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Research Article

Finite-Time H_∞ Control of Affine Nonlinear Singular Systems Subject to Actuator Saturation

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This paper discusses the finite-time stable and finite-time H_∞ control problems of affine nonlinear singular systems subject to actuator saturation. Some sufficient conditions, to guarantee the system is finite-time stable, are established for the affine nonlinear singular systems subject to actuator saturation. First, the finite-time stable problem is investigated by the state undecomposed method, and then the finite-time robust H_∞ control law is presented for the system. Finally, the effectiveness of the designed controllers is shown by an example of a nonlinear singular circuit system in this paper.

1. Introduction

Singular system, also called descriptor system, is applied to many areas, such as engineering, economic, and biological systems [1]. In the past decades, the singular system has attracted interest of more and more researchers, and a lot of results are proposed for the linear singular system [1–10]. However, due to the complexity of the structure, few explore the nonlinear singular systems [11–15]. With the linear matrix inequality (LMI) method, the author in [11] has studied the robust control problem in connection with a set of stochastically nonlinear singular jump systems, while the guaranteed cost control and stabilization problems have been investigated for a set of time-delay nonlinear singular systems in [12, 13], respectively. Based on the approach of state undecomposed, scholars have considered the asymptotic stabilization of the nonlinear singular system in [14], including systems subject to actuator saturation in [15]. In [16], the control design of singular discrete-time systems has been studied based on simulation relations and behavioral theory.

It is well known that actuator saturation can compromise the functions of the closed loop system and cause instability of the system. Hence, scholars have been studying the stabilization problem of systems subject to actuator saturation for two decades [2–5, 17, 18]. Scholars have studied the stability problem in connection with the singular system with input saturation via the Lyapunov method in [2]. For the linear singular system with actuator saturation, the studies are proposed based on the stabilization conditions of the closed loop system, and the domain of attraction has been estimated by LMI technique in [3], while the authors in [4] proposed the estimate of the domain of attraction by the saturated state feedback method. In [5], the stochastic stability has been investigated for singular discrete-time Markov jump systems subject to input saturation by LMI approach. The stabilization controller has been designed based on the adaptive dynamic programming algorithm on nonlinear systems with input saturation in [17]. Fridman et al. [18] have studied local stabilization and H_∞ control for the system with time delay and input saturation via the LMI and Lyapunov–Krasovskii functional method. The robust output regulation problem has been studied for discrete-time singular systems subject to actuator saturation with an additional control term to the nonlinear feedback in [19]. Zong et al. [20] investigated the decentralized adaptive output feedback saturated control problem for interconnected nonlinear systems with strong interconnections.

For nonlinear systems with actuator fault and saturation, the authors in [21] applied the surface control technique to address the finite-time adaptive output feedback control and the authors in [22] used the sliding mode and neutral networks methods to design the adaptive fault-tolerant controller.

Unlike the asymptotical stability [23–26], the finite-time stable (FTS) is that the state (weighted) norm does not exceed a certain boundary within a fixed time T. In many practical applications, FTS plays an important role, such as analyzing the transient behavior of the controlled system within a finite interval. Due to its extensive engineering application background, the FTS problem has attracted much scholarly interest [6, 7, 27–42]. In [27, 28], the FTS problem of the linear system has been investigated. To guarantee FTS of switched linear systems subject to actuator saturation, the authors in [29] designed the FTS controllers with the time domain approach. The authors in [30, 31] gave the sufficient and necessary conditions of FTS for the impulsive linear system by using the LMI method. Ma et al. and Wang and Feng [6, 32] considered the FTS for singular discrete-time Markov jump systems subject to input saturation by using the Lyapunov–Krasovskii functional method and using the mode-dependent parameter approach, respectively. Feng et al. [7]investigated the FTS for the linear singular system with the LMI method. Based on the sliding mode control design, FTS and input-output FTS problems are, respectively, dealt with in [33–35] for a class of nonlinear systems. In [36–38], the finite-time asynchronous dissipative filtering, finite-time asynchronous L_2-gain control, and finite region asynchronous H_∞ control have been, respectively, studied for nonlinear Markov jump systems while an annular finite-time H_∞ filter has been considered for networked switched systems in [39]. The finite-time H_∞ controller has been given for the nonlinear singular discrete-time system in [40, 41] and for the nonlinear singular continuous time system in [42], respectively. It is worth pointing out that there is another definition of finite-time stability, where all states of the system reach the equilibrium point within a fixed time T and stay at the equilibrium point permanently [21, 43–47]. Based on the Hamiltonian function method, the authors in [43] studied the observer design problem of general nonlinear time-delay systems and gave the finite-time robust stabilization results; the authors in [44] investigated the finite-time stabilization problem for a class of singular systems by the constructed new Lyapunov functional while the finite-time robust simultaneous stabilization and adaptive robust simultaneous stabilization have been investigated for nonlinear systems with time delay in [45, 46], respectively. In [47], the finite-time stabilization problem has been considered for a class of high-order stochastic nonlinear systems by using the backstepping method.

Because the control input is limited by the saturation nonlinear function, it is more difficult to design a control for nonlinear singular systems with input saturation compared with the case without actuator saturation. To the best of the knowledge, the authors are only aware of few results related to the FTS of the nonlinear singular systems with input saturation. Compared with the mentioned results, the main contributions of the paper are highlighted as follows:

(1) The state feedback controllers designed in the paper have simple form, so it has low computational complexity

(2) The design method proposed in the paper has low conservative criteria, and the singular matrix E does not need to satisfy any restriction conditions

(3) The nonlinear function $\phi(x)$ does not need to satisfy Lipschitz conditions

This paper, in Section 2, introduces the definition of FTS and provides the design method of FTS controller for affine nonlinear singular systems subject to actuator saturation (ANSSAS). With the state undecomposed method, Section 3 discusses the finite-time H_∞ control problem and designs a corresponding controller for the ANSSAS with external disturbance. In Section 4, an example of a circuit system is given to illustrate the effectiveness of the proposed controllers, and the simulation curves are presented. Section 5 provides a brief conclusion.

Notations. In the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $A \in \mathbb{R}^{n\times n}$ implies that A is an $n \times n$-matrix in real number field. A^T is the transpose of matrix A. $\lambda_{\max}(Q_1)$ and $\lambda_{\min}(Q_1)$ are the maximum and minimum eigenvalues of square matrix Q_1, respectively. $Q_1 > 0$ $(Q_1 \geq 0)$ implies that square matrix Q_1 is positive definite (positive semidefinite). The Euclidean norm of vectors z is denoted by $\|z\|$.

2. FTS of ANSSAS

This section discusses the FTS problem for ANSSAS.

Consider the ANSSAS as follows:

$$\begin{aligned} E\dot{x}(t) &= \phi(x(t)) + B(x(t))\mathrm{sat}(u(t)),\\ Ex(0) &= Ex_0,\\ \phi(0) &= 0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $E \in \mathbb{R}^{n\times n}$, $0 <$ rank $E = r < n$; $B(x(t)) \in \mathbb{R}^{n\times m}$, $\phi(x(t)) \in \mathbb{R}^n$ is a sufficiently smooth vector field; $\mathrm{sat}(u(t)) \in \mathbb{R}^m$ is the saturation nonlinearity control input, and

$$\mathrm{sat}(u_i(t)) = \begin{cases} l\iota_i, & u_i(t) > \iota_i,\\ u_i(t), & -\iota_i \leq u_i(t) \leq \iota_i, i = 1, 2, \ldots, m,\\ -\iota_i, & u_i(t) < \iota_i. \end{cases} \tag{2}$$

To study system (1), we present the following definition and lemmas.

Definition 1 (see [48]). For any initial condition Ex_0, if the resulted closed loop singular system is impulsive free, then the control law $u(x(t))$ is said to be admissible, and the original system is said to be impulse controllable.

Lemma 1 (see [49]). *If a vector function $S(x(t))$ with $S(0) = 0$ ($x(t) \in \mathbb{R}^n$) has continuous n th-order partial derivatives, then $S(x(t))$ can be rewritten as*

$$S(x(t)) = a_1(x(t))x_1(t) + \cdots + a_n(x(t))x_n(t) = A(x(t))x(t), \tag{3}$$

where $A(x(t)) = [a_1(x(t))\, a_2(x(t)) \cdots a_n(x(t))] \in \mathbb{R}^{n\times n}$.

Lemma 2 (see [15]). *Denote*

$$\mathrm{sat}(u(t)) = u(t) - \delta(t). \tag{4}$$

Then, there exists a positive real number ζ such that

$$\delta^T(t)\delta(t) \leq \zeta u^T(t)u(t), \tag{5}$$

where $0 < \zeta < 1$, $\delta(t) = [\delta_1(t), \delta_2(t), \ldots, \delta_m(t)]^T \in \mathbb{R}^m$, and $\delta_i(t)$ is the dead-zone nonlinearity function, $i = 1, 2, \ldots, m$.

According to [7, 27, 41], we introduce the definition as follows.

Definition 2. ANSSAS (1) is called FTS with respect to (c_1, c_2, T, R), with $0 < c_1 < c_2$ and $R > 0$ if ANSSAS (1) is impulse controllable and $x^T(0)E^T REx(0) \leq c_1$ such that $x^T(t)E^T REx(t) < c_2$, $\forall t \in [0, T]$.

According to Lemma 1, system (1) can be transformed into

$$E\dot{x}(t) = A(x(t))x(t) + B(x(t))\mathrm{sat}(u(t)). \tag{6}$$

To facilitate the analysis of system (6), we provide an assumption and a lemma.

Assumption 1. $\mathrm{Rank}\begin{bmatrix} 0 & E & 0 \\ E & A(x(t)) & B(x(t)) \end{bmatrix} = n + \mathrm{rank}\, E$, $\forall x(t) \in \mathbb{R}^n$.

Lemma 3 (see [14]). *Assume Assumption 1 holds, then system (6) is impulse controllable.*

Under Assumption 1, the following result is given.

Theorem 1. *Consider ANSSAS (1) and its equivalent system (6). If Assumption 1 holds, there exist two positive real numbers β and ζ and three matrices $K(x(t)) \in \mathbb{R}^{m\times n}$, $Q_1 \in \mathbb{R}^{n\times n}$, and $P \in \mathbb{R}^{n\times n}$ such that*

$$(A(x(t)) - B(x(t))K(x(t)))^T PE + E^T P(A(x(t)) - B(x(t))K(x(t))) + E^T PBB^T PE + \zeta K^T K - \beta E^T PE \leq 0, \tag{7}$$

$$\lambda_{\max}(Q_1)c_1 e^{\beta T} < c_2 \lambda_{\min}(Q_1), \tag{8}$$

then the FTS controller of system (1) can be given as follows:

$$u = -K(x(t))x(t), \tag{9}$$

where $c_2 > c_1 > 0$, $0 < \zeta < 1$, $R > 0$, $Q_1 > 0$, $P > 0$, and $P = R^{1/2}Q_1 R^{1/2}$.

Proof. Applying (4) and (9) to system (6), it has

$$E\dot{x}(t) = (A(x(t)) - B(x(t))K(x(t)))x(t) - B(x(t))\delta(t). \tag{10}$$

According to Lemma 3, we know that system (10) has no impulsive solution under Assumption 1.

Based on system (10), we construct a proper Lyapunov function $V(x(t)) = x^T(t)E^T PEx(t) \geq 0$; according to inequality (5) and condition (7), we have

$$\begin{aligned}
&\dot{V}(x(t)) - \beta V(x(t)) \\
&\quad = (E\dot{x}(t))^T PEx(t) + x^T(t)E^T PE\dot{x}(t) - \beta x^T(t)E^T PEx(t) \\
&\quad = x^T(t)(A(x(t)) - B(x(t))K(x(t)))^T PEx(t) + x^T(t)E^T P(A(x(t)) - B(x(t))K(x(t)))x(t) \\
&\qquad - 2x^T(t)E^T PB(x(t))\delta - \beta x^T(t)E^T PEx(t) \\
&\quad \leq x^T(t)\Big((A(x(t)) - B(x(t))K(x(t)))^T PE + E^T P(A(x(t)) - B(x(t))K(x(t)))\Big)x(t) \\
&\qquad + x^T(t)E^T PB(x(t))B^T(x(t))PEx(t) + \delta^T\delta(t) - \beta x^T(t)E^T PEx(t) \\
&\quad \leq x^T\Big((A(x(t)) - B(x(t))K(x(t)))^T PE + E^T P(A(x(t)) - B(x(t))K(x(t))) \\
&\qquad + E^T PB(x(t))B^T(x(t))PE + \zeta K^T(x(t))K(x(t)) - \beta E^T PE\Big)x(t) \\
&\quad \leq 0,
\end{aligned} \tag{11}$$

which is

$$\dot{V}(x(t)) \le \beta V(x(t)), \quad \forall t \in [0,T]. \tag{12}$$

Next, we prove that system (10) is FTS. By integrating inequality (12) between 0 and T with $t \in [0,T]$, it follows that

$$\ln \frac{V(x(t))}{V(x(0))} \le \beta t. \tag{13}$$

It is clear that

$$V(x(t)) \le e^{\beta t} V(x(0)). \tag{14}$$

Given the chain of inequalities as follows:

$$\begin{aligned} V(x(t)) &= x^T(t)E^T R^{1/2} Q_1 R^{1/2} Ex(t) \\ &\ge \lambda_{\min}(Q_1) x^T(t) E^T REx(t), \end{aligned} \tag{15}$$

$$\begin{aligned} V(x(0))e^{\beta t} &= x^T(0)E^T R^{1/2} Q_1 R^{1/2} Ex(0)e^{\beta t} \\ &\le \lambda_{\max}(Q_1) x^T(0)E^T REx(0)e^{\beta T}. \end{aligned} \tag{16}$$

According to $x^T(0)E^T REx(0) \le c_1$, putting together (14)–(16), we have

$$x^T(t)E^T REx(t) \le \frac{\lambda_{\max}(Q_1)c_1 e^{\beta T}}{\lambda_{\min}(Q_1)}. \tag{17}$$

From (8) and (17), it can be obtained that $x^T(t)E^T REx(t) < c_2$, $\forall t \in [0,T]$. So, system (1) is FTS with respect to (c_1, c_2, T, R). □

3. Finite Time H_∞ Control of ANSSAS

Based on Section 2, this section studied the finite-time H_∞ control law for the ANSSAS.

Consider ANSSAS as follows:

$$\begin{cases} E\dot{x}(t) = \phi(x(t)) + B(x(t))\mathrm{sat}(u(t)) + E\, d(x(t))w(t), \\ Ex(0) = Ex_0, \phi(0) = 0, \\ y(t) = h_2(x(t)), \\ z(t) = h_1(x(t)), \end{cases} \tag{18}$$

where $y(t) \in \mathbb{R}^s$ is the output, $z(t) \in \mathbb{R}^q$ is the penalty signal, $w(t) \in \mathbb{R}^s$ is the external disturbance, and $d(x(t)) \in \mathbb{R}^{n\times s}$, E, $x(t)$, $\phi(x(t))$, $\mathrm{sat}(u(t))$, and $B(x(t))$ are the same as those in ANSSAS (1).

Choose $h_1(x(t)) = L(x(t))B^T(x(t))x(t), h_2(x(t)) = d^T(x(t))E^T x(t)$, where $L(x(t))$ is full column rank. From Section 2, we know that we can design an admissible finite-time H_∞ control law $u(t)$ for system (18) under Assumption 1. The design steps of the finite-time H_∞ controller are as follows. First, we design an admissible control law u such that the L_2 gain of the closed loop system is not greater than γ, where $\gamma > 0$ is a given disturbance attenuation level. Next, we demonstrate that the resulted closed loop system is FTS when $w(t) = 0$. To design the finite-time H_∞ controller for the ANSSAS, we recall the following lemma:

Lemma 4 (see [50]). *Consider an affine nonlinear system:*

$$\begin{cases} \dot{x} = f(x) + g(x)w, \quad f(x_0) = 0, \\ z = h(x), \end{cases} \tag{19}$$

where $x \in \mathbb{R}^n$, $w \in \mathbb{R}^s$, and $z \in \mathbb{R}^q$ are the state, disturbance, and penalty signal of the system, respectively. If there exists a function $V(x) \ge 0\ (V(x_0) = 0)$ such that the Hamiltonian–Jacobian inequality

$$\frac{\partial^T V}{\partial x} f(x) + \frac{1}{2\gamma^2}\frac{\partial^T V}{\partial x} g g^T \frac{\partial V}{\partial x} + \frac{1}{2} h^T h \le 0, \tag{20}$$

holds, then the L_2 gain of system (19) (from w to z) is bounded by γ, i.e.,

$$\int_0^{\mathcal{T}} \|z(t)\|^2 \mathrm{d}t \le \gamma^2 \int_0^{\mathcal{T}} \|w(t)\|^2 \mathrm{d}t, \quad \forall w \in L_2[0,\mathcal{T}], \tag{21}$$

where γ is a positive number.

Based on with, we give the following theorem.

Theorem 2. *Consider ANSSAS (18). Suppose that Assumption 1 holds. Let*

$$\begin{aligned} u(t) = &-\left(K(x(t)) + \frac{1}{2}\left(L^T(x(t))L(x(t)) + \frac{1}{\gamma^2} I_m\right) B^T(x(t))\right) \\ &x =: -K_1(x(t))x(t). \end{aligned} \tag{22}$$

If

$$\begin{aligned} &(A(x(t)) - B(x(t))K_1(x(t)))^T PE + E^T P(A(x(t)) - B(x(t))K_1(x(t))) + \frac{2}{\gamma^2} E^T PE\, d(x(t))d^T(x(t))E^T PE \\ &+ \frac{1}{2} B(x(t))L^T(x(t))L(x(t))B^T(x(t)) + \zeta K_1^T(x(t))K_1(x(t)) + E^T PBB^T PE \le 0, \end{aligned} \tag{23}$$

$$\lambda_{\max}(Q_1)c_1 e^{\beta T} < c_2 \lambda_{\min}(Q_1), \tag{24}$$

then controller (22) is the finite-time H_∞ control law of ANSSAS (18), where P, Q_1, β, ζ, c_1, and c_2 are the same as those in Theorem 1.

Proof. Based on Section 2 and controller (22), we can give

$$\begin{cases} E\dot{x}(t) = \left(A(x(t)) - B(x(t))K_1(x(t))\right)x(t) - B(x(t))\delta(t) + E\ d(x(t))w(t), \\ y(t) = d^T(x(t))E^T x(t), \\ z(t) = L(x(t))B^T(x(t))x(t). \end{cases} \tag{25}$$

Choose a proper Lyapunov function $V(x(t)) = x^T(t)E^T PEx(t)$; according to (23), we have

$$\begin{aligned} &\dot{V}(x(t)) - \frac{\partial^T V(x(t))}{\partial x(t)} d(x(t))w(t) + \frac{1}{2\gamma^2}\frac{\partial^T V(x(t))}{\partial x(t)} d(x(t))d^T(x(t))\frac{\partial V(x(t))}{\partial x(t)} + \frac{1}{2}h_1^T(x(t))h_1(x(t)) \\ &\quad = x^T(t)\Big(\left(A(x(t)) - B(x(t))K_1(x(t))\right)^T PE + E^T P\left(A(x(t)) - B(x(t))K_1(x(t))\right)\Big)x(t) \\ &\qquad - 2x^T(t)E^T PB(x(t))\delta(t) + w^T(t)d^T(x(t))E^T PEx(t) + x^T(t)E^T PE\ d(x(t))w(t) - 2x^T(t)E^T PE\ d(x(t))w(t) \\ &\qquad + \frac{2}{\gamma^2}x^T(t)E^T PE\ d(x(t))d^T(x(t))E^T PEx(t) + \frac{1}{2}x^T(t)B(x(t))L^T(x(t))L(x(t))B^T(x(t))x(t) \\ &\quad \le x^T(t)\Big(\left(A(x(t)) - B(x(t))K_1(x(t))\right)^T PE + E^T P\left(A(x(t)) - B(x(t))K_1(x(t))\right) \\ &\qquad + \frac{2}{\gamma^2}E^T PE\ d(x(t))d^T(x(t))E^T PE + \frac{1}{2}B(x(t))L^T(x(t))L(x(t))B^T(x(t)) \\ &\qquad + \zeta K_1^T(x(t))K_1(x(t)) + E^T PB(x(t))B^T(x(t))PE\Big)x(t) \le 0. \end{aligned} \tag{26}$$

By Lemma 4, the L_2 gain of system (25) is not more than γ.

Next, we prove that the system is FTS if $w(t) = 0$.

$$\begin{aligned} \dot{V}(x(t)) - \beta V(x(t)) &= \frac{\partial^T V(x(t))}{\partial x(t)}\dot{x}(t) - \beta x^T(t)E^T PEx(t) \\ &= x^T(t)\Big(\left(A(x(t)) - B(x(t))K_1(x(t))\right)^T PE + E^T P\left(A(x(t)) - B(x(t))K_1(x(t))\right) \\ &\quad -\beta E^T PE\Big)x(t) - 2x^T(t)E^T PB(x(t))\delta \\ &\le x^T(t)\Big(\left(A(x(t)) - B(x(t))K_1(x(t))\right)^T PE + E^T P\left(A(x(t)) - B(x(t))K_1(x(t))\right) \\ &\quad +\zeta K_1^T(x(t))K_1(x(t)) + E^T PB(x(t))B^T(x(t))PE\Big)x(t) \\ &\le x^T(t)\Big(\left(A(x(t)) - B(x(t))K_1(x(t))\right)^T PE + E^T P\left(A(x(t)) - B(x(t))K_1(x(t))\right) \\ &\quad + \frac{2}{\gamma^2}E^T PE\ d(x(t))d^T(x(t))E^T PE + \frac{1}{2}B(x(t))L^T(x(t))L(x(t))B^T(x(t)) \\ &\quad +\zeta K_1^T(x(t))K_1(x(t)) + E^T PB(x(t))B^T(x(t))PE\Big)x(t) \le 0. \end{aligned} \tag{27}$$

The following proof is the same as that in Theorem 1. So, the closed loop system (25) is FTS if $w(t) = 0$.

4. A Circuit Example

This section proposes a circuit example to show the effectiveness of the finite-time H_∞ controller designed in Theorem 2.

Example 1. Consider the circuit system as Figure 1, where i_w is a disturbance signal, $u_1 = \varphi_1(q_1), u_2 = \varphi_2(q_2)$.

From Kirchhoff's current law and voltage law, the circuit system can be expressed as

$$\begin{cases} \dot{q}_1 + \dot{q}_2 = \mathrm{sat}(I_s) - \dfrac{\varphi_1(q_1)}{R_3} - i_w, \\ 0 = \mathrm{sat}(U_s) + \varphi_1(q_1) - \varphi_2(q_2). \end{cases} \tag{28}$$

We introduce $\varphi_1(q_1) = q_1, \varphi_2(q_2) = q_2^3$, $R_3 = (1/2)\Omega$, and $|U_s| \le 8\,V, |I_s| \le 4\,A$. Let $v = [v_1, v_2]^T =: [U_s, I_s]^T$, $x(t) = [x_1(t), x_2(t)]^T =: [q_1, q_2]^T$, and $w(t) = i_w$. Then, system (28) can be rewritten as

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -3x_1(t) - x_1(t)x_2^2(t) \\ x_1(t) - 6x_2(t) - 2x_2^3(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathrm{sat}(v) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} w(t). \tag{29}$$

Choose the penalty signal $z = (1/2)[q_2, q_1]^T$. Thus, system (29) and z can be combined to

$$\begin{cases} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -3x_1(t) - x_1(t)x_2^2(t) \\ x_1(t) - 6x_2(t) - 2x_2^3(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathrm{sat}(v) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} w(t), \\ z = \dfrac{1}{2} \begin{bmatrix} cx_2(t) \\ x_1(t) \end{bmatrix}, \end{cases} \tag{30}$$

where $L(x(t)) = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$.

According to $\phi(0) = 0$, we have $A(x(t)) = \begin{bmatrix} -3 - x_2^2(t) & 0 \\ 1 & -6 - 2x_2^2(t) \end{bmatrix}$. It is not difficult to verify that Assumption 1 holds.

Let

$$\begin{gathered} P = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \\ Q_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \\ R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ K(x(t)) = \begin{bmatrix} \dfrac{5}{2} & 0 \\ 0 & \dfrac{8}{5} \end{bmatrix}, \end{gathered} \tag{31}$$

for given disturbance attenuation $\gamma = 1$; choose some parameter values $c_1 = 0.5, c_2 = 3, T = 6, \beta = 0.1$, and $\zeta = 0.1$, then

$$\begin{aligned} &(A(x(t)) - B(x(t))K_1(x(t)))^T PE \\ &\quad + E^T P(A(x(t)) - B(x(t))K_1(x(t))) \\ &\quad + \frac{2}{\gamma^2} E^T PE\, d(x(t)) d^T(x(t)) E^T PE \\ &\quad + \frac{1}{2} B(x(t)) L^T(x(t)) L(x(t)) B^T(x(t)) \\ &\quad + \zeta K_1^T(x(t)) K_1(x(t)) + E^T PBB^T PE \\ &= \begin{bmatrix} -3.711 - 4x_2^2(t) & -5.319 - 4x_2^2(t) \\ -5.319 - 4x_2^2(t) & -6.230 - 4x_2^2(t) \end{bmatrix} \le 0, \\ &\frac{\lambda_{\max}(Q_1)c_1}{\lambda_{\min}(Q_1)} e^{\beta T} = 3c_1 e^{0.6} < c_2, \end{aligned} \tag{32}$$

hold.

Obviously, it is illustrated that all conditions of Theorem 2 can be satisfied.

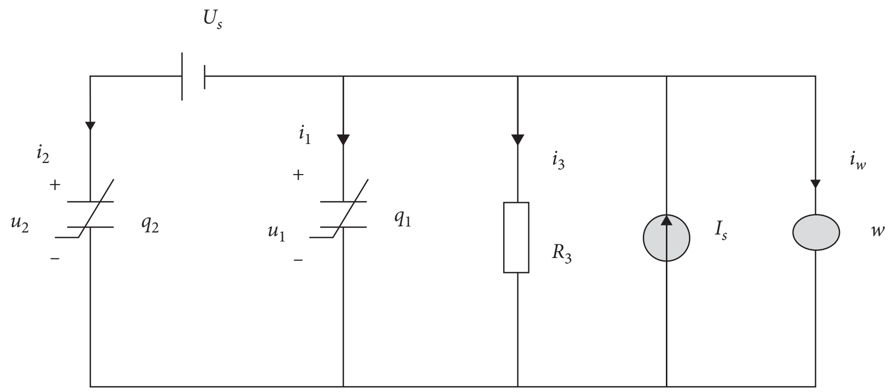

Figure 1: Nonlinear singular circuit system.

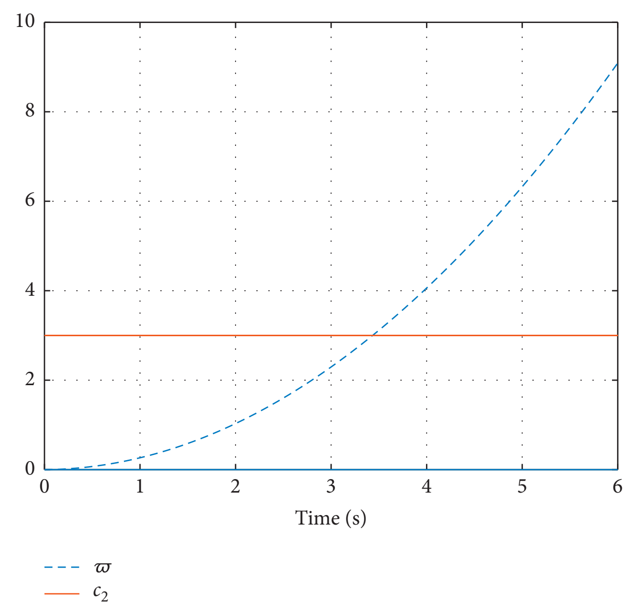

Figure 2: Response of $\varpi = x^T(t)E^T REx(t)$ for the open loop system.

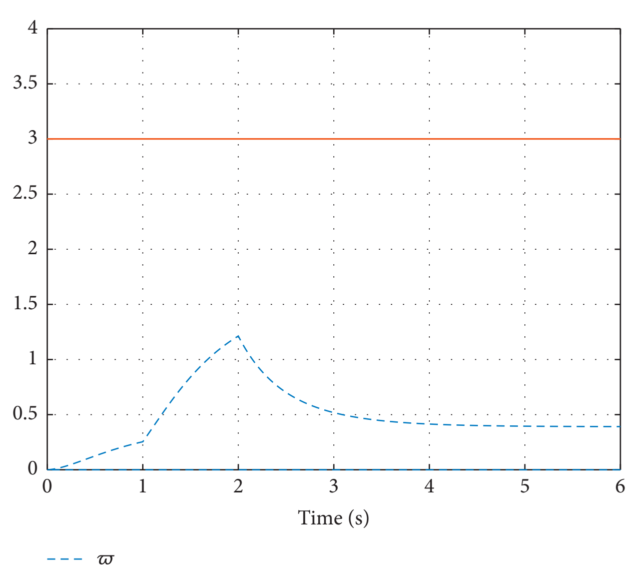

Figure 3: Response of $\varpi = x^T(t)E^T REx(t)$ for the closed loop system.

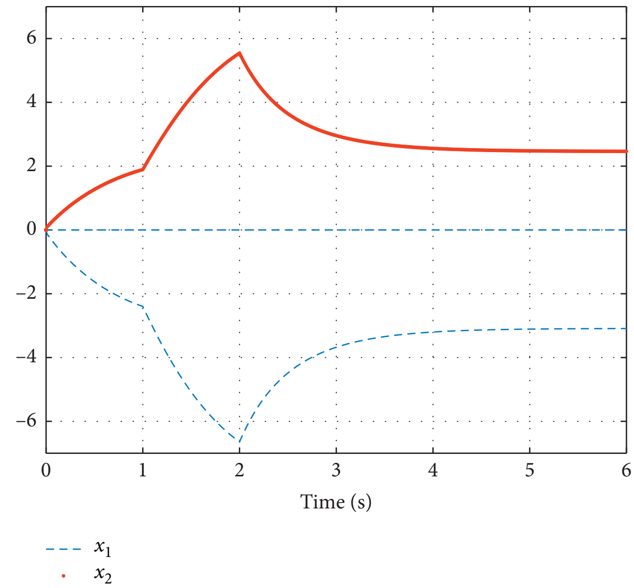

Figure 4: Response of the state $x(t)$ for the closed loop system.

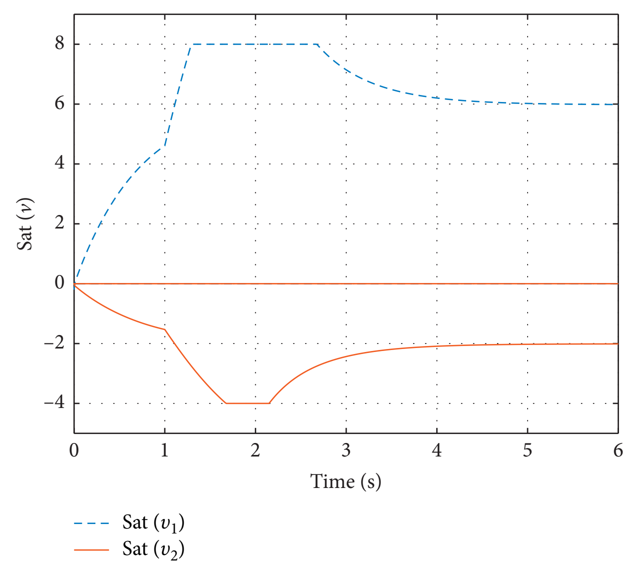

Figure 5: Saturation control signal sat (v) for the closed loop system.

Thus, we can give the following finite-time H_∞ controller of system (29):

$$v = -\begin{bmatrix} \frac{5}{2} & \frac{5}{8} \\ \frac{5}{8} & \frac{8}{5} \end{bmatrix} x(t). \tag{33}$$

To check the effectiveness of the control law (33), give $Ex(0) = [0.5, 0]^T$ and input a square-wave disturbance of amplitude $[0, 2]^T$ in the time duration $[1\,\mathrm{s} - 2\,\mathrm{s}]$ for the system. The response of $\varpi = x^T(t)E^T REx(t)$ is presented in Figures 2 and 3 for the open-loop system and the closed loop system, respectively. It is clear that $x^T(t)E^T REx(t) > 3$ in the open loop system $\forall t \in [3.5, 6]$, whereas $x^T(t)E^T REx(t) < 3$ in the closed loop system $\forall t \in [0, 6]$. The responses of state x and saturation input $\mathrm{sat}(v)$ are given in Figures 4 and 5, respectively. According to Figures 2–5, it is clear that the circuit system (29) is FTS with respect to $(0.5, 3, 6, I)$ under the admissible H_∞ control law (33).

5. Conclusion

This paper investigates the finite-time control problem for affine nonlinear singular systems subject to actuator saturation by using the state undecomposed method. First, saturation input is represented by control input and dead-zone nonlinear compensation. Then, the finite-time control law has been designed under sufficient condition of the system impulsive controllable. Based on with, the finite-time

H_∞ control problem is solved via the suitable state feedback. New results on the finite-time control and finite-time H_∞ control problems have been presented for affine nonlinear singular systems subject to actuator saturation. In the future, the input-output finite-time control problems can be studied for affine nonlinear singular systems subject to actuator saturation.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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