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Research Article

Novel Robust Control of Stochastic Nonlinear Switched Fuzzy Systems

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This paper addresses the problem of designing novel switching control for a class of stochastic nonlinear switched fuzzy systems with time delay. Firstly, a stochastic nonlinear switched fuzzy system can precisely describe continuous and discrete dynamics as well as their interactions in the complex real-world systems. Next, novel control algorithm and switching law design of the state-dependent form are developed such that the stability is guaranteed. Since convex combination techniques are used to derive the delay independent criteria, some subsystems are allowed to be unstable. Finally, various comparisons of the elaborated examples are conducted to demonstrate the effectiveness of the proposed control design approach. All results illustrate good control performances as desired.

1. Introduction

As a typical hybrid system, switched systems have attracted much attention in the past decades because many physical systems can use this multimode system for mathematical modeling. Considering the wide application of switching power supply, the research on the switching system has never stopped, and some good results have been achieved [1–5].

One of the bridge of communication between the system and the fuzzy linear system is the T-S fuzzy dynamic model; the system makes the fuzzy system which has been greatly enriched; the research methods of nonlinear systems can be used to study the stability of linear system theory. The research literature [6–14] based on this kind of system has a representative, and in real-life problems, a lot has been applied to obtain considerable social and economic benefits.

From the middle of the 1980s, a number of analysis problems about T-S fuzzy control have appeared. And recently, switched systems have been extended further to encompass fuzzy control too. A novel fuzzy adaptive design is constructed for HFVs in spite of asymmetric time-varying constraints and actuator constraints. It can be assured finite-time convergence with the aid of a smooth switch between a fractional and a linear control [15]. Then, a Nussbaum function-based adaptive control method is proposed for high-order nonlinear systems with mixed control directions and dead-zone input [16].

Very notably, the stabilizability conditions and smoothness conditions for fuzzy switching control systems were reported. For the continuous-time case, a combination of hybrid systems and fuzzy multiple model systems was described and an idea of the fuzzy switched hybrid control was put forward [17]. Based on the T-S fuzzy systems, Tanaka et al. [18–20] introduced new switching fuzzy systems for more complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second-order nonholomonic systems. Such a switching fuzzy system model has two levels of structure, in which the first level is the region rule level and the second level is the local fuzzy rule level. This model is switching in local fuzzy rule level of the second level according to the premise variable in the region rule level of the first level. In fact, it is switching according to the same premise variable. Stability conditions are given. In [21–23], some extension based on [19, 20] are given.

Because stochastic systems have many applications in the real world, including nuclear, biological, socio-economic, and chemical processes, the stability of stochastic systems has been widely concerned in the past decades. Moreover, the study of stochastic systems has been of great interest since stochastic modeling has come to play an important role in many branches of engineering applications [24]. Many results on stochastic systems can be found in the literature. To mention a few, the control problem is studied in [25–27]. The filtering problem is investigated in [28], and the model reduction problem is considered in [29].

The switched stochastic system, as the product of the combination of the switched system and the stochastic system, has important theoretical research significance and practical application value. In recent years, switched stochastic control theory has become an important research tool in a large class of practical engineering systems, such as switching transmission control, state estimation of distributed network control protocol, robust control of solar devices, and forced linear dynamic systems with communication network sequence [30, 31].

A new model of class of stochastic switched fuzzy time-delay systems is proposed in this paper. A system of this class is a switched system whose subsystems are all stochastic fuzzy time-delay systems. A switching law is designed to give robust stability. In contrast with the existing results, we study the stochastic switched fuzzy system without levels of the structure. The method provides a kind of different premise variable switching directly. In [18–20], the same premise variable switching with two levels of the structure is considered.

In addition, we propose a novel switching controller to tackle stochastic switched fuzzy time-delay systems. This switching controller consists of a number of simple subcontrollers. One of the subcontrollers will be chosen to control the plant based on a derived switching scheme. We shall formulate the design problem of the parameters of the switching controller into the LMI problem. These LMIs can be solved readily by employing existing LMI tools.

This paper is organized as follows. In Section 2, we describe the model of a SF system of time-delay case. In Section 3, sufficient conditions for stability are derived by using the method of Lyapunov–Krasovskii function and the switching controller scheme as well as the stabilizing state-dependent switching laws. Two example simulations compared with the switching fuzzy systems on stabilizing the SF time-delay systems will be presented in Section 4. Finally, a conclusion will be drawn in Section 5.

2. Stochastic Nonlinear Switched Fuzzy System Model with Time Delay

A stochastic switched fuzzy model with time delay is considered:

$$\begin{aligned} &R^l_{\sigma(t)}\colon \text{ if } z_{\sigma(t)1}(t) \text{ is } M^l_{\sigma(t)1} \ldots \text{ and } z_{\sigma(t)p}(t) \text{ is } M^l_{\sigma(t)p}, \text{ then} \\ &\mathrm{d}x(t) = \left[A_{\sigma(t)l}x(t) + A_{1\sigma(t)l}x(t-\tau) + B_{\sigma(t)l}u_{\sigma(t)}(t) + F_{\sigma(t)l}f_{\sigma(t)}(t)\right]\mathrm{d}t \\ &\qquad + D_{\sigma(t)l}x(t)\mathrm{d}\omega(t), \quad x_{t_0}(\theta) = \varphi(\theta), \theta \in [-\tau, 0], l = 1, 2, \ldots, N_{\sigma(t)}, \end{aligned} \tag{1}$$

with

$$\sigma(t)\colon \overline{M} = \{1, 2, \ldots, m\}, \tag{2}$$

where $\sigma(t)$ is a piecewise constant function, called a switching signal.

$A_{\sigma(t)l}$ and $A_{1\sigma(t)l} \in \mathbb{R}^{n\times n}$ are known system matrices, and $B_{\sigma(t)l} \in \mathbb{R}^{n\times q}$ is the input matrix. $R^l_{\sigma(t)}$ denotes the fuzzy inference rule, $N_{\sigma(t)}$ is the number of inference rules, τ is the constant bounded time delay in the state, $u_{\sigma(t)}(t)$ is the input variable, and $\omega(t)$ is the Brownian motion that satisfies $E\{\mathrm{d}\omega(t)\} = 0$ and $E\{\mathrm{d}\omega^2(t)\} = \mathrm{d}t$. $z = \begin{bmatrix} z_1 & z_2 & \cdots & z_p \end{bmatrix}$ is the vector of premise variables. $f_{\Gamma(t)}(t) \triangleq f_{\Gamma(t)}(x(t), txn(t-\tau))$ denote nonlinear known functions.

Assumption 1 (see [12]). There is an appropriate dimension real matrix Φ and Δ such that

$$\begin{aligned} &\|f(\partial(t), \partial(t-\tau) - f(\kappa, \kappa(t-\tau)))\| \\ &\quad \leq \|\Phi(\partial(t) - \kappa(t))\| + \|\Delta(\partial(t-\tau) - \kappa(t-\tau))\|. \end{aligned} \tag{3}$$

Then, the ith substochastic fuzzy control system can be expressed as follows:

$$\begin{aligned} &R^l_i\colon \text{ if } z_{i1}(t) \text{ is } M^l_{i1} \ldots \text{ and } z_{ip}(t) \text{ is } M^l_{ip}, \text{ then} \\ &\mathrm{d}x(t) = \left[A_{il}x(t) + A_{1il}x(t-\tau) + B_{il}u_i(t) + F_{il}f_i(t)\right]\mathrm{d}t \\ &\qquad + D_{il}x(t)\mathrm{d}\omega(t), \quad l = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m. \end{aligned} \tag{4}$$

A_{il} and A_{1il} are known system matrices of the ith substochastic fuzzy control system, and B_{il} is the input matrix of the ith substochastic fuzzy control system. R^l_i denotes the fuzzy inference rule of the ith substochastic fuzzy control system, N_i is the number of inference rules, $u_i(t)$ is the input variable, and $\omega(t)$ is the Brownian motion. When the ith substochastic fuzzy control system satisfies the switching law, we switch to the ith subsystem to ensure stability of the stochastic nonlinear switched fuzzy system.

Therefore, the ith substochastic fuzzy control system of the global model is described by means of

$$\mathrm{d}x(t) = \sum_{l=1}^{N_i} \eta_{il}(z(t))\left\{\left[A_{il}x(t) + B_{il}u_i(t) + F_{il}f_i(t) + A_{1il}x(t-\tau)\right]\mathrm{d}t + D_{il}x(t)\mathrm{d}\omega(t)\right\}, \tag{5}$$

along with

$$0 \leq \eta_{il}(z(t)) \leq 1, \quad \sum_{l=1}^{N_i} \eta_{il}(z(t)) = 1, \tag{6}$$

where

$$\varepsilon_{il}(z(t)) = \prod_{\rho=1}^{p} \Omega_{i\rho}^{l}(z_\rho(t)), \eta_{il}(z(t)) = \frac{\varepsilon_{il}(z(t))}{\sum_{l=1}^{N_i} \varepsilon_{il}(z(t))}, \tag{7}$$

where $\Omega_{i\rho}^{l}(z_\rho(t))$ represents the membership function that $z_\rho(t)$ belongs to the fuzzy set $\Omega_{i\rho}^{l}$.

Now, our purpose is to design a controller $u_i = u_i(t)$ and a switching law such that system (5) is asymptotically stable.

A switching controller is employed to control the stochastic switched fuzzy time-delay model of (5). The switching controller consists of some simple subcontrollers. These subcontrollers will switch among each other to control the system of (5) according to an appropriate switching scheme. The switching controller is described by

$$u_i(t) = -\sum_{a=1}^{N_i} \rho_{ia}(x(t)) R B_{ia}^T P x(t), \tag{8}$$

where $\rho_{ia}(x(t))$ takes the value of 0 or 1 according to a switching scheme discussed later, $R \in \mathbb{R}^{q\times q}$ and $P \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices to be designed, and $(\cdot)^T$ denotes the transpose of a matrix or a vector. Combining (5) with (8), we get the augmented system:

$$\mathrm{d}x(t) = \left[\sum_{l=1}^{N_i} \eta_{il} \widetilde{A}_{il} x(t) + \sum_{l=1}^{N_i} \eta_{il} A_{1il} x(t-\tau) + F_{il} f_i(t)\right]\mathrm{d}t + \sum_{l=1}^{N_i} \eta_{il} D_{il} x \,\mathrm{d}\omega(t), \tag{9}$$

where $\widetilde{A}_{il} = A_{il} + B_{il}(-\sum_{a=1}^{N_i} \rho_{ia} R B_{ia}^T P)$.

We have the following result.

Remark 1. For P in equation (8), we consider the single Lyapunov function method. The single Lyapunov function method is to find the same positive definite function for all subsystems of the switched system, which decreases along the trajectory of each working subsystem, so as to ensure the stability of the switched system.

3. Main Results

Theorem 1. *Suppose there exist positive definite symmetric matrices P, R, and S and convex combination constants $\lambda_{ij_i} > 0$ such that the following matrix inequalities (10) are satisfied; then, system (9) is asymptotically stable via the state-feedback controller (13) under the switching law (11):*

$$\sum_{l=1}^{m} \lambda_{ij_i} \begin{bmatrix} -PB_{ia}RB_{ij_i}^T P - PB_{ij_i}RB_{ia}^T P + A_{ij_i}^T P + PA_{ij_i} + \frac{1}{4}\left(D_{i\vartheta_i} + D_{ij_i}\right)^T P\left(D_{i\vartheta_i} + D_{ij_i}\right) + S & PA_{1ij_i} \\ A_{1ij_i}^T P & -S \end{bmatrix} < 0, \quad j_i, \vartheta_i, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m, \tag{10}$$

$$\sigma(t) = \arg\min \left\{ \begin{array}{c} \overline{V}_i(t) \triangleq \max_{j_i} \\ \left\{ \Xi^T(t) \begin{bmatrix} -PB_{ia}RB_{ij_i}^T P - PB_{ij_i}RB_{ia}^T P + \frac{1}{4}\left(D_{i\vartheta_i} + D_{ij_i}\right)^T P\left(D_{i\vartheta_i} + D_{ij_i}\right) + A_{ij_i}^T P + PA_{ij_i} + S & PA_{1ij_i} \\ A_{1ij_i}^T P & -S \end{bmatrix} \Xi(t) < 0, \right\} \\ j_i, \vartheta_i, a = 1, 2, \ldots, N_i, i = 1, 2, \ldots, m \end{array} \right\}. \tag{11}$$

Proof. From (11), we know that, for any $\Xi(t) \neq 0$, it holds that

$$\sum_{i=1}^{m} \lambda_{ij_i} \Xi^T(t) \begin{bmatrix} -PB_{ia}RB_{ij_i}^T P - PB_{ij_i}RB_{ia}^T P \\ +\frac{1}{4}\left(D_{i\vartheta_i} + D_{ij_i}\right)^T P\left(D_{i\vartheta_i} + D_{ij_i}\right) + A_{ij_i}^T P + PA_{ij_i} + S & PA_{1ij_i} \\ A_{1ij_i}^T P & -S \end{bmatrix} \Xi(t) < 0. \quad (12)$$

Note that (12) holds for any $j_i, \vartheta_i \in \{1, 2, \ldots, N_i\}$ and $\lambda_{ij_i} > 0$; then, there exists at least an i such that, for any j_i,

$$\Xi^T(t) \begin{bmatrix} -PB_{ia}RB_{ij_i}^T P - PB_{ij_i}RB_{ia}^T P \\ +\frac{1}{4}\left(D_{i\vartheta_i} + D_{ij_i}\right)^T P\left(D_{i\vartheta_i} + D_{ij_i}\right) + A_{ij_i}^T P + PA_{ij_i} + S & PA_{1ij_i} \\ A_{1ij_i}^T P & -S \end{bmatrix} \Xi(t) < 0. \quad (13)$$

Thus, the switching law defined by (11) is well defined. Now, define a quadratic Lyapunov–Krasovskii functional candidate:

$$V(x(t)) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(\theta)Sx(\theta)\mathrm{d}\theta, \quad (14)$$

which is positive definite since P and S are positive definite matrices.

The partial differential operator of the system is as follows:

$$\begin{aligned} LV(x(t)) &= 2x^T(t)P\sum_{l=1}^{N_i} \eta_{il}\left[\widetilde{A}_{il}x(t) + A_{1il}x(t-\tau)\right] \\ &\quad + \frac{1}{2}\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}\left(D_{il}x(t)\right)^T 2P\left(D_{ij}x(t)\right) \\ &\quad + x^T(t)Sx(t) - x^T(t-\tau)Sx(t-\tau). \end{aligned} \quad (15)$$

On the other case, we can obtain

$$\begin{aligned} \widetilde{A}_{il}^T P + P\widetilde{A}_{il} &= \left(-\sum_{a=1}^{N_i}\rho_{ia}RB_{ia}^T P\right)^T B_{il}^T P + A_{il}^T P \\ &\quad + PB_{il}\left(-\sum_{a=1}^{N_i}\rho_{ia}RB_{ia}^T P\right) + PA_{il}. \end{aligned} \quad (16)$$

From (16), we have

$$x^T(t)\left(\widetilde{A}_{il}^T P + P\widetilde{A}_{il} + S\right)x(t) = x^T(t)\left\{\begin{array}{c} -\sum_{a=1}^{N_i}\eta_{ia}\left(B_{ij_i}RB_{ia}^T P\right)^T P \\ -\sum_{a=1}^{N_i}\left(\rho_{ia} - \eta_{ia}\right)\left(B_{il}RB_{ia}^T P\right)^T P - \sum_{a=1}^{N_i}\eta_{ia}P\left(B_{ia}RB_{ia}^T P\right) \\ -\sum_{a=1}^{N_i}\left(\rho_{ia} - \eta_{ia}\right)P\left(B_{il}RB_{ia}^T P\right) + A_{il}^T P + PA_{il} + S \end{array}\right\}x(t). \quad (17)$$

Let

$$\rho_{ia}=\frac{1+\operatorname{sgn}\left(x^T(t)PB_{il}RB_{ia}^TPx(t)\right)}{2},$$
$$\operatorname{sgn}(z)=\begin{cases}1, & \text{if } z>0,\\ -1, & \text{otherwise.}\end{cases} \tag{18}$$

From (17), we have

$$\begin{aligned}
&x^T(t)\left(\widetilde{A}_{il}^TP+P\widetilde{A}_{il}+S\right)x(t)\\
&=x^T(t)\left\{\begin{array}{c}\sum_{a=1}^{N_i}\eta_{ia}\begin{pmatrix}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+S\end{pmatrix}\\ -2\sum_{a=1}^{N_i}\begin{pmatrix}\frac{1+\operatorname{sgn}\left(x^T(t)PB_{il}RB_{ia}^TP_{11}x(t)\right)}{2}\\ -\frac{1}{2}-\left(\eta_{ia}-\frac{1}{2}\right)\end{pmatrix}\left(B_{il}RB_{ia}^TP\right)^TP\end{array}\right\}x(t)\\
&=x^T(t)\left\{\begin{array}{c}\sum_{a=1}^{N_i}\eta_{ia}\begin{pmatrix}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+S\end{pmatrix}\\ -2\sum_{a=1}^{N_i}\begin{pmatrix}\frac{\operatorname{sgn}\left(x^T(t)PB_{il}RB_{ia}^TPx(t)\right)}{2}\\ -\left(\eta_{ia}-\frac{1}{2}\right)\end{pmatrix}\left(B_{il}RB_{ia}^TP\right)^TP\end{array}\right\}x(t)\\
&\leq\sum_{a=1}^{N_i}\eta_{ia}x^T(t)\begin{pmatrix}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+S\end{pmatrix}x(t)\\
&\quad-2\sum_{a=1}^{N_i}\left(\frac{1}{2}-\left(\eta_{ia}-\frac{1}{2}\right)\right)\left|x^T(t)PB_{il}RB_{ia}^TPx(t)\right|.
\end{aligned} \tag{19}$$

As $\eta_{ia}-(1/2)\in[-(1/2)\ \ (1/2)]$, due to the property of the switched fuzzy model with time-delay, it can be shown that (19) satisfies the following inequality:

$$\begin{aligned}
&x^T(t)\left(\widetilde{A}_{il}^TP+P\widetilde{A}_{il}\right)x(t)\\
&\quad\leq\sum_{a=1}^{N_i}\eta_{ia}x^T(t)\begin{pmatrix}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+S\end{pmatrix}x(t).
\end{aligned} \tag{20}$$

Thus, it is easy to see that

$$\begin{aligned}
LV(x(t))\leq{}&\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}\sum_{a=1}^{N_i}\eta_{ia}x^T(t)\\
&\begin{bmatrix}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+\frac{1}{2}\left(D_{il}^TPD_{ij}+D_{il}^TPD_{ij}\right)+S\end{bmatrix}x(t)\\
&+\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}x^T(t)P_iA_{1il}x(t-\tau)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}x^T(t)A_{1il}^TPx(t-\tau)\\
&-x^T(t-\tau)Sx(t-\tau)+2\varepsilon x^T(t)\Phi_i^T\Phi_i x(t)\\
&+2\varepsilon_i x^T(t-\tau)\Delta_i^T\Delta_i x(t-\tau)\\
&+\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}\varepsilon_i^{-1}x^T(t)P_iF_{il}F_{il}^TP_ix(t).
\end{aligned}
\tag{21}
$$

Considering (21), we have

$$
LV(x(t))\le\sum_{l=1}^{N_i}\eta_{il}\sum_{j=1}^{N_i}\eta_{ij}\sum_{a=1}^{N_i}\eta_{ia}\begin{bmatrix}x(t)\\ x(t-\tau)\end{bmatrix}^T
\begin{bmatrix}
\begin{array}{l}-PB_{ia}RB_{il}^TP-PB_{il}RB_{ia}^TP\\ +A_{il}^TP+PA_{il}+2\varepsilon_i\Phi_i^T\Phi_i+\varepsilon_i^{-1}PF_{il}F_{il}^TP+S\\ +\frac{1}{4}\left(D_{il}+D_{ij}\right)^TP\left(D_{il}+D_{ij}\right)\end{array} & PA_{1il}\\
A_{1il}^TP & -S+2\varepsilon_i\Delta_i^T\Delta_i
\end{bmatrix}
\begin{bmatrix}x(t)\\ x(t-\tau)\end{bmatrix}.
\tag{22}
$$

In order words, we formulate the finding of R and P of (22) into the LMI problem. Considering (22) and multiplying both sides of (22) by the matrix $\mathrm{diag}\{P^{-1},I\}$, we restate as follows.

Theorem 2. *Suppose there exist positive definite matrices Q, S, and R and convex combination constants $\lambda_{ij_i}>0$ such that the following matrix inequalities are satisfied; then, system (9) is asymptotically stable via the state-feedback controller (8) under the switching law (11)*

Once we have $Q=P^{-1}$, $\Psi=S^{-1}$, and S and R from (23), the switching controllers (8) can be constructed.

Hence, we can conclude that the closed-loop system of (9) is asymptotically stable if the following stability condition of (23) is satisfied and the switching scheme of (8) is applied:

$$
\sum_{i=1}^{m}\lambda_{ij_i}\begin{bmatrix}
\Lambda_{iaj_i} & \frac{1}{2}Q\left(D_{i\vartheta_i}+D_{ij_i}\right)^T & Q & A_{1ij_i}\\
\frac{1}{2}\left(D_{i\vartheta_i}+D_{ij_i}\right)Q & -Q & 0 & 0\\
Q & 0 & -\Psi & 0\\
A_{1ij_1} & 0 & 0 & -S+2\varepsilon_i\Delta_i^T\Delta_i
\end{bmatrix}<0,\quad j_i,\vartheta_i,a=1,2,\dots,N_i,\ i=1,2,\dots,m.
\tag{23}
$$

Here,

$$
\Lambda_{iaj_i}=-B_{ia}RB_{ij_i}^T-B_{ij_i}RB_{ia}^T+QA_{ij_i}^T+A_{ij_i}Q+\varepsilon_i^{-1}F_{ij_i}F_{ij_i}^T+S.
\tag{24}
$$

4. Illustrative Examples and Results

In recent years, the research results show that the chaotic system has a great applying value in practical engineering system. In [32], the chaotic system model is designed. The system is two different chaotic systems, switching between different chaotic systems, so the switch system is simple and easy to operate.

This kind of chaotic system equation is expressed as follows:

$$\begin{cases} \dot{x} = a(z - x), \\ \dot{y} = bx - \mathrm{d}xz, \\ \dot{z} = xy - cy - gz, \end{cases}$$
$$\begin{cases} \dot{x} = a((z - y), \\ \dot{y} = bx - \mathrm{d}x^2, \\ \dot{z} = xy - cy - gz. \end{cases} \tag{25}$$

Security is a very important performance of the modern synchronous communication research. Chaotic synchronization communication system using multiple correlation system switching has a huge role for the safety of aging performance of chaotic synchronization and secure communication in the field of communications. The actual engineering control systems also have a large number of such switching system and good control effects.

The established stochastic nonlinear switched fuzzy system model is as follows:

$$\begin{aligned}
&R_1^1\text{: if } z_2(t) \text{ is } M_{11}^1, \text{ then} \\
&\mathrm{d}x(t) = [A_{11}x(t) + A_{111}x(t-\tau) + B_{11}u_1(t) + F_{11}f_1(t)]\mathrm{d}t + D_{11}x(t)\mathrm{d}\omega(t), \\
&R_1^2\text{: if } z_2(t) \text{ is } M_{11}^2, \text{ then} \\
&\mathrm{d}x(t) = [A_{12}x(t) + A_{112}x(t-\tau) + B_{12}u_1(t) + F_{12}f_1(t)]\mathrm{d}t + D_{12}x(t)\mathrm{d}\omega(t), \\
&R_2^1\text{: if } z_2(t) \text{ is } M_{21}^1, \text{ then} \\
&\mathrm{d}x(t) = [A_{21}x(t) + A_{121}x(t-\tau) + B_{21}u_2(t) + F_{21}f_2(t)]\mathrm{d}t + D_{21}x(t)\mathrm{d}\omega(t), \\
&R_2^2\text{: if } z_2(t) \text{ is } M_{21}^2, \text{ then} \\
&\mathrm{d}x(t) = [A_{22}x(t) + A_{122}x(t-\tau) + B_{22}u_2(t) + F_{22}f_2(t)]\mathrm{d}t + D_{22}x(t)\mathrm{d}\omega(t),
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
A_{11} &= \begin{bmatrix} -11 & 3 \\ 1 & -1 \end{bmatrix}, & D_{11} &= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}, \\
A_{12} &= \begin{bmatrix} -21 & 4 \\ 3 & -2 \end{bmatrix}, & D_{12} &= \begin{bmatrix} -5 & 7 \\ -6 & 4 \end{bmatrix}, \\
A_{21} &= \begin{bmatrix} -9 & 4 \\ 5 & -1 \end{bmatrix}, & D_{21} &= \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}, \\
A_{22} &= \begin{bmatrix} -15 & 7 \\ 8 & -2 \end{bmatrix}, & D_{22} &= \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}, \\
B_{11} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, & A_{111} &= \begin{bmatrix} 2 & 5 \\ 2 & -2 \end{bmatrix}, \\
B_{12} &= \begin{bmatrix} -1 \\ 1 \end{bmatrix}, & A_{112} &= \begin{bmatrix} 5 & 3 \\ -6 & -2 \end{bmatrix}, \\
B_{21} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix}, & A_{121} &= \begin{bmatrix} 2 & 5 \\ 7 & -8 \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} 3 \\ 2 \end{bmatrix}, & A_{122} &= \begin{bmatrix} 6 & 8 \\ -2 & -3 \end{bmatrix}.
\end{aligned} \tag{27}$$

The membership functions are as follows:

$$
\begin{aligned}
M_{11}^{1}(z_2(t)) &= 1-\frac{1}{1+e^{-2z_2(t)}},\\
M_{11}^{2}(z_2(t)) &= \frac{1}{1+e^{-2z_2(t)}},\\
M_{21}^{1}(z_2(t)) &= 1-\frac{1}{1+e^{(-2(z_2(t)-0.3))}},\\
M_{21}^{2}(z_2(t)) &= \frac{1}{1+e^{(-2(z_2(t)-0.3))}}.
\end{aligned}
\tag{28}
$$

The nonlinearities $f(t)$ in (1) are

$$
\begin{aligned}
f_1(t) &= \begin{bmatrix} 0.4x_1(t)+0.5x_2(t) \\ 0.7x_1(t) \end{bmatrix}\sin t \\
&\quad + \begin{bmatrix} 0.4x_1(t-\tau)+0.4x_2(t-\tau) \\ 0.4x_1(t-\tau)+0.7x_2(t-\tau) \end{bmatrix}\sin t,\\
f_2(t) &= \begin{bmatrix} 0.4x_1(t)+0.5x_2(t) \\ 0.7x_1(t) \end{bmatrix}\cos t \\
&\quad + \begin{bmatrix} 0.4x_1(t-\tau)+0.4x_2(t-\tau) \\ 0.4x_1(t-\tau)+0.7x_2(t-\tau) \end{bmatrix}\cos t,
\end{aligned}
\tag{29}
$$

which satisfy Assumption 1 with

$$
\begin{aligned}
\Phi_1 = \Phi_2 &= \begin{bmatrix} 0.4 & 0.5 \\ 0.7 & 0 \end{bmatrix},\\
\Delta_1 = \Delta_2 &= \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.7 \end{bmatrix}.
\end{aligned}
\tag{30}
$$

Using MATLAB to solve (23) with parameters $j_i, \vartheta_i, a = 1, 2, i = 1, 2$, $\tau = 1$, and $\lambda_{ij_i} = 1$, the following matrices are obtained:

$$
\begin{aligned}
P &= \begin{bmatrix} 0.5232 & 0.1451 \\ 0.1451 & 0.2512 \end{bmatrix},\\
Q &= \begin{bmatrix} 0.2816 & 0.0261 \\ 0.0261 & 0.2134 \end{bmatrix},\\
S &= \begin{bmatrix} 0.4133 & 0.3241 \\ 0.3241 & 0.1341 \end{bmatrix},\\
R &= 5.2.
\end{aligned}
\tag{31}
$$

The simulation result under initial condition $[\,10\ \ -15\,]^T$ is depicted in Figure 1.

To show the advantages of the proposed method, we now compare the method with the traditional PDC fuzzy controller. Obviously, the global control of the traditional PDC fuzzy controller is

$$
u_i(t) = \sum_{l=1}^{N_i} \eta_{il} K_{il} x(t). \tag{32}
$$

The state-feedback gains of subsystems are obtained as

$$
\begin{aligned}
K_{11} &= [\,0.4623\ \ 0.6826\,],\\
K_{12} &= [\,-2.1751\ \ 3.2715\,],\\
K_{21} &= [\,1.2815\ \ 0.8361\,],\\
K_{22} &= [\,-2.3813\ \ 1.2642\,].
\end{aligned}
\tag{33}
$$

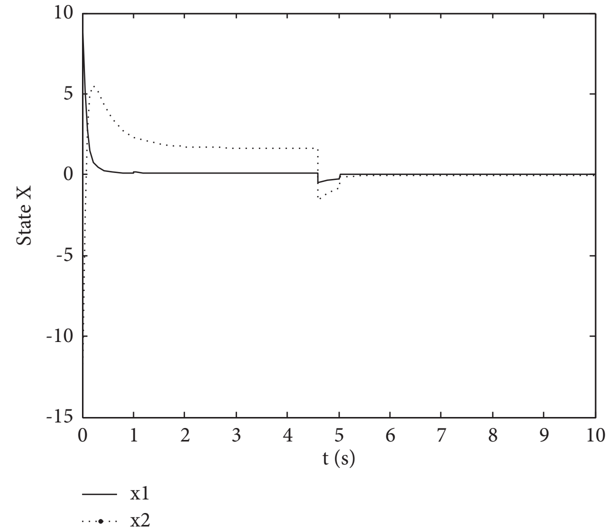

Figure 1: The state response of the system according to the switching controller.

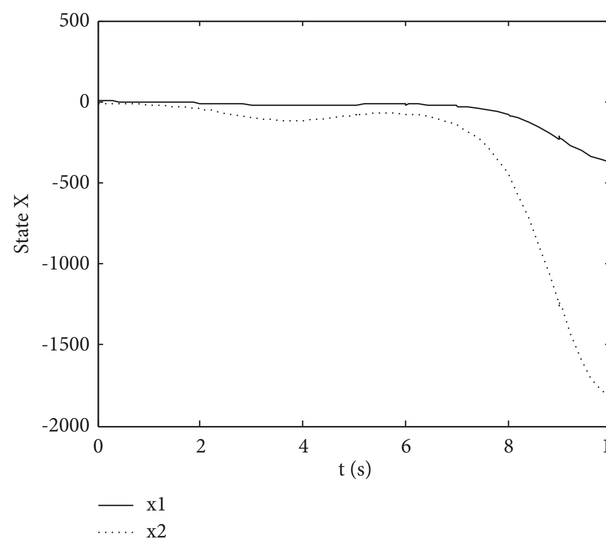

Figure 2: The state response of the system according to the traditional PDC fuzzy controller.

For the stochastic switched fuzzy system, the simulation result under the same initial condition $[\,10\ \ -15\,]^T$ is depicted in Figure 2. Figures 1 and 2 indicate that the proposed method gives better results.

5. Conclusion

In this paper, the stability of stochastic nonlinear switched fuzzy with input time-delay systems is studied. The switched fuzzy control system with stochastic factors has good control effect. Based on the Lyapunov function method, the stability condition can be given in the form of LMI in solution, and a switching control strategy is proposed when the system has large fluctuation. The ideal stability results can be made to the global switched fuzzy stochastic systems. Finally, the feasibility and effectiveness of the method are verified by simulation experiments.

Data Availability

All data included in this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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