[g(p - \gamma(r - p)) + (p - s)(1 + \gamma)g'(p - \gamma(r - p))\right],
$$

(A.1)

it follows from Assumption 2 that $(\partial L_1(p,r)/\partial p)<0$, $(\partial L_3(p,r)/\partial p)<0$, $(\partial L_1(p,r)/\partial r)>0$ and $(\partial L_3(p,r)/\partial r)>0$, which yields the results. □

Proof of Proposition 1. Since

$$
\begin{aligned}
\frac{\partial \Pi(\text{P},\text{E})}{\partial p_i^2} = {} & z^{i-1}\frac{\partial L_2(p_i,r_i)}{\partial p_i}\left\{\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)}-E_i\right\} \\
& - z^{i-1}\left\{2(\beta_1+\lambda)TL_2(p_i,r_i)\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}\right. \\
& \left. + a(\beta_1+\lambda)^2T^2\left[\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2}\right\},
\end{aligned}
\tag{A.2}
$$

noting that $\phi^\lambda(p_i,r_i,E_i)^{(1/1-a)}-E_i>0$ and $(\partial L_2(p_i,r_i)/\partial p_i)<0$, we thus, obtain $(\partial \Pi(\text{P},\text{E})/\partial p_i^2)<0$.

Moreover,

$$
\begin{aligned}
\frac{\partial \Pi(\text{P},\text{E})}{\partial E_i^2} = {} & z^{i-1}aE^{-a-1}\left[\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2} \\
& \times\left[E^{1-a}-\phi^\lambda(p_i,r_i,E_i)\right]<0.
\end{aligned}
\tag{A.3}
$$

Hence, Proposition 1 can be obtained. □

Proof of Lemma 4.

$$
\begin{aligned}
\phi^\lambda(p_i,r_i,E_i) = {} & E_i^{1-a}+(1-a)T\left[\beta_0-\beta_1 p_i-\lambda(p_i-r_i)\right] \\
> {} & (1-a)T\left[\beta_0-\beta_1\overline{p}-\lambda\left(\overline{p}-\underline{p}\right)\right] \\
> {} & a(\beta_1+\lambda)T\frac{\lambda_1 A(\overline{p},r_i)+\lambda_2 B(\overline{p},r_i)-(c-s)-(hT/2)}{L_2(\overline{p},tr_i)} \\
> {} & a(\beta_1+\lambda)T\frac{\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-(hT/2)}{L_2(p_i,r_i)},
\end{aligned}
\tag{A.4}
$$

where the last inequality follows from the fact that $(\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-(hT/2)/L_2(p_i,r_i))$ is increasing in p_i. Moreover, since $([\lambda_1(p_i-s)g(p_i-m-\lambda(r_i-p_i))+\lambda_2(p_i-s)g(p_i-\lambda(r_i-p_i))]+aT[\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-$

$(hT/2)]/L_2(p_i,r_i))$ is also increasing in p_i, the second inequality is similar to provable. □

Proof of Proposition 2.

$$
\begin{aligned}
\frac{\partial^2\Pi(\text{P},\text{E})}{\partial p_i\partial r_1} = {} & z^{i-1}\left\{\frac{\partial L_2(p_i,r_i)}{\partial r_i}\alpha^{i-1}\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1-a/1)}+\lambda\alpha^{i-1}TL_2(p_i,r_i)\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}\right\} \\
& - z^{i-1}\left\{[\lambda_1(p_i-s)g(p_i-m-\lambda(r_i-p_i))+\lambda_2(p_i-s)g(p_i-\lambda(r_i-p_i))]\right. \\
& \left. \times\lambda\alpha^{i-1}(\beta_1+\lambda)T\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}-\left[\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-\frac{hT}{2}\right]\right.
\end{aligned}
$$

$$\times\, a\lambda\alpha^{i-1}(\beta_1+\lambda)T^2\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2} - \frac{\partial L_2(p_i,r_i)}{\partial r_i}\alpha^{i-1}E_i\Bigg\}$$

$$+\sum_{n=i+1}^{\infty}z^{n-1}\lambda^2(1-\alpha)\alpha^{2n-2-i}T^2\left[\lambda_1 A(p_n,r_n)+\lambda_2 B(p_n,r_n)-(c-s)-\frac{hT}{2}\right]$$

$$\times\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2}+\sum_{n=i+1}^{\infty}z^{n-1}\lambda(1-\alpha)\alpha^{2n-2-i}T\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}$$

$$\times\left[\lambda_1(p_n-s)g(p_n-m-\lambda(r_n-p_n))+\lambda_2(p_n-s)g(p_n-\lambda(r_n-p_n))\right]$$

$$=z^{i-1}\alpha^{i-1}\frac{\partial L_2(p_i,r_i)}{\partial r_i}\left\{\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)}-E_i\right\}+z^{i-1}\lambda\alpha^{i-1}T\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2}$$

$$\times\Big\{L_2(p_i,r_i)\phi^\lambda(p_i,r_i,E_i)-(\beta_1+\lambda)$$

$$\left[\lambda_1(p_i-s)g(p_i-m-\lambda(r_i-p_i))+\lambda_2(p_i-s)g(p_i-\lambda(r_i-p_i))\right]$$

$$-a(\beta_1+\lambda)T\left[\lambda_1 A(p_n,r_n)+\lambda_2 B(p_n,r_n)-(c-s)-\frac{hT}{2}\right]$$

$$+\sum_{n=i+1}^{\infty}z^{n-1}\lambda^2(1-\alpha)\alpha^{2n-2-i}T^2\left[\lambda_1 A(p_n,r_n)+\lambda_2 B(p_n,r_n)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2}$$

$$+\sum_{n=i+1}^{\infty}z^{n-1}\lambda(1-\alpha)\alpha^{2n-2-i}T\;\left[\lambda_1(p_n-s)g(p_n-m-\lambda(r_n-p_n))+\lambda_2(p_n-s)\times g(p_n-\lambda(r_n-p_n))\right]\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}$$

$$>0,$$

$$\frac{\partial\Pi(P,E)}{\partial E_i\partial r_1}=z^{i-1}\lambda\alpha^{i-1}\left[\lambda_1(p_i-s)g(p_i-m-\lambda(r_i-p_i))+\lambda_2(p_i-s)g(p_i-\lambda(r_i-p_i))\right]$$

$$\times\left\{E_i^{-a}\left[\varphi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}-1\right\}$$

$$+z^{i-1}a\lambda\alpha^{i-1}TE_i^{-a}\left[\lambda_1 A(p_i,r_i)+\lambda_2 B(p_i,r_i)-(c-s)-\frac{hT}{2}\right]\left[\varphi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-2}>0,$$

$$\frac{\partial\Pi(P,E)}{\partial p_i\partial p_j}=\frac{\partial}{\partial p_j}\left\{\begin{array}{c}\displaystyle\sum_{n=i+1}^{\infty}z^{n-1}\lambda(1-\alpha)\alpha^{n-1-i}T\left[\lambda_1 A(p_n,r_n)+\lambda_2 B(p_n,r_n)-(c-s)-\frac{hT}{2}\right]\\[6pt]\times\left[\phi^\lambda(p_n,r_n,E_n)\right]^{(1/1-a)-1}\end{array}\right\},$$

$$=z^{j-1}\lambda(1-\alpha)\alpha^{j-1-i}TL_2\left(p_j,r_j\right)\left[\phi^\lambda(p_i,r_i,E_i)\right]^{(1/1-a)-1}-z^{j-1}a\lambda(1-\alpha)\alpha^{j-1-i}(\beta_1+\lambda)$$

$$\times T^2\left[\lambda_1 A\left(p_j,r_j\right)+\lambda_2 B\left(p_j,r_j\right)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda\left(p_j,r_j,E_j\right)\right]^{(1/1-a)-2}$$

$$+\sum_{n=j+1}^{\infty}z^{n-1}a\lambda^2(1-\alpha)^2\alpha^{2n-i-j-2}T^2\left[\lambda_1 A(p_n,r_n)+\lambda_2 B(p_n,r_n)-(c-s)-\frac{hT}{2}\right]$$

$$\times\left[\phi^\lambda(p_n,r_n,E_n)\right]^{(1/1-a)-2}+\sum_{n=j+1}^{\infty}z^{n-1}\lambda(1-\alpha)^2\alpha^{2n-i-j-2}TL_2(p_n,r_n)\left[\phi^\lambda(p_n,r_n,E_n)\right]^{(1/1-a)-1}$$

$$=z^{j-1}\lambda(1-\alpha)\alpha^{j-1-i}T$$

$$\left\{ L_2\left(p_j,r_j\right)\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right] - a\left(\beta_1+\lambda\right)T \times \left[\lambda_1 A\left(p_j,r_j\right)+\lambda_2 B\left(p_j,r_j\right)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-2} \right\}$$

$$+ \sum_{n=j+1}^{\infty} z^{n-1} a\lambda^2(1-\alpha)^2\alpha^{2n-i-j-2}T^2\left[\lambda_1 A\left(p_n,r_n\right)+\lambda_2 B\left(p_n,r_n\right)-(c-s)-\frac{hT}{2}\right]$$

$$\times \left[\phi^\lambda\left(p_n,r_n,E_n\right)\right]^{(1/1-a)-2} + \sum_{n=j+1}^{\infty} z^{n-1}\lambda(1-\alpha)^2\alpha^{2n-i-j-2}TL_2\left(p_n,r_n\right)\left[\phi^\lambda\left(p_n,r_n,E_n\right)\right]^{(1/1-a)-1}$$

$$> 0,$$

$$\frac{\partial \Pi(P,E)}{\partial p_i \partial E_j} = \begin{cases} 0, & i > j, \\[2mm] -z^{i-1}L_2\left(p_i,r_i\right) + z^{i-1}L_2\left(p_i,r_i\right)\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-1}E_i^{-a} - z^{i-1}a\left(\beta_1+\lambda\right) \\[2mm] \quad \times T\left[\lambda_1 A\left(p_i,r_i\right)+\lambda_2 B\left(p_i,r_i\right)-(c-s)-\dfrac{hT}{2}\right]\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-2}E_i^{-a}, & i = j, \\[2mm] z^{j-1}a\lambda(1-\alpha)\alpha^{j-1-i}T\left[\lambda_1 A\left(p_j,r_j\right)+\lambda_2 B\left(p_j,r_j\right)-(c-s)-\dfrac{hT}{2}\right] \\[2mm] \quad \times \left[\phi^\lambda\left(p_j,r_j,E_j\right)\right]^{(1/1-a)-2}E_j^{-a}. & i < j, \end{cases} \tag{A.5}$$

and $(\partial\Pi(P,E)/\partial E_i E_j) = 0$, respectively. Because

$$- z^{i-1}L_2\left(p_i,r_i\right) + z^{i-1}L_2\left(p_i,r_i\right)\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-1}E_i^{-a} - z^{i-1}a\left(\beta_1+\lambda\right)T$$

$$\times \left[\lambda_1 A\left(p_i,r_i\right)+\lambda_2 B\left(p_i,r_i\right)-(c-s)-\frac{hT}{2}\right]\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-2}E_i^{-a}$$

$$\geq - z^{i-1}L_2\left(p_i,r_i\right) + z^{i-1}L_2\left(p_i,r_i\right)\left[\phi^\lambda\left(p_i,r_i,E_i\right)\right]^{(1/1-a)-2} \tag{A.6}$$

$$E_i^{-a}\left\{\phi^\lambda\left(p_i,r_i,E_i\right) - a\left(\beta_1+\lambda\right)T\left[\lambda_1 A\left(p_i,r_i\right)+\lambda_2 B\left(p_i,r_i\right)-(c-s)-\frac{hT}{2}\right]\right\}$$

$$\geq - z^{i-1}L_2\left(p_i,r_i\right) + z^{i-1}L_2\left(p_i,r_i\right)E_i^{-1+2a}E_i^{1-2a}$$

$$= 0,$$

$(\partial\Pi(P,E)/\partial p_i E_j) \geq 0$. In summary, $\Pi(P,E)$ is supermodular in (P,E,r_1). Hence, $\{p_i^*\}_{i=1}^\infty$ and $\{E_i^*\}_{i=1}^\infty$ are both increasing in r_1. □

Proof of Proposition 3.

(i) Problem (10) can be rewritten in the following form:

$$\max_{(P,E)\in\Omega}\Pi(P,E) = \pi\left(p_1^*,r_1^*,E_1^*\right)$$

$$+ z \max_{(P,E)\in\Omega}\sum_{i=2}^{\infty} z^{i-2}\pi\left(p_i,r_i,E_i\right). \tag{A.7}$$

If $p_1^* > r_1$, then $r_2^* = \alpha r_1^* + (1-\alpha)p_1^*(r_1^*) > r_1^*$. It follows from the supermodularity of $\Pi(P,E)$ in (P,E,r_1) that there

are $p_2^*(r_2^*) \geq p_2^*(r_1^*) = p_1^*(r_1^*)$ and $E_2^*(r_2^*) \geq E_2^*(r_1^*) = E_1^*(r_1^*)$. Similarly, we have $p_3^*(r_3^*) \geq p_2^*(r_2^*) = p_2^*(r_2^*)$ and $E_3^*(r_3^*) \geq E_3^*(r_2^*) = E_2^*(r_2^*)$ according to $r_3^* = \alpha r_2^* + (1-\alpha)p_2^*(r_2^*)$. By induction, we can obtain that $p_1^* \leq p_2^* \leq p_3^* \leq \cdots \leq p_\infty^*$ and $E_1^* \leq E_2^* \leq E_3^* \leq \cdots \leq E_\infty^*$. Similarly, we can get (ii) and (iii). □

Proof of Proposition 4. Suppose the selling price reaches the equilibrium p_e in the nth period; we have $p_e - r_i = p_e - \alpha^{i-n}r_n - (1-\alpha)p_e\sum_{j=0}^{i-n}\alpha^{i-n-j} = \alpha^{i-n}(p_e - r_n)$ for all $i \geq n$, which implies that the reference price converges to p_e as $i \longrightarrow \infty$. Thus, the discounted total profit from period n to infinite can be rewritten as

$$\sum_{i=n}^{\infty} z^{i-1} \pi\left(p_e, r_i, E_i\right) = \sum_{i=n}^{\infty} z^{i-1}\left[\lambda_1 A\left(p_e, r_n\right) + \lambda_2 B\left(p_e, r_n\right) - (c-s) - \frac{hT}{2}\right]$$

$$\times \left\{(1-a)T\left[\beta_0 - \beta_1 p_e - \lambda\alpha^{i-n}\left(p_e - r_n\right)\right] + E_i^{1-a}\right\}^{(1/1-a)} \tag{A.8}$$

$$- \sum_{i=n}^{\infty} z^{i-1} E_i\left[\lambda_1 A\left(p_e, r_n\right) + \lambda_2 B\left(p_e, r_n\right) + \frac{hT}{2}\right].$$

The problem is simplified to derive p_e and $\{E_i\}_{i=n}^{\infty}$ in order to maximize $\Pi(P_e, E)$. The first-order conditions for an interior maximum are as follows:

$$\frac{\partial \sum_{i=n}^{\infty} z^{i-1} \pi\left(p_e, r_i, E_i\right)}{\partial p_e} = \sum_{i=n}^{\infty} z^{i-1} L_2\left(p_e, r_n\right)\left[(1-a)T\left[\beta_0 - \beta_1 p_e - \lambda\alpha^{i-n}\left(p_e - r_n\right)\right] + E_i^{1-a}\right]^{(1/1-a)}$$

$$- \sum_{i=n}^{\infty} z^{i-1}\left(\beta_1 + \lambda\alpha^{i-n}\right)T\left[\lambda_1 A\left(p_e, r_n\right) + \lambda_2 B\left(p_e, r_n\right) - (c-s) - \frac{hT}{2}\right]$$

$$\times \left\{(1-a)T\left[\beta_0 - \beta_1 p_e - \lambda\alpha^{i-n}\left(p_e - r_n\right)\right] + E_i^{1-a}\right\}^{(1/1-a)-1}$$

$$- \sum_{i=n}^{\infty} z^{i-1} L_2\left(p_e, r_n\right) E_i = 0,$$

$$\frac{\partial \sum_{i=n}^{\infty} z^{i-1} \pi\left(p_e, r_i, E_i\right)}{\partial E_i} = \sum_{i=n}^{\infty} z^{i-1} E_i^{-a}\left[\lambda_1 A\left(p_e, r_n\right) + \lambda_2 B\left(p_e, r_n\right) - (c-s) - \frac{hT}{2}\right]\left\{(1-a)T\left[\beta_0 - \beta_1 p_e - \lambda\alpha^{i-n}\left(p_e - r_n\right)\right] + E_i^{1-a}\right\}^{(1/1-a)-1}$$

$$- \sum_{i=n}^{\infty} z^{i-1}\left[\lambda_1 A\left(p_e, r_n\right) + \lambda_2 B\left(p_e, r_n\right) + \frac{hT}{2}\right] = 0, \quad i = n, n+1, n+2, \ldots,$$

$$\tag{A.9}$$

respectively. Since $\lim_{n\to\infty}\left(p_e - r_n\right) = 0$, we have $r_n = p_e$ if n is sufficiently large. Thus, substituting $p_e = r_n$ into the first-order conditions, we get

$$\frac{\partial \sum_{i=n}^{\infty} z^{i-1} \pi\left(p_e, r_i, E_i\right)}{\partial p_e} = \sum_{i=n}^{\infty} z^{i-1} L_2\left(p_e, p_e\right)\left[(1-a)\left(\beta_0 - \beta_1 p_e\right)T + E_i^{1-a}\right]^{(1/1-a)}$$

$$- \sum_{i=n}^{\infty} z^{i-1}\left(\beta_1 + \lambda\alpha^{i-n}\right)T\left[\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - \frac{hT}{2}\right]$$

$$\times \left\{(1-a)\left(\beta_0 - \beta_1 p_e\right)T + E_i^{1-a}\right\}^{(1/1-a)-1} - \sum_{i=n}^{\infty} z^{i-1} L_2\left(p_e, p_e\right) E_i = 0,$$

$$0 = \sum_{i=n}^{\infty} z^{i-1} E_i^{-a}\left[\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) - (c-s) - \frac{hT}{2}\right]\left\{(1-a)\left(\beta_0 - \beta_1 p_e\right)T + E_i^{1-a}\right\}^{(1/1-a)-1}$$

$$- \sum_{i=n}^{\infty} z^{i-1}\left[\lambda_1 A\left(p_e, p_e\right) + \lambda_2 B\left(p_e, p_e\right) + \frac{hT}{2}\right].$$

$$\tag{A.10}$$

From the last equation, we can see that $E_e = E_n = E_{n+1} = E_{n+2} = \cdots$. Substituting this result into previous equations and after rearranging terms, we get

$$L_2(p_e, p_e)\left[\phi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)} = \left(\beta_1 + \lambda\frac{1-z}{1-\alpha z}\right)T\left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - \frac{hT}{2}\right]$$

$$\times\left[\phi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)-1} + E_e,$$

$$\left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + \frac{hT}{2}\right]E_e^a = \left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - \frac{hT}{2}\right]$$

$$\times\left[\phi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)-1},$$

(A.11)

or equivalently,

$$\left(\frac{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)}{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)}\right)^{(1-a/a)} = 1 + \frac{(1-a)(\beta_0 - \beta_1 p_e)T}{E_e^{1-a}}.$$

(A.12)

After some algebraic manipulations, we get

$$L_2(p_e, p_e)\left(\frac{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)}{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)}\right)^{(1/a)} - 1$$

$$= \frac{(\beta_1 + \lambda(1 - z/1 - \alpha z))\left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)\right]}{(1-a)(\beta_0 - \beta_1 p_e)}$$

$$\times\left[\left(\frac{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)}{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)}\right)^{(1/a)} - \left(\frac{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)}{\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)}\right)\right],$$

$$E_e = \left[\frac{(1-a)T(\beta_0 - \beta_1 p_e)}{(\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) + (hT/2)/\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2))^{(1-a/a)} - 1}\right]^{(1/1-a)}.$$

(A.13)

This completes the proof.   □   (i) Taking   the   cross-partial   derivatives   of
                                                       $\sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)$ with respect to p_e, E_i, and λ
Proof of Proposition 5   yields

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial p_e \partial \lambda} = -\sum_{i=n}^{\infty} z^{i-1} \alpha^{i-n} (p_e - r_n) T L_2(p_e, r_n) \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)}$$

$$- \sum_{i=n}^{\infty} z^{i-1} \alpha^{i-n} T \left[\lambda_1 A(p_e, r_n) + \lambda_2 B(p_e, r_n) - (c-s) - \frac{hT}{2}\right]$$

$$\times \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-1} + \sum_{i=n}^{\infty} z^{i-1} a\alpha^{i-n} (p_e - r_n)(\beta_1 + \lambda \alpha^{i-n}) T^2$$

$$\times \left[\lambda_1 A(p_e, r_n) + \lambda_2 B(p_e, r_n) - (c-s) - \frac{hT}{2}\right] \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-2},$$

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial E_i \partial \lambda} = -\sum_{i=n}^{\infty} z^{i-1} a\alpha^{i-n} (p_e - r_n) T E_i^{-a} \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-2}$$

$$\times \left[\lambda_1 A(p_e, r_n) + \lambda_2 B(p_e, r_n) - (c-s) - \frac{hT}{2}\right] < 0, \tag{A.14}$$

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial p_e \partial E_i} = \sum_{i=n}^{\infty} z^{i-1} E_i^{-a} L_2(p_e, r_n) \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-1} - \sum_{i=n}^{\infty} z^{i-1} L_2(p_e, r_n)$$

$$- \sum_{i=n}^{\infty} z^{i-1} a E_i^{-a} (\beta_1 + \lambda \alpha^{i-n}) T \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-2}$$

$$\times \left[\lambda_1 A(p_e, r_n) + \lambda_2 B(p_e, r_n) - (c-s) - \frac{hT}{2}\right]$$

$$\geq z^{i-1} L_2(p_e, r_n) + z^{i-1} L_2(p_e, r_n) E_i^{-1+2a} E_i^{1-2a}$$

$$= 0.$$

Since $r_n = p_e$ when n is sufficiently large,

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial p_e \partial \lambda} = -\sum_{i=n}^{\infty} z^{i-1} \alpha^{i-n} T \left[\lambda_1 A(p_e, r_n) + \lambda_2 B(p_e, r_n) - (c-s) - \frac{hT}{2}\right]$$

$$\times \left[\phi^\lambda(p_e, r_n, E_i)\right]^{(1/1-a)-1} < 0. \tag{A.15}$$

In summary, $\sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)$ is supermodular   (ii) Let
in $(-p_e, -E_i, \lambda)$, which implies the equilibrium
selling price p_e and ending inventory level E_e both
decrease with λ.

$$\psi(p_e, E_e) = L_2(p_e, p_e) \left[\varphi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)} - \left(\beta_1 + \lambda \frac{1-z}{1-\alpha z}\right) T$$

$$\cdot \left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - \frac{hT}{2}\right] \left[\varphi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)-1} + E_e. \tag{A.16}$$

From the implicit function theorem, we have

$$\frac{dp_e}{d\alpha} = -\frac{(\partial \psi(p_e, E_e)/\partial \alpha)}{(\partial \psi(p_e, E_e)/\partial p_e)},$$

$$= \frac{-\left((1-z)z/(1-\alpha z)^2\right)T\left[\lambda_1 A(p_e, p_e) + \lambda_2 B(p_e, p_e) - (c-s) - (hT/2)\right]\left[\phi^\lambda(p_e, p_e, E_e)\right]^{(1/1-a)-1}}{-(\partial \psi(p_e, E_e)/\partial p_e)},$$

(A.17)

it follows from Proposition 2 that $-(\partial \psi(p_e, E_e)/\partial p_e) > 0$; then, we have $(dp_e/d\alpha) < 0$. Therefore, p_e is decreasing in α.

Moreover, because $\sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)$ is supermodular in (p_e, E_e), we have

$$\frac{dE_e}{d\alpha} = \frac{\partial E_e}{\partial \alpha} + \frac{\partial E_e}{\partial p_e}\frac{dp_e}{d\alpha} = \frac{\partial E_e}{\partial p_e}\frac{dp_e}{d\alpha} < 0. \qquad (A.18)$$

Hence, E_e is decreasing in α.

(iii) From the implicit function theorem, we get

$$\frac{dp_e}{dm} = -\frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial p_e \partial m\right)}{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial p_e^2\right)}$$

$$= \frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial p_e \partial m\right)}{-\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial p_e^2\right)}.$$

(A.19)

Since

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)}{\partial p_e \partial m} = \sum_{i=n}^{\infty} z^{i-1}\lambda_1\left[g(p_e - m) + (p_e - s)(1+\lambda)g'(p_e - m)\right]$$

$$\times \left\{\left[(1-a)(\beta_0 - \beta_1 p_e)T + E_i^{1-a}\right]^{(1/1-a)} - E_i\right\}$$

$$> 0,$$

(A.20)

$(dp_e/dm) > 0$, where p_e is increasing in m.

$$\frac{dE_e}{dm} = -\frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial E_e \partial m\right)}{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial E_e^2\right)} = \frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial E_e \partial m\right)}{-\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)/\partial E_e^2\right)}. \qquad (A.21)$$

Since

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1}\pi(p_e, r_i, E_i)}{\partial E_i \partial m} = \sum_{i=n}^{\infty} z^{i-1}E_i^{-a}\lambda_1\left[g(p_e - m) + (p_e - s)(1+\lambda)g'(p_e - m)\right]$$

$$\times \left\{(1-a)(\beta_0 - \beta_1 p_e)T + E_i^{1-a}\right\}^{(1/1-a)-1}$$

$$- \sum_{i=n}^{\infty} z^{i-1}\lambda_1\left[g(p_e - m) + (p_e - s)(1+\lambda)g'(p_e - m)\right] > 0,$$

(A.22)

$(dE_e/dm) > 0$, where E_e is increasing in m.

(iv) From the implicit function theorem, we get

$$\frac{dp_e}{\mathrm{d}l} = -\frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial p_e \partial l\right)}{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial p_e^2\right)} = \frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial p_e \partial l\right)}{-\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial p_e^2\right)}. \tag{A.23}$$

Since

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial p_e \partial l} = \sum_{i=n}^{\infty} z^{i-1} \lambda_2 \left\{ \left[(1-a)(\beta_0 - \beta_1 p_e)T + E_i^{1-a}\right]^{(1/1-a)} - E_i \right\} > 0, \tag{A.24}$$

$(dp_e/\mathrm{d}m) > 0$, where p_e is increasing in l.

$$\frac{dE_e}{\mathrm{d}l} = -\frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial E_e \partial l\right)}{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial E_e^2\right)} = \frac{\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial E_e \partial l\right)}{-\left(\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)/\partial E_e^2\right)}. \tag{A.25}$$

Since

$$\frac{\partial^2 \sum_{i=n}^{\infty} z^{i-1} \pi(p_e, r_i, E_i)}{\partial E_i \partial m} = \sum_{i=n}^{\infty} z^{i-1} \lambda_2 E_i^{-a} \left\{ (1-a)(\beta_0 - \beta_1 p_e)T + E_i^{1-a} \right\}^{(1/1-a)-1} - \sum_{i=n}^{\infty} z^{i-1} \lambda_2 > 0, \tag{A.26}$$

$(dE_e/\mathrm{d}m) > 0$, where E_e is increasing in l. This completes the proof. □

Proof of Proposition 7. It follows from Proposition 1 that $\pi^{\lambda}(p_i, r_i, E_i)$ is concave in p_e and E_e. Similarly, it can be proved that $\pi^{\gamma}(p_i, r_i, E_i)$ is concave in p_e and E_e. Since minimization preserves the concavity, this thus, gives the proof. □

Proof of Proposition 8. The proof is similar to that of Theorem 7 in [24], which we omit here. □

Data Availability

Data used to support the findings of this study can be obtained from the corresponding author on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

References

[1] D. Rigby, "The future of shopping," *Harvard Business Review*, vol. 89, no. 12, pp. 64–75, 2011.

[2] S. Saghiri, R. Wilding, C. Mena, and M. Bourlakis, "Toward a three-dimensional framework for omni-channel," *Journal of Business Research*, vol. 77, pp. 53–67, 2017.

[3] Statista, "B2C e-commerce sales worldwide from 2012–2018," 2019.

[4] H. F. Peng and M. Y. Huang, "Retail business model innovation and its path analysis under the new technology environment-Taking Suning cloud business as an example (in Chinese)," *Macroeconomic Research*, vol. 2, pp. 108–115, 2014.

[5] J. Zhang, Q. Xu, and Y. He, "Omnichannel retail operations with consumer returns and order cancellation," *Transportation Research Part E: Logistics and Transportation Review*, vol. 118, no. 4, pp. 308–324, 2018.

[6] UPS, *UPS Online Shopping Study: Empowered Consumers Changing the Future of Retail*, United Parcel Service of America, Atlanta, GA, USA, 2015.

[7] G. Kalyanaram and R. S. Winer, "Empirical generalizations from reference price research," *Marketing Science*, vol. 14, no. 3, pp. 161–169, 1995.

[8] X. Chen, P. Hu, S. Shum, and Y. Zhang, "Dynamic stochastic inventory management with reference price effects," *Operations Research*, vol. 64, no. 6, pp. 1529–1536, 2016.

[9] L. R. Liu and Q. Xu, "Joint decisions on pricing and ordering for omnichannel BOPS retailers: considering online returns," *Sustainability*, vol. 12, no. 4, pp. 1–18, 2020.

[10] M. G. Güler, T. Bilgiç, and R. Güllü, "Joint inventory and pricing decisions with reference effects," *IIE Transactions*, vol. 46, no. 4, pp. 330–343, 2014.

[11] M. G. Güler, T. Bilgiç, and R. Güllü, "Joint pricing and inventory control for additive demand models with reference effects," *Annals of Operations Research*, vol. 226, no. 1, pp. 255–276, 2015.

[12] P. Rosenblum and B. Kilcourse, *Omni-channel 2013: The Long Road to Adoption*, Retail System Research, London, UK, 2013.

[13] F. Gao and X. Su, "Omnichannel retail operations with buy-online-and-pick-up-in-store," *Management Science*, vol. 63, no. 8, pp. 2478–2492, 2017.

[14] S. Halzack, "From new mobile offerings to a potential off-price business," 2015.

[15] P. Zhang, Y. He, and X. Zhao, ""Preorder-online, pickup-in-store" strategy for a dual-channel retailer," *Transportation Research Part E: Logistics and Transportation Review*, vol. 122, pp. 27–47, 2019.

[16] P. Harsha, S. Subramanian, and J. Yichanco, "Dynamic pricing of omnichannel inventories," *Manufacturing & Service Operations Management*, vol. 21, no. 1, pp. 47–65, 2019.

[17] C. Fan, Y. M. Liu, X. H. Yang, X. H. Chen, and J. H. Hu, "Online and offline cooperation under buy-online, pick-up-in-store: pricing and inventory decisions," *Journal of Industrial and Management Optimization*, vol. 15, no. 3, pp. 1455–1472, 2019.

[18] M. Jin, G. Li, and T. C. E. Cheng, "Buy online and pick up in-store: design of the service area," *European Journal of Operational Research*, vol. 268, no. 2, pp. 613–623, 2018.

[19] L. Gimpl-Heersink, C. Rudloff, M. Fleischmann, and A. Taudes, "Integrating pricing and inventory control: is it worth the effort?" *Business Research*, vol. 1, no. 1, pp. 106–123, 2008.

[20] T. L. Urban, "Coordinating pricing and inventory decisions under reference price effects," *International Journal of Manufacturing Technology and Management*, vol. 13, no. 1, pp. 78–94, 2008.

[21] A. Taudes and C. Rudloff, "Integrating inventory control and a price change in the presence of reference price effects: a two-period model," *Mathematical Methods of Operations Research*, vol. 75, no. 1, pp. 1–37, 2012.

[22] Y. Zhang, *Essays on Robust Optimization, Integrated Inventory and Pricing, and Reference Price Effect*, University of Illinois at Urbana-Champaign, Illinois, IL, USA, 2010.

[23] X. Chen, P. Hu, and S. He, "Technical note-preservation of supermodularity in parametric optimization problems with nonlattice structures," *Operations Research*, vol. 61, no. 5, pp. 1166–1173, 2013.

[24] R. Li and J.-T. Teng, "Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks," *European Journal of Operational Research*, vol. 270, no. 3, pp. 1099–1108, 2018.

[25] H. Ren and T. Huang, "Modeling customer bounded rationality in operations management: a review and research opportunities," *Computers & Operations Research*, vol. 91, pp. 48–58, 2018.

[26] T.-M. Choi and S. Guo, "Responsive supply in fashion mass customisation systems with consumer returns," *International Journal of Production Research*, vol. 56, no. 10, pp. 3409–3422, 2017.

[27] Z. Pei, L. Toombs, and R. Yan, "How does the added new online channel impact the supporting advertising expenditure?" *Journal of Retailing and Consumer Services*, vol. 21, no. 3, pp. 229–238, 2014.

[28] P. Harsha and S. Subramanian, "U.S. Patent Application No. 15/199," 2018.

[29] C.-Y. Dye, "Optimal joint dynamic pricing, advertising and inventory control model for perishable items with psychic stock effect," *European Journal of Operational Research*, vol. 283, no. 2, pp. 576–587, 2020.

[30] P. He, Y. He, and H. Xu, "Channel structure and pricing in a dual-channel closed-loop supply chain with government subsidy," *International Journal of Production Economics*, vol. 213, pp. 108–123, 2019.