Research Article

Trade Credit and Revenue Sharing of Supply Chain with a Risk-Averse Retailer

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In this paper, we develop three supply chain game models, i.e., the basic model, the single trade credit model, and the trade credit and revenue sharing collaboration model. Conditional value-at-risk (CVaR) criterion is used as the measure of risk assessment in these models. We analyze the optimal decisions in the centralized and decentralized situations, respectively, and verify that single trade credit cannot coordinate the supply chain. However, the collaboration contract can coordinate the supply chain. Furthermore, the results show that the decisions and profits of both the manufacturer and the retailer rely on the degree of the risk aversion, the trade credit period, and the revenue sharing coefficient. The collaborative contract effectively improves supply chain performance and achieves a ‘win-win’ situation for the supply chain members. In addition, we also consider two extensions for our research. One extension shows that the collaborative contract of trade credit and buyback can also coordinate the supply chain in a certain range. The other extension considers the optimal decision of a risk-averse manufacturer with CVaR.

1. Introduction

Supply chain finance (SCF) is considered to be an effective means to hedge risks and optimize the cash flow and financial structure among members of the supply chain [1]. It has attracted great attention from the business and government. For example, the UK Prime Minister announced a supply chain finance scheme in 2012 [2] and the Chinese government released a guideline to promote the development of SCF in 2017 [3]. Accordingly, scholars have done a lot of researches on SCF. Researches are two major perspectives in the extant literature: the finance-oriented perspective and the supply chain-oriented perspective. From the perspective of supply chain management, the trade credit can be used as an internal financing means to solve the problem of capital constraints. The trade credit is a kind of credit relationship between upstream enterprises and downstream enterprises, where the upstream enterprises allow downstream enterprises to delay payment or advance payment. At present, the trade credit is widely used in economic activities, such as the suppliers deliver the goods to retail supermarkets without payment, presale houses by real estate developers, and presale membership cards by consumer clubs. According to a white paper published by the CGI group in 2007, about 85% of the global commercial trade uses the trade credit [4]. More than 80% of the B2B transactions in the UK are made in the form of credit sales, and approximately 80% of the companies in the United States provide trade credit [5].

Trade credit can improve the efficiency of capital and increase revenue in the supply chain. The effective cooperation of members in the supply chain can win competitive advantages and obtain extra revenues. The increased revenue should be shared by the supply chain members. The revenue sharing contract can well realize distribution of revenues in the supply chain. As a classic contract, the revenue sharing plays an important role in supply chain management and has been successfully applied to e-commerce video rental and
other industries, such as the sales of iPhone, AdSense advertising, and blockbuster.

Although the trade credit can increase profits for supply chain members, it also increases risks to members that provide trade credit. Most members of supply chain are risk-averse [6]. It is well known that the conditional value-at-risk (CVaR) is an important risk measurement in financing risk management. CVaR was proposed by Rockafellar and Uryasev [7] and widely applied in economics, finance, and insurance. As a measurement criterion, CVaR has many good properties, such as subadditivity, positive homogeneity, monotonicity, and transitivity.

Under the background of supply chain member risk-averse, we will try to answer the following questions:

(i) What is the capital constrained firms’ optimal operational decision and financing decision?

(ii) How does trade credit contract affect each member’s optimal decision? Furthermore, whether the revenue sharing collaboration contract could coordinate the supply chain? Besides, whether the collaborative contract of trade credit and buyback could coordinate the supply chain?

(iii) What if the manufacturers were risk-averse?

In this paper, we develop three supply chain game models, i.e., the basic model, single trade credit model, trade credit, and revenue sharing collaboration model, respectively. We consider the revenue sharing collaboration contract to coordinate such a supply chain. In addition, two extensions for our study can be carried out. One extension shows that the collaborative contract of trade credit and buyback can also coordinate the supply chain in a certain range. The other extension considers the case of a risk-averse manufacturer’s optimal decision with CVaR. The existing literature on trade credit has either analyzed operations decisions in the premise of risk neutrality or analyzed trade credit without considering the time value of capital. Our contributions to the existing literature lie in incorporating the time value and risk attitude, both of them take into account the firm’s risk profit-maximizing decisions and using the collaborative contract to coordinate the supply chain.

The rest of this paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the description of the problem and assumptions. Section 4 introduces three models, i.e., single trade credit model, trade credit, and revenue sharing collaboration model. Section 5 discusses numerical results and sensitivity analysis. Section 6 extends to our model with considering the case of both buyback contract and revenue sharing collaboration and the case with a risk-averse manufacturer’s optimal decision with CVaR. We summarize the major contributions of this paper in Section 7. All proofs of propositions proposed in this paper are presented in the Appendix section.

2. Literature Review

This paper is related to the literature that focused on trade credit, revenue sharing contract, and conditional value-at-risk criterion.

With regard to trade credit, the literature mainly focuses on two aspects: the financial and economic fields and the operation management field. The financial and economic fields pay more attention to the research on the causes, functions, and influencing factors of delay payment trade credit, as well as the practice of trade credit in developing countries through empirical methods. For example, Bougeas et al. [8] used empirical methods to illustrate the substitution relationship between the delay payment trade credit and the prepayment trade credit in the international trade. Mieut and Zanchettin [9] focused on the credit of the delay payment trade in the form of accounts receivable. The operation management field began to research about the trade credit from delay payment trade credit. Goyal [10] proposed that under the credit policy of delay payment trade provided by the upstream enterprise, the downstream enterprise’s order quantity is larger than that in the normal situation. Chen et al. [11] studied the optimal inventory decision problem of perishable products with full delay payment trade credit provided by upstream enterprises and partial delay payment trade credit provided by downstream enterprises for their customers. Some scholars take the delay payment credit of supply chain as a decision variable. For example, Zhong and Zhou [12] studied the supply chain composed of an upstream enterprise and a downstream enterprise. The downstream enterprise is faced with the demand that depends on the inventory and its inventory holding is limited. It obtains the optimal trade credit period decision that the upstream enterprise can encourage the downstream enterprise to increase the order quantity. Wu et al. [13] studied the problem of inventory decision and trade credit period decision of perishable products when the upstream enterprise in the supply chain provides downstream enterprise with delay payment trade credit, the downstream enterprise also provides delay payment trade credit for their customers. Chen and Kang [14] and Chung and Liao [15] also studied supply chain operations under trade credit from the perspective of supply chain cooperation.

The issue of profit distribution under the trade credit of supply chain member also attracted the attention of scholars. Only by reasonably distributing the benefits of supply chain cooperation can establish and maintain a good supply chain cooperation relationship. The revenue sharing contract is a way of coordinating the performance of supply chain operations, providing lower wholesale prices through suppliers and sharing the improvement of sales revenue with retailers. Wang et al.[16] found that the revenue sharing contract can realize the equal division of the revenue in the entire supply chain. A win-win situation for supply chain members can also be achieved through sharing contracts [17]. Ma et al. [18] studied the redistribution of extra profits in the supply chain cooperation system based on trade credit and realized the reasonable allocation of extra profits by introducing a joint contract of repurchase and revenue sharing.

All of the above studies are carried out in the premise of risk neutrality. However, the decision maker who is completely rational does not conform to the reality. In recent
years, scholars consider decision makers’ risk preference. The scholars Rockafellar and Uryasev [7] proposed to measure the degree of risk aversion using CVaR. CVaR is not only used to estimate tail risk but also easy to calculate; it has been widely used in various fields, such as securities investment, power bidding, risk management, and supply chain management. In particular, there are many studies on supply chain management. For example, Chen et al. [19] used CVaR as the decision criterion to investigate the optimal pricing and ordering decisions in the risk-averse newsvendor model and proved that the optimal order quantity is smaller than that in the risk-neutral model. Zhu et al. [20] proposed a decision-making model where the CVaR criterion is used to measure risk of assessment in a dual-channel supply chain, both retailer and manufacturer are risk-averse under uncertainties of manufacturing yield and market demand, and verified that the joint contracts of revenue sharing contract and buyback contract can coordinate the dual-channel supply chain. Fan et al. [21] considered option contract application where both the buyer and supplier are risk-averse simultaneously. The effects of option price and option exercise price are investigated via CVaR minimization and proved that the supply chain can be coordinated under option contract.

So far, research on trade credit based on risk preference model is not sufficient. Based on the prospect theory, Zhang and Luo [22] studied the coordination of trade credit and revenue sharing measures under the risk preference of retailers, ultimately all members of the supply chain achieved Pareto optimization. To the best of our knowledge, there are only two papers combining trade credit with conditional value-at-risk. Chen and Zhou [23] considered a supplier and a risk-averse retailer under trade credit contract. The retailer faces a random demand and the supplier provides the trade credit for the risk-averse retailer with budget constraints. Chen et al. [24] supposed default happens with a random probability and considered supply chain coordination with trade credit under the CVaR criterion in the perspective of risk control. Chen and Zhou [23] do not consider the time value of capital while our paper does. Chen et al. [24] mainly considered default probability of trade credit to optimize the lot-sizing decision for the creditor, while our paper mainly discusses the changes that trade credit brings towards the decisions of both the manufacturer and the retailer. Furthermore, both of these two papers do not consider the issue of supply chain coordination, whereas we do. We consider the trade credit and revenue sharing collaborative contract to coordinate supply chain. In addition, we discuss a risk-averse manufacturer’s optimal decision with CVaR and consider the collaborative contract of trade credit and buyback contract to coordinate the supply chain.

3. Problem Description and Assumptions

3.1. Model Description. We develop the supply chain game model consisting of a manufacturer and a risk-averse retailer under stochastic demand, in which the manufacturer is the leader and the retailer acts as a follower. Three models, i.e., the basic model, single trade credit model, trade credit, and revenue sharing collaboration model are developed. All members of the supply chain are completely rational and aim at maximizing their own interests.

Based on CVaR, it was proposed by Rockafellar and Uryasev [7] and has been widely applied to measure the degree of risk in theoretical research and practical areas. The CVaR value is defined as

$$CVaR_{\eta}(\pi(x, y)) = \max_{u \in R} \{u + \eta^{-1} E[\min(\pi(x, y) - u, 0)]\},$$

where $E$ represents the expected value of a decision variable, $\eta(0 < \eta \leq 1)$ represents the risk-averse coefficient of the decision maker. A risk-averse coefficient is specified by the decision maker for attaining a certain level of profit $u$. The retailer’s risk-aversion decreases as $\eta$ increases. $\pi(x, y)$ represents the profit function of deterministic variable $x$ and random variable $y$.

3.2. Assumptions and Notations. The variables and parameters used in this paper are described as shown in Table 1.

Without losing generality, this paper assumes that $p > w > c_m > c_r w > c_m + c_r h t$. For the three models proposed in our paper, three channel models are represented by subscripts $c1, c2, c3$, respectively. Similarly the subscripts of the retailer and the manufacturer are expressed by subscripts $r1, r2, r3$ and $m1, m2, m3$, respectively.

4. Model Analysis

4.1. The Basic Model ($\eta, w$). In the basic model, the manufacturer supplies the products to the retailer at the wholesale price $w$, and then the retailer decides the order quantity. At this time, the profit function of the centralized channel supply chain defined in this paper can be expressed as

$$E\Pi_{c1} = \rho \left( q - \int_0^q c(x)dx \right) + \nu \int_0^q (q - x)f(x)dx - s \int_0^q (x - q)f(x)dx - (c_r + c_m)q.$$  

Equation (2) describes how to calculate channel supply chain’s profit function by using its sales revenue $p(q - \int_0^q F(x)dx)$, salvage revenue $\nu \int_0^q (q - x)f(x)dx$, shortage cost $s \int_0^q (x - q)f(x)dx$, purchase cost $c_m q$, and sales cost $c_r q$. The structure of the expected profit function is the same in the following channels.

From equation (2), we can get the optimal order quantity of the channel:

$$F(q_{c1}^*) = \frac{p + s - (c_r + c_m)}{p + s - \nu}.$$  

The optimal order quantity of the channel is related to the retail price, salvage value, shortage cost, sales cost, and production cost, but it is independent of the wholesale price.

In decentralized supply chain, the profits of the manufacturer and the retailer are as follows:
**Table 1: Notations.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>Retail price per unit</td>
</tr>
<tr>
<td>( w )</td>
<td>Wholesale price per unit</td>
</tr>
<tr>
<td>( v )</td>
<td>Salvage price per unit of the retailer</td>
</tr>
<tr>
<td>( s )</td>
<td>Unit shortage cost</td>
</tr>
<tr>
<td>( c_m )</td>
<td>Production cost per unit</td>
</tr>
<tr>
<td>( c_r )</td>
<td>Process cost per unit of the retailer</td>
</tr>
<tr>
<td>( i_m )</td>
<td>Capital cost ratio of the manufacturer</td>
</tr>
<tr>
<td>( i_r )</td>
<td>Average return on investment of retailer</td>
</tr>
<tr>
<td>( t )</td>
<td>Trade credit period</td>
</tr>
<tr>
<td>( x )</td>
<td>Stochastic market demand</td>
</tr>
<tr>
<td>( F(\cdot) )</td>
<td>Cumulative distribution function of market demand</td>
</tr>
<tr>
<td>( f(\cdot) )</td>
<td>Probability density function of market demand</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Risk-aversion coefficient of the retailer ((0 &lt; \eta \leq 1))</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Revenue sharing coefficient</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Mean of the stochastic market demand</td>
</tr>
<tr>
<td>( q )</td>
<td>Order quantity of the retailer</td>
</tr>
<tr>
<td>( q^* )</td>
<td>Optimal quantity from the basic model, single trade credit model, trade credit, and revenue sharing collaboration model, respectively, (i = 1, 2, 3).</td>
</tr>
</tbody>
</table>

\[ II_{m1} = (w - c_m)q. \]
\[ II_{r1}(q, x) = p \min(q, x) + v(q - x)^+ - s(x - q)^+ - (c_r + w)q. \]  
(4)

Equation (4) describes the retailer’s profit consisting of its sales revenue \( p \min(q, x) \), salvage revenue \( v(q - x)^+ \), shortage cost \( s(x - q)^+ \), wholesale cost \( wq \), and sales cost \( c_r q \). The retailer has the same revenue structure.

Due to the uncertainty of demand, we assume that the retailer is usually risk-averse. The maximum of the conditional value-at-risk criterion for the risk-averse retailer can be expressed as

\[ \max_q \text{CVaR}_{\eta}(II_{r1}(q, x)) = \max_q \max_{u_1 \in R} g(u_1, q), \]  
(5)

where

\[ g(u_1, q) = u_1 + \eta^{-1}E[\min(II_{r1}(q, x) - u_1, 0)] \]
\[ = u_1 - \eta^{-1} \int_0^q [u_1 - px + v(q - x) + (c_r + w)q]^+ dF(x) \]
\[ - \eta^{-1} \int_q^U [u_1 - px + s(x - q) + (c_r + w)q]^+ dF(x). \]  
(6)

Based on equation (6), we can derive the following results.

**Proposition 1**

(1) The optimal threshold \((u^*_1)\) is as follows:

\[ u^*_1 = pq - (c_r + w)q. \]  
(7)

(2) Retailer’s conditional value-at-risk criterion is expressed as

\[ \text{CVaR}_{\eta}(II_{r1}(q, x)) = pq - (c_r + w)q - \frac{s}{\eta} (\mu - q) \]
\[ - \frac{p + s - v}{\eta} \int_0^q F(x)dx. \]  
(8)

(3) Retailer’s optimal order quantity \(q^*_1\) satisfies the following equation:

\[ F(q^*_1) = \eta \frac{p + (s/\eta) - (c_r + w)}{p + s - v} \]  
(9)

The detailed proof of Proposition 1 is given in the Appendix section.

In Proposition 1, it is observed that \(u^*_1\) increases with the retail price \(p\) and decreases with the process cost of the retailer \(c_r\) and the wholesale price of the manufacturer. Obviously, the retailer’s value is related to risk-aversion coefficient \(\eta\). Besides, the final market price and salvage price also have impact on the retailer’s value.

It is obvious that \(q^*_1\) is also the solution of the classic newsvendor model when the retailer is risk neutral (i.e., \(\eta = 1\)) and the optimal order quantity is \(F^{-1}(p + s - (c_r + w)/p + s - v)\). We can find that \(q^*_1 < F^{-1}(p + s - (c_r + w)/p + s - v)\), that is, the retailer will reduce order quantity due to the risk aversion.

Based on the retailer’s optimal response function equation (9), let \(\partial II_{m1}/\partial w = 0\), we can obtain the manufacturer’s optimal wholesale price \(w^*_1\).

According to Proposition 1, we calculate the first-order derivatives of equations (8) and (9) with respect to \(w\) to obtain the optimal decision results for \(\eta, v, s\), and \(w\) obtain Corollary 1.

**Corollary 1**

\[ (1) \frac{\partial q^*_1}{\partial \eta} > 0, \frac{\partial q^*_1}{\partial v} > 0, \frac{\partial q^*_1}{\partial s} > 0, \frac{\partial q^*_1}{\partial w} < 0. \]
\[ (2) \frac{\partial \text{CVaR}_{\eta}(II_{r1}(q, x))}{\partial \eta} > 0, \frac{\partial \text{CVaR}_{\eta}(II_{r1}(q, x))}{\partial v} > 0, \frac{\partial \text{CVaR}_{\eta}(II_{r1}(q, x))}{\partial s} < 0, \frac{\partial \text{CVaR}_{\eta}(II_{r1}(q, x))}{\partial w} < 0. \]

In the decentralized supply chain, it can be figured out from Corollary 2 that both the optimal order quantity and the risk profit of the retailer increase with the risk-aversion coefficient and the residual value, but decrease with the wholesale price. The optimal order quantity increases with the shortage cost, but risk profit of the retailer decreases with the shortage cost. The reason is that the larger the wholesale price, the smaller the marginal revenue \(p - w\), and then the retailer will reduce the order. The larger the residual value, the smaller the unit of unsold loss \(v - w\), and then the retailer will increase the order. These reflected business situations are in line with reality.

**Corollary 2.** When the retailer is risk-averse, the double marginalization effect decreases as the risk-aversion coefficient increases.
Proposition 2 Suppose the differences between optimal order quantity in the channel and retailer’s optimal order quantity in the decentralized is \( F = F(q^*_{c1}) - F(q^*_{m2}) = (p - (c_r + c_m) - \eta p + \eta (c_r + w)/p + s - v) \), then \( (\partial F/\partial \eta) = -(p - (c_r + w)/p + s - v) < 0 \).

Corollary 2 implies the effect of the double marginalization effect on the supply chain decisions. From Corollary 2, we find that with the increase of retailer’s risk aversion, the double marginalization effect decreases. 

4.2. The Single Trade Credit Model (\( \eta, w, i_m, i_r, t \)). In the context of the basic model, the manufacturer provides the retailer with a trade credit period and allows the retailer to postpone payment for goods. By saving the capital occupation of retailer, the manufacturer can induce the retailer to increase order quantity and then expand sales market-scale level. In this way, it is equivalent to that the manufacturer provides interest-free loans to the retailer, and the retailer can use the funds to obtain the corresponding investment income. If sales are expanded, the manufacturer’s profit will also increase, but the manufacturer’s funds will be occupied by the retailer. At this time, the manufacturer considers the time value of capital. When the manufacturer provides the retailer with a trade credit period \( t \) (that is, the trade credit period is not necessarily equal to the selling period, it is determined by the manufacturer’s market forecast or affordability), the manufacturer’s increased capital costs are \( c_m i_m t \), the retailer’s increased sales revenue is \( i_r t px \) when \( q > x \), and the retailer’s increased sales revenue is \( i_r t pq \) when \( x > q \). Correspondingly the retailer’s increased sales expected revenue is \( i_r t E[p \min(x, q)] \). Furthermore, the residual value also increases the revenue and the retailer’s increased residual revenue is \( \nu(1 + i_r t) \int_0^q (q - x)f(x)dx \). Our paper calls this situation a single trade credit model.

In the single trade credit model, the expected profit of the channel is

\[
\begin{align*}
\text{ELI}_{c2} = p & \left(1 + i_r t\right) \int_0^q F(x)dx \\
+ & \nu(1 + i_r t) \int_0^q \left(q - x\right)f(x)dx \\
- & s \int_0^q \left(x - q\right)f(x)dx - (c_r + c_m + c_m i_m t)q.
\end{align*}
\]

From equation (10), we can obtain the optimal order quantity of the channel:

\[
F(q^*_{c2}) = \frac{p \left(1 + i_r t\right) + s - (c_r + c_m + c_m i_m t)}{(p - \nu)(1 + i_r t) + s}.
\]

In the decentralized supply chain, with the manufacturer capital cost increased, the profit of the manufacturer will be written as follows:

\[
\Pi_{m2} = (w - c_m - c_m i_m t)q.
\]

In the decentralized supply chain, with the retailer sales revenue increased, the profit of the retailer will be written as follows:

\[
\Pi_{r2} = p \left(1 + i_r t\right) \min(q, x) + \nu(1 + i_r t) \left(q - x\right)^+ \\
- s \left(x - q\right)^+ - (c_r + w)q.
\]

The manufacturer provides a trade credit period to maximize the expected profit and the risk-averse retailer seeks to maximize the risk value. Similar to the basic model, the retailer’s risk profit is as follows:

\[
\max_{q} \text{CVaR}_{\eta}(\Pi_{r2}(q, x)) = \max_{q} \max_{u_r \in \mathbb{R}} g(u_r, q),
\]

where

\[
g(u_r, q) = u_r + \eta^{-1} E\left[\min(\Pi_{r2}(q, x) - u_r, 0)\right] \\
= u_r - \eta^{-1} E\left[u_r - \Pi_{r2}(q, x)\right]^+
\]

\[
= u_r - \eta^{-1} \int_0^q \left[u_r - p \left(1 + i_r t\right)x - \nu(1 + i_r t)(q - x) + (c_r + w)q\right]^+ \frac{dF(x)}{F(x)} \\
- \eta^{-1} \int_q^\infty \left[u_r - p \left(1 + i_r t\right)q + s \left(x - q\right) + (c_r + w)q\right]^+ \frac{dF(x)}{F(x)}.
\]

From equation (15), we derive the following results.

**Proposition 2**

(1) The optimal threshold under single trade credit model is as follows:

\[
u^*_2 = p \left(1 + i_r t\right)q - (c_r + w)q.
\]

(2) Retailer’s conditional value-at-risk criterion is expressed as

\[
\text{CVaR}_{\eta}(\Pi_{r2}(q, x)) = p \left(1 + i_r t\right)q - (c_r + w)q \\
- \frac{s}{\eta} \left(u - q\right) - \frac{(p - \nu)(1 + i_r t) + s}{\eta} \int_0^q F(x)dx.
\]

(3) Retailer’s optimal order quantity \( q^*_{c2} \) satisfies the following equation:
The detailed proof of Proposition 2 is given in the Appendix section.

In Proposition 2, it is observed that \( u^*_t \) increases with the trade credit period \( t \). Obviously, both the retailer’s value and the optimal order quantity are related to the trade credit period \( t \). In particular, when the retailer is risk neutral (\( \eta = 1 \)), the optimal order quantity is \( F^{-1}(p(1+i,t) + s - (c_r + w))/(p - v) (1+i,t) + s) \).

Based on the retailer’s optimal response function (18), and \( \partial^2 u^*_t/\partial u = 0 \), we can obtain the manufacturer’s optimal wholesale price \( w^*_t \).

According to Proposition 2, we calculate the first-order derivatives of equations (17) and (18) with respect to \( t, i, \) and comparing equations (9) and (18), we obtain the following corollary.

**Corollary 3**

1. \( \partial q^*_t/\partial t > 0 \), \( \partial q^*_t/\partial i > 0 \).
2. \( \partial \text{CVaR}_q^*(\Pi_{12}(q,x))/\partial t > 0 \), \( \partial \text{CVaR}_q^*(\Pi_{12}(q,x))/\partial i > 0 \).
3. \( q^*_t > q^*_t \)

In the single trade credit model, it can be figured out from Corollary 3 that the optimal order quantity and the risk profit of the retailer increase with the trade credit period and average return on the investment of retailer. The optimal order quantity of the retailer in the single trade credit model is larger than that in the basic model; it can be seen that the manufacturer’s trade credit has a promoting effect on the retailer’s order. The longer the credit period, the more the interest-free loans will attract the retailer to increase orders.

**Corollary 4.** In the single trade credit model, the optimal order quantity of the retailer is less than the optimal order quantity of the channel \( q^*_t < q^*_t \). Namely, the single trade credit cannot achieve supply chain coordination.

Proof. Comparing equations (11) and (18), we have \( \eta(p(1 + i,t) + (s/\eta) - (c_r + w)/(p - v) (1+i,t) + s < (p(1+i,t) + (s/\eta) - (c_r + w)/(p - v) (1+i,t) + s < (p(1+i,t) + s - (c_r + c_m + c_{m,t})/(p - v) (1+i,t) + s) \), so we can obtain \( q^*_t > q^*_t \). Substituting equation (11) into \( \partial \text{CVaR}_q^*(\Pi_{12}(q,x))/\partial q = p(1+i,t) - (c_r + w)/(p - v) (1+i,t) + s/F(q) \), we obtain \( \partial \text{CVaR}_q^*(\Pi_{12}(q,x))/\partial q^*_t < 1 - (1/\eta) \) \([p(1+i,t) - c_r - c_m - c_{m,t}] < 0 \). Since the retailer’s value-at-risk is a concave function of the order quantity and the maximum value-at-risk is taken at \( q^*_t \), the single trade credit contract is impossible to coordinate the supply chain.

Trade credit contract has a stimulating effect on the retailer’s order but the manufacturer provides a trade credit contract and will lose revenue partially. Therefore, the premise for both the manufacturer and the retailer to adopt a trade credit contract is to achieve Pareto improvement.

Namely, the profit of the manufacturer and the risk profit of the retailer should be not lower than the profit of the basic model, which are given as follows:

\[
\left\{ \begin{array}{l}
\text{CVaR}_q^*(\Pi_{12}(q^*_t,x)) \geq \text{CVaR}_q^*(\Pi_{12}(q^*_t,x)), \\
\text{EII}_m(\rho(q^*_t)) \geq \text{EII}_m(\rho(q^*_t)).
\end{array} \right. \tag{19}
\]

According to equation (19), the range of the optimal credit period \( t \) needs to satisfy the following conditions:

\[
\begin{align*}
\text{CVaR}_q^*(\Pi_{12}(q^*_t,x)) - \text{CVaR}_q^*(\Pi_{12}(q^*_t,x)) & \leq i_z(q^*_t - q^*_t) \\
& \leq \frac{(w - c_m)(q^*_t - q^*_t)}{c_m q^*_t}.
\end{align*}
\]

The existence of the range of the optimal credit period \( t \) depends mainly on the exogenous parameter. \( \square \)

4.3. The Revenue Sharing Collaboration Model \((\eta, w, i, t, \phi)\). In order to achieve the goal of mutual benefit, both the manufacturer and the retailer need to cooperate effectively. It is known that a single trade credit contract cannot coordinate the supply chain. To handle this problem, this paper introduces a revenue sharing collaborative contract, i.e., the combination of a wholesale price contract, a revenue sharing contract, and a trade credit contract. The collaborative contract coordination works in the following manner. (1) The manufacturer and the retailer negotiate to determine the sharing coefficient \( \phi \); (2) the manufacturer determines the contract \((w, t)\) as the leader of supply chain; (3) the retailer determines the optimal order quantity \( q \).

Since the revenue sharing contract only changes the ratio of revenue distribution between two supply chain members, it does not change the overall profit of the supply chain. The expected profit and the optimal order quantity of the channel are consistent with the single trade credit model \( \text{EII}_3 = \text{EII}_3 \), \( q^*_3 = q^*_3 \).

In the decentralized supply chain, the profit of the manufacturer is expressed as

\[
\Pi_{m3} = (1 - \phi) \left\{ p(1 + i,t) E[\min(q,x)] + \nu(1 + i,t)(q - x)^+ - s(x - q)^+ \right\} - (c_m + c_m q^*_m - w)q.
\]

In the decentralized supply chain, the profit of the retailer is expressed as

\[
\Pi_{r3} = \phi \left\{ p(1 + i,t) E[\min(q,x)] + \nu(1 + i,t)(q - x)^+ - s(x - q)^+ \right\} - (c_r + w)q.
\]

The retailer’s risk profit is as follows:

\[
\max_{q} \text{CVaR}_q^*(\Pi_{r3}(q,x)) = \max_{q} \max_{\rho \in R} q(\rho, q),
\]

where
According to equation (23), we have the following results.

**Proposition 3**

1. The optimal threshold under the revenue sharing collaboration model $u_3^*$ satisfies the condition:
   \[ u_3^* = \phi p(1 + i, t)q - (c_r + w)q. \]  
   \[ (25) \]
2. Retailer’s conditional value-at-risk criterion is expressed as
   \[ \text{CVaR}_q(\Pi_3(q, x)) = \frac{\phi p(1 + i, t)q - (c_r + w)q}{\eta} (\mu - q) \]
   \[ - \phi \frac{(p - \mu)(1 + i, t) + s}{\eta} \int_0^q F(x)dx. \]
   \[ (26) \]
3. Retailer’s optimal order quantity $q_{r3}^*$ satisfies the following equation:
   \[ F(q_{r3}^*) = \frac{\phi p(1 + i, t) + (\phi/s)\eta - (c_r + w)}{\phi([p - \mu](1 + i, t) + s)}. \]
   \[ (27) \]

Here, the condition $\phi p(1 + i, t) - (c_r + w) > 0$ is required to ensure the existence of the optimal solution.

The detailed proof of Proposition 3 is given in the Appendix section.

In Proposition 3, it is observed that $u_3^*$ increases with the revenue sharing coefficient $\phi$. Obviously, both retailer’s value and optimal order quantity are related to revenue sharing coefficient $\phi$.

Based on the retailer’s optimal order quantity $q_{r3}^*$, the expected profit of the manufacturer is as follows:

\[ \text{EII}_{m3} = (1 - \phi) \left\{ p(1 + i, t) \left( q_{r3}^* - \int_0^{q_{r3}^*} F(x)dx \right) \right. 
   + \left. \nu(1 + i, t) \int_0^{q_{r3}^*} F(x)dx - s \int_{q_{r3}^*}^V (x - q_{r3}^*)dF(x) \right\} 
   - (c_r + c_m + c_{m't}) - \nu q_{r3}^*. \]

\[ (28) \]

Let $(\partial \text{EII}_{m3}/\partial w) = 0$; we can obtain the manufacturer’s best wholesale price $w_3^*$. It is obvious that the sharing coefficient affects profit of the manufacturer; the profit of manufacturer decreases when the sharing coefficient increases.

According to Proposition 3, we calculate the first-order derivatives for equations (26) and (27) with respect to $\phi, w$ and comparing equations (18) and (27), we obtain the following corollary.

**Corollary 5**

1. $(\partial q_{r3}^*/\partial \phi) > 0, (\partial q_{r3}^*/\partial w) < 0$
2. $(\partial \text{CVaR}_q(\Pi_3(q, x))/\partial \phi) > 0, (\partial \text{CVaR}_q(\Pi_3(q, x))/\partial w) < 0$
3. $q_{r2}^* > q_{r3}^*$

In the collaborative model, it can be figured out from Corollary 5 that both the optimal order quantity and the risk profit of the retailer increase with the revenue sharing coefficient and decrease with the wholesale price. Compared with the single model, the order quantity of the retailer is reduced. The reason might be that the retailer shares a part of the revenue with the manufacturer. The revenue sharing coefficient affects the profit distribution. However, the determination of the sharing coefficient is the result of negotiation between the two parties, and therefore, the negotiation is a very important factor in revenue sharing contracts.

**Proposition 4.** To achieve supply chain coordination, the conditions $q_{r3}^* = q_{r3}^* = q_{m2}^*$ must be satisfied, that is,

\[ \eta w = (\eta - 1)\phi p(1 + i, t) + \phi (c_r + c_m + c_{m't}) - \eta c_r, \]

\[ (2!9) \]

where $w \leq \phi (c_r + c_m + c_{m't}) - c_r$.

Proof. To achieve supply chain coordination, the following conditions should be satisfied: $q_{r3}^* = q_{r3}^* = q_{m2}^*$; we can obtain equation (29) after deformation. Equation (29) can be transformed to $\eta = (\phi p(1 + i, t) - \phi (c_r + c_m + c_{m't})/\phi p(1 + i, t) - (c_r + w))$; moreover, $0 < \eta \leq 1$, so that $w \leq \phi (c_r + c_m + c_{m't}) - c_r$.

In particular, when the retailer is risk neutral $(\eta = 1)$, $w = \phi (c_r + c_m + c_{m't}) - c_r < c_m + c_{m't}$. It can be seen that the wholesale price of the manufacturer is less than the cost price under the joint contract; the reason is that the manufacturer’s revenue mainly comes from part of the revenue shared by the retailer.

Under the collaborative contract, the revenue of each supply chain member should not be lower than the revenue under a single trade credit contract. That is, the supply chain members realize Pareto improvement based on the following conditions:
According to equation (30), the range of the sharing coefficient $\phi$ should satisfy the following conditions:

$$\phi \geq \frac{\text{CVaR}_\eta(\Pi_{22}(q_{r,2}^*, x)) + (c_r + w)q_{r,3}^*}{p(1 + i, t)q_{r,3}^* - (s/\eta)\mu(q_{r,3}^*) - ((p - v)(1 + i, t) + s/\eta)\int_0^{\mu} F(x)dx}$$

$$\phi \leq 1 - \frac{(w - c_m - c_t i, mt)(q_{r,2}^* - q_{r,3}^*)}{p(1 + i, t)(q_{r,3}^* - \int_0^{\mu} F(x)dx) + v(1 + i, t)\int_0^{\mu}(q_{r,3}^* - x)dF(x) - s\int_{q_{r,3}^*}^{U}(x - q_{r,3}^*)dF(x)}$$  \hspace{1cm} (31)

The existence of the range of the sharing coefficient $\phi$ depends mainly on the exogenous parameter. It can be seen that the collaborative contract not only increases the retailer’s risk profit within a certain range but also increases the manufacturer’s expected profit. It can achieve a win-win situation between these two supply chain members.

5. Numerical Experiments

In this paper, we use MATLAB as a tool to analyze the factors affecting the supply chain. For the convenience of analysis, we assume that random demand $x \sim E(0.01)$; the parameters are as follows: $p = 20, c_r = 2, c_m = 4, i_r = 12\%$, $i_m = 10\%, s = 1,$ and $v = 1.$

5.1. The Effects of Risk-Aversion Coefficient. Here, we give numerical comparative analysis to investigate the impact of the risk-aversion coefficient on the optimal decisions of supply chain members and the channel profits. From the basic model, we can obtain $w_1^* = 10.8168, q_{r,1}^* = 41.1475.$ The effects of the risk-aversion coefficient on the retailer’s optimal order, the retailer’s risk profit, and the manufacturer’s profit are shown in Figures 1 and 2, respectively.

It can be seen from Figure 1 that the optimal order quantity of the retailer increases as the risk-aversion coefficient in the decentralization; the optimal order quantity in the decentralization is smaller than the optimal order quantity in the centralization. Figure 2 shows that the retailer’s risk profit and the manufacturer’s expected profit increase as the retailer’s risk-aversion coefficient increases; the profit of the decentralized retailer and manufacturer is significantly smaller than the channel profit. It can be seen that the degree of risk aversion affects the profits of members. To maximize the profitability of channel members, the manufacturer can take measures to reduce the risk-aversion degree of the retailer and wholesale price, such as the trade credit contract, the revenue sharing contract, and other measures.

5.2. The Effects of Credit Period. The length of the trade credit period provided by the manufacturer has different effects on the profitability of the manufacturer and the retailer. When $\eta = 0.8$ and $t = 1,$ we can obtain $w_2^* = 12.09, q_{r,2}^* = 42.05.$ The impact of the credit period on the retailer’s risk profit and the manufacturer’s expected profit in the basic model and the single trade credit model is shown in Figure 3.

It can be seen from Figure 3 that with the increase of the credit period, the retailer’s risk profit increases significantly and the manufacturer’s profit first rises and then reduces. On one hand, with the increase of the credit period, the retailer’s capital occupancy is reduced and the retailer’s order quantity is stimulated, so that the risk profit is increased. On the other hand, the manufacturer’s return on funds is reduced as the credit period increases and the optimal order quantity of the retailer is an increasing function of the trade credit period. The two sides together make the manufacturer’s profit increase firstly and then reduce. It can also be seen from Figure 3 that the credit period of the manufacturer’s profit is no less than the profit of the basic model which should be between 0 and 13; the trade credit period that makes the manufacturer’s profit at the highest is 6.

Management enlightenment: if the manufacturer feels that the credit period for obtaining the highest profit is too long, the manufacturer can abandon part of the profit and choose a credit period that he can accept. As shown in Figure 3, the manufacturer can choose a credit period between [0, 6]. According to equation (20), the trade credit period of the members of the supply chain without loss of profit should be in [0, 13]. To achieve Pareto improvement, the credit range must meet certain constraints, in particular, when the credit period $t = 0$ is the basic model.

5.3. The Effect of Revenue Sharing Coefficient. To achieve Pareto improvement, the range of revenue sharing should be constrained by equation (31). In the collaborative contract model, when the revenue sharing coefficient and the trade period are assumed to be $\phi = 0.8, t = 1,$ we can obtain $w_3^* = 8.46, q_{r,2}^* = 47.8.$ Obviously when $w_3^* < w_1^*, q_{r,2}^* < q_{r,3}^*$, the revenue sharing expands the optimal order. The effect of the sharing coefficient and credit period on the optimal order is shown in Figure 4. The effect of the sharing coefficient on risk profit and expected profit is shown in Figure 5.

In Figure 4, the retailer’s optimal order quantity increases as the credit period increases and it increases as the revenue sharing coefficient increases within a certain range. In Figure 5, the profit of the manufacturer increases first and
then decreases with the sharing coefficient. In fact, the profit of the manufacturer is determined by the shared revenue and the sales revenue. The prerevenue sharing ratio is small but the order quantity almost rises linearly; the overall profit of the two interactions is increasing after reaching a certain peak. The order quantity increases slowly and the profit decreases accordingly. Similar to the credit period, the revenue sharing coefficient in the model is constrained. According to equation (31), it should be roughly between $[0.33, 0.6]$ in Figure 4. When the revenue sharing coefficient is higher than 0.33, the retailer will use the cooperative contract with the manufacturer, and its risk profit increases with the increase of the revenue sharing coefficient. However, when the revenue sharing coefficient is less than 0.33, the retailer will no longer use the contract. This is because the higher the retailer’s revenue sharing coefficient, the more the retained part of its sales revenue, and the higher the benefit shared, the higher the value-at-risk profit.

When the credit period is $t = 1$, the optimal order quantity of the channel supply chain is $q_{c2}^* = q_{c3}^* = 142.86$. In the joint model, when each parameter satisfies Proposition 4, supply chain coordination can be achieved. Figure 6 shows the effect of risk-aversion coefficient on the revenue sharing coefficient. We can figure out that when the retailer’s risk-aversion coefficient is smaller, the retailer is more afraid of risks. At this time, the retailer is more cautious and the sharing coefficient should be reduced, but there is an increase, which is not consistent with the reality. The reason may be that the retailer will not adopt revenue sharing measures when the risk-aversion coefficient is small. Figure 7 shows the effect of credit period on the revenue sharing coefficient. We can obtain that with the credit period...
Figure 5: The effect of the revenue sharing coefficient on expected profit and risk profit.

Figure 6: The effect of risk-aversion coefficient on the revenue sharing coefficient.

Figure 7: The effect of trade credit period on the revenue sharing coefficient.
increase, which is provided by the manufacturer, the retailer will share more profits with the manufacturer in the joint coordinated state.

6. Extensions

6.1. Buyback Collaboration Model \((\eta, w, i_m, i_r, t, b)\). If the manufacturer reduces the wholesale price and provides a buyback contract based on a trade credit contract, the unsold products of the retailer are repurchased at the price \(b > v\); the manufacturer’s residual price is \(v_m (w \geq b \geq v_m \geq v)\). The paper calls this situation the buyback collaboration model. In this model, the channel revenue is as follows:

\[
E_{II} = p(1 + i, t) (q - \int_0^q F(x)dx) + v_m \int_0^q (q - x)f(x)dx \\
- s \int_0^q (x - q)f(x)dx - (c_r + c_m + c_m i_m t)q,
\]

\[
F(q_{c4}) = \frac{p(1 + i, t) + s - (c_r + c_m + c_m i_m t)}{p(1 + i, t) + s - v_m}
\]

The profit of the manufacturer and the retailer are expressed as

\[
\Pi_m = (w - c_m - c_m i_m t)q - (b - v_m)(q - x)^{+},
\]

\[
\Pi_r = p(1 + i, t) \min(q, x) + b(q - x)^{-} - s(x - q)^{+} - (c_r + w)q.
\] (34)

According to the buyback collaboration model, we have the following results.

**Proposition 5**

1. The optimal threshold is as follows:

\[
u_{c4}^* = p(1 + i, t)q - (c_r + w)q.
\] (37)

2. Substituting (36) into \(\text{CVaR}_\eta(\Pi_r(q, x))\), we get

\[
\text{CVaR}_\eta(\Pi_r(q, x)) = p(1 + i, t)q - (c_r + w)q
\]

\[
- \frac{s}{\eta} \mu - q - \frac{p(1 + i, t) + s - b}{\eta} \int_0^q F(x)dx.
\] (38)

3. Retailer’s optimal order quantity \(q_{r4}^*\) satisfies the following equation:

\[
F(q_{r4}^*) = \frac{\eta p(1 + i, t) + s}{p(1 + i, t) + s - b}.
\] (39)

The detailed proof of Proposition 5 is given in the Appendix section.
Corollary 6

1. \( \frac{\partial q_{r2}}{\partial \omega} > 0, \frac{\partial q_{r2}}{\partial \omega} < 0 \)
2. \( \frac{\partial q_{r2}}{\partial v_m} > 0 \)
3. \( q_{r1} > q_{r2} \)
4. \( \frac{\partial \text{CVaR}_\eta (I_{r1}, q, x)}{\partial b} > 0 \)

The risk-averse retailer’s order quantity and risk profit increase the buyback price. The optimal order quantity in the channel increases with the manufacturer’s residual value. Consequently, the contract on the basis of trade credit further encourages the retailer to order more products. It can be seen that buyback contract under trade credit can promote the order of the retailer.

Proposition 6. To achieve supply chain coordination, the condition \( q_{r2}^* = q_{r4}^* \) must be satisfied, that is,

\[
p\frac{(1+i,t) + s - (c_r + c_m + c_m t)}{p(1+i,t) + s - v_m} = \eta \frac{p(1+i,t) + (s/\eta) - (c_r + w)}{p(1+i,t) + s - b},
\]

(41)

where \( w \geq b \geq v_m \geq v \).

In particular, when \( \eta = 1 \), the coordination can be satisfied as long as \( b = v_m, w = c_m + c_m t \).

In the buyback collaboration model, in order to achieve Pareto improvement of supply chain members, the following conditions should be satisfied:

\[
\begin{align*}
\text{CVaR}_\eta (I_{r4}(q_{r2}, x)) & \geq \text{CVaR}_\eta (I_{r2}(q_{r2}, x)), \\
\text{EII}_{m4}(q_{r4}) & \geq \text{EII}_{m2}(q_{r2}).
\end{align*}
\]

(42)

According to equation (42), the range of buyback price, wholesale price, and credit period can be roughly determined.

Comparing the revenue sharing collaboration contract with the buyback collaboration contract, it can be seen that the two collaboration models can achieve channel coordination, which mainly depends on the collaborative contract. For example, the revenue sharing collaboration model is mainly determined by the wholesale price, trade credit period, and revenue sharing coefficient. The buyback collaboration model is mainly determined by the wholesale price, trade credit period, and buyback price. Secondly, due to the retailer’s revenue sharing, the wholesale price may be lower than the cost price \( w \leq \phi (c_r + c_m + c_m t) - c_r \) or even subsidies \( w \leq 0 \) in the revenue sharing collaboration model. However, the manufacturer has been rationally constrained in the buyback collaboration model \( v < v_m < b < w \). Finally, the contracts that supply chain members choose to take in actual economic activities mainly depending on the characteristics of the commodities themselves. Time-critical products such as newspapers, magazines, computer software, and greeting cards are generally coordinated by buyback contract. Orders such as DVD rentals and videotapes are generally coordinated by revenue sharing measures.

6.2. Optimal Decision of a Risk-Averse Manufacturer with CVaR. In this section, we extend our supply chain game model with a risk-averse manufacturer, and the manufacturer who provides trade credit to the retailer considers the optimal production quantity to maximize his profit in decentralized system. Based on equation (21), the conditional value-at-risk for the risk-aversion manufacturer can be expressed as follows:

\[
\max_{q \geq 0} \text{CVaR}_{\eta_2}(\Pi_m(q)) = \max_{q \geq 0} \max_{u_5 \in \mathbb{R}} g(u_5, q),
\]

(43)

where

\[
\begin{align*}
g(u_5, q) & = u_5 + \eta_2^{-1} E[\min[\Pi_m(q) - u_5, 0]] \\
& = u_5 - \eta_2^{-1} E[\Pi_m(q) - u_5]^+ \\
& = u_5 - \eta_2^{-1} \int_0^q \left[ u_5 - (1 - \phi) [p(1+i,t)x + \nu (1+i,t) (q-x)] + (c_m + c_m t - w) q \right]^+ dF(x) \\
& - \eta_2^{-1} \int_q^u \left[ u_5 - (1 - \phi) p(1+i,t) q + (c_m + c_m t - w) q \right]^+ dF(x),
\end{align*}
\]

(44)

where \( \eta_2 (0 < \eta_2 < 1) \) is the risk aversion of the manufacturer and the shortage cost \( s = 0 \).

According to the above equation, we have the following results.

Proposition 7. In a decentralized supply chain, the optimal production quantity of risk-averse manufacturer and the conditional value-at-risk criterion are given as follows:
\[
F(q_m^*) = \eta_2 \frac{(1 - \phi)p(1 + i,t) + w - c_m - c_m^t}{(1 - \phi)[p(1 + i,t) - v]},
\]

\[
\text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) = \left[ \frac{(1 - \phi)p(1 + i,t) + (w - c_m - c_m^t)}{(1 - \phi)[p(1 + i,t) - v]} \right] - \frac{(1 - \phi)[p(1 + i,t) - v]}{\eta_2} \cdot F^{-1} \left[ \eta_2 (1 - \phi)p(1 + i,t) + w - c_m - c_m^t \right] / (1 - \phi)[p(1 + i,t) - v]
\]

The detailed proof of Proposition 7 is given in the Appendix section.

**Corollary 7.** In a decentralized supply chain, we assume that stochastic variable \( x \) follows uniform distributions on \([0, \bar{x}]\). The manufacturer’s optimal production quantity \( q_m^* \) and the conditional value-at-risk criterion are given as follows:

\[
q_m^* = \bar{x} \eta_2 \frac{(1 - \phi)p(1 + i,t) + w - c_m - c_m^t}{(1 - \phi)[p(1 + i,t) - v]},
\]

\[
\text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) = \frac{1}{2} \frac{\bar{x} \eta_2}{(1 - \phi)[p(1 + i,t) - v]} \left[ (1 - \phi)p(1 + i,t) + (w - c_m - c_m^t) \right] \cdot (1 - \phi)[p(1 + i,t) - v] \cdot \frac{(1 - \phi)p(1 + i,t) + w - c_m - c_m^t}{(1 - \phi)[p(1 + i,t) - v]}. \tag{47}
\]

According to Corollary 7, we calculate first-order derivatives for equations (26) and (45) with respect to \( \eta_2, w, \phi, \) and we have the following results.

**Corollary 8**

1. \( (\partial q_m^*/\partial \eta_2) > 0, (\partial q_m^*/\partial w) > 0, (\partial q_m^*/\partial t) < 0 \)
2. \( (\partial \text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) / \partial \eta_2) > 0, (\partial \text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) / \partial w) > 0, (\partial \text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) / \partial t) < 0 \)
3. \( (\partial q_m^*/\partial \phi) > 0, w - c_m - c_m^t > 0 \),

where the condition \( (1 - \phi)p(1 + i,t) + w - c_m - c_m^t > 0 \) is required.

In a decentralized supply chain, it can be figured out from Corollary 8 that the risk-aversion manufacturer’s optimal production quantity and CVaR value \( \text{CVaR}_{\eta_2} (\Pi_m (q_m^*)) \) are increasing with risk-aversion coefficient \( \eta_2 \) and wholesale price \( w \) increase under the conditions of \( (1 - \phi)p(1 + i,t) + w - c_m - c_m^t > 0 \). From the manufacturer’s perspective, it is expected that the manufacturer’s wholesale price is as high as possible, and the revenue sharing coefficient is as small as possible.

**7. Conclusions**

Trade credit period can effectively reduce the capital occupancy rate and increase the retailer’s order quantity. More and more researchers have begun to pay attention on this issue. However, most of the existing researches are based on the assumption that the supply chain members are risk neutral. In addition, the distribution of revenue is also the focus of existing supply chain management research. Our paper adopts the CVaR criterion to study the supply chain issues; revenue sharing joint contracts are adopted to coordinate risk aversion of the retailers and the manufacturer providing trade credit. We propose three supply chain game models, i.e., the basic model, single trade credit model, and trade credit and revenue sharing collaboration model, respectively, and explore the influence of risk-aversion factor, trade credit period, and revenue sharing coefficient on retailer decision-making, CVaR value, and the expected profits of the manufacturer. We find that the decisions and profits of both the manufacturer and the retailer depend on the degree of the risk aversion, the trade credit period, and the revenue sharing coefficient. The retailer’s risk profit and the manufacturer’s expected profit increase as the retailer’s risk-aversion coefficient increases. With the increase of the credit period, the retailer’s risk profit increases significantly and the manufacturer’s profit first rises and then reduces. The trade credit and the revenue sharing collaborative contract effectively improves supply chain performance and achieves a ‘win-win’ situation for the supply chain members.

Our research can be further extended in different directions to gain broader perspectives. For example, further studies might discuss a more complex situation with multiple manufacturers and multiple retailers to study the interactions and differences in supply chain decision. This study considered only trade credit and revenue sharing collaboration contract as one of the coordinate contracts in the supply chain. Other contracts might also be coordinated. For instance, one might introduce quantity discounts contract and trade credit collaboration into the supply chain to find the different decision strategy. This aspect can also be integrated into an information update model to analyze their influence on channel coordination.
Appendix

Proof of Proposition 1

(1) Discuss the domain of definition into 3 intervals.

\[ g(u_1, q) = u_1 - \eta^{-1} \left[ \int_{0}^{u_1 + (c_r + w)q - pq / p - \nu} [u_1 - px - \nu(q - x) + (c_r + w)q] dF(x) \right] \]

\[ \frac{\partial g(u_1, q)}{\partial u_1} = 1 - \eta^{-1} \frac{d}{dq} \left[ \left( \int_{0}^{u_1 + (c_r + w)q - pq / p - \nu} [u_1 - px - \nu(q - x) + (c_r + w)q] dF(x) \right) \right] \]

In particular, \((\frac{\partial g(u_1, q)}{\partial u_1})|_{u_1 \rightarrow -pq - (c_r + w)q} = 1\) and \((\frac{\partial g(u_1, q)}{\partial u_1})|_{u_1 \rightarrow -pq - (c_r + w)q} = 1 - (1/\eta)F(q)\).

(i) When \(u_1 \leq pq - (c_r + w)q\), we have \(g(u_1, q) = u_1 \frac{\partial g(u_1, q)}{\partial u_1} = 1 > 0\). In particular, \((\frac{\partial g(u_1, q)}{\partial u_1})|_{u_1 \rightarrow -(c_r + w)q} = 1\).

(ii) When \(pq - (c_r + w)q \leq u_1 \leq pq - (c_r + w)q\), we have \(1 - \eta^{-1} \frac{d}{dq} \left[ \left( \int_{0}^{u_1 + (c_r + w)q - pq / p - \nu} [u_1 - px - \nu(q - x) + (c_r + w)q] dF(x) \right) \right] = 0\).

(iii) When \(u_1 \geq pq - (c_r + w)q\), we have \(g(u_1, q) = (p - \nu)F^{-1}(\eta) - (c_r + w)q + pq - (c_r + w)q\), \(u_1\) should satisfy

\[ 1 - \eta^{-1} \frac{d}{dq} \left[ \left( \int_{0}^{u_1 + (c_r + w)q - pq / p - \nu} [u_1 - px - \nu(q - x) + (c_r + w)q] dF(x) \right) \right] = 0, \]

\[ u_1^* = (p - \nu)F^{-1}(\eta) - (c_r + w)q + pq - (c_r + w)q, \]

\[ g(u_1^*, q) = (p - \nu)F^{-1}(\eta) - (c_r + w)q + pq - (c_r + w)q + (p - \nu)\eta^{-1} \int_{0}^{\eta} F(x) dx, \]

\[ \frac{\partial g(u_1^*, q)}{\partial q} = pq - (c_r + w)q, \]

which is inconsistent with reality, and it should be removed.

When \(F^{-1}(\eta) \geq q\), we have \(u_1^* = pq - (c_r + w)q\). (A.3)

Therefore, the optimal solution is \(u_1^* = pq - (c_r + w)q\).

(2) Substituting equation (A.4) into equation (A.2), we can obtain equation (8).

(3) Taking the first-order and second-order partial derivatives with respect to \(q\), we obtain

\[ \frac{\partial \text{CVaR}_q}{\partial q} = \frac{p - (c_r + w) + \frac{s}{\eta} \frac{p + s - \nu}{f(q)}}{\eta} F(q), \]

\[ \frac{\partial^2 \text{CVaR}_q}{\partial q^2} = \frac{p + s - \nu}{\eta} f(q) < 0. \]

(A.5)
So, there is a unique order quantity $q^*_r$. Let \( \partial \text{CVaR}_q(P_{q,x})/\partial q = 0 \), and we have equation (9), and then Proposition 1 is proved.

The proofs of Propositions 2, 3, 5, and 7 are similar to Proposition 1 and are omitted here. \( \square \)

**Data Availability**

The figures, tables, and other data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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**References**


