Research Article

One-Dimensional Nonlinear Consolidation Analysis Using Hansbo's Flow Model and Rebound-Recompression Characteristics of Soil under Cyclic Loading

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Hansbo's flow model for one-dimensional consolidation analysis of saturated clay has been widely recognized as being the most representative for soft soils. Many studies have used the model to examine the characteristics of soil under various conditions. However, very few studies have considered soil under cyclic loading. In this study, using a Hansbo's flow model and assuming known characteristics for soft clay deformation and rebound and recompression of soil, the one-dimensional consolidation model of soft clay under cyclic loading is established. A FlexPDE solution scheme with excess pore pressure u and void ratio e as variables is also given. The reliability of the proposed method is verified by comparing the obtained results with existing results. On this basis, the consolidation characteristics of soft clay foundations under unilateral drainage and cyclic loading are studied. The effects of soil rebound and recompression characteristics, Hansbo's flow parameters, cyclic loading period, and cyclic loading form on the consolidation characteristics of soft clay foundation are analyzed. The results show that under cyclic loading, the effective stress, void ratio, and average consolidation degree of the foundation all present a cyclic state and gradually enter a stable cyclic state with the increase in cycles. The peak of effective stress lags behind the peak of cyclic load. The rebound and recompression characteristics of soil have little effect on the effective stress of soil but a great effect on the void ratio. In contrast to its characteristic under linear loading, the average consolidation degree of the foundation under cyclic loading finally enters a stable cyclic state. The results of the analysis can be used as a reference in the analysis of real life highways, railways, subway tunnels built on soft soil foundations subjected to periodic cyclic loading.

1. Introduction

The traditional Terzaghi consolidation theory, which states that when stress is applied to a porous material, it is opposed by the fluid pressure filling the pores in the material, is based on the assumption that the soil skeleton is characterized by linear elasticity, small strain, and Darcy's flow. The mathematical form of the theory is simple and it is easy to obtain the analytical solution. Thus, it is widely used in practical engineering. However, with the deepening of research, more and more results show that there are sometimes large errors between the predicted foundation settlement and the measured value. On the one hand, this is mainly because seepage in soft clay deviates from Darcy's law [1–3]. On the other hand, soft clay deformation presents significant

nonlinear deformation characteristics [4]. Therefore, the study of nonlinear consolidation of soft clay considering a non-Darcy's flow has strong practical significance in engineering.

Among various non-Darcy's flow models, Hansbo's flow model has been widely recognized by the research community [5, 6]. In order to study the influence of non-Darcy's flow characteristics on the consolidation process of soft clay, Hansbo et al. carried out a series of model observations and theoretical studies on one-dimensional consolidation and sand well foundation consolidation problems. Hansbo's flow model for one-dimensional consolidation analysis of saturated clay was introduced in [7, 8]. It was solved using the finite difference method. The results under Hansbo's flow model show that the dissipation rate of pore water pressure

is slower than that of Darcy's flow. In order to consider the influence of nonlinear deformation characteristics of soft soil, one-dimensional nonlinear consolidation of soft soil based on Hansbo's flow under variable load was studied in [9] using the finite difference method and analyzed for a constant load in [10]. A semianalytical solution was found in [11]. In addition, the model was coupled to a Merchant rheological model in [12] in order to study the one-dimensional rheological consolidation characteristics of saturated clay.

None of the aforementioned studies considered cyclic loading. However, in real life, highways, railways, subway tunnels built on soft soil foundations can usually be regarded as foundations subjected to periodic cyclic loading. At this point, the consolidation characteristics of soil are obviously different from those of foundations under constant load. For the problem of cyclic loading, one-dimensional consolidation of homogeneous foundations, layered foundations, and semipermeable boundary conditions were studied in [13–15], respectively. The one-dimensional nonlinear consolidation of soft clay under cyclic loading was studied in [16–18]. An analytical solution of one-dimensional rheological consolidation of soft soil under cyclic loading was found in [19]. The Laplace transform was used to derive the solution considering the rebound-recompression characteristics of soil under cyclic loading in [20]. The one-dimensional consolidation characteristics of viscoelastic foundations were studied in [21]. The one-dimensional consolidation characteristics of saturated clay considering Hansbo's flow under cyclic loading were determined in [22].

In this article, Hansbo's flow model and nonlinear deformation characteristics of soft clay are introduced. Considering the unloading rebound and reloading effect of soil, the nonlinear consolidation equation of soft clay under cyclic loading is established. The consolidation equation is solved by FlexPDE software (a flexible solution system for partial differential equations), and the solution scheme of the bivariate of excess pore pressure and the void ratio is given. On this basis, the one-dimensional consolidation characteristics of soil under different cyclic loads are studied.

2. Basic Assumptions and Model Creation

2.1. Mathematical Description of the Problem. As shown in Figure 1, we consider a saturated soft clay layer of thickness H_0 . Its top is permeable and the bottom is impervious (or impermeable). We also assume that the soft clay has been consolidated under the action of soil weight and original uniform load q_p . The top surface of the soft clay layer is the same height as that of the groundwater table. A uniformly distributed cyclic loading ($q(t)$) is applied on the top of the soft clay layer. The period of the load is T_c and the peak is $q_{\max}$. Soil gradually begins consolidating under the uniform cyclic loading.

2.2. Fundamental Assumption. Soft soil deformation usually presents obvious nonlinear characteristics in saturated soft clay foundations. Therefore, the following assumptions are

made to establish an analytical model for the flow and aforementioned consolidation of the soft clay foundation .

- (1) The flow of water in the soil satisfies Hansbo's flow model shown in Figure 2. Moreover, Hansbo's flow model parameters (m and i_1) remain constant in the consolidation process. Hansbo's flow model can be expressed as follows:

$$v = \begin{cases} \tilde{K} i^m, & (i \leq i_1), \\ K(i - i_0), & (i > i_1), \end{cases} \quad (1)$$

where v is the seepage rate of pore water in soil; m is a constant determined by experiments; i is hydraulic gradient; i_1 is the initial hydraulic gradient in the linear seepage stage; i_0 is the calculated initial hydraulic gradient of linear seepage; $\tilde{K}$ is the permeability coefficient at the beginning of the exponential seepage stage.

$$\left. \begin{aligned} i_0 &= \frac{i_1(m-1)}{m} \\ \tilde{K} &= \frac{K}{(mi_1^{m-1})} \end{aligned} \right\}. \quad (2)$$

- (2) Soil particles and pore water in the soil layer are incompressible and the soil layer is in the normal consolidation state.
- (3) Nonlinear deformation and flow law of soft soil can be expressed in the classical form as $e - \lg \sigma'$ and $e - \lg K$ [4]. The format is as follows:

$$e - e_1 = c_c \lg \left(\frac{\sigma'_1}{\sigma'} \right), \quad (3)$$

$$e - e_1 = c_k \lg \left(\frac{K}{K_1} \right), \quad (4)$$

where e is the pore ratio of soil; σ'_1 is the reference effective stress; K_1 and e_1 are the permeability coefficient and void ratio corresponding to the reference effective stress (σ'_1); c_c is the compression index of soil; c_k is the permeability index of soil; σ' is effective stress; K is the permeability coefficient corresponding to the void ratio (e).

- (4) Considering the influence of unloading rebound and recompression of soil, the unloading curve and recompression curve are a straight line in the $e - \lg \sigma'$ coordinate system and the two straight lines overlap. As shown in Figure 3, the rebound index c_e (or recompression index) remains unchanged during consolidation.

3. Mathematical Model Formulation

3.1. Initial Effective Stress and Initial Void Ratio. According to [23], effective stress at any time can be expressed as follows:

FIGURE 1: One-dimensional consolidation under cyclic loading.

FIGURE 2: Hansbo's flow model.

$$\sigma' = q(t) - u + \sigma'_0, \quad (5)$$

where u is excess pore water pressure; $q(t)$ is the cyclic uniform load acting on the soil surface; σ'_0 is initial amplitude effective stress.

$$\sigma'_0 = \sigma'_0(z, 0) = \int_0^z \frac{\gamma_w (G_s - 1)}{1 + e_0} dz' + q_p, \quad (6)$$

where G_s is the relative density of particle; γ_w is the gravity of water; e_0 is the pore ratio corresponding to initial effective stress (σ'_0); q_p is original uniform load.

According to the aforementioned assumptions, the initial effective stress (σ'_0) and initial void ratio (e_0) meet the requirements of (3). Bringing (3) into (6) and differentiating results in the following equation:

$$\frac{d\sigma'_0}{dz} = \frac{\gamma_w (G_s - 1)}{1 + e_1 + c_c (\lg \sigma'_1 - \lg \sigma'_0)}. \quad (7)$$

Equation (7) is a complex nonlinear ordinary differential equation that cannot be solved by conventional methods. In

engineering practice, a numerical method is often used to find an approximate solution; boundary conditions $\sigma'_0(t = 0, z = 0) = q_p$ are added for solution analysis.

3.2. Consolidation Process of Soil under Cyclic Loading. Taking the triangular cyclic load given in Figure 4 as an example, the soil layer is fully consolidated under the action of self-weight stress and original uniform load (q_p), that is, point a on the compression curve shown in Figure 3. Cyclic uniform load ($q(t)$) began to act on the soil surface, and under this load, the effective stress (σ') gradually increased, whereas the void ratio (e) gradually decreased. As the soil is normally consolidated clay, σ' and e follow the normal compression curves shown in Figure 3. When the time t exceeds T_1 but is less than T_c , the external load decreases gradually from peak $q_{\max}$ and the excess pore pressure u decreases. However, due to the drainage consolidation effect, the effective stress σ' does not decrease immediately; it begins to decrease after a period of time exceeding T_1 . It decreases from point b in Figure 3 to point c along the rebound recompression curve. At this time, σ' and e fall on the rebound recompression curve. When the cyclic load enters the second cycle, the external load gradually increases. At this point, with the drainage consolidation of soil, the effective stress σ' changes from point c along the rebound-recompression curve to point b. When the effective stress σ' is greater than σ'_b , σ' and e fall along the normal compression curve.

Therefore, when considering the rebound-recompression characteristics of soil, it is necessary to distinguish the normal compression section from the rebound-recompression section and establish the corresponding consolidation differential equations.

3.3. Consolidation Equation of the Normal Compression Section. In order to simplify the analysis, the following equation can be obtained based on the continuity condition of seepage with reference to the derivation process of the traditional Terzaghi one-dimensional consolidation equation.

$$\frac{\partial v_z}{\partial z} = -\frac{1}{1 + e_0} \frac{\partial e}{\partial t}, \quad (8)$$

where v_z is vertical seepage velocity of pore water.

The following equation can be obtained by combining (1) and (2) and referring to literature [7]:

$$\frac{\partial v_z}{\partial z} = \frac{\partial}{\partial z} \left[\operatorname{sgn} \frac{K}{M \gamma_w i_1^{M-1}} \left| \frac{\partial u}{\partial z} \right|^M \right], \quad (9)$$

where M is the design conditions; when $|i|$ is greater than i_1 , $M = 1$; when $|i|$ is less than or equal to i_1 , $M = m$. sgn is the symbolic function, when i is less than 0, sgn is negative; conversely, sgn is positive when i is greater than 0. Owing to the negative hydraulic gradient, the Hansbo seepage equation becomes ambiguous; hence, this paper extends the equation to include the case of the negative hydraulic gradient by introducing the symbolic function.

Substituting (5) into (3) and differentiating time, we get the following equation:

$$\frac{\partial e}{\partial t} = \frac{c_c}{\sigma' \ln 10} \left(\frac{\partial u}{\partial t} - \frac{dq}{dt} \right). \quad (10)$$

According to (3) and (4), the relationship between the permeability coefficient K and the effective stress σ' is obtained.

$$K = K_1 \left(\frac{\sigma'_1}{\sigma'} \right)^{c_c/c_k}. \quad (11)$$

By substituting (9)–(11) into (8), the consolidation differential equation of the normal compression curve segment can be obtained.

$$\frac{1}{1+e_0} \frac{c_c}{\sigma' \ln 10} \left(\frac{\partial u}{\partial t} - \frac{dq}{dt} \right) + \frac{\partial}{\partial z} \left[\operatorname{sgn} \frac{K_1}{M \gamma_w^{M-1} i_1^{M-1}} \left| \frac{\partial u}{\partial z} \right|^M \left(\frac{\sigma'_1}{\sigma'} \right)^{c_c/c_k} \right] = 0. \quad (12)$$

3.4. One-Dimensional Consolidation Equation of the Rebound-Recompression Section. The relationship between effective stress σ' and void ratio e in the rebound-recompression section can be expressed using (3) as follows:

$$e - e_x = c_e \lg \left(\frac{\sigma'_x}{\sigma'} \right), \quad (13)$$

where σ'_x and e_x is a point on the springback recompression curve.

The effective stress levels are different when the soil at different positions starts unloading at different times. Therefore, σ'_x and e_x in (13) cannot be expressed explicitly.

Similar to the normal compression section, the following equation can be obtained by taking (5) into (13) and differentiating time on both sides of the equation:

$$\frac{\partial e}{\partial t} = \frac{c_e}{\sigma' \ln 10} \left(\frac{\partial u}{\partial t} - \frac{dq}{dt} \right). \quad (14)$$

The one-dimensional consolidation equation of the rebound-recompression section can be obtained by introducing (14) and (11) into (8). Its form is similar to that of (12) if c_c is replaced with c_e .

3.5. Initial and Boundary Conditions. To solve the aforementioned consolidation equation, the corresponding initial and boundary conditions are given as follows:

Initial condition:

$$u = 0, \quad (t = 0, 0 \leq z \leq H_0). \quad (15)$$

Boundary condition:

$$\left. \begin{aligned} u &= 0, & (t > 0, z = 0), \\ \frac{\partial u}{\partial z} &= 0, & (t > 0, z = H_0), \end{aligned} \right\}. \quad (16)$$

3.6. Simulation of Consolidation Process under Cyclic Loading. Because the consolidation equations of the normal compression section and rebound-recompression section are different, corresponding judgment criteria should be set in the calculation process; hence, the values of the soil parameters at different positions can be calculated by different consolidation equations at different time steps.

The judgment criteria of soil unloading is simple: according to $d\sigma'/dt < 0$, the consolidation equation of the rebound-recompression section is used to calculate unloading.

However, the judgment criteria of soil when the load is applied is relatively complex. It is necessary to distinguish the rebound-recompression section and normal compression section, which can be judged according to the following method:

Normal compression stage:

$$\frac{d\sigma'}{dt} \geq 0, \quad \sigma' < \sigma'_{\max}. \quad (17)$$

Recompression stage:

$$\frac{d\sigma'}{dt} \geq 0, \quad \sigma' = \sigma'_{\max}, \quad (18)$$

where $\sigma'_{\max}$ is the maximum effective stress of the soil element before the current moment.

4. Solving Consolidation Equation Based on FlexPDE

In contrast to the methods employed by other studies for solving consolidation equations; in this paper, FlexPDE software is used to solve the consolidation equation. FlexPDE software uses the finite element method to solve partial differential equations. It is widely used to solve one-dimensional to three-dimensional development problems, steady-state problems, and eigenvalue problems; high-precision numerical solutions of partial differential equations can be obtained. In addition, FlexPDE software usually uses a very simple programming language to define the partial differential control equation, solution domain, and boundary conditions of the analyzed problem, which greatly reduces the difficulty of obtaining mathematical solutions to complex problems.

However, FlexPDE software cannot record the maximum effective stress $\sigma'_{\max}$ of each finite element mesh node before the current calculation step. To address this issue, the following approaches are available:

- (1) Define two variables of excess pore pressure u and void ratio e .
- (2) Add the following initial and boundary conditions.

FIGURE 3: Compression and springback recompression characteristics of soil.

FIGURE 4: Triangle cycle loading.

The initial condition is $e = e_0$, ($t = 0, 0 \leq z \leq H_0$) and the boundary condition is $(\partial e / \partial z) = 0$, ($t > 0, z = H_0$).

- (3) The unloading criteria remain unchanged; the loading criteria are defined by the following equations:

Normal compression stage:

$$\begin{aligned} \frac{d\sigma'}{dt} &\geq 0, \\ \left| \frac{e_t - e}{e} \right| &< D. \end{aligned} \quad (19)$$

Recompression stage:

$$\begin{aligned} \frac{d\sigma'}{dt} &\geq 0, \\ \left| \frac{e_t - e}{e} \right| &\geq D, \end{aligned} \quad (20)$$

where D is the threshold value. It can be selected according to the calculation accuracy; e_t is the pore ratio calculated by (3).

5. Algorithm Verification

The saturated clay samples were prepared in advance; each group was subjected to three repeated tests, and the weighted average was taken. As shown in Figure 5, the

saturated clay consolidation test was carried out by a WG single lever consolidation instrument (main technical: 1. Loading pressure is 4000 kPa/30 cm² or 2000 kPa/50 cm²; 2. Load arm is 20 : 1 or 24 : 1; 3. Specimen area is 30 cm² or 50 cm²; especially, the sample area of this paper is 50 cm²). The WG-type series consolidation instrument is used for soil compression test to detect the relationship between soil pressure and deformation, calculate the unit deposition of soil, compression index, rebound index, and consolidation coefficient; the cyclic loading and unloading process was simulated by a Kamoer peristaltic pump (Its flow rate is 2.6–41.5 ml/min). The loading period can be modified by changing the pumping speed of the peristaltic pump. The relevant test parameters are as follows: the linear load loading period is 10,800 s, which remains unchanged after reaching the peak; the loading and unloading cycle of the triangle cyclic load is 2700 s, and the test is conducted for six cycles; the loading time of the trapezoidal cyclic load is 1440 s, the peak load is 480 s, the unloading time is 1440 s, and the no-load time is 480 s. The test is carried out for 5 cycles. The peak value of the cyclic load is 24,000 s, and the sedimentation is assumed to be the final sedimentation.

As shown in Figure 6, the test results are in good agreement with the simulation results; the average consolidation degree of the test results is higher than that of the simulation. The reason is that the test instrument is limited, and the quality of the permeable stone and the metal cover above the soil sample is large, resulting in the error of the test results. Under linear load, the mass effect of the metal cover will be more obvious, resulting in the settlement in the early stage of the test being significantly higher than that of the simulation analysis. During triangular and trapezoidal cyclic loading, the experimental results are basically consistent with the simulation results, indicating that the algorithm has high reliability.

We can define the time factor T_v as follows:

$$T_v = \frac{K_1 (1 + e_1) \sigma'_1 \ln 10}{\gamma_w c_c H^2} t. \quad (21)$$

If there is no special description below, the dimensionless time factor T_v is used as the measurement of time. Reference [10] used the finite difference method to calculate and analyze the one-dimensional consolidation problem considering the nonlinear deformation characteristics of soil under Hansbo's flow condition. This study does not consider the deposition effect of the soil layer. The initial void ratio e_0 and initial effective stress σ'_0 of the whole soil layer are considered equal, which is suitable for the case of small soil thickness. The relevant calculation parameters are as follows: $m = 2.0$, $I_1 = 2.0$ or 0.5 ; an external load $q = 4\sigma'_1$ is applied instantaneously and remains unchanged; the ratio of the compression index to permeability index is 1.2. The calculation results are shown in Figure 7. It can be seen from the figure that the calculation results in this paper have a high coincidence degree with the calculation results in [10]. This shows that the calculation results in this paper have high reliability.

FIGURE 5: WG single lever consolidation instrument (a), the type of F01A-STP Kamoer peristaltic pump (b), and manufacturing details (c).

FIGURE 6: Comparison of the results by the FlexPDE software to those of experimental results.

However, our study considers the soil layer deposition effect and nonlinear deformation characteristics, as well as Hansbo's flow and variable load conditions. In order to verify the reliability of the algorithm under the aforementioned conditions, we consider a verification example that involves large consolidation deformation of soft soil considering the influence of Hansbo's flow and variable load found in [24]. In order to obtain results that correspond with the calculation of large consolidation deformation in [24], the program in the current study is modified simply to consider the case of large consolidation deformations. In fact, if $1+e$ replaces $1+e_0$ and $\partial/\partial\xi$ replaces $\partial/\partial z$ in equation (12), the expression for nonlinear consolidation equation in the flow coordinate system can be obtained. By

transforming this equation into the Lagrangian coordinate system, the nonlinear consolidation equation under large deformation can be obtained. When verifying the calculation, the external load is loaded in single-stage loading mode. The time factor of loading duration is $T_{vc} = 0.1$ (the actual loading time is about 1000 d), the time factor $T_v = 0.4$. The other indexes of soil are the same as those in [24], and the calculation results are shown in Figure 8.

It can be seen from Figure 8 that the numerical solution in this paper is consistent with the calculation results in [24]. This shows that the one-dimensional consolidation algorithm considering Hansbo's flow condition, soft soil deposition effect, and nonlinear deformation characteristics of soil in this research is reliable.

FIGURE 7: Comparison of the results by FlexPDE software to those in [10].

6. Analysis of Consolidation Behavior

Based on the aforementioned one-dimensional consolidation analysis program under cyclic loading, the consolidation characteristics of soft soil foundation under cyclic loading and under different conditions are studied by taking unilateral drainage as an example. The physical and mechanical parameters of the soil are shown in Table 1. Some typical forms of cyclic loading are shown in Figures 9(a)–9(c); among them, T_c is a dimensionless period. At the same time, for the comparative analysis, the linear load is given, as shown in Figure 9(d). The relevant parameter of the cyclic load is $q_{\max} = 100$ kPa and the Hansbo flow parameters are $m = 1.5$, $I_1 = 2.0$. The soil rebound index is $c_e = 0.52$ and its thickness is 20 m.

There are two ways to define the average degree of consolidation, that is, average consolidation degree U_p defined by excess pore pressure u and the average consolidation degree U_s defined by surface settlement s_t , as follows:

$$\begin{aligned} U_p(t) &= \int_0^{H_0} \frac{(q(t) - u)}{q_{\max}} dz, \\ U_s(t) &= \int_0^{H_0} \frac{(e_0 - e)}{(e_0 - e_f)} dz, \end{aligned} \quad (22)$$

where e is the void ratio at t time and e_f is the final thin void ratio. Other parameters are the same as before.

6.1. Consolidation Characteristics of Soil under Cyclic Loading. Taking the triangular cyclic load shown in Figure 9(a) as an example with $T_c = 0.2$, we analyze the consolidation behavior at different positions of the soft soil foundation and the variation of average consolidation degree U with time under cyclic loading. The calculation results are shown in Figures 10–13.

Figure 10 shows the variation curves of effective stress σ' with time factor T_v at different positions of the foundation under cyclic loading. It can be seen from the figure that the effective stress in the foundation also has a cyclic effect under cyclic loading. Moreover, due to the drainage consolidation on the surface of the foundation, the cyclic variation range of effective stress at different depths of the foundation is different under the condition of unilateral drainage. The closer to the surface, the greater is the impact of surface drainage and the greater the amplitude of σ' cycle change. The peak of cyclic load does not appear simultaneously with the peak of effective stress, and the peak of effective stress has a certain degree of hysteresis. In addition, with the increase of cycle times, the effective stress σ' gradually enters a stable cycle state. At this point, the peak effective stress almost does not increase with the increase of cycle times.

Figures 10–12 show the variation curves of effective stress σ' and c_c or c_e with time factor T_v at $z = H_0/4$ and $z = 3H_0/4$, respectively. It can be seen from the figure that under cyclic loading, the effective stress σ' of the shallow foundation reaches its maximum $\sigma'_{\max}$ quickly after the previous several cycles. Subsequently, the soil (σ', e) falls on the rebound-recompression curve in Figure 3. The deep part of the foundation is less affected by surface drainage; hence, at the early stage of the cycle, with the increase of the cycle number, the peak value of the effective stress σ' of the soil increases gradually. (σ', e) of the soil constantly changes from the compression curve to the rebound compression curve and then to the compression curve.

Figure 13 is the curve of average consolidation degree U versus time factor T_v under cyclic loading. It can be seen from the figure that there are some similarities and differences between the average consolidation degree U_p defined by excess pore pressure u and the average consolidation degree U_s defined by surface settlement s_t . Under cyclic loading, U_p and U_s also present cyclic characteristics. At the early stage of the cycle, the wave peaks of U_p and U_s increase with the increase of cycle

FIGURE 8: Comparison of the results by FlexPDE to those of the literature [24].

TABLE 1: Physical and mechanical properties of soils.

σ'_1 (kPa)	e_1	c_c	c_k	H_0 (m)	G_s	K_1 (m/s)	q_p (kPa)
50	1.57	0.64	0.75	20	2.75	1×10^{-8}	10

FIGURE 9: Typical forms of loading.

times; wave peaks of U_p and U_s tend to be stable after more than a certain number of cycles. In addition, U_s is greater than U_p from the beginning of cyclic loading. This shows that the

consolidation degree defined by the surface settlement is higher than that defined by excess pore pressure, which should be considered in practical engineering.

FIGURE 10: $\sigma' - T_v$ curves for triangle cyclic loading of different z .

FIGURE 11: c_c or $c_e - T_v$ curves for $z = H_0/4$.

6.2. Influence of Cyclic Load Parameters. This section mainly analyzes the influence of the cyclic loading cycle and loading form on the consolidation characteristics of soft soil foundations. When analyzing the influence of the cyclic loading period, cyclic loading is the same as presented in the previous section. The cycle periods T_c are 0.4, 0.6, 0.8, and 1.0, respectively. When the influence of the cyclic loading form is analyzed, the cyclic loading period is 0.6. The form of cyclic load is shown in Figures 9(a)–9(c); the linear load is also considered for comparative analysis.

Figures 14 and 15 show the variation curves of average consolidation degree with time factor T_v at different cyclic periods of triangular cyclic loading. The figure shows that the cyclic loading period T_c has a great influence on the average consolidation degree of foundation. With the increase of cyclic loading period, the variation range of average consolidation degrees U_p and U_s (the difference between peak and trough) of the foundation within the same cyclic cycle

increases. The greater the cycle period of cyclic loading is, the fewer cycle times the average consolidation degree U_p and U_s need to enter the stable cycle state. In addition, the average consolidation degree U_s is more affected by the cyclic loading period than U_p . Moreover, for the average consolidation degree U_s , the wave peak and wave trough of U_s in the long cycle of the same cycle are higher than those in the short cycle.

Figures 16 and 17 show the variation curves of average consolidation degree U_p and U_s with time factor T_v under different cyclic loading forms. It can be seen from the figure that the form of cyclic load also has a great influence on the average consolidation degree of foundation. The variation range of the average consolidation degree of the foundation under trapezoidal cyclic loading is the largest in the same cycle. The average consolidation degree under cyclic loading of the triangle is similar to that under semisinusoidal cyclic loading. However, the peak and trough values of the average

FIGURE 12: c_c or $c_e - T_v$ curves for $z = 3H_0/4$. FIGURE 13: $U - T_v$ curves for triangle cyclic loading.

consolidation degree under semisinusoidal cyclic loading are greater than those under triangular cyclic loading. That is, the consolidation degree of the foundation under semisinusoidal cyclic loading is higher than that under triangular cyclic loading. Compared with cyclic loading, the average consolidation degree of foundation under linear loading increases gradually and will reach 100% at some time. Under cyclic loading, the average consolidation degree of the foundation gradually tends to be stable after several cycles, but it can never reach 100%.

6.3. Effect of the Rebound Recompression. The cyclic load adopts the triangular cyclic load in Figure 9(a), and $T_c = 0.6$, $T_1 = 0.3$. When the effect of rebound compression and recompression is not considered and $c_c = c_e$, the calculation results are shown in Figures 16–18.

Figure 18 shows the variation curve of effective stress σ' with time factor T_v when considering and not considering the influence of rebound and recompression. The figure

shows that the effective stress difference between the two cases is very small and can be ignored.

Figure 19 shows the variation curves of void ratio e with time factor T_v when considering and not considering the influence of rebound and recompression. It can be seen from the figure that compared with the effective stress, there is a certain difference in the void ratio in the two cases. In particular, the wave peaks are quite different. The void ratio is relatively small when rebound and recompression are considered. This is mainly because when considering rebound and recompression, the slope c_e of the rebound-recompression section is less than c_c , and the increment of void ratio generated during unloading is less than that without considering rebound and recompression.

Figure 20 shows the variation curves of average consolidation degree with time factor T_v when considering and not considering the influence of rebound and recompression. From the aforementioned analysis, it can be seen that the effect of rebound and recompression on effective stress σ' is much smaller than that on the void ratio e .

FIGURE 14: $U_p - T_v$ curves for different T_c under triangular loading. FIGURE 15: $U_s - T_v$ curves for different T_c under triangular loading. FIGURE 16: $U_p - T_v$ curves for different cycle loadings.

FIGURE 17: $U_s - T_v$ curves for different cycle loadings. FIGURE 18: $\sigma' - T_v$ curves when taking into account the swelling characteristic of soil vs. disregarding it.

Therefore, on the average degree of consolidation U_p , the difference between the two cases is very small. The difference of U_s is relatively large when considering the effect of springback and recompression.

6.4. Influence of Hansbo Flow Parameters. This section mainly studies the influence of Hansbo seepage parameters on the consolidation characteristics of soft clay foundations under cyclic loading. The cyclic load is the same as specified in the section “Influence of cyclic load parameters”: m is 1.0, 1.2, 1.5, and 1.8, respectively; I_1 is 0.5, 1.0, 2.0, and 5.0,

respectively. The calculation results are shown in Figures 17 and 18.

Figures 21 and 22 show the curves of average consolidation degree versus time factor T_v when m or I_1 is different. It can be seen from the figures that when the seepage in soil satisfies Darcy flow ($m = 0$), the drainage consolidation rate of the foundation is the fastest. The wave peak value is the largest in the same cycle, whereas the wave trough is the smallest. That is, the variation range is the largest in the same cycle. With the increase of m , the variation range of the average consolidation degree gradually decreases in the same cycle. It can be seen from Figures 23 and 24 that the influence

FIGURE 19: $e - T_v$ curves when taking into account the swelling characteristic of soil vs. disregarding it.

FIGURE 20: $U - T_v$ curves when taking into account the swelling characteristic of soil vs. disregarding it.

FIGURE 21: $U_p - T_v$ curves for different m . FIGURE 22: $U_s - T_v$ curves for different m . FIGURE 23: $U_p - T_v$ curves for different I_1 .

FIGURE 24: $U_s - T_v$ curves for different I_1 .

of I_1 on the consolidation characteristics of the foundation is similar to that of m . With the increase of I_1 , the variation range of the average consolidation degree decreases gradually in the same cycle. In addition, the influence of Hansbo flow parameters on U_p and U_s is basically the same.

7. Conclusion

In this paper, the nonlinear consolidation equation of soft clay under cyclic loading is established by considering Hansbo's flow model and the nonlinear deformation characteristics of soft clay, as well as the deposition effect of the soil layer and the unloading-rebound-reloading effect of soil. The numerical solution of the consolidation equation was solved by FlexPDE software, and the one-dimensional nonlinear consolidation characteristics of soil under cyclic loading were studied. The main results are as follows:

- (1) Under cyclic loading, the effective stress σ' , void ratio e , and average consolidation degree U of the foundation show a cyclic state. They gradually enter a stable cycle state as the cycle number increases. The wave peak of effective stress σ' does not occur simultaneously with the wave peak of cyclic load but shows a corresponding hysteresis phenomenon.
- (2) Under the action of periodic cyclic loading, considering or not considering the rebound-recompression characteristics of soil has little effect on the effective stress σ' and the average consolidation degree U_p of soil. However, it has a great influence on the soil void ratio e and the average consolidation degree U_s defined by surface settlement. Without considering the rebound-recompression characteristics of soil, the average consolidation degree of the foundation will be underestimated.
- (3) When the flow law conforms to Hansbo's flow, the variation amplitude of average consolidation of the

foundation in the same cycle is smaller than that of Darcy's flow. With the increase of seepage parameters m and I_1 , the variation range of the average consolidation degree of the foundation decreases gradually.

(4) With the increase of load cycle period, the average consolidation degree of the foundation requires fewer cycle times to enter the stable cycle state. Moreover, the variation range of the average consolidation degree of the foundation also gradually increases within the same cycle. In addition, within the same cycle, the variation range of average consolidation degree of foundation under trapezoidal cyclic load is the largest; the variation range of average consolidation degree under semi-sinusoidal load and triangular load is basically the same. However, the wave peak and wave trough of the average consolidation degree under semi-sinusoidal load are greater than those under triangular cyclic load. It will eventually enter a stable cyclic state.

(5) The results of the analysis can be used as a reference in the analysis of real life highways, railways, subway tunnels built on soft soil foundations subjected to periodic cyclic loading. Specifically, the analysis shows that the consolidation degree defined by the surface settlement is higher than that defined by excess pore pressure, which should be considered in practical engineering.

Data Availability

The data files of all charts and original program data used to support the findings of this study are included within the supplementary information file. In addition, the reference data in Figures 7 and 8 are obtained by GetData Graph Digitizer software. References [10, 24] are originally from the following websites: <https://rockmech.whrsm.ac.cn/EN/volumn/home.shtml> and http://manu31.magtech.com.cn/Jwk_ytgxcb/EN/volumn/home.shtml.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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Supplementary Materials

The data files of all charts and original program data used to support the findings of this study are included within the supplementary materials. (*Supplementary Materials*)

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