Research Article

The Staged Financing Selection Mechanism for Government to Maximize the Green Benefits of Start-Ups

Wenke Yang,1 Qianting Ma2, Meile Tian,3 Lei Wang,1 and Jianmin He1

1School of Economics and Management, Southeast University, Nanjing 211189, China
2College of Finance, Nanjing Agricultural University, Nanjing 210095, China
3Faculty of Management and Economics, Kunming University of Science and Technology, Kunming 650093, China

Correspondence should be addressed to Qianting Ma; mqt2626@njau.edu.cn

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In this study, we investigate the most common forms of government grant in green start-ups, which are appropriation, interest-free bank loans, and tax subsidies. These mechanisms are used to mitigate the problem of higher research costs and sunk costs of start-ups on green innovation and help venture investors better monitor the business plan, asset use, and agency cost and regularly collect information of start-ups to retain the right to terminate financing projects and improve the efficiency of them. The aim of this work is to develop a theoretical model of the agency among the government, the venture capitalists who only pursue monetary income, the strategy investors who pursue strategic objectives and monetary income, and the entrepreneur who takes into account both the influence of different forms of government grant on entrepreneur financing at a different stage and the improved monitoring process of venture investors owe to the staged capital infusion of government. The model shows that the optimal staged financing decision is given when the first target of the government is to achieve social welfare optimization and the secondary goal of maximizing green benefits. Moreover, the model explains the optimal staged financing decision of venture investors and equity stake share in different rounds. Ultimately, we find the optimal staged financing portfolios for green start-ups to acquire venture investment, reduce the staged financing uncertainty, and help the government realize a national green innovation strategy.

1. Introduction

In the long run, accelerating green technology research and development (R&D) and transferring incubation have been the focus of innovation and entrepreneurship policy-making. However, with the rapid development of big data and cloud computing technology, more studies focus on the superficial phenomena and methods to accelerate green innovation efficiency and to help develop green start-ups in a short time. These studies, to some extent, improve the energy efficiency and ecological environment and solve the financing problems of green start-ups but also mean stringent follow-up production processes and standards, which makes the theoretical research relatively backward [1, 2]. To put it differently, green start-ups in the area of energy efficiency are different from other start-ups concerning financing needs and optimal staging of investments. Particularly, there are higher risks because of insufficient theoretical supports and proofs for local governments to issue green policy leads to investments, which is vital for start-ups to break the policy-induced constraint at lower cost and achieve the common aspiration [3, 4]. For example, we knew little about the internal mechanism of why there are higher research costs and sunk costs on green innovation than on general innovation research and how it would inhibit the enthusiasm of enterprises to develop green technology, transfer, and incubate green new products because of the popular creative destruction effect, “Schumpeter effect” [5].

Obviously, the acquisition and efficient management of capital are more difficult for small enterprises in the process of technology development, product development, and technology transfer to the market, which have attracted wide
attention from enterprises, governments, and the academic world still. For example, the external financing channels of small- and medium-sized enterprises (SMEs) in developing countries are mainly bank financing due to immature stock and venture capital markets [6], and thus high risky green start-ups would not survive. In contrast, capital resources in developed stock markets are more abundant, are more dependent on external finance, and have a greater innovation level [7]. Therefore, the sudden break of the external capital chain will have a devastating impact on the enterprise that is highly dependent on external finance. For instance, a large number of international private companies went bankrupt due to the financial crisis of 2008, and most venture investors turned to the gold market for safety, which led to a sharp decrease in R&D expenditure of start-ups and then greatly suppressed the green innovation drive [8].

In this study, we establish the monitoring intensity function to show the existent asymmetric information in green innovation activities like transforming tangible assets into intangible assets and external financing process of small- and medium-sized enterprises after deriving three classic pieces of literature of Chemmanur et al. [9], Freeman [10], and Mina et al. [11] to some extent. Basically speaking, the problem of existent asymmetric information is relevant to the impact of institutional conditions on entrepreneurial motivation. Back et al. [12] consider the institutional voids which make start-ups often struggle to get enough finance. In particular, empirical evidence suggests that, under the condition of information asymmetry, even if entrepreneurs open external financing, it is difficult for them to obtain enough funds to achieve rapid growth. Even in non-transparent predatory government, start-ups are less affected by lower institutional quality in the short term, but the impact will continue to expand over time because start-ups are relatively independent due to their strong autonomy, resources, capabilities, and social network in the initial period [13].

Besides, several authors have shown that internal factors such as human capital, market share, and financial capital and external factors such as social capital of enterprises have a heterogeneous effect on entrepreneurship under different institutional conditions [14–16]. Alam et al. [17] argue that the start-ups in the bank-based financial dominated systems are more dependent on external capital, while they are mainly dependent on internal R&D capital under the market-based financial dominated system. These institutional uncertainties undermine entrepreneurial action and create insurmountable obstacles to engage in productive business entrepreneurship, which is consistent with the North’s institutional theory [18–20]. To retain the right to terminate financing projects and improve the efficiency of start-ups, venture investors will monitor the business plan, asset use, and agency cost and regularly collect information as long as the institutional conditions have asymmetric information [21–23]. Concretely, Chemmanur et al. [9] have contributed to the research of the existent asymmetric information by focusing on the screening and monitoring procedures in the venture-capital-backed firms and the non-venture-capital-backed firms and illustrated the more obvious efficiency advantage and better product market performance of enterprises generated by venture capital backing. Thus, we combine strands of institutional theory with the resource-based view and reckon that institutional voids are existing in emerging markets of start-ups.

In fact, the government grant can improve asymmetric information problems which tend to be most severe in the financing process of high risky start-ups [11]. As such, relying solely upon government subsidies can no longer meet the needs of emerging ICT innovations for large developing countries but can establish a dynamic relationship between the government and enterprise partners, upgrading it from the project level to the system level, and then forming sustainable partnerships between governments and enterprises is conducive to jointly explore market demands and deal with risks [24]. Urpelainen and Van de Graaf [25] document new evidence about the importance of the participation of government financing involved in stimulating innovation by using the case of the International Renewable Energy Agency (IRENA).

The government mainly provides three forms of green subsidies: green appropriation, green product cost subsidies, and green tax incentives or green tax credits [26]. Green appropriation means that the government provides green start-up funds to entrepreneurs with scientific achievements, but the effects of this kind of direct subsidies rely on an ex ante screening mechanism [27–29]. Green product cost subsidies and green tax incentives are mainly to help small- and medium-sized enterprises to innovate their products, expand their scale of production, and have positive impacts on R&D activities. However, we do not consider the form of government equity investment, since such enterprises generally meet the requirements of national strategy, have high technology content and strong innovation ability, and are full of market potential, which is in contrast with start-ups [30].

In this study, we reckon that the function of the government grant is related to the trend of world economic growth mode change which is pushed by natural resources turn to be pulled by the resources of knowledge and technology [31, 32]. The government grant, firstly, can address the market failure problem of underinvestment in the R&D financing process [33]. Secondly, it can also encourage small enterprises to increase their R&D activities for innovation [34]. Ideally, the government invokes an “additional effect” on the R&D activities; that is, the purpose of government subsidies is to lower the R&D cost of enterprises, encourage R&D activities, and promote knowledge spillover. More specifically, the small amount of grant for enterprise R&D will help enterprises finance, while excessive grant will crowd out investors for financing and hinder the development of start-ups [7, 35, 36].

The model presented in this study includes three features which are important differences from those references cited: (i) We divided the subject of venture capital into strategic investors and venture investors according to the pecking order financing theory because strategic investors have stronger risk tolerance and weaker risk aversion characteristics than venture investors to obtain more equity, to
achieve strategic goals, and to maximize expected returns, which are more conducive to the development of enterprise innovation activities [37–39]. (ii) Compared to the studies of Giudici and Paleari [40] and Hellmann and Puri [41], we take the cost of monitoring of venture capitalists into account during the stage of green R&D and green products expansion for further reproduction. (iii) Based on the availability of government grants, we also analyze the maximization of green benefits of financing and calculate the optimal social welfare of start-ups.

The structure of the remainder of the study is as follows: Section 2 introduces the methodology. Section 3 describes the optimal choice of government. Section 4 concludes the study.

2. The Model

The target of this section is to understand the financing strategy taken of green start-ups at different stages. Based on the financial growth cycle paradigm, we create a green staged financing model by achieving social welfare optimization and maximizing green benefits (note that we define the green benefits as the raised funds which are used as financing tools for green projects such as energy conservation, environmental protection and pollution prevention, resource conservation, and recycling).

Briefly, we consider three issues: the optimal staged financing portfolio to maximize social welfare and green benefits [38, 42–44], the optimal staged financing portfolio for entrepreneur, and the optimal monitoring intensity for venture investors to reduce information asymmetry with the participation of government grant.

2.1. The Assumptions Based on the Existence of the Green Government Grant. The optimal level of monitoring is preferable to no intervention in a symmetric information environment, which means maximizing the value of the venture without monitoring cost.

Consider that an entrepreneur who is financially constrained has a new green technology only in a risk-neutral world and fully competitive market with no discounting. The goal of these parties is to realize industrialization, commercialization, and marketization of new green technologies; and R&D takes place in the first stages \([0, T_1]\), such as green conceptual product design, followed by the green product expansion reproduction period \([T_1, T_2]\) which is no grounds for terminating. Assume that minimum financing capital is available at each stage for start-ups. Hence, in this model, we divide two periods. Suppose that the entrepreneur has no capital in the state \(t = 0\) and seeks external financing. The external channels of financing of this stage include special appropriation funds \(K_{0G}^G\) for green start-ups from government and venture investments \(K_{0V}^G\) [31, 45, 46]. Due to the high risk of green R&D activities, we simply define the venture capitalists who solely pursue monetary income as VIs and the venture capitalists who pursue strategic objectives and monetary income as SIs. The venture investments \(K_{0V}^G\) and \(K_{0S}^G\) occur, respectively, from VIs and SIs. At the state of \(t = T_2\), the value of the asset of the start-up is \(\bar{A}\) in the case of success and \(\bar{A}\) in the case of failure. We define \(\theta = \bar{A} - \bar{A}\) and it measures the net impact of enterprises on SIs’ own assets. Besides, it means the complementary function on SIs’ own assets if \(\theta > 0\) and substituted function if \(\theta < 0\) [41]. Naturally, we have \(E\) homogeneous strategic investors with complementary function if \(\theta > 0\), \(F\) homogeneous strategic investors with substituted function if \(\theta < 0\), and \(H\) homogeneous VIs, so that there are \(E_0\) + \(F_0\) + \(H_0\) investors for the entrepreneur to select. Note that the entrepreneur is able to choose different portfolios in each financing stage. Thus, the entrepreneur is willing to sign contracts with \(m(m = 1, 2, \ldots, E_0 + F_0 + H_0)\) investors and each of them invests capital \(k\) at the state of \(t = 0\). Moreover, the contract stipulates that the entrepreneur and the investor have only two choices: accept or reject. The total capitals from strategic investors are \(K_0^S = (E_0 + F_0)k\) and VIs are \(K_0^V = H_0k\). Namely, the total raised capitals at the time \(t = 0\) are \(K_{0G} = K_{0G}^G + K_{0V}^G + K_{0S}^G\), so the entrepreneur holds an asset stake that is equal to \(a_0\). In return, the entrepreneur will provide investors equity stakes \(a_{0G}, a_{0V}, a_{0S}\). Later, the period \((0, T_1)\) involves establishing new plants, purchase equipment, green technology incubation, employment expenditure, and other activities. Suppose that the green start-up follows the Cobb–Douglas production function \(Y(K, L) = aK^\alpha L^{1-\alpha}\); the factor of labor production \(L\) is simply decided by the efforts of the entrepreneur, while the factor of capital production \(K\) is decided by the number of capital investments [47]. \(a > 0\) corresponds to the elasticity coefficient of labor output and \(1 - \alpha > 0\) corresponds to the elasticity coefficient of capital output. This simple specification ignores the effort cost of the entrepreneur as the sunk cost, for the assumption of the absolutely responsible entrepreneur.

Besides, we assume that investors merely monitor the start-up to prevent entrepreneurs from opportunism, while the government only regulates the green appropriation funds it provides in this period (note that we have not considered the role of direct support for start-ups (e.g., through consultancy, networks)). Thus, VIs and SIs, respectively, determine the monitoring intensity \(\xi_{0G}^V\) (\(\xi_{0V}^G > 0\)) and \(\xi_{0S}^G\) (\(\xi_{0S}^G > 0\)) (note that government regulation of green appropriation funds, to some extent, promotes the disclosure of financial information of green start-up, which significantly helps investors reduce the cost of monitoring and avoid unnecessary monitoring). Let \(\mu\) be the maximum influence of monitoring intensity on the output value of the green start-up, and the value growth diminishes at the monitoring intensity \(\xi_{0G}^V = e^{\xi_{0G}^V}\). So, we hypothesized that the output value of green start-up is positively correlated with monitoring intensity; namely,

\[
f(\xi_{0G}^V) = b(e^{\xi_{0G}^V})^2 + c\xi_{0G}^V + d. \tag{1}
\]

We also set that the start-up has the minimum output value of the enterprise regardless of whether or not it is monitored by investors; thus, \(f_{\text{min}}(\xi_{0G}^V) = 1\).

The output value of green start-up is as follows:
\[ Y(\xi_0^G, L_0, K_{0G}) = f(\xi_0^G)Y(L_0, K_{0G}), \]  
where \(L_0\) is an exogenous model parameter, \(a\) captures productivity parameter, and \(Y(L_0, K_{0G}) = aL^{1-a}K_{0G}^{a} \) denotes the original output value of green start-up.

The investors’ wealth gain stems from their financing and monitoring activities (note that we use the production function to describe the process of capital appreciation at the period \((0, T_1)\)). Thus, according to the equity allocation plan in the contract, the wealth of VIs and SIs is equal to \(\alpha^V_G Y(\xi_0^G, K_{0G})\) and \(\alpha^S_G Y(\xi_0^G, K_{0G})\), respectively.

Based on the principal-agent theory [48], we assume that the green grant from the government has no effect on the cost of monitoring; thus, we use the effort cost function proposed by the compensation contract model to compute the cost of monitoring \(C_0(\xi_0^G, T_1)\) (note that the capital and the cost of monitoring are fully deductible due to the high cost of Big Data) which can be expressed as

\[
C_0(\xi_0^G, T_1) = C_0(\xi_0^{GV}, T_1) + C_0(\xi_0^{GS}, T_1),
\]

\[
C_0(\xi_0^{GV}, T_1) = \frac{1}{2} \xi_0^{GV}(K_0^V)^2,
\]

\[
C_0(\xi_0^{GS}, T_1) = \frac{1}{2} \xi_0^{GS}(K_0^S)^2.
\]

Taking into account a tax rate \(\sigma\), we may now compute the total net profit \(\pi(\xi_0^G, K_{0G})\) at the time of \(t = T_1\):

\[
\pi(\xi_0^G, K_{0G}) = (1 - \sigma)\left(Y(\xi_0^G, K_{0G}) - C_0(\xi_0^G, T_1) - K_{0G}\right).
\]

Thus we are able to obtain the net profit of VIs and SIs:

\[
\pi_V(\xi_0^G, K_{0G}) = \left(\frac{H_0}{(E_0 + F_0) + H_0}\right)\alpha^V_G \pi(\xi_0^G, K_{0G}),
\]

\[
\pi_S(\xi_0^G, K_{0G}) = \left(\frac{(E_0 + F_0)}{(E_0 + F_0) + H_0}\right)\alpha^S_G \pi(\xi_0^G, K_{0G}),
\]

where \(H_0/(E_0 + F_0) + H_0\) and \((E_0 + F_0)/(E_0 + F_0) + H_0\) denote the investment proportion between VIs and SIs, respectively.

In the time \(T_1\), to make the green product marketization, the government provides green product sales growth subsidies \(\eta\) per item at this time. We assume that there are only two possible states in the times \(T_1\) and \(T_2\), that is, success and failure. Moreover, the VIs and SIs are myopic in the sense that they do not care about the information they obtained in the period of \([0, T_1]\) as long as the first stage is able to succeed. Then investors will decide whether to continue investing in this start-up. Besides, venture investments \(K_1^V\) and \(K_1^S\) occur from VIs and SIs. Thus, the total capital of \(T_1\) is

\[
K_1 = K_1^V + K_1^S.
\]

Thus, the entrepreneur holds an equity stake that is equal to \(\alpha_1\). In return, the entrepreneur will provide investors equity stakes \(\alpha_i^V, i = S, V\). However, the entrepreneur is the absolute decision-maker; thus, we assume that \(\alpha_1^V + \alpha_1^S < 1/2\alpha_0\).

Besides, the fixed and variable costs of the green product are denoted as follows:

\[
c_{fG} = K_1^G + C_0(\xi_0^G, T_1),
\]

\[
c_{vG} = c_{vG},
\]

\[
C(G) = c_{vG} + c_{vG},
\]

where \(q_G\) represents sale quantity, \(c_f\) captures the fixed cost of the green product, \(r\) expresses unit variable cost, and \(c_{va}\) denotes the variable cost of the green product.

\[
\pi_1^V = \left[pq_G - (1 - \eta)C(G)\right][1 - \sigma(1 - \delta)],
\]

where \(p\) captures retail price of the green product and \(\delta\) represents tax rate of the green product.

Then we can obtain the net profit of VIs and SIs of time \(T_2\) as follows:

\[
\pi_1^V = \alpha_1^V \left[pq_G - (1 - \eta)C(G)\right][1 - \sigma(1 - \delta)],
\]

\[
\pi_1^S = \alpha_1^S \left[pq_G - (1 - \eta)C(G)\right][1 - \sigma(1 - \delta)].
\]

Figure 1 shows the staged financing process among government, VIs, and SIs between two subsequent R&D and expanded reproduction periods.

2.2. The Assumptions Based on the Absence of the Green Government Grant. Similarly, we now assume that the external financing channels of green start-ups of this stage come from venture investments only. For simplicity, we define that venture investments \(K_0^V\) and \(K_0^S\) occur from VIs and SIs. Thus, the total raised capital at time \(t = 0\) is \(K_0 = K_0^V + K_0^S\), so the entrepreneur holds an equity stake that is equal to \(\alpha_0\). In return, the entrepreneur will provide investors equity stakes \(\alpha_i^0, i = S, V\). However, the entrepreneur is the absolute decision-maker; thus, we assume that \(\alpha_0^V + \alpha_0^S < 1/2\alpha_0\). Likewise, the investors merely monitor the start-up to prevent entrepreneurs from opportunism. Therefore, VIs and SIs determine the monitoring intensity \(\xi_0^V(0 < \xi_0^G < \xi_0^V)\) and \(\xi_0^S(0 < \xi_0^G < \xi_0^S)\), respectively (note that the green capital support of the government will improve the probability of investors to finance; however, the impact of the monitoring on the output value between venture investors and strategy investors remains steady). Then, we are able to rewrite equation (1) as \(f(\xi_0^G) = b(\xi_0^G)^2 + c\xi_0^G + d\). Thus, the output value can be rewritten as \(Y(\xi_0^G, K_0) = f(\xi_0^G)Y(\xi_0^G, K_0)\). To put it in another way, the investors’ wealth gain stems from their financing and monitoring activities (also note that monitoring intensity of investors will be much higher without the green capital support of the government, for the reason of information asymmetry). Thus, according to the equity allocation plan in the contract, the wealth of VIs and SIs is equal to \(\alpha_0^V Y(\xi_0^G, K_0)\) and \(\alpha_0^S Y(\xi_0^G, K_0)\), respectively. Besides, the cost of monitoring is \(C_0(\xi_0^G, T_1) = C_0(\xi_0^G, T_1) + C_0(\xi_0^G, T_1)\),

\[
\begin{align*}
C_0(\xi_0^G, T_1) &= 1/2(\xi_0^{GV}(K_0^V)^2), \\
C_0(\xi_0^G, T_1) &= 1/2(\xi_0^{GS}(K_0^S)^2).
\end{align*}
\]
In addition, we similarly compute the total net profit \( \pi(\xi_0^V, \xi_0^S, K_0) \) at the time \( t = T_1 \):

\[
\pi(\xi_0^V, \xi_0^S, K_0) = \left(1 - \sigma\right) \left[ Y(\xi_0^V, \xi_0^S, K_0) - C_0(\xi_0, T_1) - K_0\right].
\]

Then we can obtain the net profit of VIs and SIs:

\[
\pi_V(\xi_0^V, \xi_0^S, K_0) = \left(\frac{H_0}{(E_0 + F_0) + H_0}\right) \alpha_0^V \pi(\xi_0^V, \xi_0^S, K_0),
\]

\[
\pi_S(\xi_0^V, \xi_0^S, K_0) = \left(\frac{(E_0 + F_0)}{(E_0 + F_0) + H_0}\right) \alpha_0^S \pi(\xi_0^V, \xi_0^S, K_0),
\]

where \( H_0/(E_0 + F_0) + H_0 \) and \( (E_0 + F_0)/(E_0 + F_0) + H_0 \) denote the investment proportion between VIs and SIs, respectively.

In the time \( T_1 \), to make the green product marketization, we assume that the VIs and SIs are myopic in the sense that they do not care about the information they obtained in the period of \( [0, T_1] \) as long as the first stage was successful when they decide whether to continue investing on this start-up. Besides, venture investments \( K_1^V \) and \( K_1^S \) occur from VIs and SIs.

Besides, the fixed and variable costs of the green product are denoted as follows:

\[
c_f = K_0 + K_1 + C_0(\xi_0, T_1) + C_1(\xi_1, T_2),
\]

\[
c_{va} = \tau q_1,
\]

\[
C(K) = c_f + c_{va},
\]

where \( q_1 \) represents sale quantity, \( c_f \) captures the fixed cost of the green product, \( \tau \) expresses unit variable cost, and \( c_{va} \) denotes the variable cost of the green product.

\[
\pi_1 = [pq - C(K)][1 - \sigma],
\]

where \( p \) captures retail price of the green product and \( \delta \) represents tax refund rate of green product.

Then we can obtain the net profit of VIs and SIs of time \( T_2 \) as follows:

\[
\pi_1 = \alpha_1^V [pq_G - C(K_G)][1 - \sigma],
\]

\[
\pi_1 = \alpha_1^S [pq_G - C(K_G)][1 - \sigma].
\]

Table 1 presents the main notations to be used throughout this study.

Figure 2 shows the staged financing process between VIs and SIs in two subsequent R&D and expanded reproduction periods.


3.1. The Optimal Choice for Government Based on the Data Simulation. To achieve the government strategic goals of green development, we will first consider what happens whether or not the green government grant is made with the start-up in each stage. In this case, we investigate the government optimal choices to maximize the green benefits \( GP_1 \) from the first-stage conditions:

\[
GP_1 = \pi(\xi_0^G, K_{0G}) - \pi(\xi_0, K_0).
\]

The remaining constraint is a bound on the utilities of government. The green benefits of the start-up must satisfy the relation

\[
GP_1 = \pi(\xi_0^G, K_{0G}) - \pi(\xi_0, K_0) > 0.
\]

Apparently, the green benefits of the analysis are highly relevant to the intensity of monitoring, so the problem is

\[
\max GP_1 = \max \{\pi(\xi_0^G, K_{0G}) - \pi(\xi_0, K_0)\}.
\]

Proposition 1. Green benefits increase as \( \xi_0^{GV} \) increases first, and then, from the monitoring intensity of \( \xi_0^{GV} \), green benefits decrease as \( \xi_0^{GV} \) increases. There only exit venture investors with the monitoring intensity of \( \xi_0^{GV} \), and thus, the function of monitoring intensity is represented as follows:

\[
f(\xi_0^{GV}) = b_1(\xi_0^{GV})^2 + c_1\xi_0^{GV} + d_1.
\]
where $b_1$, $c_1$, and $d_1$ are constants, $b_1 < 0$, and $c_1 > 0$. Besides, since $f_{GV}(0) = 1$ at a given monitoring intensity $\xi^G_0 = 0$, which denotes that the monitoring is not considered in the production process, we obtain that $d_1 = 1$, and the output value of the green start-up is as follows:

$$Y(\xi^G_0, K_{0GV}) = f_{GV}(\xi^G_0)Y(I_0, K_{0GV}).$$

(19)

The total net profit at the time $t = T_1$ is as follows:

$$\pi(\xi^G_0, K_{0GV}) = (1 - \sigma)\{Y(\xi^G_0, K_{0GV}) - C_0(\xi^G_0, T_1) - K_{0GV}\}.$$  

(20)

Thus, the problem can be rewritten as follows:

$$\max\{\pi(\xi^G_0, K_{0GV}) - \pi(\xi^V_0, K_{0V})\},$$

(21)

where the monitoring intensities $\xi^G_0$ and $\xi^V_0$ are the venture investors’ choice variable. Specifically, we assume that the maximum influence of monitoring intensity on an output value of the green start-ups only depends on the venture investors. Meanwhile, there is no effect on the monitoring intensity of venture investors whether government exists. To obtain the optimal analytic solution, we thus suppose $\xi^G_0 = \xi^V_0$. Then, $(\xi^G_0^*, \mu)$ denotes the efficient choices.

**Table 1: Summary of notations.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>Receive government support</td>
</tr>
<tr>
<td>VIs (V)</td>
<td>Venture capitalists who solely pursue monetary income</td>
</tr>
<tr>
<td>SIs (S)</td>
<td>Venture capitalists who pursue strategic objectives and monetary income</td>
</tr>
<tr>
<td>$K$</td>
<td>Amount of capital</td>
</tr>
<tr>
<td>$p$</td>
<td>The probability of venture investments occurrence</td>
</tr>
<tr>
<td>$A$</td>
<td>The value of asset of the start-up in the case of success</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The value of asset of the start-up in the case of failure</td>
</tr>
<tr>
<td>$E$</td>
<td>The net impact of enterprises on SIs’ own assets</td>
</tr>
<tr>
<td>$F$</td>
<td>Amount of homogeneous strategic investors with complementary function if $\theta &gt; 0$</td>
</tr>
<tr>
<td>$H$</td>
<td>Amount of homogeneous VIs</td>
</tr>
<tr>
<td>$k$</td>
<td>Amount of capital invested of each investor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Equity ratio</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The elasticity coefficient of capital output</td>
</tr>
<tr>
<td>$L$</td>
<td>The factor of labor production</td>
</tr>
<tr>
<td>$\xi$</td>
<td>The monitoring intensity</td>
</tr>
<tr>
<td>$C(\xi, T)$</td>
<td>The maximum influence of monitoring intensity on the output value of the green start-up</td>
</tr>
<tr>
<td>$a$</td>
<td>The cost of monitoring</td>
</tr>
<tr>
<td>$\pi$</td>
<td>The tax rate for green start-ups</td>
</tr>
<tr>
<td>$\eta$</td>
<td>The green product sales growth subsides</td>
</tr>
<tr>
<td>$q$</td>
<td>The sale quantity</td>
</tr>
<tr>
<td>$c_f$</td>
<td>The fixed cost of the green product</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The unit variable cost</td>
</tr>
<tr>
<td>$c_{va}$</td>
<td>The variable cost of the green product</td>
</tr>
<tr>
<td>$p$</td>
<td>The retail price of the green product</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The tax refund rate of green product</td>
</tr>
</tbody>
</table>

**Figure 2: The financing process without the green grant of the government.**

Venture investments from VIs and SIs only

Monitored by venture investors

0

$T_1$

$T_2$

R&D

Expanded reproduction

New venture investments from VIs and SIs only
Let \( \partial GP_{IV}/\partial \xi_0 \) = 0, and then we can obtain the optimal analytic solution as follows:
\[
\left\{ \begin{array}{l}
\xi_{GV}^* = \frac{c}{2b_0}, \\
\end{array} \right. \tag{22}
\]

To determine whether the solution is a minimum or a maximum, we take the second derivative of equation (A.5), and we have these equations for force here.
\[
\frac{\partial GP_{IV}}{\partial \xi_0} = (1 - \sigma) \left( aL^{1-a} \left( 2b_1 \xi_0^V + c_1 \left( K_0^G + K_0^V \right)^a - \frac{1}{2} \left( K_0^{GV} \right)^2 \right) \right),
\]
\[
\frac{\partial GP_{IV}}{\partial \xi_0^V} = (1 - \sigma) \left( -aL^{1-a} \left( 2b_1 \xi_0^V + c_1 \left( K_0^V \right)^a + \frac{1}{2} \left( K_0^{GV} \right)^2 \right) \right),
\]
\[
\frac{\partial GP_{IV}}{\partial^2 \xi_0^V} = 2ab_1 (1 - \sigma) L^{1-a} \left( K_0^G + K_0^V \right)^a < 0,
\]
\[
\frac{\partial GP_{IV}}{\partial^2 \xi_0^V} = -2ab_1 (1 - \sigma) L^{1-a} \left( K_0^V \right)^a > 0.
\]

According to equations (23)–(26), we can learn that green benefits monotonically increase with the rising of monitoring intensity (0, \( \xi_0^{GV} \)) and then with \( \xi_0^{GV} \) growing and \( GP_{IV} \) decreasing. Meanwhile, green benefits monotonically decrease with the rising of monitoring intensity (0, \( \xi_0^{GV} \)) and then with \( \xi_0^{GV} \) growing and \( GP_{IV} \) increasing.

Additionally, we clearly presented the impacts of the monitoring intensity on the green benefits in Figure 3. This figure also depicts the optimal monitoring intensity and green benefits do exist, which means the desire of the government.

Figure 3 shows the impacts of the monitoring intensity on the green benefits (\( a = 550; L = 1; \alpha = 0.3 \) [49]; \( b = -1; c = 3; K_0 = 10; K_1 = 100 \)).

**Proposition 2.** Green benefits increase as \( \xi_0^{GS} \) increases first, and then, from the monitoring intensity of \( \xi_0^{GS} \), green benefits decrease as \( \xi_0^{GS} \) increases. There only exist strategy investors with the monitoring intensity of \( \xi_0^{GS} \), and thus, the function of monitoring intensity can be expressed as follows:
\[
f_{GS}(\xi_0^{GS}) = b_2(\xi_0^{GS})^2 + c_2\xi_0^{GS} + d_2.
\]

Similarly, we can acquire the same results of Proposition 1. Thus, we only presented the impacts of the monitoring intensity on the green benefits in Figure 4 (note that the actual monitoring intensity of venture investors is usually different from strategy investors; however, the impacts of monitoring on green benefits are the same).

Figure 4 shows the impacts of the monitoring intensity on the green benefits (\( a = 550; L = 1; \alpha = 0.3 \) [49]; \( b = -1; c = 3; K_0 = 10; K_1 = 100 \)).

**Proposition 3.** There both exist venture investors and strategic investors with the monitoring intensities of \( \xi_0^{GV} \) and \( \xi_0^{GV} \), and thus, we obtain the monitoring intensity function:
\[
\max \left\{ f_{GV}(\xi_0^{GV}), f_{GS}(\xi_0^{GS}) \right\} = \max \left\{ b_1(\xi_0^{GV})^2 + c_1\xi_0^{GV} + d_1, b_2(\xi_0^{GS})^2 + c_2\xi_0^{GS} + d_2 \right\},
\]
\[
\max \left\{ f_{GV}(\xi_0^{GV}), f_{GS}(\xi_0^{GS}) \right\} = Z[\xi_0^{GV}, \xi_0^{GS}]
\]
\[
s.t. Z[\xi_0^{GV}, \xi_0^{GS}] \geq f_{GV}(\xi_0^{GV})
\]
\[
Z[\xi_0^{GV}, \xi_0^{GS}] \geq f_{GS}(\xi_0^{GS})
\]
\[
\xi_0^{GV} \geq \xi_0^{GS} > 0
\]
\[
d_1 = d_2 = 1.
\]

**Figure 3:** The green benefits when monitored by venture investors only.
Then, the objective function can be reexpressed as follows:

\[
\begin{align*}
\Pi_{G}(\xi_0^0, \xi_0^{GS}, K_{oG}) &= (1 - \sigma) \left\{ f_{GS}(\xi_0^{GS})Y(L_0, K_{oG}) - C_o(\xi_0^{OV}, T_1) - C_o(\xi_0^{GS}, T_1) - K_{oG} \right\} \\
&\quad \left( \frac{c}{2b} \right) - \left( -\frac{c}{2b} - \xi_0^{GS} \right) \\
\Pi_{F}(\xi_0^{G'}, \xi_0^{S'}, K_{oF}) &= (1 - \sigma) \left\{ f_{GV}(\xi_0^{GV})Y(L_0, K_{oF}) - C_o(\xi_0^{GV}, T_1) - C_o(\xi_0^{GS}, T_1) - K_{oF} \right\} \\
&\quad \left( \frac{c}{2b} \right) - \left( -\frac{c}{2b} - \xi_0^{GS} \right)
\end{align*}
\]

To calculate the optimal green benefits, the output value of the green start-up becomes

\[
\Pi(\xi_0^{GV}, \xi_0^{GS}, K_{oG}) = (1 - \sigma) \left\{ f_{GS}(\xi_0^{GS})Y(L_0, K_{oG}) - C_o(\xi_0^{GV}, T_1) - C_o(\xi_0^{GS}, T_1) - K_{oG} \right\} \\
\left( \frac{c}{2b} \right) - \left( -\frac{c}{2b} - \xi_0^{GS} \right)
\]

\[
\Pi(\xi_0^{G'}, \xi_0^{S'}, K_{oF}) = (1 - \sigma) \left\{ f_{GV}(\xi_0^{GV})Y(L_0, K_{oF}) - C_o(\xi_0^{GV}, T_1) - C_o(\xi_0^{GS}, T_1) - K_{oF} \right\} \\
\left( \frac{c}{2b} \right) - \left( -\frac{c}{2b} - \xi_0^{GS} \right)
\]

Therefore, the problem must be of the following form:

\[
\max \{ \Pi(\xi_0^{GV}, \xi_0^{GS}, K_{oG}) - \Pi(\xi_0^{G'}, \xi_0^{S'}, K_{oF}) \} \quad (33)
\]

To directly show the relationship between green benefits and the monitoring intensity, we presented the impacts of the monitoring intensity on the green benefits in Figure 5. Figure 5 shows the impacts of the monitoring intensity on the green benefits (\(a = 550; L = 1; \alpha = 0.3; b = -1; c = 3; K_0 = 10; K_1 = 100, K_2 = 100\)).

As is shown in Figure 5, we can learn that green benefits GP1 decrease first and then increase with monitoring at the point of \(\xi_0^{G'N} \), where \(\xi_0^{G'V} = \xi_0^{G'} \) and \(\xi_0^{GS} = \xi_0^{S} \). Clearly, the government thus may be willing to improve and promote the monitoring intensity of venture investors and strategic investors to maximize the green benefits at the point of \(\xi_0^{G'N} \). Besides, the results also imply that making an optimal investment portfolio is not the decisive factor in maximizing green benefits, which means that the monitoring intensity of investors should be drawn attention to realize optimal social welfare. In the Appendix, we examine it at the proof of Proposition 3.

**Proposition 4.** Then, we consider the optimal choices of government to maximize the green benefits GP2 at the time \(T_1 \); we assume that \(k_0 = 10q c \), \(\xi_0^{G'} = \xi_0^{G}, \xi_0^{GS} = \xi_0^{GS}, \xi_0^{G'V} = \xi_0^{G'} \), and \(\xi_0^{S} = \xi_0^{S'} \), where \(\lambda > 1 \), and thus, the problem can be expressed in the following form:

\[
GP_2 = \pi_1^G - \sigma_1 > 0
\]

The remaining constraint is a bound on the utilities of society. To realize optimal social welfare, we maximize the total net profits:
maximize \( \pi \left( \xi_0^{GV}, \xi_0^{GS}, K_{0G} \right) \),
maximize \( \pi_i^G \).

In the Appendix, we thus examine it at the proof of Proposition 4. For \( p (\lambda - 1)q > K_0^T \), we can understand that the optimal green benefits happen when \( \xi_1^{GP} = \max \xi_1^{GP} \) and \( \xi_1^{GV} = 0 \) and when \( \xi_3^{GP} = \max \xi_1^{GP} \) and \( \xi_1^{GV} = 0 \) in the meantime.

4. Conclusions

In this study, we establish a theoretical model based on the financing selection mechanism of green start-ups under government grants and investigate the subject of venture capital into strategic investors and venture investors according to the pecking order financing theory. We restrict the equity control of investors and take the cost of monitoring of venture investors into account during the stage of green R&D and green products expansion for reproduction combining data simulation with empirical data to show the entire financing process of green start-ups with the help of the government to achieve green benefits, to reduce the staged financing uncertainty, and to help government realize national green innovation strategy. Our findings are as follows: Firstly, expanding the scale of green start-ups with the help of government grants is indispensable to the development of sustainable economic. Secondly, moderate monitoring intensity is beneficial to enhance green benefits. Specifically, the impact of monitoring intensity on green benefits exists in an inverted u-shaped relationship. Thirdly, both venture investors and strategic investors have the same influence on green benefits through monitoring intensity.

Appendix

For the optimal choice for the government to maximize the green benefits of stage \( T_1 \), only the venture investors or strategic investors exist.
For the optimal choice for the government to maximize the green benefits of stage $T_1$, both venture investors and strategy investors exist.

\[
\max \left\{ \pi^{GV}_0, \pi^{GS}_0, K_{CG} \right\}
\]

\[
\left[ aL^{1-a} \left( b_1 \left( \xi^{GV}_0 \right)^2 + c_1 \xi^{GV}_0 + 1 \right) \right]
\left[ \left( b_1 \left( \xi^{GV}_0 \right)^2 + c_1 \xi^{GV}_0 + 1 \right) \right] - \left( b_1 \left( \xi^{GV}_0 \right)^2 + c_1 \xi^{GV}_0 + 1 \right)
\]

\[
= (1 - \sigma)
\]

\[
\left[ aL^{1-a} \left( b_1 \left( \xi^{GS}_0 \right)^2 + c_1 \xi^{GS}_0 + 1 \right) \right] - \left( b_1 \left( \xi^{GS}_0 \right)^2 + c_1 \xi^{GS}_0 + 1 \right)
\]

\[
= (1 - \sigma)
\]

\[
\text{(A.7)}
\]

The optimal choice for the government to maximize the green benefits of stage $T_2$ is
\[ C_G = K_0^G + K_1^V + K_0^S + K_1^S + \frac{1}{2}G_G(K_0^V)^2 + \frac{1}{2}S_G(K_0^S)^2 + \frac{1}{2}G_S(K_1^V)^2 + \frac{1}{2}S_S(K_1^S)^2 + \tau q, \]

\[ C = K_0^V + K_1^S + K_0^S + K_1^V + \frac{1}{2}G_G(K_0^V)^2 + \frac{1}{2}S_G(K_0^S)^2 + \frac{1}{2}G_S(K_1^V)^2 + \frac{1}{2}S_S(K_1^S)^2 + \tau q, \]

\[ \pi_1^G = [pq_G - C_G][1 - \sigma] \]

\[ \pi_1 = [pq - C][1 - \sigma] \]

\[ \pi_1^G = \pi^G_1 - \pi_1 \]

\[ = \left[ pq_G - \left( K_0^G + K_1^V + K_0^S + K_1^S + \frac{1}{2}G_G(K_0^V)^2 + \frac{1}{2}S_G(K_0^S)^2 + \frac{1}{2}G_S(K_1^V)^2 + \frac{1}{2}S_S(K_1^S)^2 + \tau q \right) \right][1 - \sigma], \]

\[ = \left[ pq - \left( K_0^V + K_0^S + K_1^V + K_1^S + \frac{1}{2}G_G(K_0^V)^2 + \frac{1}{2}S_G(K_0^S)^2 + \frac{1}{2}G_S(K_1^V)^2 + \frac{1}{2}S_S(K_1^S)^2 + \tau q \right) \right][1 - \sigma], \]

\[ \pi_1 = [pq - C][1 - \sigma] \]

\[ = [1 - \sigma] \left[ p(q_G - q) - K_0^G - \frac{1}{2}G_G(\left(K_0^V\right)^2 + \left(K_1^V\right)^2) - \frac{1}{2}S_G(\left(K_0^S\right)^2 + \left(K_1^S\right)^2) + \frac{1}{2}G_S(\left(K_0^V\right)^2 + \left(K_1^V\right)^2) + \frac{1}{2}S_S(\left(K_0^S\right)^2 + \left(K_1^S\right)^2) \right] \]

We assume that \( q_G = \lambda q \), and thus, equation (A.12) can be rewritten as

\[ \pi_1^G = \pi^G_1 - \pi_1 \]

\[ = [1 - \sigma] \left[ p(\lambda - 1)q - K_0^G - \frac{1}{2}G_G(\left(K_0^V\right)^2 + \left(K_1^V\right)^2) - \frac{1}{2}S_G(\left(K_0^S\right)^2 + \left(K_1^S\right)^2) + \frac{1}{2}G_S(\left(K_0^V\right)^2 + \left(K_1^V\right)^2) + \frac{1}{2}S_S(\left(K_0^S\right)^2 + \left(K_1^S\right)^2) \right] \]

Then, to calculate the optimal social welfare at \( T_2 \), we take the first derivative of equation (A.12):

\[ \frac{\partial \pi_2}{\partial K_0^G} = [1 - \sigma] \left[ -\frac{1}{2}(K_0^V)^2 - \frac{1}{2}(K_1^V)^2 \right] < 0, \]

\[ \frac{\partial \pi_2}{\partial K_0^S} = [1 - \sigma] \left[ -\frac{1}{2}(K_0^S)^2 - \frac{1}{2}(K_1^S)^2 \right] < 0, \]

\[ \frac{\partial \pi_2}{\partial K_1^V} = [1 - \sigma] \left[ \frac{1}{2}(K_0^V)^2 + \frac{1}{2}(K_1^V)^2 \right] > 0, \]

\[ \frac{\partial \pi_2}{\partial K_1^S} = [1 - \sigma] \left[ \frac{1}{2}(K_0^S)^2 + \frac{1}{2}(K_1^S)^2 \right] > 0. \]

Besides, we assume that \( \xi_0^{GV} = \xi_0^{GS} = \xi_0^{S} = \xi_0^{S} \); namely, the green benefits can be represented as follows:

\[ [1 - \sigma] \left[ p(\lambda - 1)q - K_0^G \right] > 0. \]

Therefore, \( p(\lambda - 1)q > K_0^G \), and then we can understand that the optimal green benefits happen when \( \xi_0^V = \max \xi_0^V \) and \( \xi_0^{GV} = 0 \) and when \( \xi_0^S = \max \xi_0^S, \xi_0^{GS} = 0 \) in the meantime.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.
Disclosure
Wenke Yang and Lei Wang are co-first authors.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

Authors’ Contributions
Wenke Yang and Lei Wang contributed equally to this work.

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