Research Article

Approximations of $(\epsilon, \epsilon \vee q_k)$ -Fuzzy Hyperideals in Ordered LA-Semihypergroups

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This paper concerns the relationship between rough sets, $(\epsilon, \epsilon \vee q_k)$ -fuzzy sets, and the LA-semihypergroups. We proved that the lower approximation (l -approx) and the upper approximation (u -approx) of different hyperideals of LA-semihypergroups become again hyperideal and also provided some examples. We also provided a result which shows that, if $\mathfrak{H}$ is not a hyperideal of an LA-semihypergroup P , then its lower approximation is also not a hyperideal.

1. Introduction

Hyperstructures represent a natural extension of classical algebraic structures. Algebraic hyperstructure theory depends on hyperoperation and their properties. One of these theories is the semihypergroup theory. Semihypergroup theory was first introduced by French Mathematician Marty in 1934 [1]. The concept of semihypergroup is the generalization of the concept of semigroup. In semigroup, the composition of two elements is an element, while in a semihypergroup the composition of two elements is a nonempty set. In semihypergroup, closure and associative law hold. Many authors studied different aspects of semihypergroup (see [2–4]). Recently, Hila and Dine [5] introduced the notion of LA-sm-hyp-group (LA-semihypergroup), and then, Yaqoob, Corsini, and Yousafzai [6] extended the work of Hila and Dine and characterized intraregular left almost semihypergroups by their HP-ideals

(hyperideals) using pure left identity. The concept of ordered semihypergroup was studied by Heidari and Davvaz [7] where they used a binary relation $\leq$ on semihypergroup $(Y, \otimes)$ such that the binary relation is a partial order and the structure $(Y, \otimes, \leq)$ is known as ordered semihypergroup [8].

The concept of fuzzy sets was first introduced by Zadeh [9] in 1964. Fuzzy set is a set having degrees of membership between 1 and 0. Fuzzy set theory is a classical application of set theory. Applications of fuzzy sets in some algebraic structures and other structures can be seen in [10–13]. Zdzisław Pawlak [14] introduced the notion of rough sets, in the year 1982. Rough set theory constitutes a sound basis for knowledge discovery databases. It offers mathematical tools to discover pattern hidden in data. It can be used for feature selection, feature extraction, decision rule generation, and paper extraction. Applications of rough sets in some algebraic structures and algebraic hyperstructures can be seen in [15–36].

2. Preliminaries and Notations

Let Y be a nonempty set. Then, the map $\otimes: Y \times Y \longrightarrow \rho^*(Y)$ is called HP-operation (hyperoperation) or join operation on the set Y , where $\rho^*(Y) = (\rho(Y)/\{\emptyset\})$ denotes the set of all nonempty subsets of Y . A hypergroupoid is a set Y together with a (binary) HP-operation. For any nonempty subsets of Ω_1, Ω_2 of Y , we denote

$$\Omega_1 \otimes \Omega_2 = \bigcup_{a \in \Omega_1, b \in \Omega_2} a \otimes b. \quad (1)$$

Instead of $\{a\} \otimes \Omega_1$ and $\{\Omega_2\} \otimes a$, we write $a \otimes \Omega_1$ and $\Omega_2 \otimes a$, respectively.

Definition 1 (see [5, 6]). A hypergroupoid $(Y, \otimes)$ is called an LA-sm-hyp-group if for every $p, q, r \in Y$, we have $(p \otimes q) \otimes r = (r \otimes q) \otimes p$.

The law $(p \otimes q) \otimes r = (r \otimes q) \otimes p$ is called a left invertive law.

Definition 2 (see [8]). Let Y be a nonempty set and $\leq$ be a partially ordered relation on Y . The triplet $(Y, \otimes, \leq)$ is called a partially ordered LA-sm-hyp-group if the following conditions are satisfied:

- (i) $(Y, \otimes)$ is an LA-sm-hyp-group
- (ii) $(Y, \leq)$ is a partially ordered set
- (iii) For every $a, b, c \in Y$, $a \leq b$ implies $a \otimes c \leq b \otimes c$ and $c \otimes a \leq c \otimes b$, where $a \otimes c \leq b \otimes c$ means that for $p \in a \otimes c$, there exist $q \in b \otimes c$ such that $p \leq q$

Definition 3 (see [8]). If $(Y, \otimes, \leq)$ is a partially ordered LA-sm-hyp-group and $\Omega \subseteq Y$, then (Ω) is the subset of Y defined as follows:

$$(\Omega) = \{t \in Y: t \leq a, \text{ for some } a \in \Omega\}. \quad (2)$$

If $p \in Y$ and Ω is a nonempty subset of Y , then $\Omega_p = \{(q, r) \in Y \times Y | p \leq q \otimes r\}$.

Definition 4 (see [8]). A nonempty subset Ω of a partially ordered LA-sm-hyp-group is called a LAS-sm-hyp-group (LA-subsemihypergroup) of Y if $(\Omega \otimes \Omega) \subseteq (\Omega)$.

Definition 5 (see [8]). A nonempty subset Ω of a partially ordered LA-sm-hyp-group $(Y, \otimes, \leq)$ is called a right (resp., left) HP-ideal (hyperideal) of Y if

- (i) $\Omega \otimes Y \subseteq \Omega$, (resp., $Y \otimes \Omega \subseteq \Omega$)
- (ii) For every $a \in Y$ and $b \in \Omega$, $a \leq b$ implies $a \in \Omega$

If Ω is both right HP-ideal and left HP-ideal of LA-sm-hyp-group Y , then Ω is called a HP-ideal (or two-sided HP-ideal) of LA-sm-hyp-group Y .

Definition 6 (see [8]). A nonempty subset Ω of a partially ordered LA-sm-hyp-group $(Y, \otimes, \leq)$ is called an interior HP-ideal of Y , if

- (i) $(Y \otimes \Omega) \otimes Y \subseteq \Omega$
- (ii) For every $a \in Y$ and $b \in \Omega$, $a \leq b$ implies $a \in \Omega$

Definition 7 (see [8]). A nonempty subset Ω of a partially ordered LA-sm-hyp-group $(Y, \otimes, \leq)$ is called a generalized bi-HP-ideal of Y , if

- (i) $(\Omega \otimes Y) \otimes \Omega \subseteq \Omega$
- (ii) For every $a \in Y$ and $b \in \Omega$, $a \leq b$ implies $a \in \Omega$

Definitions on $(\epsilon, \epsilon \vee q_k)$ -fuzzy HP-ideals in LA-sm-hyp-group can be found in [10].

3. Rough $(\epsilon, \epsilon \vee q_k)$ -Fuzzy LAS-sm-hyp-Groups

Definition 8. Let Y be a nonempty set and δ be a binary relation on Y . By $\wp(Y)$, we mean the power set of Y . For all $\Omega \subseteq Y$, we define $\overline{\delta}^{\text{upper}}$ and $\underline{\delta}^{\text{lower}}: \wp(Y) \longrightarrow \wp(Y)$ by

$$\begin{aligned} \overline{\delta}^{\text{upper}}(\Omega) &= \{p \in Y: \exists q \in \Omega, \text{ such that } p \delta q\} \\ &= \{p \in Y: \delta C(p) \cap \Omega \neq \emptyset\}, \\ \underline{\delta}^{\text{lower}}(\Omega) &= \{p \in Y: \forall q, p \delta q \longrightarrow q \in \Omega\} \\ &= \{p \in Y: \delta C(p) \subseteq \Omega\}, \end{aligned} \quad (3)$$

where $\delta C(p) = \{q \in Y: p \delta q\}$. $\overline{\delta}^{\text{upper}}(\Omega)$ and $\underline{\delta}^{\text{lower}}(\Omega)$ are called the u -approx and the l -approx operations, respectively.

Definition 9. Let δ be a PH-order relation (pseudohyper-order relation [36]) on a partially ordered LA-sm-hyp-group Y . Let ϑ be a fuzzy subset of LA-sm-hyp-group Y . For every p in Y , we define δ - u -approx and the δ - l -approx of ϑ by

$$\begin{aligned} \overline{\delta}^{\text{upper}}(\vartheta)(p) &= \bigvee_{p' \in \delta C(p)} \vartheta(p'), \\ \underline{\delta}^{\text{lower}}(\vartheta)(p) &= \bigwedge_{p' \in \delta C(p)} \vartheta(p'). \end{aligned} \quad (4)$$

Theorem 1. Let δ be a PH-order relation on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y . Then, $\overline{\delta}^{\text{upper}}(\vartheta)$ is an LAS-sm-hyp-group of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y and δ be a PH-order relation on Y . We have to show that $\overline{\delta}^{\text{upper}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y .

- (i) For each p, q in Y and $p \leq q$, we have $\delta C(q) \subseteq \delta C(p)$.
Now

$$\begin{aligned} \overline{\delta}^{\text{upper}}(\vartheta)(p) &= \bigvee_{p' \in \delta C(p)} \vartheta(p') \\ &\geq \bigvee_{q' \in \delta C(q)} \min \left\{ \vartheta(q'), \frac{1-k}{2} \right\} \\ &= \min \left\{ \bigvee_{q' \in \delta C(q)} \vartheta(q'), \frac{1-k}{2} \right\} \\ &= \min \left\{ \overline{\delta}^{\text{upper}}(\vartheta)(q), \frac{1-k}{2} \right\}. \end{aligned} \quad (5)$$

Thus, $\overline{\delta^{\text{upper}}}(\vartheta)(p) \geq \min\{\overline{\delta^{\text{upper}}}(\vartheta)(q), (1 - k/2)\}$. we get

(ii) Now, let p, q, r in Y . Let

$$\begin{aligned}
 \bigwedge_{r \in p \oplus q} \overline{\delta^{\text{upper}}}(\vartheta)(r) &= \bigwedge_{r \in p \oplus q} \bigvee_{t \in \delta C(r)} \vartheta(t) \\
 &\geq \bigvee_{t \in \delta C(p \oplus q)} \vartheta(t) \geq \bigvee_{t \in \delta C(p) \oplus \delta C(q)} \vartheta(t) \\
 &\geq \bigwedge_{t \in p_I \oplus q_I} \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)} \vartheta(t) \\
 &= \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)} \bigwedge_{t \in p_I \oplus q_I} \vartheta(t) \\
 &\geq \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)} \min\left\{\vartheta(p_I), \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &\geq \min\left\{\bigvee_{p_I \in \delta C(p)} \vartheta(p_I), \bigvee_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), \frac{1-k}{2}\right\}.
 \end{aligned} \tag{6}$$

Thus, we get $\bigwedge_{r \in p \oplus q} \overline{\delta^{\text{upper}}}(\vartheta)(r) \geq \min\{\overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), (1 - k/2)\}$. Hence $\overline{\delta^{\text{upper}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y . $\square$

Theorem 2. Let δ be a complete and symmetric PH-order relation on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y . Then, $\overline{\delta^{\text{lower}}}(\vartheta)$ is an LAS-sm-hyp-group of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y and δ be a complete and symmetric PH-order relation on Y . We have to show that $\overline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y .

(i) For each p, q in Y and $p \leq q$, we have $\delta C(p) = \delta C(q)$. Now

$$\begin{aligned}
 \overline{\delta^{\text{lower}}}(\vartheta)(p) &= \bigwedge_{p_I \in \delta C(p)} \vartheta(p_I) \\
 &= \bigwedge_{q_I \in \delta C(q)} \min\left\{\vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\bigwedge_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\overline{\delta^{\text{lower}}}(\vartheta)(q), \frac{1-k}{2}\right\}.
 \end{aligned} \tag{7}$$

Thus, $\overline{\delta^{\text{lower}}}(\vartheta)(p) = \min\{\overline{\delta^{\text{lower}}}(\vartheta)(q), (1 - k/2)\}$. we get

(ii) Now let p, q, r in Y . Let

$$\begin{aligned}
 \bigwedge_{r \in p \oplus q} \overline{\delta^{\text{lower}}}(\vartheta)(r) &= \bigwedge_{r \in p \oplus q} \bigwedge_{t \in \delta C(r)} \vartheta(t) \\
 &= \bigwedge_{t \in \delta C(p \oplus q)} \vartheta(t) = \bigwedge_{t \in \delta C(p) \oplus \delta C(q)} \vartheta(t) \\
 &\geq \bigwedge_{t \in p_I \oplus q_I}, \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q)} \vartheta(t) \\
 &= \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q)} \bigwedge_{t \in p_I \oplus q_I} \vartheta(t) \\
 &\geq \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q)} \min\left\{\vartheta(p_I), \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &\geq \min\left\{\bigwedge_{p_I \in \delta C(p)} \vartheta(p_I), \bigwedge_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\overline{\delta^{\text{lower}}}(\vartheta)(p), \overline{\delta^{\text{lower}}}(\vartheta)(q), \frac{1-k}{2}\right\},
 \end{aligned} \tag{8}$$

Thus, we get $\bigwedge_{r \in p \oplus q} \overline{\delta^{\text{lower}}}(\vartheta)(r) \geq \min\{\overline{\delta^{\text{lower}}}(\vartheta)(p), \overline{\delta^{\text{lower}}}(\vartheta)(q), (1 - k/2)\}$. Hence $\overline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y . $\square$

4. Rough $(\epsilon, \epsilon \vee q_k)$ -Fuzzy HP-Ideals

Theorem 3. Let δ be a PH-order on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y . Then, $\overline{\delta^{\text{upper}}}(\vartheta)$ is a left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y and δ be a PH-order relation on Y . We have to show that $\overline{\delta^{\text{upper}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y .

(i) For each p, q in Y and $p \leq q$, we have $\delta C(q) \subseteq \delta C(p)$. Now

$$\begin{aligned}
 \overline{\delta^{\text{upper}}}(\vartheta)(p) &= \bigvee_{p_I \in \delta C(p)} \vartheta(p_I) \\
 &\geq \bigvee_{q_I \in \delta C(q)} \min\left\{\vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\bigvee_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\overline{\delta^{\text{upper}}}(\vartheta)(q), \frac{1-k}{2}\right\}.
 \end{aligned} \tag{9}$$

Thus, we get $\overline{\delta^{\text{upper}}}(\vartheta)(p) \geq \min\{\overline{\delta^{\text{upper}}}(\vartheta)(q), (1 - k/2)\}$.

(ii) Now, let p, q, r in Y . Let

$$\begin{aligned}
\bigwedge_{r \in p \otimes q} \overline{\delta^{\text{upper}}}(\vartheta)(r) &= \bigwedge_{r \in p \otimes q}, \bigvee_{t \in \delta C(r)} \vartheta(t) \\
&\geq \bigvee_{t \in \delta C(p \otimes q)} \vartheta(t) \geq \bigvee_{t \in \delta C(p) \otimes \delta C(q)} \vartheta(t) \\
&\geq \bigwedge_{t \in p_I \otimes q_I}, \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)} \vartheta(t) \\
&= \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)}, \bigwedge_{t \in p_I \otimes q_I} \vartheta(t) \\
&\geq \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q)} \min \left\{ \vartheta(p_I), \vartheta(q_I), \frac{1-k}{2} \right\} \\
&\geq \min \left\{ \bigvee_{p_I \in \delta C(p)} \vartheta(p_I), \bigvee_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2} \right\} \\
&= \min \left\{ \overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), \frac{1-k}{2} \right\}.
\end{aligned} \tag{10}$$

Thus, we get $\bigwedge_{r \in p \otimes q} \overline{\delta^{\text{upper}}}(\vartheta)(r) \geq \min \{ \overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), (1-k/2) \}$.
 (iii) Now, let p, q, r, w in Y . Let

$$\begin{aligned}
\bigwedge_{w \in (p \otimes q) \otimes r} \overline{\delta^{\text{upper}}}(\vartheta)(w) &= \bigwedge_{w \in (p \otimes q) \otimes r}, \bigvee_{t \in \delta C(w)} \vartheta(t) \\
&\geq \bigvee_{t \in \delta C((p \otimes q) \otimes r)} \vartheta(t) \geq \bigvee_{t \in (\delta C(p) \otimes \delta C(q)) \otimes \delta C(r)} \vartheta(t) \\
&\geq \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I}, \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \vartheta(t) \\
&= \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)}, \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I} \vartheta(t) \\
&\geq \bigvee_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \min \left\{ \vartheta(p_I), \vartheta(q_I), \vartheta(r_I), \frac{1-k}{2} \right\} \\
&\geq \min \left\{ \bigvee_{p_I \in \delta C(p)} \vartheta(p_I), \bigvee_{q_I \in \delta C(q)} \vartheta(q_I), \bigvee_{r_I \in \delta C(r)} \vartheta(r_I), \frac{1-k}{2} \right\} \\
&= \min \left\{ \overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), \overline{\delta^{\text{upper}}}(\vartheta)(r), \frac{1-k}{2} \right\}.
\end{aligned} \tag{11}$$

Thus, we get $\bigwedge_{w \in (p \otimes q) \otimes r} \overline{\delta^{\text{upper}}}(\vartheta)(w) \geq \min \{ \overline{\delta^{\text{upper}}}(\vartheta)(p), \overline{\delta^{\text{upper}}}(\vartheta)(q), \overline{\delta^{\text{upper}}}(\vartheta)(r), (1-k/2) \}$. Hence, $\overline{\delta^{\text{upper}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y . The other cases can be proved in a similar way.

The following example shows that the converse of Theorem 3 is not true in general. $\square$

Example 1. Consider a set $Y = \{a, b, c, d\}$ with the following hyperoperation “ $\otimes$ ” defined in Table 1.

The order “ $\leq$ ” is $\leq: = \{(a, a), (b, b), (c, c), (d, d), (d, a)\}$. Then, $(Y, \otimes, \leq)$ becomes a partially ordered LA-sm-hyp-group. Let us define a fuzzy subset ϑ of LA-sm-hyp-group Y by $\vartheta = \{(0.5/a), (0.7/b), (0.9/c), (0.2/d)\}$ and take $t = 0.4$ and $n = 0.3$. Then, it is clear that ϑ is not an $(\epsilon, \epsilon \vee q_k)$ -fuzzy HP-ideal of LA-sm-hyp-group Y because for $d \leq a$, a_t in ϑ but $d_t \notin \vartheta$ and $\vartheta(d) + t + n = 0.9 < 1$. Now define a PH-order relation δ on Y by

$$\delta = \{(a, a), (a, d), (b, b), (b, c), (c, b), (c, c), (d, a), (d, d)\}. \tag{12}$$

TABLE 1: Tabular form of the hyperoperation “ $\otimes$ ” defined in Example 1.

$\otimes$	a	b	c	d
a	a	$\{a, d\}$	$\{a, d\}$	d
b	$\{a, d\}$	$\{b, c\}$	$\{b, c\}$	d
c	$\{a, d\}$	b	b	d
d	d	d	d	d

By the routine calculations, we get

$$\begin{aligned}
 \overline{\delta^{\text{upper}}}(\vartheta)(a) &= \bigvee_{t \in \delta C(a)} \vartheta(t) = 0.5, \\
 \overline{\delta^{\text{upper}}}(\vartheta)(b) &= \bigvee_{t \in \delta C(b)} \vartheta(t) = 0.9, \\
 \overline{\delta^{\text{upper}}}(\vartheta)(c) &= \bigvee_{t \in \delta C(c)} \vartheta(t) = 0.9, \\
 \overline{\delta^{\text{upper}}}(\vartheta)(d) &= \bigvee_{t \in \delta C(d)} \vartheta(t) = 0.5.
 \end{aligned} \tag{13}$$

Thus, we get $\overline{\delta^{\text{upper}}}(\vartheta) = \{(0.5/a), (0.9/b), (0.9/c), (0.5/d)\}$. One can see that $\overline{\delta^{\text{upper}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy HP-ideal of LA-sm-hyp-group Y . Similarly, we can find the example for other cases.

Theorem 4. Let δ be a complete and symmetric PH-order relation on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y . Then, $\underline{\delta^{\text{lower}}}(\vartheta)$ is a left HP-

ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y and δ be a complete and symmetric PH-order relation on Y . We have to show that $\underline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y .

(i) For each p, q in Y and $p \leq q$, we have $\delta C(p) = \delta C(q)$. Now

$$\begin{aligned}
 \underline{\delta^{\text{lower}}}(\vartheta)(p) &= \bigwedge_{p' \in \delta C(p)} \vartheta(p') \\
 &= \bigwedge_{q' \in \delta C(q)} \min \left\{ \vartheta(q'), \frac{1-k}{2} \right\} \\
 &= \min \left\{ \bigwedge_{q' \in \delta C(q)} \vartheta(q'), \frac{1-k}{2} \right\} \\
 &= \min \left\{ \underline{\delta^{\text{lower}}}(\vartheta)(q), \frac{1-k}{2} \right\}.
 \end{aligned} \tag{14}$$

Thus, we get $\underline{\delta^{\text{lower}}}(\vartheta)(p) = \min \left\{ \underline{\delta^{\text{lower}}}(\vartheta)(q), (1-k)/2 \right\}$.

(ii) Now, let p, q, r in Y . Let

$$\begin{aligned}
 \bigwedge_{r \in p \otimes q} \underline{\delta^{\text{lower}}}(\vartheta)(r) &= \bigwedge_{r \in p \otimes q} \bigwedge_{t \in \delta C(r)} \vartheta(t) \\
 &= \bigwedge_{t \in \delta C(p \otimes q)} \vartheta(t) = \bigwedge_{t \in \delta C(p) \otimes \delta C(q)} \vartheta(t) \\
 &\geq \bigwedge_{t \in p' \otimes q'} \bigwedge_{p' \in \delta C(p), q' \in \delta C(q)} \vartheta(t) \\
 &= \bigwedge_{p' \in \delta C(p), q' \in \delta C(q)} \bigwedge_{t \in p' \otimes q'} \vartheta(t) \\
 &\geq \bigwedge_{p' \in \delta C(p), q' \in \delta C(q)} \min \left\{ \vartheta(p'), \vartheta(q'), \frac{1-k}{2} \right\} \\
 &\geq \min \left\{ \bigwedge_{p' \in \delta C(p)} \vartheta(p'), \bigwedge_{q' \in \delta C(q)} \vartheta(q'), \frac{1-k}{2} \right\} \\
 &= \min \left\{ \underline{\delta^{\text{lower}}}(\vartheta)(p), \underline{\delta^{\text{lower}}}(\vartheta)(q), \frac{1-k}{2} \right\}.
 \end{aligned} \tag{15}$$

TABLE 2: Tabular form of the hyperoperation “ $\otimes$ ” defined in Example 2.

$\otimes$	a	b	c
a	$\{a, c\}$	c	$\{b, c\}$
b	$\{b, c\}$	c	c
c	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$

Thus, $(\vartheta)(r) \geq \min\left\{\delta_{\text{lower}}^{\text{we}}(\vartheta)(p), \delta_{\text{lower}}^{\text{get}}(\vartheta)(q), \bigwedge_{r \in p \otimes q} \delta_{\text{lower}}^{\text{LA}}(\vartheta)(r)\right\}.$

$$\begin{aligned}
\bigwedge_{w \in (p \otimes q) \otimes r} \delta_{\text{lower}}^{\text{LA}}(\vartheta)(w) &= \bigwedge_{w \in (p \otimes q) \otimes r} \bigwedge_{t \in \delta C(w)} \vartheta(t) \\
&= \bigwedge_{t \in \delta C((p \otimes q) \otimes r)} \vartheta(t) = \bigwedge_{t \in (\delta C(p) \otimes \delta C(q)) \otimes \delta C(r)} \vartheta(t) \\
&\geq \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I} \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \vartheta(t) \\
&= \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I} \vartheta(t) \\
&\geq \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \min\left\{\vartheta(p_I), \vartheta(r_I), \frac{1-k}{2}\right\} \\
&\geq \min\left\{\bigwedge_{p_I \in \delta C(p)} \vartheta(p_I), \bigwedge_{r_I \in \delta C(r)} \vartheta(r_I), \frac{1-k}{2}\right\} \\
&= \min\left\{\delta_{\text{lower}}^{\text{LA}}(\vartheta)(p), \delta_{\text{lower}}^{\text{LA}}(\vartheta)(r), \frac{1-k}{2}\right\}.
\end{aligned} \tag{16}$$

Thus, we get $\bigwedge_{w \in (p \otimes q) \otimes r} \delta_{\text{lower}}^{\text{LA}}(\vartheta)(w) \geq \min\left\{\delta_{\text{lower}}^{\text{LA}}(\vartheta)(p), \delta_{\text{lower}}^{\text{LA}}(\vartheta)(r), \frac{1-k}{2}\right\}$. Hence $\delta_{\text{lower}}^{\text{LA}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy bi-HP-ideal of LA-sm-hyp-group Y . The other cases can be proved in a similar way.

The following theorem shows that, if ϑ is not a left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y , then the l -approx is also not a left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y . $\square$

Theorem 5. Let δ be a complete and symmetric PH-order relation on a partially ordered LA-sm-hyp-group Y and ϑ is not an $(\epsilon, \epsilon \vee q_k)$ -fuzzy left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y . Then, $\delta_{\text{lower}}^{\text{LA}}(\vartheta)$ is also not a left HP-ideal (resp., right HP-ideal, HP-ideal, generalized bi-HP-ideal, bi-HP-ideal, and interior HP-ideal) of LA-sm-hyp-group Y .

(iii) Now, let p, q, r, w in Y . Let

Proof. Let δ be a complete and symmetric PH-order relation on Y . Suppose that ϑ is not an $(\epsilon, \epsilon \vee q_k)$ -fuzzy interior HP-ideal of LA-sm-hyp-group Y , that is,

- (i) If $p \leq q$ implies that $\vartheta(p) \not\geq \min\{\vartheta(q), (1-k/2)\}$
- (ii) $\bigwedge_{w \in (p \otimes q) \otimes r} \vartheta(w) \not\geq \min\{\vartheta(q), (1-k/2)\}$.
- (iii) Let $p \leq q$, then $\delta C(p) = \delta C(q)$. Now,

$$\begin{aligned}
\delta_{\text{lower}}^{\text{LA}}(\vartheta)(p) &= \bigwedge_{p_I \in \delta C(p)} \vartheta(p_I) \\
&\not\geq \bigwedge_{q_I \in \delta C(q)} \min\left\{\vartheta(q_I), \frac{1-k}{2}\right\} \\
&= \min\left\{\bigwedge_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
&= \min\left\{\delta_{\text{lower}}^{\text{LA}}(\vartheta)(q), \frac{1-k}{2}\right\}.
\end{aligned} \tag{17}$$

Thus, we get $\delta^{\text{lower}}(\vartheta)(p) \not\geq \min\{\delta^{\text{lower}}(\vartheta)(q), (1 - k/2)\}$.

(iv) Now, let p, q, r, w in Y . Let

$$\begin{aligned}
 \bigwedge_{w \in (p \otimes q) \otimes r} \delta^{\text{lower}}(\vartheta)(w) &= \bigwedge_{w \in (p \otimes q) \otimes r} \bigwedge_{t \in \delta C(w)} \vartheta(t) \\
 &= \bigwedge_{t \in \delta C((p \otimes q) \otimes r)} \vartheta(t) = \bigwedge_{t \in (\delta C(p) \otimes \delta C(q)) \otimes \delta C(r)} \vartheta(t) \\
 &\geq \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I} \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \vartheta(t) \\
 &= \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \bigwedge_{t \in (p_I \otimes q_I) \otimes r_I} \vartheta(t) \\
 &\not\geq \bigwedge_{p_I \in \delta C(p), q_I \in \delta C(q), r_I \in \delta C(r)} \min\left\{\vartheta(q_I), \frac{1-k}{2}\right\} \\
 &\geq \min\left\{\bigwedge_{q_I \in \delta C(q)} \vartheta(q_I), \frac{1-k}{2}\right\} \\
 &= \min\left\{\delta^{\text{lower}}(\vartheta)(q), \frac{1-k}{2}\right\}.
 \end{aligned} \tag{18}$$

Thus, we get $\bigwedge_{w \in (p \otimes q) \otimes r} \delta^{\text{lower}}(\vartheta)(w) \not\geq \min\{\delta^{\text{lower}}(\vartheta)(q), (1 - k/2)\}$. Hence $\delta^{\text{lower}}(\vartheta)$ is not an interior HP-ideal of LA-sm-hyp-group Y . The other cases can be proved in a similar way. $\square$

5. Rough $(\epsilon, \epsilon \vee q_k)$ -Fuzzy Quasi-HP-Ideals

In this section, we first provide some results on $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideals and rough $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideals. For a partially ordered LA-sm-hyp-group Y , the fuzzy subset ϑ of LA-sm-hyp-group Y is defined as follows:

$$\vartheta: Y \longrightarrow [0, 1] | p \longrightarrow \vartheta(p) := 1. \tag{19}$$

Definition 10. Let ϑ, η be the fuzzy subsets of a set Y . The product of the fuzzy subset ϑ and η is defined as $(\vartheta \boxtimes \eta)(p) = \bigvee_{p \in q \otimes r} \min\{\vartheta(q), \eta(r)\}$.

Definition 11. Let Ω be a nonempty subset of an ordered LA-sm-hyp-group Y . Then, the characteristic function ϑ_Ω of Ω is a fuzzy subset of LA-sm-hyp-group Y , defined as follows:

$$\vartheta_\Omega(p) = \begin{cases} 1, & \text{if } p \in \Omega, \\ 0, & \text{if } p \notin \Omega. \end{cases} \tag{20}$$

Definition 12. Let ϑ be a fuzzy subset of LA-sm-hyp-group Y . Then, ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y if and only if

(i) $p \leq q \Rightarrow \vartheta(p) \geq \min\{\vartheta(q), (1 - k/2)\}$, for all p, q in Y

(ii) $\vartheta(p) \geq \min\{(\vartheta \boxtimes \vartheta)(p), (\vartheta \boxtimes Y)(p), (1 - k/2)\}$, for all p in Y

Example 2. Consider a set $Y = \{a, b, c\}$ with the following hyperoperation " $\otimes$ " defined in Table 2.

The order " $\leq$ " is $\leq: \{(a, a), (b, a), (b, b), (b, c), (c, c)\}$. Then, $(Y, \otimes, \leq)$ becomes a partially ordered LA-sm-hyp-group. Now define a fuzzy subset of LA-sm-hyp-group Y as $\vartheta(a) = 0.5$ and $\vartheta(b) = \vartheta(c) = 0.7$. If we take $t = 0.4$ and $k \in [0.3, 1]$, then ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

Theorem 6. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of a partially ordered LA-sm-hyp-group Y . Then the set $\vartheta_t = \{p \in Y | \vartheta(p) \geq t\}$ is a quasi-HP-ideal of LA-sm-hyp-group Y , where $t \in (0, (1 - k/2)]$.

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

(i) Let p in Y , $p \leq q$ and q in ϑ_t . Then, $\vartheta(q) \geq t$. As ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y and $p \leq q$, so

$$\vartheta(p) \geq \min\left\{\vartheta(q), \frac{1-k}{2}\right\} \geq \min\left\{t, \frac{1-k}{2}\right\} = t. \tag{21}$$

This implies that p in ϑ_t .

(ii) Now, we have to show that $Y \boxtimes \vartheta_t \cap \vartheta_t \boxtimes Y \subseteq \vartheta_t$. Let a in $Y \boxtimes \vartheta_t \cap \vartheta_t \boxtimes Y$. Thus, a in $Y \boxtimes \vartheta_t$ and a in $\vartheta_t \boxtimes Y$. So a in $s \otimes p$ and a in $q \otimes t$ for some s, t in Y and p, q in ϑ_t . Thus, $\vartheta(p) \geq t$ and $\vartheta(q) \geq t$. Since

$$\begin{aligned}
(Y \boxtimes \vartheta)(a) &= \bigvee_{a \in s \boxtimes p} \{Y(s) \wedge \vartheta(p)\} \\
&\geq \{Y(s) \wedge \vartheta(p)\} \\
&= \{Y \wedge \vartheta(p)\} \\
&= \vartheta(p).
\end{aligned} \tag{22}$$

Similarly, $(\vartheta \boxtimes Y)(a) \geq \vartheta(q)$. Thus,

$$\begin{aligned}
\vartheta(a) &\geq \min \left\{ (Y \boxtimes \vartheta)(a), (\vartheta \boxtimes Y)(a), \frac{1-k}{2} \right\} \\
&\geq \min \left\{ \vartheta(p), \vartheta(q), \frac{1-k}{2} \right\} \\
&\geq \min \left\{ t, \frac{1-k}{2} \right\} \\
&= t.
\end{aligned} \tag{23}$$

Thus, a in ϑ_t . Hence, ϑ_t is a quasi-HP-ideal of LA-sm-hyp-group Y . $\square$

Corollary 1. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of a partially ordered LA-sm-hyp-group Y . Then the set $\vartheta_0 = \{p \in Y \mid \vartheta(p) > 0\}$ is a quasi-HP-ideal of LA-sm-hyp-group Y .

Corollary 2. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of a partially ordered LA-sm-hyp-group Y . Then, the set $\vartheta_t^S = \{p \in Y \mid \vartheta(p) > t\}$ is a quasi-HP-ideal of LA-sm-hyp-group Y .

Lemma 1. A nonempty subset Q of a partially ordered LA-sm-hyp-group Y is a quasi-HP-ideal of LA-sm-hyp-group Y if and only if the characteristic function ϑ_Q of Q is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

Theorem 7. Every $(\epsilon, \epsilon \vee q_k)$ -fuzzy left (right) HP-ideal ϑ of LA-sm-hyp-group Y is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy left HP-ideal of LA-sm-hyp-group Y and p in Y . Then

$$(Y \boxtimes \vartheta)(p) = \bigvee_{p \in q \boxtimes r} \{Y(q) \wedge \vartheta(r)\} = \bigvee_{p \in q \boxtimes r} \vartheta(r). \tag{24}$$

This implies that

$$\begin{aligned}
(Y \boxtimes \vartheta)(p) \wedge \frac{1-k}{2} &= \left(\bigvee_{p \in q \boxtimes r} \vartheta(r) \right) \wedge \frac{1-k}{2} \\
&= \bigvee_{p \in q \boxtimes r} \left\{ \vartheta(r) \wedge \frac{1-k}{2} \right\} \\
&\leq \bigvee_{p \in q \boxtimes r} \bigwedge_{p \in q \boxtimes r} \vartheta(p) = \vartheta(p).
\end{aligned} \tag{25}$$

Hence, $\vartheta(p) \geq (Y \boxtimes \vartheta)(p) \wedge (1 - k/2) \geq \min\{(Y \boxtimes \vartheta)(p), (\vartheta \boxtimes Y)(p), (1 - k/2)\}$. Thus, ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . The other case can be proved in a similar way. $\square$

Theorem 8. Every $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of a partially ordered LA-sm-hyp-group Y is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y .

Proof. Suppose ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . Let p, q in Y . Now for every p in $p \boxtimes q$, we have

$$\begin{aligned}
\vartheta(p) &\geq \min \left\{ (Y \boxtimes \vartheta)(p), (\vartheta \boxtimes Y)(p), \frac{1-k}{2} \right\} \\
&= \left[\bigvee_{p \in p \boxtimes q} \{Y(p) \wedge \vartheta(q)\} \right] \wedge \left[\bigvee_{a \in p \boxtimes q} \{\vartheta(p) \wedge Y(q)\} \right] \wedge \frac{1-k}{2} \\
&\geq \{Y(p) \wedge \vartheta(q)\} \wedge \{\vartheta(p) \wedge Y(q)\} \wedge \frac{1-k}{2} \\
&\geq \{1 \wedge \vartheta(q)\} \wedge \{\vartheta(p) \wedge 1\} \wedge \frac{1-k}{2} \\
&= \vartheta(q) \wedge \vartheta(p) \wedge \frac{1-k}{2}.
\end{aligned} \tag{26}$$

So $\bigwedge_{p \in p \boxtimes q} \vartheta(p) \geq \min\{\vartheta(p), \vartheta(q), (1 - k/2)\}$, for all p, q in Y . Thus, ϑ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy LAS-sm-hyp-group of LA-sm-hyp-group Y . $\square$

Theorem 9. Let δ be a PH-order on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . Then, $\delta^{\text{upper}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y and δ be a PH-order relation on Y . We have to show that $\delta^{\text{upper}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

(i) For each p, q in Y and $p \leq q$, we have $\delta C(q) \subseteq \delta C(p)$. Now

$$\begin{aligned}
\delta^{\text{upper}}(\vartheta)(p) &= \bigvee_{p' \in \delta C(p)} \vartheta(p') \\
&\geq \bigvee_{q' \in \delta C(q)} \min \left\{ \vartheta(q'), \frac{1-k}{2} \right\} \\
&= \min \left\{ \bigvee_{q' \in \delta C(q)} \vartheta(q'), \frac{1-k}{2} \right\} \\
&= \min \left\{ \delta^{\text{upper}}(\vartheta)(q), \frac{1-k}{2} \right\}.
\end{aligned} \tag{27}$$

Thus, we get $\overline{\delta^{\text{upper}}}(\vartheta)(p) \geq \min\{\overline{\delta^{\text{upper}}}(\vartheta)(q), (1 - k/2)\}$.

(ii) Now let w in Y . Let

$$\begin{aligned}\overline{\delta^{\text{upper}}}(\vartheta)(w) &= \bigwedge_{t \in \delta C(w)} \vartheta(t) \geq \bigwedge_{t \in \delta C(w)} \min\left\{(\gamma \boxtimes \vartheta)(t), (\vartheta \boxtimes \gamma)(t), \frac{1-k}{2}\right\} \\ &= \min\left\{\bigwedge_{t \in \delta C(w)} (\gamma \boxtimes \vartheta)(t), \bigwedge_{t \in \delta C(w)} (\vartheta \boxtimes \gamma)(t), \frac{1-k}{2}\right\} \\ &= \min\left\{(\gamma \boxtimes \overline{\delta^{\text{upper}}}(\vartheta))(p), (\overline{\delta^{\text{upper}}}(\vartheta) \boxtimes \gamma)(p), \frac{1-k}{2}\right\}.\end{aligned}\quad (28)$$

Thus, $\overline{\delta^{\text{upper}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . $\square$

Theorem 10. Let δ be a complete and symmetric PH-order on a partially ordered LA-sm-hyp-group Y . Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . Then, $\underline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

Proof. Let ϑ be an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y and δ be a complete and symmetric PH-order relation on Y . We have to show that $\underline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y .

(i) For each p, q in Y and $p \leq q$, we have $\delta C(p) = \delta C(q)$.
Now

$$\begin{aligned}\underline{\delta^{\text{lower}}}(\vartheta)(p) &= \bigwedge_{p' \in \delta C(p)} \vartheta(p') \\ &= \bigwedge_{q' \in \delta C(q)} \min\left\{\vartheta(q'), \frac{1-k}{2}\right\} \\ &= \min\left\{\bigwedge_{q' \in \delta C(q)} \vartheta(q'), \frac{1-k}{2}\right\} \\ &= \min\left\{\underline{\delta^{\text{lower}}}(\vartheta)(q), \frac{1-k}{2}\right\}.\end{aligned}\quad (29)$$

Thus, $\underline{\delta^{\text{lower}}}(\vartheta)(p) \geq \min\{\underline{\delta^{\text{lower}}}(\vartheta)(q), (1 - k/2)\}$.
(ii) Now let w in Y . Let

$$\begin{aligned}\underline{\delta^{\text{lower}}}(\vartheta)(w) &= \bigwedge_{t \in \delta C(w)} \vartheta(t) \geq \bigwedge_{t \in \delta C(w)} \min\left\{(\gamma \boxtimes \vartheta)(t), (\vartheta \boxtimes \gamma)(t), \frac{1-k}{2}\right\} \\ &= \min\left\{\bigwedge_{t \in \delta C(w)} (\gamma \boxtimes \vartheta)(t), \bigwedge_{t \in \delta C(w)} (\vartheta \boxtimes \gamma)(t), \frac{1-k}{2}\right\} \\ &= \min\left\{(\gamma \boxtimes \underline{\delta^{\text{lower}}}(\vartheta))(p), (\underline{\delta^{\text{lower}}}(\vartheta) \boxtimes \gamma)(p), \frac{1-k}{2}\right\}.\end{aligned}\quad (30)$$

Thus, $\underline{\delta^{\text{lower}}}(\vartheta)$ is an $(\epsilon, \epsilon \vee q_k)$ -fuzzy quasi-HP-ideal of LA-sm-hyp-group Y . $\square$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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