Research Article

Statistical Development of the V_{SQ} -Control Chart for Extreme Data with an Application to the Carbon Fiber Industry

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The normality assumption is a significant part of the development of control charts. This underlying assumption of normality most likely does not hold true in real scenarios. One of such designs usually devised to observe the target parameter σ^2 of the Maxwell quality characteristics is the V -control chart. In general, quality practitioners preferably have to observe the scale parameter σ rather than σ^2 in examined processes. The contemporary V -control chart is relying on the V -statistic which does not hold the unbiasedness property with respect to parameter σ of the Maxwell probability model. In view of this, implementation of the V -chart is not an appropriate design in monitoring a real parameter of the underlying Maxwell data. To accommodate the monitoring of the parameter σ of the Maxwell model, a novel design of the V_{SQ} -chart is mainly proposed in this work. To support a statistical understanding of the V_{SQ} -chart, power function, characteristic function, and the average run length (ARL) have been essentially established. The parameters of the V_{SQ} -chart are determined from the results of the sampling distribution of the derived statistic. Analytical findings are further applied to determine the performance of the study proposal with its existing counterpart. Substantially, the better performance of the proposed technique has been observed because of statistical power used as a performance measure. Eventually, the computational plan of the V_{SQ} -chart is considered both for the simulated and real datasets with the aim of illustrating the theory of the proposed design.

1. Introduction

There are two kinds of variations in all production processes, namely, normal and unnatural variations. Normal variations spontaneously cause harmless disparities in all systems, whereas unnatural variations are unique and trigger discrepancies because of such process defects [1]. The abnormal changes are especially important because their presence may harm the quality of the final product [2]. Regulation of the statistical process control (SPC) provides a valuable range of methods for managing irregular process variations [3]. One of the most powerful methods for process monitoring in SPC is the control chart approach. Control charts in the industry are now widely recognized and applied to learn, monitor, and improve the process sequentially [4]. The traditional control chart designs are based on the notion of the normal or

approximately normal distribution of the quality characteristics under study. The assumption of normality in the process population in many cases is, however, not appropriate. The distribution may be skewed for many real datasets [5]. Accelerated life testing observations, measurements obtained from chemical operations, semiconductor analysis, and cutting wear processes are few examples of the positively skewed data [6]. If the quality variable is distorted in its distribution, analyzing the process with typical control charts can be misleading [7]. When the skewness increases due to population variability, implementation of the conventional Shewhart charts under asymmetrical distributions increases the risk of type-I error [8]. Several researchers have investigated quality monitoring in the context of positively skewed data [9]. The Shewhart control chart has been analyzed by Raza et al. [10] under the

type-I censoring Rayleigh distributed data. The quality monitoring of the compressing strength of concrete for the log-normal model has been studied by Aichouni et al. [11]. Derya and Canan developed the design for the distorted distributions, namely, Gamma, log-normal, and Weibull [12]. Guo and Wang proposed a Weibull-based control chart using the type-II censoring data [13]. Designs of control chart rooting in mixture distribution models have been also suggested [14, 15]. Other related studies on this subject include Yang and Xie [16], Chan and Cui [17], Santiago and Smith [18], Chang and Bai [19], Aslam et al. [20], and Yang et al. [21].

In literature, the V -control chart rooted on the Maxwell model has been proposed for monitoring the target parameter σ^2 (i.e., square quantity of the monitoring quality) of the Maxwell process [22]; however, the scale parameter of the Maxwell model is occasionally desired to be estimated rather than σ^2 .

A new design called the V_{SQ} -control chart that allows monitoring of the parameter σ of the Maxwell process is presented in this study. The structure of the existing V -control is definitely not relying on an unbiased statistic of σ so unable to observe the target parameter σ of the Maxwell quality characteristics.

Discussion is structured in the following sequence. Section 2 discusses the current model of the V -chart. A novel design of the V_{SQ} -chart is discussed in Section 3. The performance study of the proposed design is conducted in Section 4. Construction of the suggested V_{SQ} -chart using simulated data is presented in Section 5. Section 6 offers a comparative analysis. In Section 7, the implementation of the proposed chart with an illustration of real-world data is given. Eventually, the results of the study are concluded in Section 8.

2. The Existing Approach of the V -Chart

The theoretical model employed in the design of the V -chart is rooted on the assumption that the data generating process supports the Maxwell model with a single scale parameter σ [22]. The quality of interest W is said to assume the single parameter Maxwell probability model if it takes the cumulative distribution function (CDF) and probability density function (PDF) as described below:

$$F(w) = \left(\frac{2}{\pi}\right) \gamma \left[\frac{3}{2}, \frac{w^2}{(2\sigma^2)} \right]; \quad \sigma > 0, w > 0, \quad (1)$$

$$f(w) = \sqrt{\left(\frac{2}{\pi}\right)} \sigma^{-3} w^2 e^{-(w^2/2\sigma^2)}, \quad \sigma > 0, w > 0,$$

where $\int_0^a y^{\alpha-1} e^{-\beta y} dy = \beta^{-\alpha} \gamma(\alpha, \beta a)$.

Assuming the scale parameter σ equal to 0.75, the graphical overview of the CDF and PDF of the Maxwell distribution is depicted in Figure 1.

It is clearly shown in Figure 1 that the Maxwell distribution is skewed particularly for a smaller value of σ . The scale parameter σ also plays a significant role in describing the shape of the curve. The PDF curve becomes symmetric

FIGURE 1: The Maxwell distributed random variable W with the scale parameter $\sigma = 0.75$.

for the greater value of σ , while CDF progressively reaches to unity for increased values of W . In certain real-life cases, the value of the characterised Maxwell distribution parameter is not known. To find the unknown quantity σ , a variety of estimation methods may be used [23]. An estimate of σ using the maximum likelihood (ML) approach has been given by Krishna et al. [24]:

$$\hat{\sigma} = \sqrt{(3m)^{-1} \sum_{i=1}^m w_i^2}. \quad (2)$$

The V -chart is now derived on the random variable as defined below:

$$V = \frac{\sum_{i=1}^m w_i^2}{3m}, \quad (3)$$

where the random variable W_i assumes the Maxwell distribution, and the description refers to the defined fixed realization $\hat{\sigma}^2 = \sum_{i=1}^m w_i^2 / 3m$.

The distribution underlying random variable V is the Gamma distribution having parameters m and σ^2/m . The V -chart parameters based on the estimator defined in (3) are given by

$$\left. \begin{aligned} \text{LCL} &= E(V) - p * \text{SD}(V), \\ \text{CL} &= \sigma^2, \\ \text{UCL} &= E(V) + p * \text{SD}(V), \end{aligned} \right\} \quad (4)$$

where V is an unbiased statistic of the target parameter σ^2 and the multiplier p is a percentile of the Gamma probability model at given value of α i.e., probability of incorrect out-of-control signals.

Note that the specified statistic V used to estimate the quantity σ^2 is not in the same measuring unit as the actual quality variable is observed. Moreover, the resulting estimator V is not unbiased with respect to parameter σ . A new proposal of the V_{SQ} -chart for monitoring the stability of

target parameter σ of the Maxwell process has been suggested in the next section.

3. Statistical Development of the V_{SQ} -Chart

The following V_{SQ} -statistic has been considered in the development of the V_{SQ} -chart:

$$V_{SQ} = \sqrt{\frac{\sum_{i=1}^m w_i^2}{3m}}. \quad (5)$$

Using the following theorems, the distribution of V_{SQ} -statistic can be determined.

Theorem 1. *The ratio of the random variable $Z = W^2/\sigma^2$ is a Gamma distribution having parameters $3/2$ and 2 if the underlying distribution of the random variable W is Maxwell with a single scale parameter σ , i.e.,*

$$Z = \frac{W^2}{\sigma^2} \sim \text{Gamma}\left(\frac{3}{2}, 2\right). \quad (6)$$

Proof. Suppose that the random variable W has Maxwell distribution with a real parameter σ ; subsequently, the PDF is given by

$$f(w) = \sqrt{\left(\frac{2}{\pi}\right)} \sigma^{-3} w^2 e^{-(w^2/2\sigma^2)}, \quad \sigma > 0, w \geq 0. \quad (7)$$

The transformation $Z = W^2/\sigma^2$ is a one-one transformation from $\Omega = \{w|w > 0\}$ to $\omega = \{z|z > 0\}$ with the inverse solution of $z = w^2/\sigma^2$ is given by

$$w = \sigma z^{1/2}. \quad (8)$$

Consequently,

$$J = \left| \frac{dw}{dz} \right| = \left| \frac{\sigma}{2\sqrt{z}} \right|, \quad (9)$$

where J is the Jacobian of the transformation.

Thus, the distribution of random variable Z can be immediately written as

$$f(z) = \sqrt{\left(\frac{2}{\pi}\right)} \sigma^{-3} (\sigma^2 z) e^{-(z/2)} \times \left| \frac{\sigma}{2\sqrt{z}} \right|. \quad (10)$$

Solving (10) gives the density of Z :

$$f(z) = \frac{1}{2^{3/2} \Gamma(3/2)} z^{(3/2)-1} e^{-(z/2)}, \quad z > 0. \quad (11)$$

Thus, $Z \sim \text{Gamma}((3/2), 2)$.

Theorem 2. *If $Z_1, Z_2, \dots, Z_m$ are independent identical distributed Gamma random variables each having the PDF as defined in (11), then*

$$H = Z_1 + Z_2 \cdots Z_m \sim \text{Gamma}\left(\frac{3}{2}m, 2\right), \quad (12)$$

where $Z_i = (W^2/\sigma^2)$, $i = 1, 2, \dots, m$.

Proof. Since each $Z_i \sim \text{Gamma}((3/2), 2)$ with independent and identical distributed property, therefore using the reproductive property of Gamma distribution [25], we can write:

$$H = \sum_{i=1}^m Z_i \sim \text{Gamma}\left(\frac{3}{2}m, 2\right), \quad (13)$$

and hence, it is proved.

Theorem 3. *The transformation $U = \sqrt{H}$ follows the chi distribution (χ -dist) with $3m$ parameter if $H \sim \text{Gamma}((3/2)m, 2)$.*

Proof. Given that $H \sim \text{Gamma}((3/2)m, 2)$, thus PDF of H is defined by

$$f(h) = \frac{1}{2^{(3/2)m} \Gamma(3m/2)} h^{(3m/2)-1} e^{-(h/2)}, \quad m, h > 0. \quad (14)$$

The transformation $U = \sqrt{H}$ is a one-one transformation from $\Omega = \{h|h > 0\}$ to $\omega = \{u|u > 0\}$ with the value of inverse solution $h = u^2$ and Jacobian:

$$|J| = \left| \frac{dh}{du} \right| = 2u. \quad (15)$$

Thus, the new random variable U gets the PDF as

$$f(u) = \frac{1}{2^{(3/2)m} \Gamma(3m/2)} (u^2)^{(3m/2)-1} e^{-(u^2/2)} \times 2u. \quad (16)$$

Further simplification provides

$$f(u) = \frac{1}{2^{(3m/2)-1} \Gamma(3m/2)} (u)^{3m-1} e^{-(u^2/2)}, \quad u > 0, \quad (17)$$

which is the PDF of χ -dist with $3m$ parameter, and hence, it is proved.

Now, the mean and variance of U can be achieved as

$$E(U) = \sqrt{2} \left(\frac{\Gamma((3m+1)/2)}{\Gamma(3m/2)} \right), \quad (18)$$

$$\text{Var}(U) = 3 \left[m - \frac{2}{3} \left(\frac{\Gamma((3m+1)/2)}{\Gamma(3m/2)} \right)^2 \right].$$

Consequently, mean and variance of the estimator V_{SQ} would be set by the following relation:

$$U = \sqrt{\frac{\sum_{i=1}^m w_i^2}{\sigma^2}} \times \frac{\sqrt{3m}}{\sqrt{3m}}. \quad (19)$$

This provides

$$V_{SQ} = \frac{\sigma}{\sqrt{3m}} U. \quad (20)$$

The relationship defined in (20) yields the mean and variance of the V_{SQ} -estimator as follows:

$$\left. \begin{aligned} E(V_{SQ}) &= \sigma\delta, \\ \text{Var}(V_{SQ}) &= \sigma^2[1 - \delta^2], \end{aligned} \right\} \quad (21)$$

where the function $\delta = \sqrt{(2/3m)}((\Gamma((3m+1)/2))/(\Gamma(3m/2)))$ depends on m . Besides, the V_{SQ} -statistic is not an unbiased estimator of the target parameter σ , as indicated in (21). The unknown parameter σ of the underlying Maxwell quality characteristic can therefore be estimated by an unbiased estimator $\bar{V}_{SQ}/\delta$:

$$\hat{V}_{SQ} = \frac{\bar{V}_{SQ}}{\delta}. \quad (22)$$

The percentiles of U -statistic are essential for probability limits (PM) of the V_{SQ} -chart. As the random variable U follows χ -dist with $3m$ degree of freedom (df), the α -percentile of the statistic U can be written as

$$F(u) = \alpha. \quad (23)$$

Here, $F(u)$ denotes the CDF of the χ -dist. Thus,

$$U = F^{-1}(\alpha). \quad (24)$$

Further simplification of (24) implies that

$$V_{SQ} = \frac{\sigma}{\sqrt{3m}} F^{-1}(\alpha). \quad (25)$$

Thus, the PM of the V_{SQ} -chart at a given level of significance α is provided by

$$\left. \begin{aligned} \text{UPL} &= \frac{\sigma}{\sqrt{3m}} F^{-1}\left(1 - \frac{\alpha}{2}\right) = \sigma\vartheta_1, \\ \text{LPL} &= \frac{\sigma}{\sqrt{3m}} F^{-1}\left(\frac{\alpha}{2}\right) = \sigma\vartheta_2, \end{aligned} \right\} \quad (26)$$

where $\vartheta_1 = F^{-1}(1 - (\alpha/2))/\sqrt{3m}$ and $\vartheta_2 = F^{-1}(\alpha/2)/\sqrt{3m}$.

In addition, the PM based on the estimated value of σ is given by

$$\left. \begin{aligned} \text{UPL} &= \frac{\hat{\sigma}}{\sqrt{3m}} F^{-1}\left(1 - \frac{\alpha}{2}\right) = \bar{V}_{SQ}\vartheta_3, \\ \text{LPL} &= \frac{\hat{\sigma}}{\sqrt{3m}} F^{-1}\left(\frac{\alpha}{2}\right) = \bar{V}_{SQ}\vartheta_4, \end{aligned} \right\} \quad (27)$$

where $\vartheta_3 = F^{-1}(1 - (\alpha/2))/\delta\sqrt{3m}$ and $\vartheta_4 = F^{-1}(\alpha/2)/\delta\sqrt{3m}$.

The coefficient pairs $(\vartheta_1, \vartheta_2)$ and $(\vartheta_3, \vartheta_4)$ are the quantiles of the χ -dist with $3m$ df multiplied by the constants $1/\sqrt{3m}$ and $1/\delta\sqrt{3m}$, respectively. These coefficient values at given the false signal probability α and various m values are calculated and given in Table 1.

A general model based on k -sigma limits may also be developed for any chart. To this end, two initial moments of the V_{SQ} -statistic are required. Utilizing properties (21) of the V_{SQ} -estimator, the k -sigma limits are given by

TABLE 1: Pairs of values $(\vartheta_1, \vartheta_2)$ and $(\vartheta_3, \vartheta_4)$ at given α and m .

Sample size m	False signal probability α							
	0.005				0.0027			
	ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_1	ϑ_2	ϑ_3	ϑ_4
1	0.122	2.184	0.133	2.371	0.099	2.283	0.108	2.477
2	0.296	1.837	0.309	1.915	0.265	1.903	0.277	1.984
3	0.401	1.682	0.413	1.729	0.371	1.735	0.382	1.784
4	0.471	1.589	0.481	1.623	0.442	1.635	0.452	1.669
5	0.521	1.526	0.529	1.552	0.494	1.566	0.502	1.593
6	0.559	1.479	0.567	1.500	0.534	1.516	0.541	1.537
7	0.589	1.443	0.596	1.461	0.565	1.477	0.572	1.494
8	0.614	1.415	0.620	1.429	0.591	1.445	0.597	1.461
9	0.635	1.390	0.641	1.403	0.613	1.419	0.618	1.433
10	0.652	1.370	0.658	1.381	0.631	1.397	0.636	1.409
11	0.668	1.352	0.673	1.363	0.647	1.379	0.652	1.389
12	0.681	1.337	0.686	1.347	0.662	1.362	0.666	1.372

$$\left. \begin{aligned} \text{LCL} &= \sigma\delta - k\sigma\sqrt{1 - \delta^2} = \sigma\delta_1, \\ \text{CL} &= \sigma C, \\ \text{UCL} &= \sigma\delta + k\sigma\sqrt{1 - \delta^2} = \sigma\delta_2, \end{aligned} \right\} \quad (28)$$

where σ is a known quantity, k is the quantile point of the χ -dist at specified α , and the customary constants are given by

$$\begin{aligned} \delta_1 &= \delta - k\sqrt{1 - \delta^2}, \\ \delta_2 &= \delta + k\sqrt{1 - \delta^2}, \end{aligned} \quad (29)$$

where factor k is calculated in Table 2 for fixed α and m .

If the in-control σ is not provided, the unbiased estimator specified in (22) can be used to estimate it. Let us assume that initially T samples are present, and let V_{SQ_i} be an ML estimator, computed for the i th sample; then, the T samples' average is

$$\bar{V}_{SQ} = \frac{\sum_{i=1}^T V_{SQ_i}}{T}. \quad (30)$$

Thus, the model of the V_{SQ} -control chart could be defined as

$$\left. \begin{aligned} \text{LCL} &= \bar{V}_{SQ} - \frac{k\bar{V}_{SQ}}{\delta} \sqrt{1 - \delta^2} = \bar{V}_{SQ}\delta_3, \\ \text{CL} &= \bar{V}_{SQ}, \\ \text{UCL} &= \bar{V}_{SQ} + \frac{k\bar{V}_{SQ}}{\delta} \sqrt{1 - \delta^2} = \bar{V}_{SQ}\delta_4. \end{aligned} \right\} \quad (31)$$

Typically, constants δ_3 and δ_4 are given by

$$\begin{aligned} \delta_3 &= 1 - \frac{k}{\delta} \sqrt{1 - \delta^2}, \\ \delta_4 &= 1 + \frac{k}{\delta} \sqrt{1 - \delta^2}. \end{aligned} \quad (32)$$

The resulting constants δ_1 , δ_2 , δ_3 , and δ_4 are listed in Table 3 for defined α and sample size m .

TABLE 2: Multiplier k at different values of m and α .

m	False signal probability α		
	0.0027	0.005	0.01
	k		
1	0.218	0.268	0.339
2	0.736	0.822	0.934
3	1.216	1.317	1.445
4	1.644	1.753	1.889
5	2.031	2.144	2.287
6	2.385	2.503	2.648
7	2.714	2.834	2.983
8	3.022	3.144	3.294
9	3.312	3.436	3.588
10	3.588	3.713	3.866
11	3.850	3.976	4.132
12	4.101	4.229	4.385

TABLE 3: Pairs of values (δ_1, δ_2) and (δ_3, δ_4) for various values of α and m .

m	Type-I error α							
	0.0027				0.01			
	δ_1	δ_2	δ_3	δ_4	δ_1	δ_2	δ_3	δ_4
1	0.836	1.006	0.908	1.091	0.789	1.053	0.857	1.143
2	0.751	1.167	0.783	1.216	0.696	1.222	0.725	1.274
3	0.690	1.255	0.709	1.290	0.637	1.308	0.654	1.345
4	0.647	1.311	0.661	1.338	0.598	1.360	0.610	1.389
5	0.615	1.351	0.626	1.373	0.569	1.397	0.579	1.420
6	0.591	1.380	0.599	1.400	0.548	1.424	0.555	1.444
7	0.571	1.404	0.578	1.421	0.530	1.445	0.537	1.462
8	0.555	1.423	0.561	1.438	0.516	1.462	0.522	1.477
9	0.542	1.439	0.547	1.452	0.504	1.476	0.509	1.490
10	0.530	1.452	0.534	1.465	0.494	1.488	0.498	1.501
11	0.520	1.464	0.524	1.475	0.485	1.499	0.489	1.510
12	0.511	1.474	0.515	1.484	0.478	1.508	0.481	1.518

4. Performance Measures

Several statistical metrics such as the power curve (PC), operating characteristic curve (OCC), and run-length (RL) distributional properties are readily available to find out the performance of a control chart. The OCC and PC are employed to identify the control chart's ability to respond against an infrequent change in the quality process observed, while the ARL metric reflects the expected number of points plotted prior to a certain change is identified for the process operating under any potential cause. As the approach to the control chart is identical to the strategy of hypothesis testing, thus the process conditions can be set as $H_0: \sigma = \sigma_0$ (i.e., the process is in-control) vs $H_1: \sigma = \sigma_1$ (i.e., the process is shifted), where $\sigma_1 = \varepsilon\sigma_0$ with a shift constant $\varepsilon \neq 1$. Let the process be undergone some special cause that led to a change from σ_0 to σ_1 in the control value. In relation to type-II error, the ability of the V_{SQ} -control to not detect this shift can thus be defined as

$$\beta = P[LPL \leq V_{SQ} \leq UPL | H_1], \quad (33)$$

where UPL and LPL represent upper and lower probability bounds of the V_{SQ} design.

Simplifying (33) leads to

$$\beta = F(\varepsilon\vartheta_1\sqrt{3m}) - F(\varepsilon\vartheta_2\sqrt{3m}), \quad (34)$$

where F is the χ -dist function with $3m$ df.

Subsequently, out-of-control ARL and power function can be obtained as

$$ARL_1 = \frac{1}{1 - F(\varepsilon\vartheta_1\sqrt{3m}) + F(\varepsilon\vartheta_2\sqrt{3m})}, \quad (35)$$

$$1 - \beta = 1 - F(\varepsilon\vartheta_1\sqrt{3m}) + F(\varepsilon\vartheta_2\sqrt{3m}). \quad (36)$$

For $\varepsilon = 1$, (35) turns into ARL_0 (in-control average run). In order to illustrate the implementation of (35), let us assume that $\varepsilon > 1$, and we are desired to know the probability of identifying a shift of the amount $\sigma_1 = \varepsilon\sigma_0$ in parameter of the Maxwell distributed quality at a specified value of ε taken after the shift has occurred. Thus, the efficiency of the proposed chart for detecting this change is summarized in the form of power curves at α equal to 0.0027 in Figure 2.

From examining power curves in Figure 2, we note that the change of various amounts is efficiently detected by the suggested V_{SQ} -chart. In addition, Figure 2 shows that the probability of identifying the change of a particular amount increases with the rise in the sample size. As an illustration, a fairly greater change of amount 4 in accordance with sample size m equals to 2, 4, and 8, and the chances of detection are 46%, 90%, and 95%, respectively.

Along with power curves, a set of ARL plots for the proposed chart have been considered. In Figure 3, the ARL curves for the same sample sizes are presented.

To demonstrate the use of Figure 3, if we desire to use a sample size of $m = 4$ to detect a shift of amount 2ε , then the average number of samples essential to determine the shift would be $ARL_1 = 3$. Note, also, that if sample size m increases to 30, ARL_1 is decreased to about 1.

5. Simulation Analysis

Simulated data has been employed in this section to establish the parameters of the V_{SQ} -chart. The Maxwell model with parameter $\sigma = 1.5$ has been used for the selection of twenty-five preliminary sample subgroups, each with 4 observations. The V_{SQ} -statistic has now been computed from each subgroup observations. Consequently, from all 25 batches, 25 V_{SQ} values have been obtained. The factors $\delta_1 = 0.615$ and $\delta_2 = 1.351$ are obtained from Table 3 for the fixed probability $\alpha = 0.0027$ and $m = 5$. The parameters of the V_{SQ} -chart are thus determined as

$$\begin{aligned} UCL &= 2.03, \\ CL &= 1.47, \\ LCL &= 0.92. \end{aligned} \quad (37)$$

Figure 4 illustrates the graphical display of the V_{SQ} values along with computed parameters of the V_{SQ} -chart. All the plotted points in Figure 4 represent a nonrandom pattern within the k -sigma limits constructed, which explicitly implies that the underlying process is stable. Similarly from

FIGURE 2: Performance of the V_{SQ} -chart at specified m and $\alpha = 0.0027$.

FIGURE 3: Average run length curves for the V_{SQ} -chart at various sample sizes.

Table 1, coefficients $\vartheta_1 = 0.494$ and $\vartheta_2 = 1.566$ are used to establish the PM. The PM thus obtained at chosen sample $m = 5$ and false alarm risk $\alpha = 0.0027$ are given below:

$$\begin{aligned} \text{LPL} &= 0.74, \\ \text{UPL} &= 2.35. \end{aligned} \quad (38)$$

These PM with calculated values of V_{SQ} estimator are depicted in Figure 5.

Figure 5 indicates again that there is no sign of an out-of-control scenario and PM for the current process are slightly wider than k -sigma limits. We deliberately implemented various magnitudes of shifts in data to determine the sensitivity of the V_{SQ} -chart against different shifts i.e., how the proposed chart would identify these shifts. After batch 15, a shift of upward amount has been incorporated, and the rest 10 subgroups with 5 observations each are generated from the Maxwell probability model with parameter of scale equal

to 2, 2.5, 3, and 3.5, respectively. The V_{SQ} -control chart with conventional k -sigma limits have been employed on all 25 subgroups. For each subgroup, the V_{SQ} -statistic has been determined and graphical displayed in Figure 6.

Figure 6 demonstrates that the last 10 points are all above the central line and in one direction, clearly showing that the observed process is in an out-of-control situation. Additionally, all points of the last 10 subgroups are not above the upper control limit, as shown in Figure 6(a), while in Figures 6(b)–6(d), all points are above the designed upper limit. Certainly, the rationale is the quantity of shift that is comparatively larger in latter cases. The proposed chart effectively identified the shift, thus demonstrating an efficient approach to detecting a shift of any amount.

6. Comparison Study

This section presents a comparative study to determine the efficacy of the proposed design compared to the alternative counterpart of the V_{SQ} -control. For this comparison, many performance metrics may be used, but PC construction is commonly employed in numerous studies [26, 27]. Therefore, the efficiency of the design proposed is assessed in the form of the power function. We have constructed power curves for the proposed chart using Monte Carlo simulations (10^5) under PM with sample sizes $m = 3$ and 7, and false alarm risk $\alpha = 0.0027$ is performed. Different values of the constant ϵ have been considered in the construction of power curves. In Figure 7, the probabilities of identifying a dramatic change in the parameter of the Maxwell distribution are graphed.

It is evident from Figure 7 that the power is very close to the fixed value α when the process is under control. If the process goes in an out-of-control situation, with the rise of ϵ , power increases for varied values of m . The proposed control chart, however, is evidently observed in Figure 7 to be more efficient and responsive to detect the change of a certain size at a specified value of m . For example, a shift of size 3σ has

FIGURE 4: The V_{SQ} -control chart with conventional k -sigma limits. FIGURE 5: The V_{SQ} -control chart with probability limits.

(a)

(b)

(c)

(d)

FIGURE 6: Out-of-control process with a shift constant. (a) $\varepsilon = 2$. (b) $\varepsilon = 2.5$. (c) $\varepsilon = 3$. (d) $\varepsilon = 3.5$.

FIGURE 7: Comparison of the V_{SQ} -design and existing design at (a) $m = 3$ and (b) $m = 7$.

been detected by the proposed design with a probability 0.821 for a sample size $m = 7$ which is greater than the respective probability of 0.397 of the existing design. In addition, the proposed chart more effectively detects the shift with increased sample size.

7. Real Application

The real dataset on carbon fiber strengths has been employed to illustrate the computational process of the V_{SQ} -control chart. The strength data was originally reported as an application example by Badar and Priest [28]. Data show the strengths of single fibers trialed under stress at various gauge levels. Here, we consider 60 measurements of the strength data tested at gauge length 20 mm. To acquire a good fit, original data is slightly modified by subtracting 1.2 from each value to shift the origin of the dataset. This modified data is then employed in further analysis. Initially, we have used the probability plotting as a graphical tool to check if the strength data are consistent with a hypothesized Maxwell distribution based on a subjective visual analysis of the given data. The general process is straightforward and can be carried out easily using *R* software. The probability plot for our sample dataset is shown in Figure 8.

In Figure 8, a straight line has been drawn through plotted points as the best fit line assuming the Maxwell model. Figure 8 shows that all plotted points are covered up by this straight line, and the Maxwell model reasonably portrays the given dataset. Following the graphical approach, the data are further analyzed with formally goodness of fit Kalmogorov–Simirnov (KS) test.

The p value of the KS test for the analyzed dataset is 0.670 which indicates that the Maxwell model may be appropriate for given data. This dataset has now been used for the proposed chart for the Maxwell scale parameter monitoring. Ignoring the last three observations in the modified dataset, all 60 observations are classified simply in 12 subgroups each of size 5, as shown in Table 4.

These subgroups are further considered in the construction of k -sigma limits and PM for the V_{SQ} -control chart. Now, B -coefficients for $m = 5$ and $\alpha = 0.0027$ are $\delta_3 = 0.626$ and $\delta_4 = 1.373$. Thus, the k -sigma limits using (31) are computed as

$$\begin{aligned} \text{UCL} &= 1.54, \\ \text{CL} &= 1.12, \\ \text{LCL} &= 0.70. \end{aligned} \quad (39)$$

These computed V_{SQ} statistics in the proposed chart are displayed in Figure 9.

Figure 9 shows that the proposed chart has not indicated any unstable state i.e., no single sample lies outside the designed limits of the chart and plotted statistic displays a random pattern. Similarly, constants $\vartheta_3 = 0.502$ and $\vartheta_4 = 1.593$ for $\alpha = 0.0027$ and sample size $m = 5$ have been utilized from Table 1 to find the PM of the V_{SQ} -chart. This provides the parameters of the proposed chart in terms of PM as $\text{UPL} = 1.79$, $\text{CL} = 1.12$, and $\text{LPL} = 0.56$. The chart with PM is depicted in Figure 10. The PM constructed in Figure 10 again justifies that the quality characteristic under investigation is in the in-control state.

FIGURE 8: Probability plot indicating Maxwell distributed data.

TABLE 4: Carbon strength data along with computed values of V_{SQ} statistic.

Sample no.	Breaking strengths					V_{SQ}
1	1.414	1.424	1.728	1.161	2.208	0.939
2	1.830	2.132	1.717	2.293	0.932	1.064
3	2.177	1.459	2.301	2.023	3.195	1.328
4	1.375	1.737	0.701	2.824	1.737	1.045
5	2.354	2.686	3.820	1.197	2.652	1.532
6	2.035	2.235	1.538	1.416	1.945	1.073
7	1.028	1.245	2.827	1.274	2.146	1.059
8	1.057	1.475	1.003	2.094	2.020	0.922
9	1.322	2.771	2.043	1.796	2.671	1.263
10	2.072	1.325	1.656	2.064	1.540	1.014
11	3.025	1.939	2.337	2.428	2.362	1.410
12	1.777	1.150	1.332	1.318	1.418	0.816

FIGURE 9: The V_{SQ} -control chart with conventional k -sigma limits.

FIGURE 10: The V_{SQ} -control chart with probability limits.

8. Conclusions

The Maxwell distribution properties are described, and the V_{SQ} -control chart as a novel design for monitoring the quality characteristic, which is best defined by the Maxwell model, has been proposed in this work. The ARL properties and power function have been derived to find out the performance of the suggested V_{SQ} control. Numerical results obtained in the simulated environment demonstrate that, with the increased process shift in addition to increment of sample size, V_{SQ} -control chart performs efficiently for identifying the shift. The V_{SQ} -chart has been compared to the contemporary design of the V -chart. The power curves produced in this comparison have shown that the suggested chart is more effective and powerful than the competitor V -chart in identifying the infrequent changes in the target parameter of the Maxwell model. On the contrary, the RL distribution also indicates that the proposed chart is more responsive with a substantial shift in the monitoring parameter. A simulated data from the Maxwell distribution has been applied to exhibit the computational methodology of the proposed chart. Finally, the implementation procedure has also been demonstrated using real example data on fiber's stress strength.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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